Wealth Transfers and Portfolio Constraints

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August 2005

Abstract

This paper examines the co-movement among stock market prices and exchange rates within a three-country Center-Periphery dynamic equilibrium model in which agents in the Center country face portfolio constraints. In our model, international transmission occurs through the terms of trade, through the common discount factor for cash flows, and, finally, through an additional channel reflecting the tightness of the portfolio constraints. Portfolio constraints are shown to generate endogenous wealth transfers to or from the Periphery countries. These implicit transfers are responsible for creating excess co-movement among the terms of trade of the Periphery countries, as well as their stock market prices. We characterize equilibrium in the model under a very general specification of the portfolio constraints imposed on the Center country and fully solve for the stock prices, terms of trade and allocations in the context of two examples.

JEL Classifications: G12, G15, F31, F36
Keywords: International finance, asset pricing, terms of trade, portfolio constraints, contagion.

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1. Introduction

The effects of portfolio constraints—either government- or institutionally-imposed—on asset prices have long preoccupied academics and policymakers. Such constraints have been argued to cause market crashes and to spread financial instability around the globe. For example, margin and collateral requirements may have sparked the propagation of the 1998 Russian crisis (Calvo (1999)) and created a threat to the worldwide financial system due to severe losses suffered by hedge funds (Edwards (1999)). Our goal in this paper is to investigate the role of portfolio constraints in the international transmission of shocks within a general equilibrium framework. The specific focus is on the impact of the constraints on the stock prices and terms of trade and their co-movement.

Of course, international propagation of shocks is a general equilibrium phenomenon that occurs for many reasons other than the presence of financial market frictions. The first reason, put forward by the international economics literature, is linkages through terms of trade. A shock to one of the countries affects its terms of trade with the rest of the world. Consequently, the trading partners of the country see their goods become more or less valuable, affecting their profits and ultimately their stock prices. The second reason, highlighted in the international asset pricing literature, is the common worldwide discount factor for cash flows (common state prices).[1] Provided that financial markets are frictionless, stock prices of all firms in the world have to be equal to their expected cash flows, discounted with the same state prices. Innovations to these state prices then have to affect stock returns worldwide, generating the co-movement in stock returns even when there is no correlation in their cash flows. However, while these two transmission channels are clearly at play, they cannot account for the full extent of international financial co-movement found in the data. For example, they have nothing to say about the surprisingly high correlation of financial instruments belonging to the same asset class found in the international data (e.g., stocks of emerging markets; see Eichengreen and Mody (2000), Kaminsky and Schmukler (2002), and Rigobon (2002) for recent evidence). It has been argued that this effect could be an outcome of portfolio constraints limiting exposure to a particular class of assets—commonly imposed on institutional investors, pension funds and mutual funds—whereby a tightening or a loosening of such constraints affects prices of all assets belonging to the class.[2]

[1] See Ammer and Mei (1996), Cochrane, Longstaff, and Santa-Clara (2004), Dumas, Harvey, and Ruiz (2003), Kodres and Pritsker (2002), and Kyle and Xiong (2001). These papers are all cast in a single-good framework, and hence highlight exclusively propagation through the common discount factor (or attribute the cross-stock spillovers to portfolio rebalancing, which is equivalent in this framework).

[2] The first work proposing this channel is Calvo (1999) which argues that limits of arbitrage (margin requirements)

We try to understand formally the effects of portfolio constraints within a unified framework which also encompasses international propagation both through the terms of trade and the common discount factor. We directly constrain a representative agent in a country, rather than model economic agents such as institutions or governments who typically impose portfolio constraints. While this makes our framework more abstract, it embeds our model into the familiar Arrow-Debreu economy, which provides a natural benchmark.[3] The main message of this paper is that financial constraints generate wealth transfers among international investors, which in turn create international co-movement. We show that while different constraints may have different implications for asset market dynamics, they all operate through their impact on investors' distribution of wealth and the ensuing wealth transfers. From the methodological viewpoint, this paper presents a tractable model which can be used to study the impact of many different constraints on the terms of trade, stock prices and their co-movement. We apply the model to study several specific constraints and show that it can generate transmission patterns observed in the international financial markets.

We consider a three-country Center-Periphery dynamic equilibrium model. We think of the Center country as a large developed economy and of the two Periphery countries as emerging markets. Each country produces its own good via a Lucas (1978) tree-type technology, where each tree's production is driven by its own supply shock. Each country consumes all three goods available in the world, albeit with a preference bias towards its own good. There are no frictions in the goods markets, but financial markets are imperfect in that agents in the Center country face portfolio constraints of a general form. We specialize countries' preferences so that, absent the portfolio constraints, the model entails (i) constant wealth distribution and (ii) identical portfolio compositions across international investors. This allows us to better disentangle the effects of portfolio constraints from those of the other two channels. The portfolio constraints alter the wealth distribution and the portfolio compositions, introducing a common stochastic factor, which reflects the tightness of the constraints, into the dynamics of the stock prices and the terms of trade.

The constraints imposed on the Center country are thus responsible for generating endogenous

generated the 1998 Russian contagion. See also Boyer, Kumagai, and Yuan (2005), Gromb and Vayanos (2002), Mendoza and Smith (2002), and Yuan (2005). For evidence on how mutual funds respond to shocks in emerging markets see Broner, Gelos, and Reinhart (2004) and references therein.

[3]See Cass, Siconolfi, and Villanacci (2001) for an analysis of a general competitive equilibrium model with portfolio constraints. They show existence and finite local uniqueness of equilibrium.

wealth transfers to or from the Periphery countries. One can then appeal to the classic Transfer Problem of international economics to pinpoint the directions of the responses of the terms of trade to a tightening of the portfolio constraints.[4] A wealth transfer to the Periphery countries improves their terms of trade; this in turn boosts their stock market prices. The effect of the transfer on the Center country is the opposite. Hence, in our model, the portfolio constraints always increase the co-movement among the stock market prices and the terms of trade of the Periphery beyond that implied by the trade and the common discount factor channels, and decrease their co-movement with the Center. We verify that these results hold even when the Periphery countries do not trade amongst themselves.

Finally, to gain further insight, we consider two examples of portfolio constraints and fully characterize the states in which they tighten (loosen) and hence the direction of the ensuing wealth transfers. Both constraints impose a limit on how much the Center can invest in the stocks of the Periphery countries, with the first constraint specifying this limit in absolute and the second one in relative terms. We find that both constraints give rise to two effects we highlight: an amplification and a flight to quality. An amplification is said to occur when a shock to one country has a larger impact on its stock market than that entailed by the unconstrained model. A flight to quality refers to the phenomenon where a negative shock to one of the Periphery countries (an emerging market) depresses stock prices throughout the Periphery, while boosting the stock price of the Center (developed) country.[5] These implications are consistent with the patterns of co-movement observed in the international financial markets.

In terms of the modeling framework, the closest to our work are the two-good two-country asset-pricing models of Helpman and Razin (1978), Cole and Obstfeld (1991), and Zapatero (1995), which feature both the trade and the common discount factor channels of international transmission. We extend their framework by introducing demand shocks and portfolio constraints. The former are responsible for overcoming the shared implication of the three models that due to trade in goods all financial market frictions, and in particular portfolio constraints, are fully alleviated in equilibrium.[6] Also related is the literature on portfolio constraints in asset pricing. Basak and

[4]The Transfer Problem stems from the argument made originally by Keynes that in a world with a home bias in consumption (like ours) an income transfer from one country to another will improve the terms of trade of the recipient country.

[5]There are other definitions of a "flight to quality" employed in macroeconomics, international economics, and finance, which differ across applications. See e.g., Bernanke, Gertler, and Gilchrist (1996), Eichengreen, Hale, and Mody (2001), and Vayanos (2004).

[6]Other recent attempts to break the financial market structure irrelevancy result of Helpman and Razin are Engel and Matsumoto (2004), Ghironi, Lee, and Rebucci (2005), Pavlova and Rigobon (2003), Serrat (2001), as well as

Croitoru (2000), Basak and Cuoco (1998), Detemple and Murthy (1997), Detemple and Serrat (2003), Gallmeyer and Hollifield (2004), Shapiro (2002), among others, all consider the effects of portfolio constraints on asset prices. While we employ a similar solution methodology, our implications are quite different because we depart from their single-good framework. We employ techniques developed in Cvitanić and Karatzas (1992) to solve the (partial equilibrium) dynamic optimization problem of the investor who is facing portfolio constraints. Our model illustrates the usefulness of these techniques in solving dynamic equilibrium models with market frictions. Finally, our framework can be applied to study international financial contagion—the excess co-movement of stock prices and exchange rates worldwide. Kaminsky, Reinhart, and Végh (2003) survey the theoretical literature on contagion. This recent literature is rapidly growing and has already produced many important insights; however, to deal with the technical difficulties involved in solving dynamic models with multiple risky assets, it has either resorted to a partial equilibrium framework or relied on behavioral assumptions.

2. The Model

Our goal is to investigate how portfolio constraints affect the co-movement of asset prices and terms of trade. Towards that end, we develop a three country Center-Periphery model in the spirit of Lucas (1982). For the purposes of exposition and interpretation, it is useful to think of the Center country as a large developed economy and of the two Periphery countries as small emerging markets. First, we present our model, designed to capture standard features of asset pricing and open economy macroeconomics models in the simplest possible setting. The only financial market imperfection we allow for in the model is that investors in the Center face portfolio constraints. Second, we solve the model in the absence of the constraints—our benchmark—and characterize the mechanism underlying the co-movement of asset prices and terms of trade. Third, we study the general constrained case and show that constraints give rise to an additional common factor driving the co-movement of the terms of trade and stock prices in the Periphery countries. This factor is proportional to the relative wealth of international investors. We then demonstrate that our main insights carry through in the setting where there is no trade among the Periphery countries. Finally, we consider two specific constraints, a concentration and a market share constraint, and demonstrate how portfolio constraints can cause amplification of shocks and a flight to quality.

Soumare and Wang (2005), which is the closest to ours in terms of the modeling framework.

2.1. The Economic Setting

We consider a continuous-time pure-exchange world economy with a finite horizon, $[0, T]$. Uncertainty represented by a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$, on which is defined a standard three-dimensional Brownian motion $w(t) = (w^0(t), w^1(t), w^2(t))^\top$, $t \in [0, T]$. All stochastic processes are assumed adapted to $\{\mathcal{F}_t; t \in [0, T]\}$, the augmented filtration generated by w. All stated (in)equalities involving random variables hold P-almost surely. In what follows, given our focus, we assume all processes introduced to be well-defined, without explicitly stating regularity conditions ensuring this.

There are three countries in the world economy, indexed by $j \in \{0,\, 1,\, 2\}$. Country 0 represents a large Center country (e.g., an industrialized economy) and countries 1 and 2 smaller Periphery countries (e.g., emerging economies). Each country j produces its own perishable good via a strictly positive output process modeled as a Lucas (1978) tree:

$$dY^j(t) = \mu_{Y^j}(t)\, Y^j(t)\, dt + \sigma_{Y^j}(t)\, Y^j(t)\, dw^j(t), \qquad j \in \{0,\, 1,\, 2\}, \tag{1}$$

where μ_{Y^j} and $\sigma_{Y^j} > 0$ are arbitrary adapted processes. The price of the good produced by country j is denoted by p^j. Since prices are not pinned done in a real model such as ours, we need to adopt a numeraire. We fix a basket containing $\beta \in (0,\, 1)$ units of the good produced in country 0 and $(1 - \beta)/2$ units of each of the remaining two goods and normalize the price of this basket to be equal to unity. We think of β as the size of the (large) Center country relative to the world economy.

Investment opportunities are represented by four securities. Each country j issues a stock S^j, a claim to its output. All stocks are in unit supply. There is also the "world" bond B, which is a money market account locally riskless in units of the numeraire.[7] The bond is in zero net supply. It is convenient to define the terms of trade from the viewpoint the Center country (country 0): $q^1 \equiv p^1/p^0$ and $q^2 \equiv p^2/p^0$ are the terms of trade of the Periphery countries 1 and 2, respectively, with the Center country.

A representative consumer-investor of each country is endowed at time 0 with a total supply of the stock market of his country; the initial wealth of agent i is denoted by $W_i(0)$. Each consumer i chooses nonnegative consumption of each good $(C_i^0(t),\, C_i^1(t),\, C_i^2(t))$, $i \in \{0,\, 1,\, 2\}$, and a portfolio

[7]There are three Brownian motions driving the evolution of the world economy. We therefore need four nonredundant financial instruments to complete markets.

of the available risky securities $x_i(t) \equiv (x_i^{S^0}(t),\, x_i^{S^1}(t),\, x_i^{S^2}(t))^\top$, where x_i^j denotes a fraction of wealth W_i invested in security j. The dynamic budget constraint of each consumer takes the standard form

$$\begin{aligned}\frac{dW_i(t)}{W_i(t)} &= x_i^{S^0}(t)\,\frac{dS^0(t)+p^0(t)Y^0(t)dt}{S^0(t)} + x_i^{S^1}(t)\,\frac{dS^1(t)+p^1(t)Y^1(t)dt}{S^1(t)} + x_i^{S^2}(t)\,\frac{dS^2(t)+p^2(t)Y^2(t)dt}{S^2(t)} \\ &\quad + (1 - x_i^{S^0}(t) - x_i^{S^1}(t) - x_i^{S^2}(t))\,\frac{dB(t)}{B(t)} - \frac{1}{W_i(t)}(p^0(t)C_i^0(t) + p^1(t)C_i^1(t) + p^2(t)C_i^2(t))\,dt\,, \quad (2)\end{aligned}$$

with $W_i(T) \geq 0$, $i \in \{0,\, 1,\, 2\}$. Preferences of consumer i are represented by a time-additive utility function defined over consumption of all three goods:

$$E\left[\int_0^T u_i\big(C_i^0(t),\, C_i^1(t),\, C_i^2(t)\big)\,dt\right],$$

where

$$\begin{aligned}u_0\big(C_0^0,\, C_0^1,\, C_0^2\big) &= \alpha_0 \log C_0^0(t) + \frac{1-\alpha_0}{2}\log C_0^1(t) + \frac{1-\alpha_0}{2}\log C_0^2(t), \\ u_1\big(C_1^0,\, C_1^1,\, C_1^2\big) &= \frac{1-\alpha_1(t)}{2}\log C_1^0(t) + \alpha_1(t)\log C_1^1(t) + \frac{1-\alpha_1(t)}{2}\log C_1^2(t), \\ u_2\big(C_2^0,\, C_2^1,\, C_2^2\big) &= \frac{1-\alpha_2(t)}{2}\log C_2^0(t) + \frac{1-\alpha_2(t)}{2}\log C_2^1(t) + \alpha_2(t)\log C_2^2(t).\end{aligned}$$

In our preferences specification, we are building on the insights from the open economy macroeconomics. In particular, we require that our specification possesses the following cornerstone properties: it must be consistent with a broader set of models incorporating nontradable goods and it must be sufficiently flexible to capture demand shifts. The presence of non-tradable goods produces a home bias in consumption, well-documented empirically and widely accepted to be responsible for many important patterns established in open economy macroeconomics. Instead of explicitly modeling the nontradable goods sector, we adopt a reduced-form approach that produces the same implications: we set the preference weight on the domestically-produced good, α_i, to be greater than 1/3 (and less than 1).[8] This assumption is responsible for the home bias in consumption occurring in our model.

The other component, demand shifts, is also an important source of uncertainty behind our theory of asset price co-movement. First, in the absence of demand uncertainty, free trade in goods may imply excessively high correlation of stock market prices and irrelevancy of a financial

[8] This assumption may be replaced by explicitly accounting for the demand of nontradables and assuming that the nontradables are produced using domestically produced inputs. The implications of both models are identical and we hence adopted the more parsimonious specification. Furthermore, note that the purpose of the assumption is to generate a home bias in *consumption*, and not in *portfolios*.

market structure (see Helpman and Razin (1978), Cole and Obstfeld (1991), and Zapatero (1995)). Second, empirical evidence indicates that demand uncertainty is of the same order of magnitude as supply uncertainty (see Pavlova and Rigobon (2003)). For these two reasons it is important to include the demand shifts into our specification. The literature offers several alternative modeling approaches to capture them. In this paper we have opted to follow the seminal contribution of Dornbusch, Fischer, and Samuelson (1977). A change in α_i in our model exactly parallels their demand shifts toward domestically produced goods. Although the interpretation we favor is the one from Dornbusch, Fischer, and Samuelson, we remark that in our reduced-form model a demand shock may also be interpreted as a shift in the demand toward nontradable goods. Formally, we assume that each α_i is a martingale (i.e., $E[\alpha_i(s)|\mathcal{F}_t] = \alpha_i(t),\ s > t$), and hence can be represented as

$$d\alpha_1(t) = \sigma_{\alpha_1}(t)^\top\, dw(t), \quad d\alpha_2(t) = \sigma_{\alpha_2}(t)^\top\, dw(t),$$

where $\sigma_{\alpha_1}(t)$ and $\sigma_{\alpha_2}(t)$ are such that our restriction that α_1 and α_2 take values between 1/3 and 1 is satisfied.[9] Since our primary focus is on the Periphery countries, for expositional clarity, we keep the preference parameter of the Center country, α_0, fixed. The log-linear specification of the preferences is adopted for tractability: it allows us to derive closed-form expressions for stock prices. These preferences also generate wealth effects driving portfolio rebalancing in our model, which are essential for understanding the portfolio constraints channel of co-movement. In Section 6 we discuss potential drawbacks of log-linear utilities.

Investment policies of the residents of Periphery countries 1 and 2 are unconstrained. However, the Center's (country 0) resident faces a portfolio constraint, which we here specify in the most general form, suggested by Cvitanić and Karatzas (1992). Namely, portfolio values x_0 are constrained to lie in a closed, convex, non-empty subset $K \in R^3$. Moreover, the deterministic subset K may be replaced by a family of stochastic constraints, so that

$$x_0(t,\,\omega) \in \{K_t(\omega);\ (t,\,\omega) \in [0,\,T] \times \Omega\}.$$

Making the constraint set stochastic, and in particular dependent on exogenous variables in the Center's optimization problem (e.g., S^i, p^i, Y^i, $i = 0,\,1,\,2$), allows for more flexibility in specifying constraints, which we exploit in Section 5.[10] Examples of portfolio constraints belonging to this

[9] An example of a martingale process that does not exist the interval (1/3, 1) is $\alpha_i(t) = E\left[\alpha_i(T)\,\middle|\,\mathcal{F}_t\right]$, with $\alpha_i(T) \in (1/3,\,1)$. We thank Mark Loewenstein for this example.

[10] See Cvitanić and Karatzas for (minor) regularity conditions imposed on the constraint set.

class include prohibitions to trade certain stocks or some less severe provisions such as limits on the fraction of the portfolio that could be invested in the emerging markets S^1 and S^2. This specification can also capture constraints on borrowing, VaR constraints, margin requirements, collateral constraints, etc. In this paper, we do not provide a model supporting the economic rationale behind imposing portfolio constraints. Typically, such constraints are either government-imposed or arise in response to an agency problem in institutional money management.[11] The analysis presented in this paper is for the case of a portfolio constraint imposed on one of the investors. We believe that it is possible, but not straightforward, to extend our model to the case in which multiple investors are constrained. Such analysis is beyond the scope of this paper, and we leave it for future research.

2.2. Countries' Optimization

Periphery countries 1 and 2 are unconstrained and are facing (potentially) dynamically complete markets.[12] This implies existence of a common state price density process ξ, consistent with no arbitrage, given by

$$d\xi(t) = -\xi(t)[r(t)dt + m(t)^\top dw(t)], \tag{3}$$

where r is the interest rate on the bond and m is the (vector) market price of risk process associated with the Brownian motions w^0, w^1, and w^2. The quantity $\xi(t, \omega)$ is interpreted as the Arrow-Debreu price per unit probability P of one unit of the numeraire delivered in state $\omega \in \Omega$ at time t.

Building on Cox and Huang (1989) and Karatzas, Lehoczky, and Shreve (1987), we convert optimization problems of consumers $i = 1, 2$ into the following static variational problem:

$$\max_{C_i^0,\, C_i^1,\, C_i^2} E\left[\int_0^T u_i\big(C_i^0(t),\, C_i^1(t),\, C_i^2(t)\big)\, dt\right] \tag{4}$$

$$\text{subject to} \qquad E\left[\int_0^T \xi(t)\left(p^0(t)C_i^0(t) + p^1(t)C_i^1(t) + p^2(t)C_i^2(t)\right) dt\right] \le W_i(0). \tag{5}$$

[11] For the latter, see, for example, Almazan, Brown, Carlson, and Chapman (2004), Basak, Pavlova, and Shapiro (2005), Dybvig, Farnsworth, and Carpenter (2001). Such constraints are particularly prevalent in developed countries, where risk management practices are more sophisticated, motivating our choice of studying the effects of portfolio constraints imposed on the Center country.

[12] Although we have three independent sources of uncertainty and four securities available for investment, market completeness is not necessarily guaranteed (see Cass and Pavlova (2004)). To ensure the validity of our solution method, we need to verify that none of the securities comprising the investment opportunity set ends up being redundant in the equilibrium we construct.

The first-order conditions for this problem are given by

$$\frac{\partial u_i\left(C_i^0(t),\, C_i^1(t),\, C_i^2(t)\right)}{\partial C_i^j(t)} = y_i\, p^j(t)\, \xi(t), \qquad i = 1,\, 2,\; j = 0,\, 1,\, 2. \tag{6}$$

where the (scalar) Lagrange multiplier y_i solves

$$E\left[\int_0^T \xi(t)\left(p^0(t)C_i^0(t) + p^1(t)C_i^1(t) + p^2(t)C_i^2(t)\right)dt\right] = W_i(0). \tag{7}$$

On the other hand, the Center country is facing financial markets with frictions, and hence, in general, the above state price density process would not appropriately reflect its investment opportunity set. Instead, the state price density faced by the Center is

$$d\xi_0(t) = -\xi_0(t)[r_0(t)dt + m_0(t)^\top dw(t)], \tag{8}$$

where the Center-specific subscript $_0$ denotes the quantities that, in general, are country-specific. These quantities reflect the impact of the portfolio constraint on the investment opportunity set of the Center country. The optimization problem of the Center subject to the portfolio constraints is formally equivalent to an auxiliary problem with no constraints but the Center facing a fictitious investment opportunity set in which the unrestricted investments are made more attractive relative to the original market and the restricted investments are made relatively less attractive (Cvitanić and Karatzas (1992)). Cvitanić and Karatzas show that the tilt in the fictitious investment opportunity set is characterized by the multipliers on the portfolio constraints. Furthermore, one can still represent the constrained consumer's problem in a static form, with the personalized state price density ξ_0 replacing ξ in (4)–(5):

$$\max_{C_0^0,\, C_0^1,\, C_0^2} E\left[\int_0^T u_0\big(C_0^0(t),\, C_0^1(t),\, C_0^2(t)\big)\,dt\right]$$

$$\text{subject to} \qquad E\left[\int_0^T \xi_0(t)\left(p^0(t)C_0^0(t) + p^1(t)C_0^1(t) + p^2(t)C_0^2(t)\right)dt\right] \leq W_0(0).$$

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where the (scalar) Lagrange multiplier y_0 solves

$$E\left[\int_0^T \xi_0(t)\left(p^0(t)C_0^0(t) + p^1(t)C_0^1(t) + p^2(t)C_0^2(t)\right)dt\right] = W_0(0). \tag{10}$$

As is to be expected in a model with log-linear preferences, the consumption expenditure on each good is proportional to wealth. This is a direct consequence of the optimality conditions

(6)–(7) and (9)–(10). However, in our economy the marginal propensity to consume out of wealth is stochastic, due to possible demand shifts.

Lemma 1. *The optimal consumption allocations and wealth are linked as follows:*

$$\begin{pmatrix} C_0^0(t) \\ C_1^0(t) \\ C_2^0(t) \end{pmatrix} = \frac{1}{p^0(t)(T-t)} \begin{pmatrix} \alpha_0 W_0(t) \\ \frac{1-\alpha_1(t)}{2} W_1(t) \\ \frac{1-\alpha_2(t)}{2} W_2(t) \end{pmatrix}, \quad \begin{pmatrix} C_0^1(t) \\ C_1^1(t) \\ C_2^1(t) \end{pmatrix} = \frac{1}{p^1(t)(T-t)} \begin{pmatrix} \frac{1-\alpha_0}{2} W_0(t) \\ \alpha_1(t) W_1(t) \\ \frac{1-\alpha_2(t)}{2} W_2(t) \end{pmatrix},$$

$$\begin{pmatrix} C_0^2(t) \\ C_1^2(t) \\ C_2^2(t) \end{pmatrix} = \frac{1}{p^2(t)(T-t)} \begin{pmatrix} \frac{1-\alpha_0}{2} W_0(t) \\ \frac{1-\alpha_1(t)}{2} W_1(t) \\ \alpha_2(t) W_2(t) \end{pmatrix}.$$

Lemma 1 allows us to easily generalize the standard implication of the single-good models that logarithmic agents follow myopic trading strategies, holding only the Merton (1971) mean-variance efficient portfolio. Let σ represent the volatility matrix of the (unconstrained) investment opportunity set.

Corollary 1. *The countries' portfolios of risky assets are given by*

$$x_0(t) = (\sigma(t)^\top)^{-1} m_0(t), \quad x_i(t) = (\sigma(t)^\top)^{-1} m(t), \quad i \in \{1, 2\}.$$

Note that the portfolio of the investor in the Center generally differs from those chosen by the investors in the Periphery because his investment opportunity set is augmented by the portfolio constraint in the sense that his effective market price of risk m_0 differs from that faced by the (unconstrained) investors in the Periphery. Only when the constraint is absent or not binding all investors in the world economy hold the same portfolio.[13]

2.3. Benchmark Unconstrained Equilibrium

To facilitate the comparisons with the economy where the Center's consumer faces a portfolio constraint, we solve for an equilibrium in a benchmark economy with no constraints. Our solution approach replies on aggregating the countries' representative consumers into a world representative agent. The representative agent is endowed with the aggregate supply of securities and consumes the aggregate output. His utility is given by

[13]This result may appear surprising because the investors in our model are heterogenous and, in particular, have a home bias in consumption. However, it follows from Lemma 1 that their total consumption expenditures constitute the same fraction of wealth. Thus the investors trade assets to achieve the maximal possible consumption expenditure (which requires the same portfolios), and then allocate this expenditure among goods through importing/exporting.

$$U(C^0, C^1, C^2; \lambda_0, \lambda_1, \lambda_2) \;=\; E\left[\int_0^T u(C^0(t), C^1(t), C^2(t); \lambda_0, \lambda_1, \lambda_2)dt\right],$$

with

$$u(C^0, C^1, C^2; \lambda_0, \lambda_1, \lambda_2) \;=\; \max_{\sum_{i=0}^2 C_i^j = C^j,\, j\in\{0,1,2\}} \sum_{i=0}^2 \lambda_i u_i(C_i^0, C_i^1, C_i^2),$$

where $\lambda_i > 0$, $i = 0, 1, 2$, are the weights on consumers 0, 1, and 2, respectively. These weights are going to be constant in the unconstrained economy, but will be stochastic in the economy with portfolio constraints. In the unconstrained case, these weights are the inverses of the Lagrange multipliers on the consumers' intertemporal budget constraints. Since in equilibrium these multipliers, and hence the weights, cannot be individually determined, we adopt a normalization $\lambda_0 = 1$. The values of λ_1 and λ_2 are reported in the Appendix.

The sharing rules for aggregate endowment, emerging from the representative agent's optimization, are given by

$$\begin{pmatrix} C_0^0(t) \\ C_1^0(t) \\ C_2^0(t) \end{pmatrix} = \frac{Y^0(t)}{\alpha_0 + \lambda_1 \frac{1-\alpha_1(t)}{2} + \lambda_2 \frac{1-\alpha_2(t)}{2}} \begin{pmatrix} \alpha_0 \\ \lambda_1 \frac{1-\alpha_1(t)}{2} \\ \lambda_2 \frac{1-\alpha_2(t)}{2} \end{pmatrix}, \quad \text{Consumption of country 0's good} \tag{11}$$

$$\begin{pmatrix} C_0^1(t) \\ C_1^1(t) \\ C_2^1(t) \end{pmatrix} = \frac{Y^1(t)}{\frac{1-\alpha_0}{2} + \lambda_1 \alpha_1(t) + \lambda_2 \frac{1-\alpha_2(t)}{2}} \begin{pmatrix} \frac{1-\alpha_0}{2} \\ \lambda_1 \alpha_1(t) \\ \lambda_2 \frac{1-\alpha_2(t)}{2} \end{pmatrix}, \quad \text{Consumption of country 1's good} \tag{12}$$

$$\begin{pmatrix} C_0^2(t) \\ C_1^2(t) \\ C_2^2(t) \end{pmatrix} = \frac{Y^2(t)}{\frac{1-\alpha_0}{2} + \lambda_1 \frac{1-\alpha_1(t)}{2} + \lambda_2 \alpha_2(t)} \begin{pmatrix} \frac{1-\alpha_0}{2} \\ \lambda_1 \frac{1-\alpha_1(t)}{2} \\ \lambda_2 \alpha_2(t) \end{pmatrix}, \quad \text{Consumption of country 2's good} \tag{13}$$

These consumption allocations are similar to familiar sharing rules arising in equilibrium models with logarithmic preferences. In the benchmark economy with perfect risk sharing, the correlation between consumption of a particular good and its aggregate output would have been perfect if not for the demand shifts.

Since consuming the aggregate output must be optimal for the representative agent, the terms of trade are given by the pertinent marginal rates of substitution processes

$$q^1(t) = \frac{u_{C^1}(Y^0(t), Y^1(t), Y^2(t); \lambda_1, \lambda_2)}{u_{C^0}(Y^0(t), Y^1(t), Y^2(t); \lambda_1, \lambda_2)} = \frac{\frac{1-\alpha_0}{2} + \lambda_1 \alpha_1(t) + \lambda_2 \frac{1-\alpha_2(t)}{2}}{\alpha_0 + \lambda_1 \frac{1-\alpha_1(t)}{2} + \lambda_2 \frac{1-\alpha_2(t)}{2}} \frac{Y^0(t)}{Y^1(t)}, \tag{14}$$

$$q^2(t) = \frac{u_{C^2}(Y^0(t), Y^1(t), Y^2(t); \lambda_1, \lambda_2)}{u_{C^0}(Y^0(t), Y^1(t), Y^2(t); \lambda_1, \lambda_2)} = \frac{\frac{1-\alpha_0}{2} + \lambda_1 \frac{1-\alpha_1(t)}{2} + \lambda_2 \alpha_2(t)}{\alpha_0 + \lambda_1 \frac{1-\alpha_1(t)}{2} + \lambda_2 \frac{1-\alpha_2(t)}{2}} \frac{Y^0(t)}{Y^2(t)}. \tag{15}$$

Since in our model the terms of trade would play a central role in linking together the countries' stock markets, we structure our benchmark economy so as to be able to capture some of their most important properties highlighted in international economics. First, the terms of trade of the Periphery countries with the Center decrease in their domestic output and increase in the Center's output. This is a standard feature of Ricardian models of international trade: terms of trade move against countries experiencing an increase in productivity or output as their goods become relatively less scarce.[14] Second, we attempt to capture the "dependent economy" effects highlighted in open economy macroeconomics: the terms of trade improve for a country, i, that has experienced a positive demand shift (an increase in α_i). The intuition for this result is that a higher demand for domestic goods increases the price of domestic relative to foreign goods, improving the terms of trade.

The key to the tractability of our model is that the stock prices can be computed in closed form. We report the resulting expressions in the following lemma.

Lemma 2. *The prices of the stocks of the Center and the Periphery countries are given by*

$$S^0(t) = \frac{1}{\beta + \frac{1-\beta}{2} q^1(t) + \frac{1-\beta}{2} q^2(t)} Y^0(t)(T-t), \tag{16}$$

$$S^1(t) = \frac{q^1(t)}{\beta + \frac{1-\beta}{2} q^1(t) + \frac{1-\beta}{2} q^2(t)} Y^1(t)(T-t), \tag{17}$$

$$S^2(t) = \frac{q^2(t)}{\beta + \frac{1-\beta}{2} q^1(t) + \frac{1-\beta}{2} q^2(t)} Y^2(t)(T-t). \tag{18}$$

Equations (11)–(18) summarize the prices and allocations which would prevail in the competitive equilibrium in our economy. At this point it is important to note that wealth distribution in the economy does not enter as a state variable in any of the above equations. This is because wealth distribution is constant, determined by the initial shareholdings:

$$\frac{W_1(t)}{W_0(t)} = \lambda_1 \quad \text{and} \quad \frac{W_2(t)}{W_0(t)} = \lambda_2. \tag{19}$$

[14] This result is independent of the wealth distribution and the consumption shares.

The equalities in (19) follow from, for example, (11), combined with Lemma 1. This is a convenient feature of our benchmark equilibrium, allowing us to easily disentangle the effects of the time-varying wealth distribution in the economy with portfolio constraints, presented in the next section.

To facilitate the comparison with the economy with portfolio constraints, we need the following proposition.

Proposition 1. *(i) The joint dynamics of the terms of trade and the three stock markets in the benchmark unconstrained economy are given by*

$$\begin{bmatrix} \frac{dq^1(t)}{q^1(t)} \\ \frac{dq^2(t)}{q^2(t)} \\ \frac{dS^0(t)}{S^0(t)} \\ \frac{dS^1(t)}{S^1(t)} \\ \frac{dS^2(t)}{S^2(t)} \end{bmatrix} = I(t)dt + \underbrace{\begin{bmatrix} a(t) & b(t) & 1 & -1 & 0 \\ \widetilde{a}(t) & \widetilde{b}(t) & 1 & 0 & -1 \\ -X_{\alpha_1}(t) & -X_{\alpha_2}(t) & \beta M(t) & \frac{1-\beta}{2}\frac{M(t)}{q^1(t)} & \frac{1-\beta}{2}\frac{M(t)}{q^2(t)} \\ a(t) - X_{\alpha_1}(t) & b(t) - X_{\alpha_2(t)} & \beta M(t) & \frac{1-\beta}{2}\frac{M(t)}{q^1(t)} & \frac{1-\beta}{2}\frac{M(t)}{q^2(t)} \\ \widetilde{a}(t) - X_{\alpha_1}(t) & \widetilde{b}(t) - X_{\alpha_2(t)} & \beta M(t) & \frac{1-\beta}{2}\frac{M(t)}{q^1(t)} & \frac{1-\beta}{2}\frac{M(t)}{q^2(t)} \end{bmatrix}}_{\Theta_u(t)} \begin{bmatrix} d\alpha_1(t) \\ d\alpha_2(t) \\ \sigma_{Y^0}(t)dw^0(t) \\ \sigma_{Y^1}(t)dw^1(t) \\ \sigma_{Y^2}(t)dw^2(t) \end{bmatrix}.$$

The drift term I and quantities X_{α_1}, X_{α_2}, M, a, $\widetilde{a}$, b, and $\widetilde{b}$ are defined in the Appendix.

Proposition 1 decomposes stock and commodity markets returns into responses to five underlying factors: demand shifts in Periphery countries 1 and 2 and output (supply) shocks in all three countries. There responses are captured in matrix Θ_u, henceforth referred to as the *unconstrained dynamics*. Some of the elements of Θ_u can be readily signed, while the signs of others are ambiguous. In particular, the directions of the transmission of the supply shocks to the stock markets and the terms of trade are unambiguous, while those for the demand shifts depends on the relative size of the countries involved.

Understanding the responses of the terms of the terms of trade to the shocks is key to understanding the transmission of the shocks to the remaining quantities. The directions of transmission of supply shocks are unambiguous and easy to sign. On the other hand, those of the demand shifts depend on the relative sizes of the countries. Our leading interpretation of the economy involves a large Center country (a developed economy) and two small and relatively similar Periphery countries (emerging markets). Such interpretation allows us to get sharper predictions for the signs of the responses of the terms of trade and therefore the stock market prices to the demand shocks. It justifies considering the following conditions:[15]

[15]In the sequel, we always specify whether a sign is unambiguous or occurs under a specific condition. Condition A1

Variable/ Effects of	$d\alpha_1(t)$	$d\alpha_2(t)$	$dw^0(t)$	$dw^1(t)$	$dw^2(t)$
$\frac{dq^1(t)}{q^1(t)}$	$+$	$-^{A_1}$	$+$	$-$	0
$\frac{dq^2(t)}{q^2(t)}$	$-^{A_1}$	$+$	$+$	0	$-$
$\frac{dS^0(t)}{S^0(t)}$	$-^{A_2}$	$-^{A_2}$	$+$	$+$	$+$
$\frac{dS^1(t)}{S^1(t)}$	$+^{A_1}$	$-^{A_2}$	$+$	$+$	$+$
$\frac{dS^2(t)}{S^2(t)}$	$-^{A_2}$	$+^{A_1}$	$+$	$+$	$+$

Table 1: Terms of trade and stock returns in the benchmark unconstrained economy. Where a sign is ambiguous, we specify a sufficient or a necessary and sufficient condition for the sign to obtain: A_1 stands for the "small country" condition A1, and A_2 stands for the "similar country" condition A2.

Condition A1. *The Periphery countries are small relative to the Center.*

$$\lambda_2 < \frac{3\alpha_0 - 1}{3\alpha_2(t) - 1}$$
$$\lambda_1 < \frac{3\alpha_0 - 1}{3\alpha_1(t) - 1}$$

Condition A2. *The Periphery countries are similar.*

$$\frac{3\alpha_0 - 1}{3\alpha_0 + 1} < \frac{Y^2(t)}{Y^1(t)} < \frac{3\alpha_0 + 1}{3\alpha_0 - 1}$$

Let us now discuss the details of the transmission mechanisms in our model and relate them to the literature. Table 1 summarizes the patterns of responses of the terms of trade and stock prices to the underlying shocks.[16] One immediate implication of Table 1 is that supply shocks create co-movement among stock market prices worldwide. The co-movement is generated by two channels of international transmission: the terms of trade and the common worldwide discount factors for cash flows (common state prices). To illustrate the workings of the former channel, consider a positive supply shock in country j. Such a shock has a direct (positive) effect on country j's stock market. Additionally, it has an indirect (also positive) effect on the remaining stock markets through the terms of trade. Indeed, as discussed earlier, a supply shock in country j creates an excess supply of good j, and hence causes a drop in its price relative to the rest of the goods. This implies that

is necessary and sufficient and Condition A2 is sufficient. Further discussion of these conditions is presented in the Appendix. The conditions affect none of the derivations, they are used only for presenting the directions of responses of the stocks and the terms of trade to the underlying shocks.

[16]In our specification, demand and supply shocks are correlated. We nonetheless find it useful to report their effects separately in Table 1. We do so because the implications of the supply shocks for the stock market co-movement are of the opposite nature as those of the demand shocks, and disentangling the effects of the two types of shocks is useful for understanding the mechanism behind international propagation.

the prices of all the other goods increase relative to good j, boosting the value of the stock markets in the rest of the world. This explanation of the transmission of shocks across countries appears to be solely based on goods markets clearing, where the terms of trade act as a propagation channel. This channel, however, is not unrelated to the second transmission vehicle: the well-functioning financial markets creating the common discount factor for all financial securities. Indeed, in our model, clearing in good markets implies clearing in stock and bond markets as well, and hence the above intuition could be restated in terms of equilibrium responses of the stock market prices. Such intuition for "financial contagion" was highlighted by Kyle and Xiong (2001), who see contagion as a wealth effect (see also Cochrane, Longstaff, and Santa-Clara (2004)). An output shock in one of the countries always increases its stock market price and hence each agent's wealth (because all agents have positive positions in each stock market). At a partial equilibrium level, a wealth increase triggers portfolio rebalancing. In particular, it is easy to show that, for diversification reasons, our agents demand more of all stocks. At an equilibrium level, of course, no rebalancing takes place because the agents have identical portfolios and they must jointly hold the entire supply of each market. Therefore, prices of all stocks move upwards to counteract the incentive to rebalance. So, the two transmission channels—the terms of trade and the common discount factor—interact and may potentially be substitutes for each other. Note that none of these arguments makes any assumption about the correlation of output shocks across countries—in fact, in our model they are unrelated. The existing literature would identify the phenomenon we described here as "contagion" (the co-movement in stock markets beyond the co-movement in fundamentals). In our personal views, this co-movement is not contagion—we view it as nothing else but a simple consequence of market clearing and hence a natural propagation that is to be expected in any international general equilibrium model. Our definition of contagion is the co-movement in excess of the natural propagation described above.

While supply shocks induce co-movement among the countries' stock markets, demand shocks potentially introduce divergence. Consider, for example, a positive demand shift occurring in country 1. Country 1 now demands more of the domestically-produced good and less of the foreign goods, which unambiguously increases the price of the domestic good. The direction of the response of the other Periphery country's terms of trade depends on its wealth relative to the Center, λ_2. If the country is small (Condition A1), it suffers disproportionately more due to a drop in demand for its good, and its terms of trade with the Center deteriorate. The impact on the stock markets, however, requires a more detailed discussion. We can represent the stock market prices of the countries in the following form: $S^0(t) = p^0(t)\,Y^0(t)(T-t)$, $S^1(t) = q^1(t)p^0(t)\,Y^1(t)(T-t)$, and $S^2(t) = q^2(t)p^0(t)\,Y^2(t)(T-t)$. A demand shift in country 1 improves its relative price q^1 and deteriorates the other Periphery countries relative price q^2, pushing S^1 up and S^2 down—this is the direct effect. However, there is also an indirect effect due to a fall in the price level in the Center country. The conditions of similar and small Periphery countries ensure that the impact of these demand shocks on the Center price p^0 are small, forcing the terms of trade effect to dominate. However small, there is a drop in the price of the Center's good p^0, and hence the stock price of the Center falls.

3. Equilibrium in the Economy with Portfolio Constraints

The previous section outlines two of the most prominent channels behind co-movement among stock markets across the world: the trade and the common discount factor channels. Although the empirical literature has shown that these two transmission mechanisms are important components of the international propagation of shocks, it has been argued that other channels could be at play, primarily those resulting from financial market imperfections.[17] In this section we explore the role of portfolio constraints—perhaps the most frequently mentioned market friction—in propagation and amplification of shocks. We stress the importance of addressing this question within a general equilibrium framework, which highlights the critical role wealth redistribution in the transmission mechanism.

3.1. The Common Factor due to Constraints

In the economy with financial markets imperfections the equilibrium allocation would not be Pareto optimal, and hence the usual construction of a representative agent's utility as a weighted sum (with constant weights) of individual utility functions is not possible. Instead, we are going to employ a representative agent with stochastic weights (introduced in an important contribution by Cuoco

[17] Kaminsky, Reinhart, and Végh (2003) presents a thorough review of the literature. See Calvo (1999) and Yuan (2005) for theories in which margin calls are responsible for the excess co-movement. See also Geanakoplos (2003). Kaminsky and Schmukler (2002) and Rigobon (2002) present empirical evidence suggesting that the co-movement of a country's stock returns with those of other countries depends on the credit rating and on an asset class its financial instruments belong to. One such example is Mexico whose correlation with other Latin American countries dropped by a half when its debt got upgraded from non-investment grade to investment grade. Other examples include Malaysia, Indonesia, and Thailand whose debt got downgraded during the 1997 Asian crisis, resulting in a sharp increase of the correlation of their stock markets amongst themselves, as well as with Latin American markets.

and He (2001)), with these stochastic weights capturing the effects of market frictions.[18] This representative agent has utility function

$$U(C^0, C^1, C^2; \lambda_0, \lambda_1, \lambda_2) = E\left[\int_0^T u(C^0(t), C^1(t), C^2(t); \lambda_0(t), \lambda_1(t), \lambda_2(t))dt\right],$$

with

$$u(C^0, C^1, C^2; \lambda_0, \lambda_1, \lambda_2) = \max_{\sum_{i=0}^2 C_i^j = C^j,\, j\in\{0,1,2\}} \sum_{i=0}^2 \lambda_i(t)\, u_i(C_i^0, C_i^1, C_i^2),$$

where $\lambda_i(t) > 0$, $i = 0, 1, 2$ are (yet to be determined) weighting processes, which may be stochastic. We again normalize the weight of the Center's consumer to be equal to one ($\lambda_0(t) = 1$). The advantage of employing this approach is that a bulk of the analysis of the previous section can be directly imported to this section. In particular, the only required modification to equations (11)–(15) is that the constant weights λ_1 and λ_2 are now replaced by their stochastic counterparts. The expressions for stock market prices (16)–(18) also continue to hold in the constrained economy (see the proof of Lemma 2 in the Appendix). Furthermore, as a consequence of the consumption sharing rules and Lemma 1, we again conclude that $\lambda_1(t) = W_1(t)/W_0(t)$ and $\lambda_2(t) = W_2(t)/W_0(t)$. So in the constrained economy the wealth distribution, captured by the quantities λ_1 and λ_2, becomes a new state variable. Finally, in the constrained economy, we also have an analog of Proposition 1, except now the weighting processes λ_1 and λ_2 enter as additional factors. These factors capture the effects of the portfolio constraint imposed on the Center's consumer.

Proposition 2. *(i) In an equilibrium with the portfolio constraint, the weighting processes λ_1 and λ_2 are the same up to a multiplicative constant.*

(ii) When such equilibrium exists, the joint dynamics of the terms of trade and three stock markets in the economy with the portfolio constraint are given by

$$\begin{bmatrix} \frac{dq^1(t)}{q^1(t)} \\ \frac{dq^2(t)}{q^2(t)} \\ \frac{dS^0(t)}{S^0(t)} \\ \frac{dS^1(t)}{S^1(t)} \\ \frac{dS^2(t)}{S^2(t)} \end{bmatrix} = I_c(t)dt + \left[\begin{array}{c|c} A(t) & \\ \widetilde{A}(t) & \\ -X_\lambda(t) & \Theta_u(t) \\ A(t) - X_\lambda(t) & \\ \widetilde{A}(t) - X_\lambda(t) & \end{array}\right] \begin{bmatrix} \frac{d\lambda(t)}{\lambda(t)} \\ d\alpha_1(t) \\ d\alpha_2(t) \\ \sigma_{Y^0}(t)dw^0(t) \\ \sigma_{Y^1}(t)dw^1(t) \\ \sigma_{Y^2}(t)dw^2(t) \end{bmatrix}$$

[18] The construction of a representative agent with stochastic weights has been employed extensively in dynamic asset pricing. See, for example, Basak and Croitoru (2000), Basak and Cuoco (1998), and Shapiro (2002). A related approach is the extra-state-variable methodology of Kehoe and Perri (2002). For the original solution method utilizing weights in the representative agent see Negishi (1960).

where $\lambda(t) \equiv \lambda_1(t)$, I_c and X_λ are reported in the Appendix, and where the unconstrained dynamics matrix $\Theta_u(t)$ is as defined in Proposition 1.[19]

Proposition 2 reveals that the same transmission channels underlying the benchmark economy are present in the economy with portfolio constraints. Ceteris paribus, the sensitivities of the terms of trade and stock prices to the demand and supply shocks are exactly the same as in Proposition 1. The only difference from the benchmark economy comes in the first, $d\lambda/\lambda$, term. This term summarizes the dynamics of the two stochastic weighting processes λ_1 and λ_2, which end up being proportional in equilibrium, and hence represent a single common factor we labeled λ.[20] Thus, the process λ should be viewed as an additional factor in stock prices and the terms of trade dynamics, arising as a consequence of the portfolio constraints.

One can already note the cross-markets effect of portfolio constraints: the constraint affects not only the Center's stock market, but also Periphery stocks, as well as the terms of trade. This finding is, of course, to be expected in a general equilibrium model. The effects of constraints in financial markets get transmitted to all other (stock, bond, and commodity) markets via pertinent market clearing equations. Our contribution is to fully characterize these spillover effects and identify their direction. The signs of responses to the supply and demand shocks are, of course, the same as in the benchmark unconstrained equilibrium. Additionally, we can sign the responses of all markets to innovations in the new factor; some signs are unambiguous, and some obtain under the following condition:

[19] Existence of equilibrium can be shown for the case in which the portfolio constraint does not bind (the unconstrained benchmark) and for the case of specific constraints considered in our examples in Section 5, but would be very difficult to show for the general specification of the constraint considered in this section. Still, we feel that our analysis in this section is important, as it characterizes properties of equilibrium that obtain for *any* constraint imposed on the Center country.

[20] This finding depends on the fact that the two Periphery countries face the same investment opportunity set: here, they are both unconstrained. If these two countries faced heterogeneous constraints, in general, one would not expect their weighting processes to be proportional, and hence both λ_1 and λ_2 would enter as relevant factors. Furthermore, in our specification the new factor λ is not independent from the existing factors—for example, an innovation to any underlying Brownian motion affects the distribution of wealth and hence λ. However, for the purposes of separating the incremental effect of the portfolio constraint relative to the dynamics occurring in the unconstrained benchmark we find it useful to treat λ as an additional factor.

Variable/ Effects of	$\frac{d\lambda(t)}{\lambda(t)}$	$d\alpha_1(t)$	$d\alpha_2(t)$	$dw^0(t)$	$dw^1(t)$	$dw^2(t)$
$\frac{dq^1(t)}{q^1(t)}$	$+$	$+$	$-^{A_1}$	$+$	$-$	0
$\frac{dq^2(t)}{q^2(t)}$	$+$	$-^{A_1}$	$+$	$+$	0	$-$
$\frac{dS^0(t)}{S^0(t)}$	$-$	$-^{A_2}$	$-^{A_2}$	$+$	$+$	$+$
$\frac{dS^1(t)}{S^1(t)}$	$+^{A_3}$	$+^{A_1}$	$-^{A_2}$	$+$	$+$	$+$
$\frac{dS^2(t)}{S^2(t)}$	$+^{A_3}$	$-^{A_2}$	$+^{A_1}$	$+$	$+$	$+$

Table 2: Terms of trade and stock returns in the economy with portfolio constraints. Where a sign is ambiguous, we specify a sufficient or a necessary and sufficient condition for the sign to obtain: A_1 stands for the "small country" condition A1, A_2 for the "similar country" condition A2, and A_3 for the "small effect on p^0" condition A3.

Condition A3. *The effect of the portfolio constraint on p^0 is small.*[21]

$$\frac{1-\beta}{2}\, q^2(t)(\widetilde{A}(t) - A(t)) \quad < \quad \beta A(t), \tag{20}$$
$$\frac{1-\beta}{2}\, q^1(t)(A(t) - \widetilde{A}(t)) \quad < \quad \beta \widetilde{A}(t). \tag{21}$$

Table 2 reveals the contribution of financial markets frictions to international co-movement. The first striking implication is that the terms of trade faced by both Periphery countries move in the *same direction* in response to an innovation in the λ factor. A movement in λ should be viewed in our model as a tightening or a loosening of the portfolio constraint. Given the definition of λ, such innovation reflects a wealth redistribution in the world economy *to* or *away* from the Periphery countries. Parallels may be drawn to the literature studying the effects of wealth transfers on the terms of trade. It is well-known from the classic "Transfer Problem" of the international economics literature that an income (wealth) transfer from one country to another improves the terms of trade of the recipient. As wealth of the recipient of the transfer goes up, his total demand increases, but because of the preference bias for his own good, the demand for the domestic good increases disproportionately more. Hence the price of the home good rises relative to the foreign goods, improving the terms of trade of the recipient.[22] In our model, a decrease in the factor λ

[21] The condition is necessary and sufficient. It is likely to be satisfied under the leading interpretation of the Center country being big. In the Appendix we investigate this condition further, representing it as a combination of two effects: (i) the impact of a change in λ (the implied wealth transfer) on the demand for good 0 and (ii) the cross-country demand reallocation in the Periphery countries. The condition affect none of the derivations, it is used only for presenting the directions of responses of the Periphery countries' stocks to innovations in λ.

[22] The original "Transfer Problem" was the outcome of a debate between Bertil Ohlin and John Maynard Keynes regarding the true value of the burden of reparations payments demanded of Germany after World War I. Keynes

is interpreted as a wealth transfer to the Center country. Just like in the Transfer Problem, it results in an improvement of its terms of trade against the world and hence a deterioration of the terms of trade of both Periphery countries—the reverse for an increase in λ. The main difference between our work and the Transfer Problem literature is that the latter considers exogenous wealth transfers, while wealth transfers are generated endogenously in our model as a result of a tightening of the portfolio constraint. The direction of such a transfer (to or from the Periphery countries) in response to a tightening or a loosening of the constraint depends on the form of a constraint.[23]

The intuition behind the occurrence of the wealth transfers in our model is simple. Assume for a moment that there is no constraint. Then each country holds the same portfolio. When a (binding) constraint is imposed on the investors in the Center, their portfolio has to deviate from the benchmark, and now the portfolios of the Center and Periphery investors differ. This means that stock market price movements will have differential effects on the investors' wealth. The movements of wealth obviously depend on the type of the constraint. For *any* constraint that binds, however, one can say that the distribution of wealth will fluctuate, becoming the additional transmission vehicle. Moreover, since the Periphery countries hold identical portfolios, their wealth shares move in tandem. That is, the resolution of uncertainty always affects the Periphery countries in the same way: they either both become poorer or both become richer relative to the Center.

The portfolio constraint also generally induces the co-movement between the stock markets of the Periphery countries. This co-movement may be partially confounded by the Center good price effect, which is of the same nature as the one encountered in the case of the demand shifts in the benchmark model (see Section 2). Consider, for example, a response to a positive shock in λ. While the improving terms of trade effect boosts the Periphery stock markets, the associated downward move in p^0 may potentially offset this. However, given our Condition A3, the latter effect is dwarfed by the improvement in the terms of trade. If we were to quote stock market prices of the Periphery in terms of the production basket of Center, rather than the world consumption basket, the two Periphery markets would always co-move in response to a tightening or a loosening of the portfolio

argued that the payments would result in a reduction of the demand for German goods and cause a deterioration of the German terms of trade, making the burden on Germany much higher than the actual value of the payments. On the other hand, Ohlin's view was that the shift in demand would have no impact on relative prices. This implication would be correct if all countries have the exact same demands (in our model this requires an assumption that $\alpha_i = 1/3$, $i = 0, 1, 2$.) See Krugman and Obstfeld (2003) for an elaboration and references.

[23]There exists ample empirical evidence documenting contagion among the exchange rates or the terms of trade of emerging markets, the Periphery countries, under our leading interpretation (see, for example, Kaminsky and Schmukler (2002) and Rigobon (2002)). We offer a simple theory in which this contagion arises as a natural consequence of wealth transfers due to financial market frictions.

constraint. On the other hand, the response of the stock market of the Center is unambiguous and goes in opposite direction of λ, reflecting the effects of an implicit wealth transfer to or from the Center. So, in summary, the implicit wealth transfers due to the portfolio constraint create an additional co-movement among the terms of trade of the Periphery countries, as well as their stock market prices, while reducing the co-movement between the Center and the Periphery stock markets.

4. "Contagion" without Trade

In the previous section we have considered a model in which each Periphery country allocates equal expenditure shares to the two goods it imports. This may appear unrealistic in the context of our leading interpretation, where the Center country represents a large developed economy and the Periphery countries two emerging markets, because emerging economies trade with industrialized economies much more than amongst themselves. Moreover, recent empirical studies of emerging markets have cast doubt on the ability of trade relationships to generate international co-movement of observed magnitudes and have documented that contagion exists even among countries with insignificant trade relationships.[24] Since the movements in the terms of trade is an essential ingredient of the contagion mechanism in our model, it is natural to ask whether our results still hold under alternative assumptions regarding the extent of trade (in goods) between the Periphery countries. In this section we take our setting to the limit and show that even when Periphery countries do not trade at all, their stock markets will co-move as described in the baseline analysis.

To examine this scenario, we modify the countries' preferences as follows:

$$\begin{aligned}
u_0\big(C_0^0,\, C_0^1,\, C_0^2\big) &= \log C_0^0(t), \\
u_1\big(C_1^0,\, C_1^1,\, C_1^2\big) &= (1-\alpha_1(t)) \log C_1^0(t) + \alpha_1(t) \log C_1^1(t), \\
u_2\big(C_2^0,\, C_2^1,\, C_2^2\big) &= (1-\alpha_2(t)) \log C_2^0(t) + \alpha_2(t) \log C_2^2(t).
\end{aligned}$$

That is, we assume that the goods produced by the Periphery countries are nontraded, and the only trade occurring in the model is that between each Periphery country and the Center. We continue to assume that there is a home bias in consumption by restricting α_i to be a martingale lying between 1/2 and 1. As before, the Center country's portfolios are constrained to lie in a closed, convex, non-empty subset $\{K_t(\omega);\ (t,\, \omega) \in [0,\, T] \times \Omega\}$.

[24] See Kaminsky, Reinhart, and Végh (2003) and Broner, Gelos, and Reinhart (2004).

Under this specification, the terms of the trade of each Periphery country with the Center are

$$q^j(t) = \frac{\alpha_j(t)\lambda_j(t)}{1 + \lambda_1(t)(1-\alpha_1(t)) + \lambda_2(t)(1-\alpha_2(t))} \frac{Y^0(t)}{Y^j(t)}, \qquad j \in \{1,\, 2\}, \tag{22}$$

where the relative weights λ_1 and λ_2 are possibly stochastic. It is straightforward to show that the expressions for the stock prices remain the same, given by (16)–(18).

Variable/ Effects of	$\frac{d\lambda(t)}{\lambda(t)}$	$d\alpha_1(t)$	$d\alpha_2(t)$	$dw^0(t)$	$dw^1(t)$	$dw^2(t)$
$\frac{dq^1(t)}{q^1(t)}$	$+$	$+$	$+$	$+$	$-$	0
$\frac{dq^2(t)}{q^2(t)}$	$+$	$+$	$+$	$+$	0	$-$
$\frac{dS^0(t)}{S^0(t)}$	$-$	$-$	$-$	$+$	$+$	$+$
$\frac{dS^1(t)}{S^1(t)}$	$+$	$+$	A	$+$	$+$	$+$
$\frac{dS^2(t)}{S^2(t)}$	$+$	A	$+$	$+$	$+$	$+$

Table 3: Terms of trade and stock returns in the economy with portfolio constraints and no trade between the Periphery countries. "A" stands for "ambiguous."

In the interest of space, we do not provide the dynamics of the terms of trade and stock prices in this economy; we just present a table (Table 3) that mimics Table 2 of Section 3. In contrast to Table 2, only two signs in Table 3 are ambiguous (related, again, to demand shifts); the remaining implications do not require any further conditions. The effects of the demand shocks on the terms of trade are now clear-cut because a demand shift in a Periphery country 1 not only increases the world demand for good 1 relative to all other goods (as before), but also decreases the demand for good 0, while leaving the demand for good 2 unchanged. Therefore, the price of good 0 drops relative to that of both goods 1 and 2. Another set of signs that becomes unambiguous is that for the effects of the innovation in the wealth shares of the Periphery countries captured by λ on the stock prices in the Periphery.

Within this economy it is easy to derive the real exchange rates faced by the Periphery countries.

Remark 1 (Real Exchange Rates). The price indexes in each country, derived from the countries' preferences, are given by

$$P^0(t) = p^0(t), \quad P^1(t) = \left(\frac{p^0(t)}{1-\alpha_1(t)}\right)^{1-\alpha_1(t)} \left(\frac{p^1(t)}{\alpha_1(t)}\right)^{\alpha_1(t)}, \quad P^2(t) = \left(\frac{p^0(t)}{1-\alpha_2(t)}\right)^{1-\alpha_2(t)} \left(\frac{p^1(t)}{\alpha_2(t)}\right)^{\alpha_2(t)}.$$

The real exchange rates, expressed as functions of the terms of trade, are then

$$e^j(t) = \frac{P^j(t)}{P^0(t)} = (1-\alpha_j(t))^{\alpha_j(t)-1}\alpha_j(t)^{-\alpha_j(t)}\Big(q^j(t)\Big)^{\alpha_j(t)}, \qquad j \in \{1,\, 2\}.$$

Our primary concern is the incremental effect of a change in λ on the real exchange rates, as in the first column of Table 3. Since the utility weights α_j are positive, the real exchange rates respond to a change in λ in the same direction the terms of trade do. This implies that the excess co-movement in the terms of trade due to the portfolio constraint translates into the excess co-movement of the real exchange rates of the Periphery countries.

5. Examples of Portfolio Constraints

The purpose of this section is to illustrate the applicability of our general framework to studying specific portfolio constraints. Under a specific constraint, we can fully characterize the countries' portfolios and hence identify the direction of the constraint-necessitated wealth transfers. This will allow us to address questions of the following nature, "Does a positive shock in the Center entail a wealth transfer to the Center?", "How does the origin of a shock affect stock returns worldwide?", "Does the constraint amplify the shocks?"

5.1. Pure Wealth Transfers: A Portfolio Concentration Constraint

Here, we return to our workhorse model presented in Section 2 and specialize the constraint set K to represent a portfolio concentration constraint. That is, the resident of the Center country now faces a constraint permitting him to invest no more than a certain fraction of his wealth γ into the stock markets of countries 1 and 2:

$$x_0^{S^1}(t) + x_0^{S^2}(t) \leq \gamma, \qquad \gamma \in \mathbb{R}. \tag{23}$$

While we think this constraint is reasonable, we do not intend to argue that such a constraint is necessarily behind the patterns of correlations observed in reality. Our goal is to merely illustrate the workings of our model. We feel that (23) is particularly well-suited for this purpose, since its impact on the portfolio composition and hence the entailed wealth transfers are very easy to understand.[25]

For the concentration constraint, we can fully characterize the process λ and hence the remaining equilibrium quantities. Note that the consumption allocations, terms and trade, and stock prices all depend on the primitives of the model and the unknown stochastic weights. Therefore, once the

[25] We concede that other constraints, especially government-imposed, may be more economically relevant, but in this section we consider only two possible constraints. Another set of restrictions absent from the model is those on the Periphery countries. The model possesses sufficient flexibility to accommodate these alternative constraints (for now, constraining one country at a time), but we leave this analysis, as well as a formal calibration, for future applications.

process λ and the constants $\lambda_1(0)$ and $\lambda_2(0)$ are determined, we would be able to pin down all of these equilibrium quantities. It follows from (6), (9), and Lemma 1 that

$$\lambda(t) = \frac{y_0 \xi_0(t)}{y_1 \xi(t)}.$$

Recall that due to the portfolio constraint, the Center country and the Periphery face different state price densities, ξ_0 and ξ, respectively. In particular, the (constrained) Center country's effective interest rate and the market price of risk, r_0 and m_0, are tilted so as to reflect the extent to which the country's investments are constrained. Applying Itô's lemma, and using the definitions of ξ and ξ_0 from (3) and (8), we obtain

$$d\lambda(t) = \lambda(t)[r(t) - r_0(t) + m(t)^\top (m_0(t) - m(t))]dt - \lambda(t)(m_0(t) - m(t))^\top dw(t). \tag{24}$$

Substituting this dynamics into the expressions in Proposition 2, we have the following representation for the volatility matrix of the stock returns, σ:

$$\sigma(t) = \begin{bmatrix} -X_\lambda(t) & -X_{\alpha_1}(t) & -X_{\alpha_2}(t) & \beta & \frac{1-\beta}{2} & \frac{1-\beta}{2} \\ A(t) - X_\lambda(t) & a(t) - X_{\alpha_1}(t) & b(t) - X_{\alpha_2}(t) & \beta & \frac{1-\beta}{2} & \frac{1-\beta}{2} \\ \widetilde{A}(t) - X_\lambda(t) & \widetilde{a}(t) - X_{\alpha_1}(t) & \widetilde{b}(t) - X_{\alpha_2}(t) & \beta & \frac{1-\beta}{2} & \frac{1-\beta}{2} \end{bmatrix} \begin{bmatrix} (m(t) - m_0(t))^\top \\ \sigma_{\alpha_1}(t)^\top \\ \sigma_{\alpha_2}(t)^\top \\ M(t)\sigma_{Y^0}(t)\, i_0^\top \\ M(t)q^1(t)\sigma_{Y^1}(t)\, i_2^\top \\ M(t)q^2(t)\sigma_{Y^2}(t)\, i_2^\top \end{bmatrix} \tag{25}$$

where $i_0 \equiv (1,\,0,\,0)^\top$, $i_1 \equiv (0,\,1,\,0)^\top$, and $i_2 \equiv (0,\,0,\,1)^\top$. The 3×3 matrix σ represents the loadings on the three underlying Brownian motions w^0, w^1, and w^2 of the three stocks: S^0 (captured by the the first row of σ), S^1 (the second row), and S^2 (the third row). In the benchmark unconstrained economy or at times when the constraint is not binding, all countries face the same state price density, and hence the market price of risk $m_0(t)$ coincides with $m(t)$, and the matrix σ coincides with its counterpart in the benchmark unconstrained economy.

The final set of equations, required to fully determine the volatility matrix in the economy with portfolio constraints, is presented in the following proposition.

Proposition 3. *When equilibrium exists, the equilibrium market price of risk processes faced by* *the Center and the Periphery are related as follows:*[26]

$$\textit{When} \qquad (i_1+i_2)^\top(\sigma(t)^\top)^{-1}m(t) \le \gamma,$$

$$m_0(t) = m(t), \quad \psi(t) = 0, \qquad \textit{(Constraint not binding)}, \tag{26}$$

otherwise,

$$\begin{aligned} m_0(t) &= m(t) - (\sigma(t))^{-1}(i_1+i_2)\psi(t), \\ \psi(t) &= -\frac{\gamma - (i_1+i_2)^\top(\sigma(t)^\top)^{-1}m(t)}{(i_1+i_2)^\top(\sigma(t)\sigma(t)^\top)^{-1}(i_1+i_2)} > 0, \qquad \textit{(Constraint binding)}, \end{aligned} \tag{27}$$

where $\sigma(t)$ is given by (25). Furthermore,

$$\begin{aligned} \sigma_{Y^0}(t)i_0 &- \frac{\left(\lambda_1(t)\frac{1-\alpha_1(t)}{2} + \lambda_2(t)\frac{1-\alpha_2(t)}{2}\right)(m(t)-m_0(t)) - \frac{\lambda_1(t)}{2}\sigma_{\alpha_1}(t) - \frac{\lambda_2(t)}{2}\sigma_{\alpha_2}(t)}{\alpha_0 + \lambda_1(t)\frac{1-\alpha_1(t)}{2} + \lambda_2(t)\frac{1-\alpha_2(t)}{2}} \\ &= X_\lambda(t)(m(t)-m_0(t)) + X_{\alpha_1}(t)\sigma_{\alpha_1}(t) + X_{\alpha_2}(t)\sigma_{\alpha_2}(t) + \frac{1-\beta}{2}M(t)(q^1(t)+q^2(t))\sigma_{Y^0}(t)\, i_0 \\ &\quad -\frac{1-\beta}{2}M(t)q^1(t)\sigma_{Y^1}(t)\, i_1 - \frac{1-\beta}{2}M(t)q^2(t)\sigma_{Y^2}(t)\, i_2 + m_0(t). \end{aligned} \tag{28}$$

Equations (26)–(27) are the complementary slackness conditions coming from the constrained portfolio optimization of the resident of the Center. At times when the constraint is not binding, the market price of risk faced by the Center coincides with that faced by the Periphery. Therefore, the portfolio of the Center is given by the same equation as the unconstrained portfolios. When the constraint is binding, however, there is a wedge between the market prices of risk faced by the Center and the Periphery (27). Equation (28) is the direct consequence of market clearing in the consumption goods. Together, (26)–(28) allow us to pin down the equilibrium market prices of risk of Center and Periphery, and hence the responses of all three stock markets to innovations in the underlying Brownian motions w^0, w^1, and w^2, as functions of the state variables in the economy. Once the market prices of risk processes m_0 and m are determined, it is straightforward to compute the effective interest rate differential faced by the Center country (Proposition 4), which completes our description of the dynamics of the process λ in (24). This, together with the countries' portfolio holdings reported in Corollary 1, concludes the full characterization of the economy.

[26] Proving existence consists of showing existence of a solution to algebraic equations (26)–(28) given our state variables, and then showing that this solution implies existence and uniqueness of a solution to the stochastic differential equation (24). The first step requires showing the invertibility of matrix σ in equation (25). There is a possibility that this matrix may not be invertible in our model, which happens when there are no demand shifts. However, existence for that case has been established in the previous literature (Cass and Pavlova (2004), Zapatero (1995)). To highlight and characterize the behavior of asset prices in our model, in the following subsections we compute the solution to our model for specific parameter values. In all of the examples the matrix σ was always invertible. The second step amounts to verifying that Lipschitz and growth conditions (see, for example, Øksendal (2003) Theorem 5.2.1) are satisfied for the drift and diffusion terms in equation (24).

β	0.9	γ	0.5		
α_0	0.75	$Y^0(t)$	1.0	$\sigma_{Y^0}(t)$	0.04
α_1	0.75	$Y^1(t)$	0.1	$\sigma_{Y^1}(t)$	0.08
α_2	0.75	$Y^2(t)$	0.1	$\sigma_{Y^2}(t)$	0.08
$\lambda_1(t) \in [0.1, 0.35]$					
$\lambda_2(t) \in [0.1, 0.35]$					
$\sigma_{\alpha_1}(t)$	$(0,\ 0.1,\ 0)$				
$\sigma_{\alpha_2}(t)$	$(0,\ 0,\ 0.1)$				

Table 4: Parameter choices

Proposition 4. *When equilibrium exists, the differential between the interest rates faced by countries 1 and 2 and that effectively faced by country 0 is given by*

$$r(t) - r_0(t) = \frac{\gamma - (i_1 + i_2)^\top (\sigma(t)^\top)^{-1} m(t)}{(i_1 + i_2)^\top (\sigma(t)\sigma(t)^\top)^{-1} (i_1 + i_2)}\, \gamma. \tag{29}$$

From (26)–(27) and (29), one can easily show that the interest rate differential is always nonpositive. That is, the interest rate effectively faced by the constrained country is higher than the world (unconstrained) interest rate. This accounts for the effects of the portfolio constraints. Recall from Section 2.2 that the optimization problem of the Center subject to a portfolio constraint is formally equivalent to an auxiliary problem with no constraints but the Center facing a fictitious investment opportunity set in which the bond and the Center's stock (the unrestricted investments) are made more attractive relative to the original market, and the stocks of the Periphery countries (the restricted investments) are made relatively less attractive. In this fictitious market, the Center optimally invests more in the bond and in the Center's stock relative to the original market, and less in the Periphery countries' stocks.

We now turn to analyzing the equilibrium prices in our economy. The solution to equations (26)–(28) is best illustrated by means of a graph. The parameters used in the analysis are summarized in Table 4. All time-dependent variables in Table 4 are the state variables in our model. In the interest of space, in our figures we fix all of them but the wealth shares of the Periphery countries $\lambda_1(t)$ and $\lambda_2(t)$. These stochastic wealth shares are behind the additional common factor driving the stock prices and terms of trade that we identify in our model, and it is of interest to highlight the dependence of the prices and portfolios in our model on these wealth shares. Hence, the horizontal axes in all the figures are λ_1 and λ_2.

The reasoning behind the choice of these parameters is the following. In our leading interpretation, the Periphery countries are small, so for the choice of the numeraire consumption basket we decided that they represented 5 percent of the world. We have chosen 75 percent as the share of expenditures on the domestic good, which is a conservative estimate, given the share of the service sector in GDP. In terms of output, the Periphery countries are one tenth of the Center, and twice as volatile. We assume that the wealth ratios relative to the Center for both Periphery countries may range from 0.1 to 0.35. Finally, we choose the parameters of the demand shocks such that the supply shocks dominate the stock price dynamics in the unconstrained economy. Recall that in our model there are only three primitive sources of uncertainty—the Brownian motions w^0, w^1, and w^2—so the supply and demand shocks are necessarily correlated. In Pavlova and Rigobon (2003) we find that in the data demand shocks are positively correlated with domestic supply innovations. Therefore, we assume that a demand shift in country j has a positive loading on w^j and zero loadings on the remaining Brownian motions.[27] Using these parameters we compute the region where the constraint is binding, the prices, and the responses of the terms of trade and stock prices to innovations in the underlying Brownian motions.

To develop initial insight into the solution we examine the region where the constraint is binding. The tightness of the constraint is measured by the multiplier ψ from equations (26) and (27). As is evident from Figure 1, for small wealth shares of the Periphery countries, the portfolio constraint is not binding, and the multiplier is zero. As their wealth shares increase, the constraint tightens: the multiplier is increasing in both λ's. In the unconstrained economy, larger λ's imply that Periphery countries constitute a larger fraction of world market capitalization, and hence, they command a larger share of the investors' portfolios. Therefore, given the same upper bound constraint on the investment in the Periphery countries, the larger these countries are, the tighter the constraint.

Let us now concentrate on how the portfolio constraint affects portfolio decisions by the investors in the Center. In our parametrization, the Periphery countries are symmetric, and therefore we only show figures for one of the Periphery countries. Figure 2 depicts the changes in portfolio weights relative to the unconstrained economy: the "excess" weight in the Center country's stock is shown in panel (a), and the "excess" weight in the Periphery country 1's stock in panel (b). For the range of λ's where the constraint is not binding, the portfolio holdings are identical to those in the unconstrained equilibrium. For the range where it becomes binding, the investor in the Center is forced to decrease his holdings of the Periphery markets. The freed-up assets get

[27] We have repeated the analysis using different realistic parameterizations and have found that the main message remained unaltered—in so far as the supply shocks dominated the dynamics of asset prices in the unconstrained economy.

invested in the stock market of the Center country and the bond, making the Center country over-weighted in the Center's stock market relative to its desired unconstrained position. Of course, the Periphery countries take the offsetting position so that the securities markets clear. In other words, the portfolio constraint forces a "home bias" on the Center and the Periphery investors. As we will demonstrate, this "home bias" implies that the wealth of the investor in the Center is more sensitive to shocks to the Center's stock market, while the wealth of the Periphery investors is relatively more susceptible to shocks to the Periphery.

5.1.1. Transfer Problem, Amplification and Flight to Quality

The next goal is to analyze how the distribution of wealth evolves in response to shocks in the three countries. From equation (24) we have computed the diffusion term in the evolution of the wealth shares λ_1 (=λ) and λ_2, which appears in Figure 3. (Recall that the two wealth shares are perfectly correlated.) Panel (a) depicts the move in these wealth shares when the Center receives a positive shock, and panel (b) shows what happens to it when a shock originates in one of the Periphery countries. Again, because of symmetry we only consider one of the Periphery countries. The response of the wealth share of the Periphery countries clearly depends on the origin of the shock: a shock in the Center depresses the share (a wealth transfer from the Periphery to the Center), while a shock in the Periphery increases it (a wealth transfer from the Center to the Periphery). To understand this effect, consider the representation of the evolution of λ in terms of the countries' portfolios:

$$\frac{d\lambda(t)}{\lambda(t)} = \text{Drift terms } dt + \left(\frac{dS^0(t)}{S^0(t)}, \frac{dS^1(t)}{S^1(t)}, \frac{dS^2(t)}{S^2(t)}\right)(x_i(t) - x_0(t)), \qquad i = 1, 2, \tag{30}$$

which follows from (24) and Corollary 1. The portfolios are the same over a range where the constraint is not binding, and hence no wealth transfers take place. In the constraint-binding range, the first component of the vector $x_i(t) - x_0(t)$ is negative, while the last two are positive. This is because the investor in the Center (Periphery) is over-weighted (under-weighted) in the Center's stock market and under-weighted (over-weighted) in the Periphery stock markets. One can verify that, although country-specific shocks spread internationally inducing co-movement, the effect of a shock on own stock market is bigger than on the remaining markets (because of divergence induced by the demand shocks).

A tighter constraint implies larger transfers, a looser constraint smaller transfers, and in the limit when the constraint is not binding, there are no wealth transfers taking place. Consequently, the effects of the transfers on the terms or trade and the stock prices become larger when the constraint is tighter. For brevity, we here omit a figure depicting the effects of the supply shocks on the terms of trade, which simply confirms the intuition we gathered from the Transfer Problem.

The incremental effect on the stock prices, brought about by the portfolio constraint, mimics the effects on the terms of trade. A country experiencing an improvement of its terms of trade enjoys an increase in its stock market, and that experiencing a deterioration sees its stock drop. Now we can fully address the issue of the co-movement among the stock markets that the portfolio constraint induces. These results are presented in Figures 4. Panel (a) demonstrates the impact that a shock to the Center has on the Center's stock market, beyond the already positive effect that takes place in the unconstrained economy. In the unconstrained region the effect is zero, but it is positive over the remainder of the state space. That is, the effect of a shock to the Center is *amplified* in the presence of the constraint. Furthermore, the magnitude of the effect increases with the λ's, which is to be expected because the higher the wealth shares of the Periphery countries are, the tighter the constraint. The exact same intuition applies to the effects of the shocks in the Periphery on domestic stock prices (panel (d)), except that here the responses are plotted for a negative shock (for later use).

The transmission of shocks across countries is depicted in panels (b), (c), and (e). The impact of a productivity shock in the Center on the Periphery stock prices is shown in panel (b), that of a shock in a Periphery country on the Center in panel (c) and, finally, that of a shock in one Periphery country on the other Periphery country in panel (e). Again, these are incremental effects due to the constraint, net of the co-movement implied by the unconstrained model. The emerging pattern is consistent with the flight to quality and contagion effects, observed in the data. The *flight to quality* and contagion refer to a transmission pattern where a negative shock to one of the Periphery countries (emerging markets) depresses stocks of other countries in the Periphery, but boosts the Center country's stock market (an industrialized economy). Panels (c)–(e) demonstrate that in our model a negative shock to one of the Periphery countries reduces its stock price, decreases the stock price of the other Periphery country (contagion), and increases the stock market price in the Center (flight to quality). A similar pattern occurs if the Center receives a positive shock.

5.2. Varying Restrictiveness: A Market Share Constraint

The previous constraint is one of the simplest that can be studied within our framework. However, it generates some counterfactual implications. For instance, a negative shock to the Periphery relaxes the constraint, instead of tightening it.[28] We therefore consider a constraint of a different nature, a *market share constraint*, which becomes more restrictive when the market share of the Periphery countries in the world drops:

$$x_0^{S^1}(t) + x_0^{S^2}(t) \leq \gamma\, F\left(\frac{S^1(t) + S^2(t)}{S^0(t) + S^1(t) + S^2(t)}\right), \qquad \gamma \in \mathbb{R}, \tag{31}$$

where F is an arbitrary function. This constraint is very similar to the concentration constraint, with the only difference that the upper bound on the investment in the Periphery is specified not in absolute, but in relative terms, reflecting the market capitalization of the Periphery.

The characterization of the equilibrium quantities of interest in the economy with the market share constraint (31) is as before, with the only difference that each entry of γ in Propositions 3–4 gets replaced with the term on the right-hand side of equation (31). This is due to the fact that logarithmic preferences induce myopic behavior, and hence the investors in the Center do not hedge against changes in the restrictiveness of their portfolio constraint.

We again describe the effects of the constraint on the economy by means of plots. We have tried several increasing linear and increasing polynomial functions F, and they all produce very similar patterns. In fact, the qualitative implications are identical. Figures 5 depicts the multiplier on the market share constraint. One can easily see that in contrast to the case of the concentration constraint, presented in Figure 1, the multiplier is zero when the wealth shares of the Periphery countries are large. As the wealth shares of the Periphery countries in the world fall, the constraint starts to bind, becoming more and more restrictive the lower the wealth shares are. The tilt in the portfolio of the Center country reflects the restrictiveness of the constraint: the highest tilt occurs when the wealth shares of the Periphery countries are small. The sign of the tilt in the asset allocation of the Center is the same as before: the Center is over-weighted in the Center's stock market and under-weighted in the Periphery stock markets, relative to the unconstrained economy. Like the concentration constraint, the market share constraint restricts the investment in the Periphery, causing wealth transfers to/from the Periphery in response to a shock in the Center or the Periphery (Figure 7). However, unlike in the case of the concentration constraint, the

[28] It has been argued in the empirical literature that recent contagious crises in emerging markets may have been caused by the tightenings of constraints in developed countries in response to a crisis in one emerging market.

restrictiveness of the constraint changes in response to a wealth transfer. For example, a wealth transfer from the Periphery to the Center makes the constraint more restrictive as the market share of the Periphery falls. These two effects—(i) a wealth transfer and (ii) a change in the restrictiveness of the constraint—interact in our model, producing rich variations in the pattern of capital flows. Now the Center withdraws funds from the Periphery when it receives a wealth transfer, because the constraint becomes more restrictive. Therefore, the flight to quality pattern emerging in Figure 8, where a negative shock to one of the Periphery depresses stock prices in the other Periphery country (panel (e)), while boosting the stock market in the Center (panel (c)), is accompanied by a capital flight from the Periphery towards the Center. That is, in response to a negative shock in a Periphery country, the Center becomes more constrained, causing it to sell shares in the Periphery and invest domestically, as well as invest in the bond. This pattern represents a more realistic model of the world, as it is consistent with recent crises in which some developed countries have been forced to withdraw funds from emerging markets in order to meet tightened constraints at home. A market share constraint is just one example of a constraint that would generate such pattern of transmission. We leave for future research analysis of other, perhaps more prevalent and realistic, constraints that could create this, as well as other, more sophisticated, transmission patterns.

6. Discussion

Our model captures (and fully characterizes) several aspects of the asset price co-movement among emerging and developed economies, in a general equilibrium framework. To do so, we had to make a number of simplifying assumptions, that might produce counterfactual implications in dimensions that we have not addressed in the paper. First, our benchmark has no home bias in portfolios, although there is ample evidence of the contrary. Our purpose is to study the incremental effect of a constraint relative to an unconstrained benchmark, and our benchmark where every investor holds the same portfolio has been particularly convenient for this purpose. The model can be extended to exhibit a home bias in portfolios in the benchmark. This can be done by changing the specification of our demand shocks (e.g., along the lines of Pavlova and Rigobon (2003)). In the modified benchmark, even if there is a home bias in portfolios, the weights λ_1 and λ_2 remain constant, and hence the transmission due to the constraint occurs through the same channel, wealth transfers (changes in λ's). The corresponding implications for the effects of constraints on the co-movement are going to be qualitatively the same as in our model. The second counterfactual implication of our framework is that the interest rate in the Center (a developed economy, in our leading interpretation) is higher than the interest rate in the Periphery (emerging economies). Clearly, this is not supported by the data. However, we have not included important determinants of interest rates, such as default risk, monetary and fiscal policy, different capital intensities, and institutional quality. Including any of these aspects in the model would have made it intractable. Finally, agents are myopic in our model since their preferences are log-linear. However, this has allowed us to solve the model in closed form, even in the presence of financial market frictions.

Throughout the analysis in this paper we maintained the assumption that only one agent is constrained in his portfolio choice. For the ease of interpretation, we have imposed such a constraint on the (large) Center country. Alternatively, a constraint could have been imposed on one of the Periphery countries. The main change would be in the signs of some of the responses of stock prices and the terms of trade to the underlying shocks; the formulae characterizing the economy would require only relabeling. Our analysis, however, leaves out an important case in which more than one investor face constraints. Such extension is possible but not straightforward. The main challenge is in proving that our closed-form expressions for the stock prices remain valid. In this extension, the wealth shares of the Periphery countries will no longer be perfectly correlated. But the effects of the constraints will still be captured by changes in these wealth shares. The economics behind the ensuing wealth transfer will be the same as in our analysis. Finally, we have presented only two examples of specific constraints. The framework developed in our paper can be easily extended to study alternative investment restrictions, government- or institutionally-imposed: for example, borrowing constraints, or special provisions such as margin requirements, VaR, and collateral constraints.

7. Conclusion

Empirical literature has highlighted the importance of financial market imperfections in the international financial co-movement. We have examined a form of such imperfections, portfolio constraints, in the context of a three-country Center-Periphery economy, where the interactions between the portfolio constraints and the traditional channels of international propagation can be fully characterized.

We have shown that a portfolio constraint gives rise to an additional common factor in the dynamics of the asset prices and the terms of trade, which reflects the tightness of the constraint. We fully describe the excess co-movement in the stock prices and the terms of trade induced by the new factor. Under our leading interpretation in which the Center country represents a large developed country where traders are constrained and the Periphery countries small emerging markets, we find that the presence of the constraints increases the co-movement among the terms of trade and the stock markets in the Periphery, and reduces the co-movement between the Center and the Periphery. These results are consistent with the empirical findings documenting contagion among the stock prices and the exchange rates or the terms of trade of countries belonging to the same asset class.

The workings of the portfolio constraint in our model are easily understood once one recognizes that portfolio constraints give rise to (endogenous) wealth transfers to or from the Periphery countries. We thus provide a theoretical framework in which changes in the wealth share of constrained investors affect stock returns and the degree of stock price co-movement. Our insight regarding the effects of wealth transfers applies more generally: any portfolio rebalancing should be associated with a wealth transfer, and hence the intuition behind the "portfolio channel of contagion" (see e.g., Broner, Gelos, and Reinhart (2004)) can be alternatively represented as the outcome of cross-country wealth transfers, like in our constrained equilibrium. Finally, our model predicts that wealth of financially constrained investors enters as a priced factor in stock returns: this prediction is yet to be tested empirically.

Appendix A: Proofs

Proof of Lemma 1. It follows from the existing literature (e.g., Karatzas and Shreve (1998)) that $W_0(t)$ and $W_i(t)$, $i = 1, 2$, have representations

$$W_0(t) = \frac{1}{\xi_0(t)} E\left[\int_t^T \xi_0(s)\left(p^0(s)C_0^0(s) + p^1(s)C_0^1(s) + p^2(s)C_0^2(s)\right) ds \middle| \mathcal{F}_t\right]$$

$$W_i(t) = \frac{1}{\xi(t)} E\left[\int_t^T \xi(s)\left(p^0(s)C_i^0(s) + p^1(s)C_i^1(s) + p^2(s)C_i^2(s)\right) ds \middle| \mathcal{F}_t\right], \quad i = 1, 2.$$

These expressions, combined with equations (6) and (9), yield

$$W_0(t) = \frac{T-t}{y_0\xi_0(t)}, \quad W_i(t) = \frac{T-t}{y_i\xi(t)}, \quad i = 1, 2.$$

Making use of the first-order conditions (6) and (9), we arrive at the statement of the lemma.

Proof of Corollary 1. This is a standard result for logarithmic preferences over a single good (e.g., Karatzas and Shreve (1998, Ch. 6, Example 4.2)). The modification of the standard argument for the case of multiple goods is simple thanks to Lemma 1. In particular, we can equivalently represent the objective function of country 0 in the form

$$E\int_0^T \left[\alpha_0 \log\left(\frac{W_0(t)}{p^0(t)(T-t)}\right) + \frac{1-\alpha_0}{2}\log\left(\frac{W_0(t)}{p^1(t)(T-t)}\right) + \frac{1-\alpha_0}{2}\log\left(\frac{W_0(t)}{p^2(t)(T-t)}\right)\right] dt$$
$$= E\int_0^T \left[\log W_0(t) - \alpha_0 \log(p^0(t)(T-t)) - \frac{1-\alpha_0}{2}\log(p^1(t)(T-t)) - \frac{1-\alpha_0}{2}\log(p^2(t)(T-t))\right] dt.$$

Since the investor of country 0 takes prices in the good markets p^j, $j = 0, 1, 2$ as given, and hence from his viewpoint the last three terms in the integrand are exogenous, this objective function belongs to the family considered by Karatzas and Shreve. A similar argument applies to investors 1 and 2. *Q.E.D.*

Weights in the Planner's Problem. To conform with the competitive equilibrium allocation, the weights λ_1 and λ_2 in the planner's problem in Section 2 are chosen to reflect the countries' initial endowments. Since we normalized the weight of Country 0, λ_0, to be equal to 1, the weights of the two remaining countries $i = 1, 2$ are identified with the ratios of Lagrange multipliers associated with the countries' Arrow-Debreu (static) budget constraint y_i/y_0, $i = 1, 2$, respectively. (This follows from the first-order conditions with respect to, for example, good 0 (6) and (9) combined with the sharing rules for good 0 (11)). The values of λ_1 and λ_2 are inferred from Lemma 1 and the sharing rules (12)–(13) combined with the model assumption that the initial endowments of countries 1 and 2 are given by $W_i(0) = S^i(0)$, $i = 1, 2$, and substituting pertinent quantities from (6), (9), and (17)–(18).

Proof of Lemma 2. It is easy to verify that an equivalent representation of the expressions in Lemma 2 is

$$S^0(t) = p^0(t)Y^0(t)(T-t), \quad S^1(t) = p^1(t)Y^1(t)(T-t), \text{ and } S^2(t) = p^2(t)Y^2(t)(T-t).$$

Absence of arbitrage implies that

$$S^j(t) = \frac{1}{\xi(t)} E\left[\int_t^T \xi(s)p^j(s)Y^j(s)ds \Big| \mathcal{F}_t\right], \qquad j = 0,\, 1\, 2. \tag{A.1}$$

It follows from (6) and (11) that

$$\frac{\frac{1-\alpha_1(t)}{2}}{y_1 p^0(t)\xi(t)} = \frac{\frac{1-\alpha_1(t)}{2}\lambda_1(t)Y^0(t)}{\alpha_0 + \lambda_1(t)\frac{1-\alpha_1(t)}{2} + \lambda_2(t)\frac{1-\alpha_2(t)}{2}}, \tag{A.2}$$

where λ_1 and λ_2 are constant weights in the unconstrained economy of Section 2 and stochastic in the constrained economy of Section 3. Hence, in equilibrium

$$\begin{aligned} p^0(t)\xi(t) &= \frac{\alpha_0 + \lambda_1(t)\frac{1-\alpha_1(t)}{2} + \lambda_2(t)\frac{1-\alpha_2(t)}{2}}{y_1\lambda_1(t)Y^0(t)} \\ &= \frac{1}{y_1Y^0(t)}\left(\alpha_0\frac{1}{\lambda_1(t)} + \frac{1-\alpha_1(t)}{2} + \frac{1-\alpha_2(t)}{2}\frac{\lambda_2(t)}{\lambda_1(t)}\right) \\ &= \frac{1}{y_1Y^0(t)}\left(\alpha_0\frac{1}{\lambda_1(t)} + \frac{1-\alpha_1(t)}{2} + \frac{1-\alpha_2(t)}{2}\frac{y_1}{y_2}\right) \end{aligned} \tag{A.3}$$

Analogous steps can be used to derive that

$$p^1(t)\xi(t) = \frac{1}{y_1Y^1(t)}\left(\frac{1-\alpha_0}{2}\frac{1}{\lambda_1(t)} + \alpha_1(t) + \frac{1-\alpha_2(t)}{2}\frac{y_1}{y_2}\right) \tag{A.4}$$

$$p^2(t)\xi(t) = \frac{1}{y_1Y^2(t)}\left(\frac{1-\alpha_0}{2}\frac{1}{\lambda_1(t)} + \frac{1-\alpha_1(t)}{2} + \alpha_2(t)\frac{y_1}{y_2}\right) \tag{A.5}$$

Making use of the the assumption that α_1 and α_2 are martingales, from (A.1)–(A.3) we obtain

$$\begin{aligned} S^0(t) &= \frac{p^0(t)y_1\lambda_1(t)Y^0(t)}{\alpha_0 + \lambda_1(t)\frac{1-\alpha_1(t)}{2} + \lambda_2(t)\frac{1-\alpha_2(t)}{2}} E\left[\int_t^T \frac{1}{y_1}\left(\alpha_0\frac{1}{\lambda_1(s)} + \frac{1-\alpha_1(s)}{2} + \frac{1-\alpha_2(s)}{2}\frac{y_1}{y_2}\right)ds\Big|\mathcal{F}_t\right] \\ &= \frac{p^0(t)\lambda_1(t)Y^0(t)}{\alpha_0 + \lambda_1(t)\frac{1-\alpha_1(t)}{2} + \lambda_2(t)\frac{1-\alpha_2(t)}{2}}\left(\alpha_0\frac{1}{\lambda_1(t)} + \frac{1-\alpha_1(t)}{2} + \frac{1-\alpha_2(t)}{2}\frac{y_1}{y_2}\right)(T-t) \\ &\quad + \frac{p^0(t)\lambda_1(t)Y^0(t)}{\alpha_0 + \lambda_1(t)\frac{1-\alpha_1(t)}{2} + \lambda_2(t)\frac{1-\alpha_2(t)}{2}}\left(E\left[\int_t^T \alpha_0\frac{1}{\lambda_1(s)}ds\Big|\mathcal{F}_t\right] - \alpha_0\frac{1}{\lambda_1(t)}(T-t)\right) \\ &= p^0(t)Y^0(t)(T-t) + \frac{\alpha_0p^0(t)\lambda_1(t)Y^0(t)}{\alpha_0 + \lambda_1(t)\frac{1-\alpha_1(t)}{2} + \lambda_2(t)\frac{1-\alpha_2(t)}{2}}\left(E\left[\int_t^T \frac{1}{\lambda_1(s)}ds\Big|\mathcal{F}_t\right] - \frac{1}{\lambda_1(t)}(T-t)\right). \end{aligned}$$

An analogous argument can be used to show that

$$S^1(t) = p^1(t)Y^1(t)(T-t) + \frac{\frac{1-\alpha_0}{2}p^1(t)\lambda_1(t)Y^1(t)}{\frac{1-\alpha_0}{2} + \lambda_1(t)\alpha_1(t) + \lambda_2(t)\frac{1-\alpha_2(t)}{2}}\left(E\left[\int_t^T \frac{1}{\lambda_1(s)}ds\Big|\mathcal{F}_t\right] - \frac{1}{\lambda_1(t)}(T-t)\right),$$

$$S^2(t) = p^2(t)Y^2(t)(T-t) + \frac{\frac{1-\alpha_0}{2}p^2(t)\lambda_1(t)Y^2(t)}{\frac{1-\alpha_0}{2} + \lambda_1(t)\frac{1-\alpha_1(t)}{2} + \lambda_2(t)\alpha_2(t)}\left(E\left[\int_t^T \frac{1}{\lambda_1(s)}ds\Big|\mathcal{F}_t\right] - \frac{1}{\lambda_1(t)}(T-t)\right).$$

Note that the term $E\left[\int_t^T \frac{1}{\lambda_1(s)}\,ds \middle| \mathcal{F}_t\right] - \frac{1}{\lambda_1(t)}(T-t)$ enters the expression for each stock symmetrically. Therefore, at any time t, the prices of all stocks in the economy are either above or below the value of their dividends, augmented by the factor $T-t$:

$$\begin{aligned} S^j(t) &\le p^j(t)Y^j(t)(T-t) \qquad \text{if} \qquad E\left[\int_t^T \frac{1}{\lambda_1(s)}\,ds \middle| \mathcal{F}_t\right] - \frac{1}{\lambda_1(t)}(T-t) \le 0, \\ S^j(t) &\ge p^j(t)Y^j(t)(T-t) \qquad \text{if} \qquad E\left[\int_t^T \frac{1}{\lambda_1(s)}\,ds \middle| \mathcal{F}_t\right] - \frac{1}{\lambda_1(t)}(T-t) \ge 0, \quad j = 0,\, 1,\, 2, \end{aligned} \tag{A.6}$$

where we have used the restrictions that $0 < \alpha_i < 1/3$, $\lambda_i > 0$, and $Y^i > 0$, $i = 0,\, 1,\, 2$, at all times.

On the other hand, from bond market clearing we have that

$$W_0(t) + W_1(t) + W_2(t) = S^0(t) + S^1(t) + S^2(t) \tag{A.7}$$

and from Lemma 1 and market clearing for goods 0, 1 and 2 that

$$\frac{1}{p^0(t)}\left(\frac{\alpha_0 W_0(t)}{T-t} + \frac{\frac{1-\alpha_1(t)}{2}W_1(t)}{T-t} + \frac{\frac{1-\alpha_2(t)}{2}W_2(t)}{T-t}\right) = Y^0(t), \tag{A.8}$$

$$\frac{1}{p^1(t)}\left(\frac{\frac{1-\alpha_0}{2}W_0(t)}{T-t} + \frac{\alpha_1(t) W_1(t)}{T-t} + \frac{\frac{1-\alpha_2(t)}{2}W_2(t)}{T-t}\right) = Y^1(t), \tag{A.9}$$

$$\frac{1}{p^2(t)}\left(\frac{\frac{1-\alpha_0}{2}W_0(t)}{T-t} + \frac{\frac{1-\alpha_1(t)}{2}W_1(t)}{T-t} + \frac{\alpha_2(t) W_2(t)}{T-t}\right) = Y^2(t). \tag{A.10}$$

Hence, by multiplying (A.8), (A.9) and (A.10) by $p^0(t)$, $p^1(t)$ and $p^2(t)$, respectively, and adding them up, we can show that

$$W_0(t) + W_1(t) + W_2(t) = p^0(t)Y^0(t)(T-t) + p^1(t)Y^1(t)(T-t) + p^2(t)Y^2(t)(T-t).$$

This, together with (A.6) yields the required result. *Q.E.D.*

Proof of Propositions 1 and 2. Since our proofs of the two propositions follow analogous steps, we present them together.

We first report the quantities $A(t)$, $\widetilde{A}(t)$, $a(t)$, $\widetilde{a}(t)$, $b(t)$, $\widetilde{b}(t)$, $M(t)$, X_λ X_{α_1}, and X_{α_2} omitted in the body Propositions 1 and 2:

$$A(t) \equiv \frac{\left(\alpha_0\alpha_1(t) - \frac{1-\alpha_0}{2}\frac{1-\alpha_1(t)}{2}\right)\lambda_1(t) + \frac{1-\alpha_2(t)}{2}\frac{3\alpha_0-1}{2}\lambda_2(t)}{\left(\alpha_0 + \lambda_1(t)\frac{1-\alpha_1(t)}{2} + \lambda_2(t)\frac{1-\alpha_2(t)}{2}\right)\left(\frac{1-\alpha_0}{2} + \lambda_1(t)\alpha_1(t) + \lambda_2(t)\frac{1-\alpha_2(t)}{2}\right)}, \tag{A.11}$$

$$\widetilde{A}(t) \equiv \frac{\left(\alpha_0\alpha_2(t) - \frac{1-\alpha_0}{2}\frac{1-\alpha_2(t)}{2}\right)\lambda_2(t) + \frac{1-\alpha_1(t)}{2}\frac{3\alpha_0-1}{2}\lambda_1(t)}{\left(\alpha_0 + \lambda_1(t)\frac{1-\alpha_1(t)}{2} + \lambda_2(t)\frac{1-\alpha_2(t)}{2}\right)\left(\frac{1-\alpha_0}{2} + \lambda_1(t)\frac{1-\alpha_1(t)}{2} + \lambda_2(t)\alpha_2(t)\right)} \tag{A.12}$$

$$a(t) \equiv \frac{\frac{\lambda_1(t)}{2}}{\alpha_0 + \lambda_1(t)\frac{1-\alpha_1(t)}{2} + \lambda_2(t)\frac{1-\alpha_2(t)}{2}} + \frac{\lambda_1(t)}{\frac{1-\alpha_0}{2} + \lambda_1(t)\alpha_1(t) + \lambda_2(t)\frac{1-\alpha_2(t)}{2}}, \tag{A.13}$$

$$\widetilde{a}(t) \equiv \frac{\lambda_1(t)}{2} \frac{\frac{1-3\alpha_0}{2} - \frac{1-3\alpha_2(t)}{2}\lambda_2(t)}{\left(\alpha_0 + \lambda_1(t)\frac{1-\alpha_1(t)}{2} + \lambda_2(t)\frac{1-\alpha_2(t)}{2}\right)\left(\frac{1-\alpha_0}{2} + \lambda_1(t)\frac{1-\alpha_1(t)}{2} + \lambda_2(t)\alpha_2(t)\right)}, \quad (A.14)$$

$$b(t) \equiv \frac{\lambda_2(t)}{2} \frac{\frac{1-3\alpha_0}{2} - \frac{1-3\alpha_1(t)}{2}\lambda_1(t)}{\left(\alpha_0 + \lambda_1(t)\frac{1-\alpha_2(t)}{2} + \lambda_2(t)\frac{1-\alpha_2(t)}{2}\right)\left(\frac{1-\alpha_0}{2} + \lambda_1(t)\alpha_1(t) + \lambda_2(t)\frac{1-\alpha_2(t)}{2}\right)}, \quad (A.15)$$

$$\widetilde{b}(t) \equiv \frac{\frac{\lambda_2(t)}{2}}{\alpha_0 + \lambda_1(t)\frac{1-\alpha_2(t)}{2} + \lambda_2(t)\frac{1-\alpha_2(t)}{2}} + \frac{\lambda_2(t)}{\frac{1-\alpha_0}{2} + \lambda_1(t)\alpha_1(t) + \lambda_2(t)\frac{1-\alpha_2(t)}{2}} \quad (A.16)$$

$$M(t) \equiv \frac{1}{\beta + \frac{1-\beta}{2}q^1(t) + \frac{1-\beta}{2}q^2(t)}, \quad X_\lambda(t) \equiv \frac{1-\beta}{2}M(t)(q^1(t)A(t) + q^2(t)\widetilde{A}(t)), \quad (A.17)$$

$$X_{\alpha_1} \equiv \frac{1-\beta}{2}M(t)(a(t)q^1(t) + \widetilde{a}(t)q^2(t)), \quad X_{\alpha_2} \equiv \frac{1-\beta}{2}M(t)(b(t)q^1(t) + \widetilde{b}(t)q^2(t)) \quad (A.18)$$

These expressions are the same across Propositions 1 and 2, except that in Proposition 1 $\lambda_i(t)$ are constant weights.

To demonstrate that $\lambda_1(t)$ and $\lambda_2(t)$ are the same up to a multiplicative constant, we use (6), (9), and Lemma 1 to conclude that

$$\lambda_1(t) = \frac{y_0\xi_0(t)}{y_1\xi(t)} \quad \text{and} \quad \lambda_2(t) = \frac{y_0\xi_0(t)}{y_2\xi(t)}.$$

The result then follows from the observation that y_1 and y_2 are constants.

Taking logs in (14) we obtain

$$\log q^1(t) = \log \frac{\alpha_0 + \lambda_1(t)\frac{1-\alpha_0}{2} + \lambda_2(t)\frac{1-\alpha_0}{2}}{\frac{1-\alpha_1(t)}{2} + \lambda_1(t)\alpha_1(t) + \lambda_2(t)\frac{1-\alpha}{2})} + \log Y^0(t) - \log Y^1(t).$$

Applying Itô's lemma to both sides and simplifying, we have

$$\begin{aligned}\frac{dq^1(t)}{q^1(t)} =& \text{Itô terms } dt + \frac{1}{\frac{1-\alpha_0}{2} + \lambda_1(t)\alpha_1(t) + \lambda_2(t)\frac{1-\alpha_0}{2}}\left(\lambda_1(t)\alpha_1(t)\frac{d\lambda_1(t)}{\lambda_1(t)} + \lambda_1(t)\,d\alpha_1(t) - \frac{\lambda_2(t)}{2}\,d\alpha_2(t)\right. \\ &\left. + \lambda_2(t)\frac{1-\alpha_2(t)}{2}\frac{d\lambda_2(t)}{\lambda_2(t)}\right) - \frac{1}{\frac{1-\alpha_1(t)}{2} + \lambda_1(t)\alpha_1(t) + \lambda_2(t)\frac{1-\alpha}{2}}\left(\lambda_1(t)\frac{1-\alpha_1(t)}{2}\frac{d\lambda_1(t)}{\lambda_1(t)}\right. \\ &\left. - \frac{\lambda_1(t)}{2}\,d\alpha_1(t) + \lambda_2(t)\frac{1-\alpha_2(t)}{2}\frac{d\lambda_2(t)}{\lambda_2(t)} - \frac{\lambda_2(t)}{2}\,d\alpha_2(t)\right) + \frac{dY^0(t)}{Y^0(t)} - \frac{dY^1(t)}{Y^1(t)}.\end{aligned}$$

Substituting $\frac{d\lambda_1(t)}{\lambda_1(t)} = \frac{d\lambda_2(t)}{\lambda_2(t)} = \frac{d\lambda(t)}{\lambda(t)}$ in the expression above, simplifying, and making use of (1) and the definitions in (A.11–(A.18), we arrive at the statement in the propositions. Of course, in Proposition 1, $d\lambda_1(t)$=$d\lambda_2(t) = 0$, and hence the terms involving $d\lambda_1(t)$ and $d\lambda_2(t)$ drop out. The dynamics of q^2 are derived analogously.

To derive the dynamics of S^0, we restate (16)–(18) as

$$\log S^0(t) = -\log\left(\beta + \frac{1-\beta}{2}q^1(t) + \frac{1-\beta}{2}q^2(t)\right) + \log Y^0(t) + \log(T-t), \quad (A.19)$$

$$\log S^j(t) = \log q^j(t) - \log\left(\beta + \frac{1-\beta}{2}q^1(t) + \frac{1-\beta}{2}q^2(t)\right) + \log Y^j(t) + \log(T-t). \quad (A.20)$$

Applying Itô's lemma to both sides of (A.19)-(A.20), we arrive at

$$\frac{dS^0(t)}{S^0(t)} = \text{Drift terms } dt - \frac{\frac{1-\beta}{2}}{\beta + \frac{1-\beta}{2}q^1(t) + \frac{1-\beta}{2}q^2(t)}\left(q^1(t)\frac{dq^1(t)}{q^1(t)} + q^2(t)\frac{dq^2(t)}{q^2(t)}\right) + \frac{dY^0(t)}{Y^0(t)},$$

$$\frac{dS^j(t)}{S^j(t)} = \text{Drift terms } dt + \frac{dq^j(t)}{q^j(t)} - \frac{\frac{1-\beta}{2}}{\beta + \frac{1-\beta}{2}q^1(t) + \frac{1-\beta}{2}q^2(t)}\left(q^1(t)\frac{dq^1(t)}{q^1(t)} + q^2(t)\frac{dq^2(t)}{q^2(t)}\right) + \frac{dY^j(t)}{Y^j(t)}.$$

Substituting the dynamics of q^1 and q^2 derived above and making use of the definitions in (A.11)–(A.18) we arrive at the statement in the propositions.

Computation of the drift term $I_c(t) \equiv (I_{c_1}, I_{c_2}, I_{c_3}, I_{c_4}, I_{c_5})$ is straightforward but tedious, so in the interest of space we report just the end result.

$$\begin{aligned}
I_{c_1}(t) &= \mu_{Y^0}(t) - \mu_{Y^1}(t) + \sigma_{Y^0}(t)^2 + \sigma_{Y^1}(t)^2 + A(t)\sigma_{Y^0}(t)\sigma_\lambda(t)^\top i_0 - A(t)\sigma_{Y^1}(t)\sigma_\lambda(t)^\top i_1 \\
&\quad + a(t)\sigma_{Y^0}(t)\sigma_{\alpha_1}(t)^\top i_0 - a(t)\sigma_{Y^1}(t)\sigma_{\alpha_1}(t)^\top i_1 + b(t)\sigma_{Y^0}(t)\sigma_{\alpha_2}(t)^\top i_0 - b(t)\sigma_{Y^1}(t)\sigma_{\alpha_2}(t)^\top i_1, \\
I_{c_2}(t) &= \mu_{Y^0}(t) - \mu_{Y_2}(t) + \sigma_{Y^0}(t)^2 + \sigma_{Y^2}(t)^2 + \tilde{A}(t)\sigma_{Y^0}(t)\sigma_\lambda(t)^\top i_0 - \tilde{A}(t)\sigma_{Y^2}(t)\sigma_\lambda(t)^\top i_2 \\
&\quad + \tilde{a}(t)\sigma_{Y^0}(t)\sigma_{\alpha_1}(t)^\top i_0 - \tilde{a}(t)\sigma_{Y^2}(t)\sigma_{\alpha_1}(t)^\top i_2 + \tilde{b}(t)\sigma_{Y^0}(t)\sigma_{\alpha_2}(t)^\top i_0 - \tilde{b}(t)\sigma_{Y^2}(t)\sigma_{\alpha_2}(t)^\top i_2, \\
I_{c_3}(t) &= \mu_{Y^0}(t) - \frac{1-\beta}{2}M(t)q^1(t)D(t) - \frac{1-\beta}{2}M(t)q^2(t)\tilde{D}(t) - \frac{1}{T-t}, \\
I_{c_4}(t) &= \mu_{Y^1}(t) + \left(\beta + \frac{1-\beta}{2}q^2(t)\right)M(t)G(t) - \frac{1-\beta}{2}M(t)q^2(t)\tilde{G}(t) - \frac{1}{T-t}, \\
I_{c_5}(t) &= \mu_{Y^2}(t) + \left(\beta + \frac{1-\beta}{2}q^1(t)\right)M(t)\tilde{H}(t) - \frac{1-\beta}{2}M(t)q^1(t)H(t) - \frac{1}{T-t},
\end{aligned}$$

where

$$\begin{aligned}
D(t) &\equiv I_{c_1}(t) + A(t)\sigma_{Y^0}(t)\sigma_\lambda(t)^\top i_0 + a(t)\sigma_{Y^0}(t)\sigma_{\alpha_1}(t)^\top i_0 + b(t)\sigma_{Y^0}(t)\sigma_{\alpha_2}(t)^\top i_0 + \sigma_{Y^0}(t)^2, \\
\tilde{D}(t) &\equiv I_{c_2}(t) + \tilde{A}(t)\sigma_{Y^0}(t)\sigma_\lambda(t)^\top i_0 + \tilde{a}(t)\sigma_{Y^0}(t)\sigma_{\alpha_1}(t)^\top i_0 + \tilde{b}(t)\sigma_{Y^0}(t)\sigma_{\alpha_2}(t)^\top i_0 + \sigma_{Y^0}(t)^2, \\
G(t) &\equiv I_{c_1}(t) + A(t)\sigma_{Y^1}(t)\sigma_\lambda(t)^\top i_1 + a(t)\sigma_{Y^1}(t)\sigma_{\alpha_1}(t)^\top i_1 + b(t)\sigma_{Y^1}(t)\sigma_{\alpha_2}(t)^\top i_1 - \sigma_{Y^1}(t)^2, \\
\tilde{G}(t) &\equiv I_{c_2}(t) + \tilde{A}(t)\sigma_{Y^1}(t)\sigma_\lambda(t)^\top i_1 + \tilde{a}(t)\sigma_{Y^1}(t)\sigma_{\alpha_1}(t)^\top i_1 + \tilde{b}(t)\sigma_{Y^1}(t)\sigma_{\alpha_2}(t)^\top i_1 - \sigma_{Y^1}(t)^2, \\
H(t) &\equiv I_{c_1}(t) + A(t)\sigma_{Y^2}(t)\sigma_\lambda(t)^\top i_2 + a(t)\sigma_{Y^2}(t)\sigma_{\alpha_1}(t)^\top i_2 + b(t)\sigma_{Y^2}(t)\sigma_{\alpha_2}(t)^\top i_2 - \sigma_{Y^2}(t)^2, \\
\tilde{H}(t) &\equiv I_{c_2}(t) + \tilde{A}(t)\sigma_{Y^2}(t)\sigma_\lambda(t)^\top i_2 + \tilde{a}(t)\sigma_{Y^2}(t)\sigma_{\alpha_1}(t)^\top i_2 + \tilde{b}(t)\sigma_{Y^2}(t)\sigma_{\alpha_2}(t)^\top i_2 - \sigma_{Y^2}(t)^2,
\end{aligned}$$

and $\sigma_\lambda(t) \equiv -\lambda(t)(m_0(t) - m(t))$, $i_0 \equiv (1, 0, 0)^\top$, $i_1 \equiv (0, 1, 0)^\top$, and $i_2 \equiv (0, 0, 1)^\top$. In Proposition 1, the weights λ_1 and λ_2 are constant, and hence the drift term I is a special case of I_c in which $\sigma_\lambda(t) = 0$. *Q.E.D.*

Proof of Proposition 3. Equations (26) and (27) are derived at a partial equilibrium level. The partial-equilibrium constrained optimization problem of country 0 closely resembles the problem solved in Teplá (2000). Teplá considers a borrowing constraint, which in our setting is equivalent to $x_0^{S^0}(t) + x_0^{S^1}(t) + x_0^{S^2}(t) \leq \gamma$. Our constraint does not contain the first, $x_0^{S^0}(t)$, term. It is straightforward to modify Teplá's derivation for the case of our constraint. Our problem is even simpler, because we consider logarithmic preferences. The fact that we consider multiple goods does not affect the objective function, as shown in the Proof of Corollary 1.

Equation (28) follows from market clearing, coupled with the investors' first-order conditions. It follows from, for example, (6) and (11) that

$$\frac{\alpha_0}{y_0 p^0(t)\xi_0(t)} = \frac{\alpha_0 Y^0(t)}{\alpha_0 + \lambda_1(t)\frac{1-\alpha_1(t)}{2} + \lambda_2(t)\frac{1-\alpha_2(t)}{2}}. \tag{A.21}$$

Applying Itô's lemma to both sides of (A.21) and equating the ensuing diffusion terms, we arrive at the statement in the Proposition. *Q.E.D.*

Proof of Proposition 4. This result again involves a modification of the derivation in Teplá (2000). *Q.E.D.*

Sign Implications in Tables 1 and 2. Due to the restrictions $\alpha_i \in (1/3, 1)$ and $\beta \in (0, 1)$, the quantities $A(t)$, $\widetilde{A}(t)$, $a(t)$, $\widetilde{b}(t)$, $M(t)$, $X_\lambda(t)$, $X_{\alpha_1}(t)$, and $X_{\alpha_2}(t)$ are all unambiguously positive. We also have

$$\begin{aligned} \widetilde{a}(t) < 0 \quad &\text{iff} \quad \tfrac{1-\alpha_0}{2} - \alpha_0 < \left(\tfrac{1-\alpha_2(t)}{2} - \alpha_2(t)\right)\lambda_2(t) \\ b(t) < 0 \quad &\text{iff} \quad \tfrac{1-\alpha_0}{2} - \alpha_0 < \left(\tfrac{1-\alpha_1(t)}{2} - \alpha_1(t)\right)\lambda_1(t), \end{aligned}$$

It follows from Propositions 1 and 2 that the effect of demand shift in country 1 (2) on the terms of trade in country 2 (1) is negative iff $\widetilde{a}(t) < 0$ ($b(t) < 0$), which are the conditions in our Condition A1. Deriving the signs of the responses of the stock prices to the demand shifts is then straightforward, given the characterization in Propositions 1 and 2. Condition A2, the rationale for which is given in the body of Section 2, provides a sufficient condition for the effects to result in the signs reported in Tables 1 and 2.

The effects of a change in λ on the terms of trade of each Periphery country are positive because $A(t) > 0$ and $\widetilde{A}(t) > 0$—guaranteed by the assumption that $\alpha_i \in (1/3, 1)$ (home bias in consumption).

We now derive an alternative form of Condition A3. It follows from Proposition 2 that the impact of the constraint (the effect of a change in λ) on the stock prices of the Periphery countries is positive iff (20)–(21) hold. Notice that if $A(t) < \widetilde{A}(t)$, condition (21) is trivially satisfied, and if $A(t) > \widetilde{A}(t)$, (20) is the one that is satisfied. This means that, in general, only one of the conditions needs to be checked. If we assume that $A(t) < \widetilde{A}(t)$, then a sufficient condition for both effects to be positive is that (20) is satisfied. The condition guaranteeing that $A(t) < \widetilde{A}(t)$ is

$$\frac{3\alpha_1(t) - 1}{3\alpha_2(t) - 1} < \frac{\lambda_2(t)}{\lambda_1(t)}. \tag{A.22}$$

After some algebra and using the fact that $\alpha_1(t) > 1/3$ one can show that (20) is satisfied when

$$\alpha_1(t)\lambda_1(t) + \frac{1-\alpha_2(t)}{2}\lambda_2(t) > \frac{1-\beta}{\beta}\frac{Y_0(t)}{Y_2(t)}\frac{1}{2}\left(\frac{3\alpha_2(t)-1}{3\alpha_0 - 1}\lambda_2(t) - \frac{3\alpha_1(t)-1}{3\alpha_0-1}\lambda_1(t)\right). \tag{A.23}$$

This is a sufficient condition guaranteeing that Condition A3 is satisfied. The left hand side of equation (A.23) represents the direct effect of lambda (the wealth transfer, as explained in Section 3) on the relative price of good 1. The terms on the right-hand side represent the two indirect effects: (i) the impact of the drop in the demand for good 0, and (ii) the impact of the cross-country demand reallocation in the Periphery countries. We discuss these effects in detail in the NBER working paper version of this paper.

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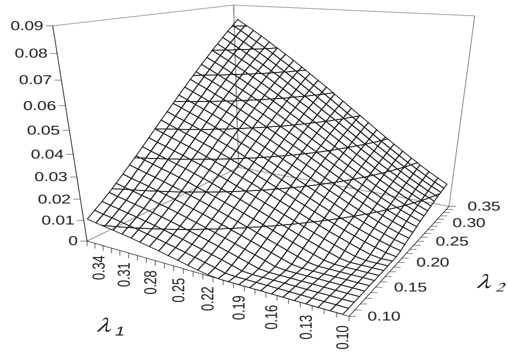
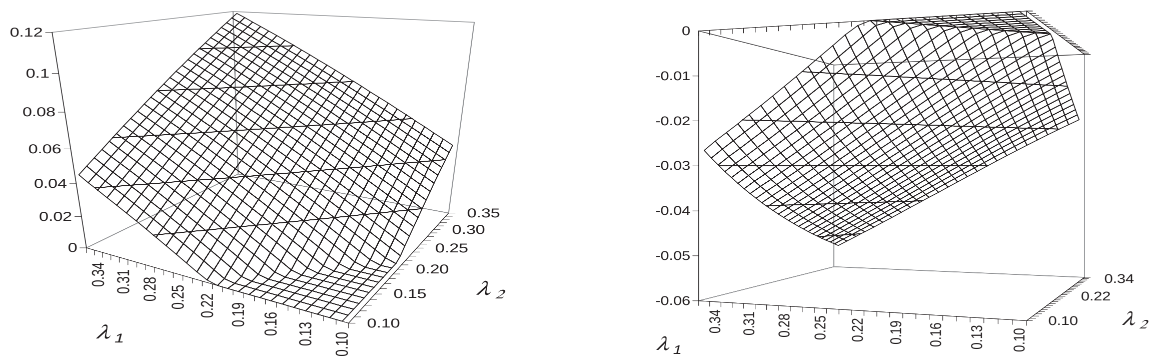


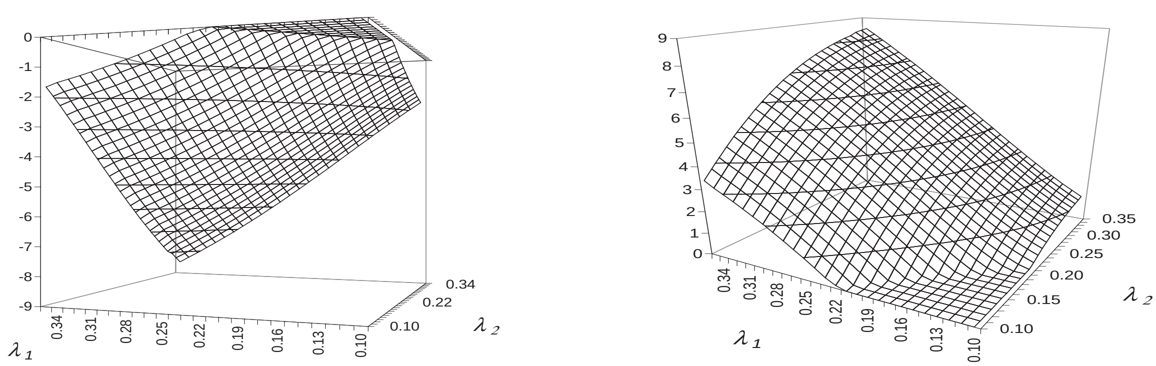
Figure 1: Value of the multiplier on the portfolio concentration constraint ψ.



(a) tilt in the investment in the Center's stock

(b) tilt in the investment in a Periphery country's stock

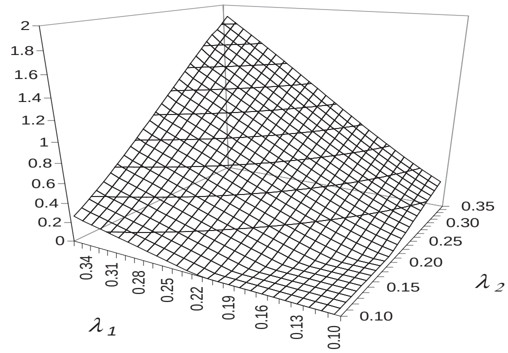
Figure 2: Tilt in the asset allocation of the Center relative to the unconstrained allocation due to the portfolio concentration constraint.



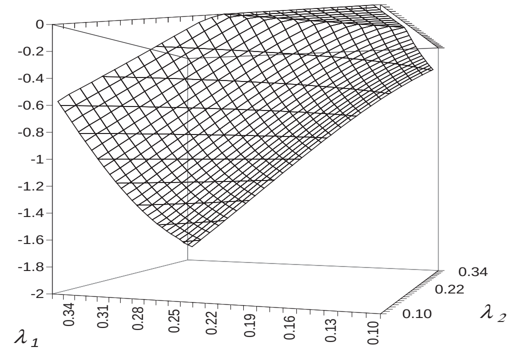
(a) a shock in the Center dw^0

(b) a shock in the Periphery dw^1

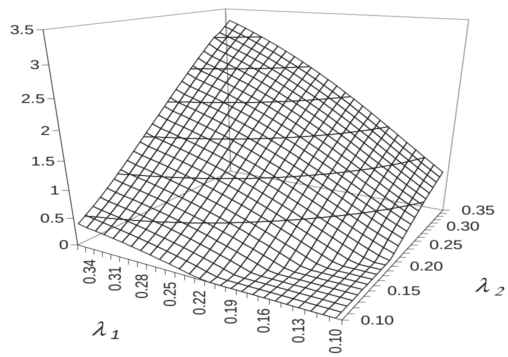
Figure 3: The effects of supply shocks on the wealth distribution, $\frac{d\lambda}{\lambda}$, in the presence of the portfolio concentration constraint.



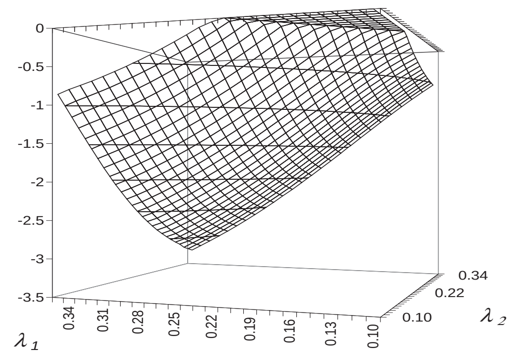
(a) of a shock in the Center, dw^0, on the Center's stock return, $\frac{dS^0}{S^0}$



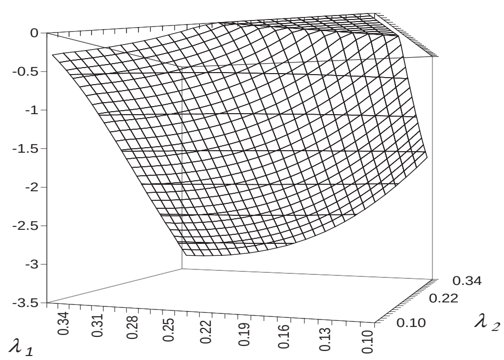
(b) of a shock in the Center, dw^0, on a Periphery country's stock return, $\frac{dS^1}{S^1}$



(c) of a negative shock in Periphery country 1, dw^1, on the Center's stock return, $\frac{dS^0}{S^0}$



(d) of a negative shock in Periphery country 1, dw^1, on the Periphery country's stock return, $\frac{dS^1}{S^1}$



(e) of a negative shock in Periphery country 1, dw^1, on the other Periphery country's stock return, $\frac{dS^2}{S^2}$

Figure 4: The incremental effects of supply shocks on stock prices in the presence of the portfolio concentration constraint.

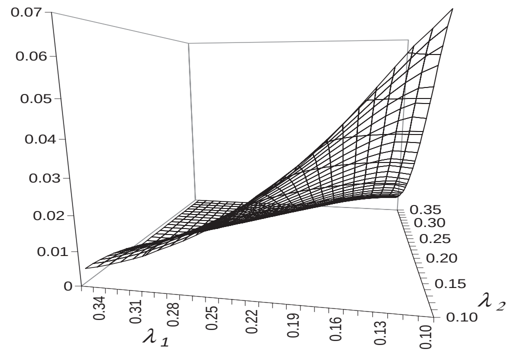
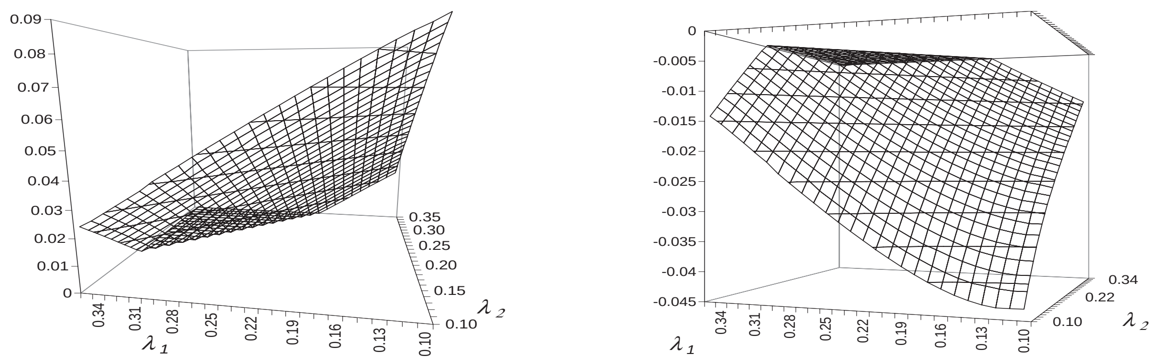


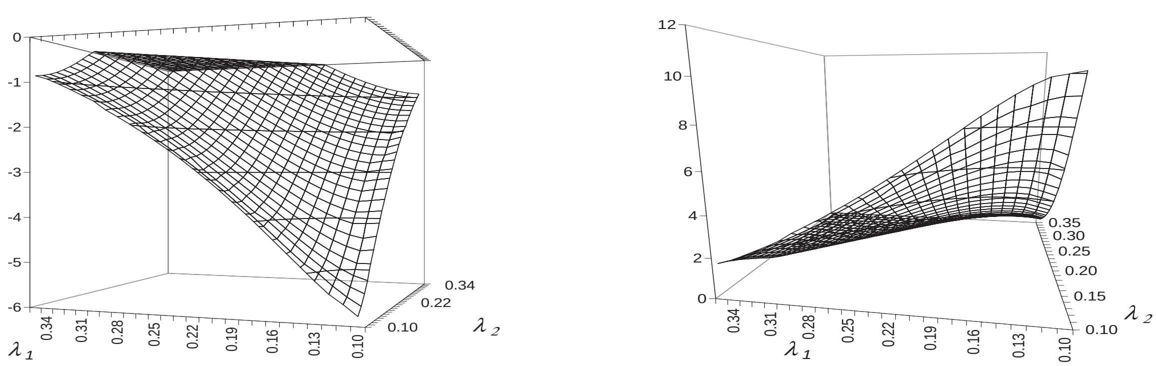
Figure 5: Value of the multiplier on the market share constraint ψ.



(a) tilt in the investment in the Center's stock

(b) tilt in the investment in a Periphery country's stock

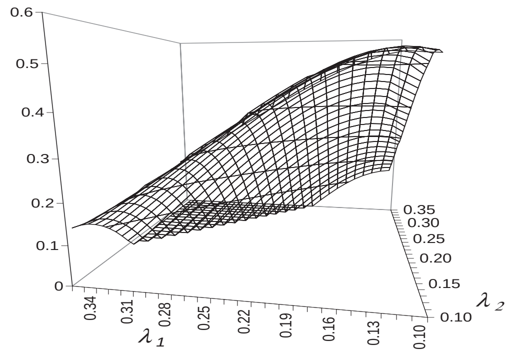
Figure 6: Tilt in the asset allocation of the Center relative to the unconstrained allocation due to the market share constraint.



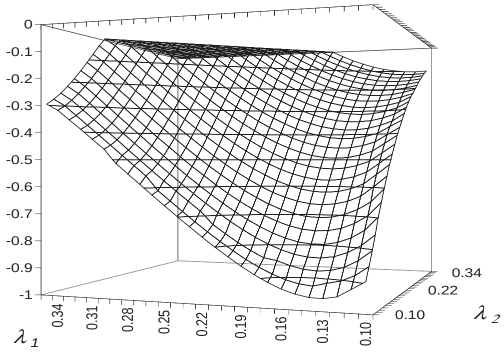
(a) a shock in the Center dw^0

(b) a shock in the Periphery dw^1

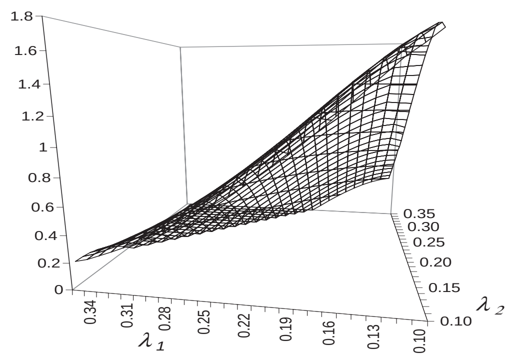
Figure 7: The effects of supply shocks on the wealth distribution, $\frac{d\lambda}{\lambda}$, in the presence of the market share constraint.



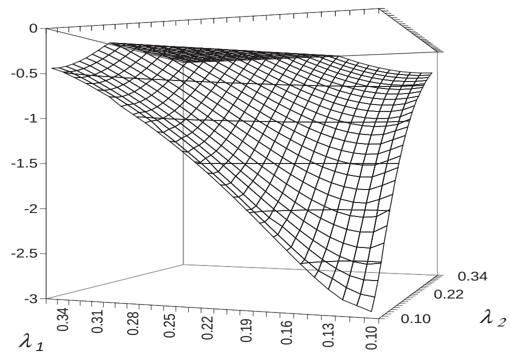
(a) of a shock in the Center, dw^0, on the Center's stock return, $\frac{dS^0}{S^0}$



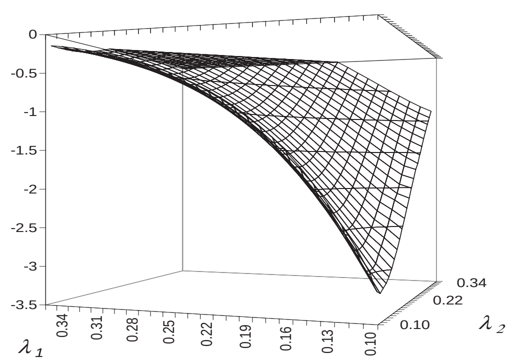
(b) of a shock in the Center, dw^0, on a Periphery country's stock return, $\frac{dS^1}{S^1}$



(c) of a negative shock in Periphery country 1, dw^1, on the Center's stock return, $\frac{dS^0}{S^0}$



(d) of a negative shock in Periphery country 1, dw^1, on the Periphery country's stock return, $\frac{dS^1}{S^1}$



(e) of a negative shock in Periphery country 1, dw^1, on the other Periphery country's stock return, $\frac{dS^2}{S^2}$

Figure 8: The incremental effects of supply shocks on stock prices in the presence of the market share constraint.