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Spillovers and Spillbacks

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Abstract

We study international monetary policy spillovers and spillbacks in a tractable two-country Heterogeneous Agent New Keynesian model. Relative to Representative Agent (RANK) models, our framework introduces a precautionary-savings channel, as households in both countries face uninsurable income risk, and a real-income channel, as households have heterogeneous marginal propensities to consume (MPC). While both channels amplify the size of spillovers/spillbacks, only precautionary savings can change their sign relative to RANK. Spillovers are likely to be larger in economies with higher fractions of high MPC households and more countercyclical income risk. Quantitatively, both channels amplify spillovers by 30-60 percent relative to RANK.

JEL classification: E50, F41, F42
Key words: monetary policy spillovers, incomplete markets, precautionary savings, real-income channel

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1 Introduction

The recent policy debate on the impact of monetary policy tightening in advanced economies has rekindled concerns about global spillovers among interdependent economies and associated spillback effects. There is a steadfast *déjà vu* factor underlying policymakers' pronouncements about how tighter policy stances and financial conditions in the U.S. and Europe keep representing a potential source of disruption and instability for their trading partners. These concerns about *beggar-thy-neighbor* effects are particularly strong in emerging market economies, where a large fraction of households and firms have imperfect access to credit and financial markets and are unable to reap the benefits of effective insurance opportunities against income and consumption risks. In turn, disruption and instability in emerging markets can have reverse *beggar-thyself* spillback effects onto the advanced economies, weighing on their outlook and affecting real activity and (dis-)inflationary dynamics. In fact, with remarkable regularity over the past decades, the assessment of size and sign of policy interdependencies has been a contentious point of controversy against the evergreen debates on the costs and benefits of flexible exchange rate regimes and policy coordination. Trade and financial integration in the global economy, despite recurrent crises and protectionist allures, make spillovers - and concerns thereof - quite unavoidable.

The classic theoretical paradigm in open-economy macroeconomics since Fleming (1962), Mundell (1963) and Dornbusch (1976) studies international monetary policy spillovers (the effect of foreign monetary policy on domestic activity) largely focusing on representative agent economies.[1] This literature has broadly identified two key channels of macroeconomic transmission which shape the size and sign of international monetary spillovers. The taxonomy of these channels tends to change from model to model, but a recurrent theme is the textbook dichotomy between expenditure-switching and expenditure-changing channels. Tighter monetary policy abroad makes foreign goods more expensive, reallocating demand toward domestic goods and reducing demand for foreign output (*expenditure-switching* channel). At the same time, tighter monetary policy also curtails foreign incomes and expenditures for all internationally traded goods, reducing demand for domestic output (*intertemporal-substitution* channel). Depending on the relative strength of these two channels, the policy spillovers onto the domestic economy can be positive or negative. The characterization of spillbacks is less frequently considered in the literature but, arguably, can be directly related to the more traditional analysis of spillovers. In what follows we will refer to monetary policy spillbacks as the effect of a change in the domestic policy stance on domestic output in excess of the effect that would arise if the economy were closed to international trade in goods and assets.[2]

In this paper, we study how market incompleteness and household heterogeneity can change the profile of global monetary policy spillovers and spillbacks. While several contributions in the classic literature have also considered the role of incomplete markets in the international transmission mechanism, they have largely focused on settings with representative agents in each country, i.e., frameworks in which markets are incomplete with respect to *aggregate* (country level) risk. In contrast, our paper contributes to the fast-growing Heterogeneous Agent New Keynesian (HANK) literature, which studies the propa-

[1]See, for example, Obstfeld and Rogoff (1995, 1996, 2002); Betts and Devereux (2000); Corsetti and Pesenti (2001); Tille (2001); Clarida et al. (2002); Benigno and Benigno (2003); Devereux and Engel (2003); Gabaix and Maggiori (2015); Bianchi and Coulibaly (2022); Akinci and Queralto (2023); Fornaro and Romei (2023); Bodenstein et al. (2023); Bianchi and Coulibaly (2024) among others.

[2]See Section 4.1 for a precise analytical definition of spillbacks.

gation of shocks focusing on the role of markets which are incomplete with respect to *idiosyncratic* risk. The HANK literature has identified two main features which distinguish its assessments and affect its conclusions relative to the ones in representative-agent frameworks. First, households face uninsurable idiosyncratic income risk resulting in demand for *precautionary savings*, which can vary over time. Second, households have heterogeneous marginal propensities to consume (MPC) and – crucially – some households have high MPCs which are close to 1. As has been highlighted by Auclert et al. (2021), the presence of high MPC households introduces a *real-income* channel in an open-economy context, as changes in the real exchange rate affect the purchasing power of these households' incomes and can result in sizable fluctuations in aggregate consumption.

We conduct our analysis by extending the tractable HANK model of Acharya and Dogra (2020) to a two-country open economy setting, which contains both aforementioned features. In our model, households in each country face uninsurable idiosyncratic income risk. Households within each country are also heterogeneous with respect to their MPCs: households who are excluded from credit markets are forced to consume their current disposable incomes and have an MPC equal to 1, while those who can access asset markets are able to borrow and save to smooth consumption, and thus have a low MPC. The virtue of our framework is that it allows us to study the contribution of these two features in isolation, thus providing a sharp distinction between how the *precautionary-savings channel* and the *real-income channel* affect both the *size* and the *sign* of international monetary spillovers and spillbacks relative to the perfect-insurance case, i.e, the Representative Agent benchmark (which we refer to as RANK in what follows). In fact, we show that while MPC heterogeneity (and hence the *real-income* channel) can amplify the size of spillovers and spillbacks relative to RANK, it cannot change the sign of the spillovers relative to RANK. In contrast, the *precautionary-savings* channel can not only amplify the size of spillovers and spillbacks, it can also flip the sign of the spillovers relative to RANK.

In addition to the general characterization described above, we also derive conditions under which spillovers and spillbacks are unaffected by MPC heterogeneity or the presence of risk. As emphasized in Corsetti and Pesenti (2001), the RANK benchmark does not feature spillovers or spillbacks under the Cole and Obstfeld (1991) parameterization. This is because the intertemporal-substitution channel is exactly offset by the expenditure-switching channel, implying that Home output is unaffected by a Foreign monetary tightening. By the same token, the decline in Foreign output due to the monetary tightening is the same as in the case if Foreign was a closed economy. In our HANK model we show that including MPC heterogeneity and hence the real-income channel does not change this conclusion. Under the Cole and Obstfeld (1991) parameterization, even though the intertemporal substitution channel is not operative for high MPC households who are unable to access credit markets, the effect of the decline in their real-income is exactly offset by the expenditure-switching effect. Consequently, their demand for Home output remains unchanged and, overall, Foreign monetary policy has no effect on Home output in equilibrium. However, even under the Cole and Obstfeld (1991) parameterization, the precautionary-savings channel delivers a contractionary depreciation, i.e., a scenario in which Home output contracts in response to a monetary tightening abroad, even though the real exchange rate depreciates. In contrast, the precautionary-savings channel has no effect on the sign and magnitude of spillovers and spillbacks relative to RANK when households face acyclical consumption risk.

Our characterization of spillovers is also relevant for the open-economy macroeconomics literature that investigates why exchange rate devaluations may be ineffective in stimulating the economy in the face of external shocks. We show in Section 4.3 that with empirically relevant elasticities, accounting for fluctuations in precautionary savings, an economy can experience a large contraction in domestic output despite a real exchange rate depreciation following a Foreign monetary tightening. These results are consistent with the empirical findings of Vicondoa (2019), who finds evidence of contractionary depreciations. Our results also suggest that spillovers are likely to be larger in emerging economies with higher fractions of high MPC households and more countercyclical income risk relative to advanced economies. While the empirical literature is far from providing definitive conclusions on the size and sign of spillovers across jurisdictions, our conclusions are broadly consistent with the conventional wisdom that US monetary policy shocks can affect advanced and emerging markets differently (Carstens, 2013; Rajan, 2015; Rey, 2015; Bernanke, 2017; Kalemli-Özcan, 2019). Thus, our model shows how the benefits of a flexible exchange rate regime may be overstated once we account for market incompleteness.

Finally, while our baseline model features full and symmetric exchange rate pass-through (i.e., a regime of Producer Currency Pricing or PCP in international pricing and invoicing), we also extend our analysis to the more empirically realistic case of limited and asymmetric pass-through, i.e., a regime of Dominant Currency Pricing (DCP) (Gopinath, 2015; Gopinath et al., 2020). Even in the DCP regime, we show that for empirically realistic level of exchange rate pass-through, our main conclusions from the baseline model continue to hold.

The open-economy HANK literature As aforementioned, our paper is closely related to the fast growing open economy HANK literature. Within this literature, closest to us is the recent work by Auclert et al. (2021), who argue that while a RANK model cannot generate a contractionary depreciation, a HANK model can do so via the real-income channel. Our paper shows that their conclusion is strongly driven by their specification of monetary policy. We clarify that, under a arguably more standard monetary policy specification, even RANK can generate a contractionary depreciation. Moreover, we show that the real-income channel cannot change the sign of the domestic output response to a Foreign monetary shock relative to RANK. In other words, the real-income channel can deliver a contractionary depreciation *if and only if* the RANK benchmark delivers one as well. In contrast, we show that the precautionary savings channel can deliver a contractionary depreciation at Home in response to a Foreign tightening even under a parameterization under which RANK does not.

Next, we discuss some other papers in the HANK literature, which touch on similar issues as our paper. De Ferra et al. (2020) find that currency mismatch in a small open economy can lead to a contraction of national income alongside a depreciation of the exchange rate. Zhou (2022) investigates the role of household heterogeneity in the transmission of adverse foreign shocks and finds that their impact is amplified when liquidity-constrained households are disproportionately leveraged in foreign currency. Ferrante and Gornemann (2022) show that currency mismatch can restrict credit supply and amplify the economic downturn. Our paper focuses on a different channel without currency mismatch, and shows that countercyclical income risk can generate a contractionary depreciation, even when the corresponding RANK economy features an expansion of output. Guo et al. (2023) study international spillovers but focus on the role of differential exposure of households to international trade and financial markets, and find that these sources of heterogeneity can create stabilization-inequality trade-offs for monetary policy. Oskolkov (2023) studies the role of exchange rate flexibility in mediating distributional and aggregate effects in a small open economy in which households are differentially exposed to international trade.[3] Finally, Acharya and Challe (2023) study the optimal conduct of monetary policy in an open-economy HANK model in which the precautionary-savings channel and real-income channel are both operative. In contrast to the papers listed above which study small open economies, we focus on a two-country model, allowing us to study *spillback* effects in addition to *spillover* effects.[4]

The rest of the paper is organized as follows. Section 2 describes a model of the world economy with heterogeneous agents. Section 3 describes the equilibrium of the world economy focusing on the roles of self-insurance and cyclical income risk in affecting the transmission of monetary policy. Section 4 describes the main channels which shape the magnitude and sign of spillovers and spillbacks. Specifically, Section 4.1 provides a brief description of the forces shaping global interdependencies in the perfect-insurance benchmark; Section 4.2 studies how the real-income channel alters spillovers and spillbacks relative to the perfect-insurance benchmark; and Section 4.3 shows how introducing uninsurable idiosyncratic risk modifies the previous findings. Section 5 presents an extension of the baseline model by incorporating a Dominant Currency Pricing (DCP) regime. Section 6 concludes.

2 Heterogeneous agents in the world economy

The world economy consists of two equally sized countries, "Home" and "Foreign". In what follows we describe the Home economy, with the understanding that the Foreign economy is similarly characterized and its variables are indexed by a star (*). For simplicity, we abstract from aggregate risk.

2.1 Households

Demographics Each economy features a perpetual-youth structure à la Blanchard-Yaari, whereby all households face a constant survival probability ϑ in any period.[5] Population is constant and normalized to 1 in each country. Each cohort starts off with zero wealth.

Asset market structure Households within a country differ in their access to asset markets. The Home economy comprises of a fraction $1-\mathcal{O}$ of unconstrained (uc) and a fraction $\mathcal{O}$ of hand-to-mouth (HtM) households, where $\mathcal{O} \in [0,1)$. HtM households do not have access to asset markets, while unconstrained households can trade nominally riskless actuarial bonds denominated in terms of the Home and Foreign currency. The date t demand for Home and Foreign bonds by household j born at date τ is $B^{\tau}_{j,H,t+1}$ and $B^{\tau}_{j,F,t+1}$ respectively, which they can purchase at a per unit price of $\vartheta/(1+i_t)$ and $\vartheta\mathcal{E}_t/(1+i^*_t)$, respectively, where i_t and i^*_t denote the nominal interest rates in Home and Foreign and $\mathcal{E}_t$ denotes the nominal exchange rate. Conditional on survival, the Home bond pays off 1 unit in terms of the Home currency while the Foreign bond pays off 1 unit in terms of the Foreign currency; $\mathcal{E}_t$ denotes the nominal exchange rate. Since we abstract from aggregate risk, it is sufficient to track the net worth of an unconstrained household, $A^{\tau}_{j,t} = B^{\tau}_{j,H,t} + \mathcal{E}_t B^{\tau}_{j,F,t}$.

[3]See also, Iyer (2016); Sunel (2018); Cugat (2019); Giagheddu (2020); Hong (2020); Druedahl et al. (2022) among others for papers that study how heterogeneity interacts with various aspects of the international transmission of shocks.

[4]Bayer et al. (2023) also study a two country model, but in the context of a currency union.

[5]The size of the cohort born at any date t is $1-\vartheta$ and the date t size of a cohort born at $s<t$ is $(1-\vartheta)\vartheta^{t-s}$.

Preferences All unconstrained and HtM Home households have identical non-time separable preferences *à-la* Epstein-Zin, with constant absolute risk version (CARA) preference kernels rather than constant relative risk aversion. The lifetime utility of a Home household j born at date τ can be written as

$$u\left(\mathbb{W}^{\tau}_{j,t}\right) = (1-\beta\vartheta)u\left(c^{\tau}_{j,t}\right) + \beta\vartheta u\left(\mathcal{R}^{-1}\left[\mathbb{E}_t\mathcal{R}\left(\mathbb{W}^{\tau}_{j,t+1}\right)\right]\right)$$

where $\mathbb{W}^{\tau}_{j,t}$ and $c^{\tau}_{j,t}$ denote the lifetime value and the date t consumption of the household, respectively. The function $u(c) = -\exp\{-c\}$ captures the household's attitude towards intertemporal substitution. Given this specification, the median Home household in steady state exhibits a unit elasticity of intertemporal substitution (IES).[6] $\mathcal{R}(x) = -\varphi^{-1}\exp\{-\varphi x\}$ describes households' attitude towards risk with φ denoting the coefficient of *absolute risk aversion*, which also happens to be equal to the coefficient of *prudence*. In the special case with $\varphi = 1$, households preferences are time-separable.

Household income A labor union directs all Home households to supply n_t hours at date t.[7] The date t effective labor supply of Home household j born at date τ is given by $z^{\tau}_{j,t}n_t$, where $z^{\tau}_{j,t} = \bar{z} + \sigma_t\xi^{\tau}_{j,t}$ denotes the stochastic idiosyncratic productivity. $\bar{z}$ denotes the mean productivity and is normalized to $\bar{z} = 1$, while $\sigma_t\xi^{\tau}_{j,t}$ is the component of productivity specific to that household. $\xi^{\tau}_{j,t}$ is an AR(1) with persistence $\lambda \in [0,1]$, capturing the fact that households may face persistent productivity risk:

$$\xi^{\tau}_{j,t} = \lambda\xi^{\tau}_{j,t-1} + v^{\tau}_{j,t}, \qquad \xi^{\tau}_{j,\tau-1} = 0, \qquad v^{\tau}_{j,t} \sim N(0,1) \tag{1}$$

σ_t captures the productivity risk that a Home household faces at date t; we allow $\sigma_t = \sigma(y_t)$ to vary with Home aggregate output to account for the fact that income risk is different in expansions vs recessions, i.e., to account for the *cyclicality of income risk*.

Nominal labor income of household (j,τ) is given by $W_tn_tz^{\tau}_{j,t}$, where W_t is the Home nominal wage. In addition to labor income, each Home household also receives taxes-net-of transfers and dividend income. For simplicity, we assume that all households receive the same dividend and transfer income at each date given by $T_t + D_t$. Overall, the date t nominal income of household (j,τ) can be written as

$$P_ty^{\tau}_{j,t} = P_{H,t}y_t + W_tn_t\left(z^{\tau}_{j,t} - \bar{z}\right) = P_{H,t}y_t + P_t\sigma_{y,t}\xi^{\tau}_{j,t} \qquad \text{where} \qquad \sigma_{y,t} = w_tn_t\sigma_t$$

where P_t is the Home *consumption good* price, $P_{H,t}$ is the price of the Home *produced good*, $w_t = Wt/P_t$ is the real wage and $P_{H,t}y_t = W_tn_t\bar{z} + P_t(D_t + T_t)$ denotes nominal Home national income.

Finally, since only labor income is stochastic, $\sigma_{y,t} = \sigma_tw_tn_t$ denotes the standard deviation of income shocks in the population at date t. To capture the fact that income risk is countercyclical (Storesletten et al., 2004; Nakajima and Smirnyagin, 2019), i.e., households face higher income risk in recessions relative to expansions, we postulate that $\sigma_t = \sigma(y_t)$ is such that income risk is given by:

$$\sigma_{y,t} = \sigma_y e^{-\Theta\hat{y}_t} \qquad \Rightarrow \qquad \Theta = -\frac{\partial \ln \sigma_{y,t}}{\partial \ln y_t}, \tag{2}$$

[6] Notice that CARA preferences imply that the IES depends on the level of consumption and is given by c for a household with consumption c. As discussed in Section 3.1, we normalize Home aggregate output to 1 in steady state, which also implies that the consumption of the median Home household in steady state is 1, i.e., its IES is 1.

[7] See section 2.4 for details.

where σ_y denotes the steady-state level of income risk faced by Home households, and Θ denotes the elasticity of income risk with respect to output, thus a measure of the *cyclicality* of income risk. $\Theta = 0$ implies that income risk is *acyclical*: households face the same level of income risk in booms and recessions. $\Theta > 0$ implies that income risk is *countercyclical*: households face greater income risk when output is below its steady state level (a recession) than when output is above its steady state level (an expansion). Foreign households are similarly characterized.

2.2 Production sector

Home consumption goods producers Home households consume all goods that are produced in Home as well as in Foreign. A representative competitive firm transforms Home goods $(c_{H,t})$ and Foreign goods $(c_{F,t})$ into the Home consumption good according to a CES aggregator:

$$c_t = \left[\alpha^{\frac{1}{\eta}} (c_{F,t})^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} (c_{H,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \qquad 0 < \alpha < 0.5,$$

where $\eta > 0$ is the elasticity of substitution between Home and Foreign goods. In what follows we will highlight the special case $\eta = 1$ as a useful benchmark. Also, $1-\alpha$ is the degree of home bias in preferences: $\alpha = 0$ implies complete home bias while $\alpha = 0.5$ implies no home bias. This yields the standard Home demand curves for Home and Foreign goods:

$$c_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} c_t \qquad \text{and} \qquad c_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} c_t$$

where c_t denotes total consumption demand by all Home households and P_t denotes the Home consumer price index. The latter is given by $P_t = \left[\alpha (P_{F,t})^{1-\eta} + (1-\alpha)P_{H,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}$, where $P_{H,t}$ denotes the Home price of goods produced in Home (Home PPI) and $P_{F,t}$ denotes the Home price of goods produced in Foreign.

Intermediate goods firms' production and pricing decisions The Home final good y_t is a CES aggregate of all domestically produced varieties, which are produced by a unit measure of monopolistically competitive producers indexed by $l \in [0,1]$:

$$y_t = \left[\int_0^1 y_t(l)^{\frac{\varepsilon-1}{\varepsilon}} dl\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $y_t(l)$ denotes the output of the variety produced by firm l at date t. Each firm l faces demand from both Home and Foreign households, and the demand curve can be written as

$$y_t(l) = \underbrace{(1-\alpha)\left(\frac{P_{H,t}(l)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} c_t}_{\substack{\text{Home consumption demand} \\ c_{H,t}(l)}} + \underbrace{\alpha\left(\frac{P^*_{H,t}(l)}{P^*_{H,t}}\right)^{-\varepsilon}\left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta} c^*_t}_{\substack{\text{Foreign consumption demand} \\ c^*_{H,t}(l)}}, \tag{3}$$

where $P_{H,t}(l)$ and $P^*_{H,t}(l)$ denote the price of Home variety l in Home and Foreign respectively, while c_t and c^*_t denote the consumption in Home and Foreign respectively.

Each firm produces its good using a linear technology $y_t(l) = n_t(l)$ where $n_t(l)$ denotes effective employment at firm l, which costs $w_t = W_t/P_t$ in real terms per unit. At any date t, firm l sets prices $P_{H,t}(l)$ at Home and $P^*_{H,t}(l)$ in Foreign to maximize per-period profits. Firm l's problem can be written as

$$\max_{P_{H,t}(l),P^*_{H,t}(l)} \left[\frac{P_{H,t}(l)}{P_t} - (1-\overline{\tau})\frac{W_t}{P_t}\right] c_{H,t}(l) + \left[\frac{\mathcal{E}_t P^*_{H,t}(l)}{P_t} - (1-\overline{\tau})\frac{W_t}{P_t}\right] c^*_{H,t}(l) \tag{4}$$

subject to (3). $\overline{\tau} = \varepsilon^{-1}$ is a payroll subsidy which has been set to to eliminate the steady state markup. Optimal pricing decisions imply that $P_{H,t}(l)/P_t = W_t/P_t$ and $\mathcal{E}_t P^*_{H,t}(l)/P_t = W_t/P_t$. Thus, in a symmetric equilibrium $P_{H,t}(l) = P_{H,t}$ and $P^*_{H,t}(l) = P^*_{H,t}$, the pricing decisions of Home and Foreign firms imply that

$$\frac{P_{H,t}}{P_t} = \mathcal{Q}_t \frac{P^*_{H,t}}{P^*_t} = \frac{W_t}{P_t} \qquad \text{and} \qquad \frac{P_{F,t}}{P_t} = \mathcal{Q}_t \frac{P^*_{F,t}}{P^*_t} = \mathcal{Q}_t \frac{W^*_t}{P^*_t},$$

where $\mathcal{Q}_t = \mathcal{E}_t P^*_t / P_t$ denotes the real exchange rate (RER), $P^*_{F,t}$ denotes the Foreign price of Foreign goods, P^*_t denotes the Foreign CPI and W^*_t denotes the Foreign wage. Appendix A.2 shows that in equilibrium, the real price of Home goods in Home, $P_{H,t}/P_t$, and Foreign goods in Foreign, $P^*_{F,t}/P^*_t$, can be written as functions of the RER $\mathcal{Q}_t$:

$$\frac{P_{H,t}}{P_t} \equiv p_H(\mathcal{Q}_t) = \left[\frac{1-\alpha-\alpha\mathcal{Q}_t^{1-\eta}}{1-2\alpha}\right]^{\frac{1}{1-\eta}} \qquad \frac{P^*_{F,t}}{P^*_t} \equiv p^*_F(\mathcal{Q}_t) = \left[\frac{1-\alpha-\alpha\mathcal{Q}_t^{\eta-1}}{1-2\alpha}\right]^{\frac{1}{1-\eta}}, \tag{5}$$

where $p'_H(\mathcal{Q}_t) < 0$ and $p^{*'}_F(\mathcal{Q}_t) > 0$. In other words, a real depreciation (higher $\mathcal{Q}_t$) lowers the relative price of Home goods and raises the relative price of Foreign goods. Imposing symmetry in (3), the market clearing conditions for Home goods and Foreign goods can be written as:

$$y_t = (1-\alpha)\left(p_H(\mathcal{Q}_t)\right)^{-\eta} c_t + \alpha\left(\frac{p_H(\mathcal{Q}_t)}{\mathcal{Q}_t}\right)^{-\eta} c^*_t \tag{6a}$$

$$y^*_t = (1-\alpha)\left(p^*_F(\mathcal{Q}_t)\right)^{-\eta} c^*_t + \alpha\left(\mathcal{Q}_t p^*_F(\mathcal{Q}_t)\right)^{-\eta} c_t \tag{6b}$$

2.3 Monetary policy

Following Galí and Monacelli (2005), we specify monetary policy as an interest rate rule which targets domestic PPI inflation. For simplicity, we consider a rule in which monetary policy in each country pegs the PPI based real interest rate to its steady state level:

$$\ln(1+i_t) = \ln(1+r_H) + \ln\Pi_{H,t+1} + u_t \qquad \ln(1+i^*_t) = \ln(1+r^*_F) + \ln\Pi^*_{F,t+1} + u^*_t$$

where r_H and r^*_F denote the steady state PPI based real interest rates in Home and Foreign respectively, $\Pi_{H,t+1}$ and $\Pi^*_{F,t+1}$ denote the PPI inflation in Home and Foreign respectively, and finally u_t and u^*_t denote the Home and Foreign *monetary policy shocks* respectively.

A few comments about our specification of monetary policy are warranted here. First, in the rest of the paper, we study the effect of a Foreign monetary tightening $\{u^*_t \geq 0\}^\infty_{t=0}$ while holding Home monetary policy fixed $u_t = 0$ for all t. As is standard in the literature on spillovers, we assume that Home monetary policy does not respond to the Foreign monetary policy shock, in order to clearly identify the effect of Foreign monetary policy on spillovers and spillbacks. If we allowed Home monetary policy to respond to the Foreign monetary policy shock, it would be incorrect to interpret the equilibrium effects on Home and Foreign output as spillovers and spillbacks. Instead, the correct interpretation would be the joint effect of changes in both Foreign and Home interest rates on Home and Foreign output.

Second, while it is common to specify monetary policy in terms of a Taylor-type rule which specify monetary policy in terms of *nominal* interest rates, the benefit of specifying monetary policy in terms of a path for the *real* interest rates is that it renders the model block-recursive: the dynamics of all variables except inflation can be computed independently of the Philips curve, and the relevant variables can then be plugged into the Phillips curve to back out the implied path of inflation. For this reason, the closed-economy HANK literature such as McKay et al. (2016) and others have modeled monetary policy as directly controlling the real interest rate. While this is innocuous in the closed economy, the open economy requires a bit more nuance. Since the producer and consumer price indices differ in the open economy, a choice has to be made regarding whether monetary policy controls the PPI-based or CPI-based real interest rate. In our model, we assume that monetary policymakers in each country control the respective PPI-based real interest rates. As we argue in Appendix I, this choice allows us to correctly interpret the equilibrium effects of an increase in $r^*_{F,t}$ on Home and Foreign output as spillovers and spillbacks, capturing all the relevant channels independently of contentious parameters such as the slope of the Phillips curve. In contrast, if we specified monetary policy in terms of a path for the CPI-based real interest rates, the effect of a higher r^*_t on Home and Foreign output would more accurately be described as the *combined* effect of an increase in the Foreign nominal interest rates and a cut in the Home nominal interest rate, as Appendix I shows. Thus, our choice of specifying monetary policy in terms of PPI based real interest rates allows us to correctly identify spillovers and spillbacks. In fact, Appendix I.1 also shows that our specification of monetary policy allows us to recover some well established benchmark results in the literature, such as the *insularity case* in Corsetti and Pesenti (2001) under the Cole and Obstfeld (1991) parameterization, while the alternative specification does not yield these benchmarks.

Finally, for simplicity, bonds issued by each country are in zero net-supply ($B_{H,t} = B^*_{F,t} = 0$), and $T_t = T^*_t = 0$ at all dates.

2.4 Nominal rigidities

As in Auclert et al. (2021), we assume a standard formulation of sticky wages for heterogeneous households. To wit, a labor union employs all Home households for an equal number of hours n_t at any date t, and penalizes the disutility of hours worked according to an increasing and convex function $v(n_t)$. The union sets nominal wages in order to maximize the welfare of the average household but faces a cost of adjusting nominal wages as in Rotemberg (1982). This yields the standard wage-Phillips curve:

$$\ln \Pi^w_t = \kappa \left[\frac{v'(n_t)/\mathrm{e}^{-c_t}}{w_t} - \mathcal{M}^{-1} \right] + \beta \ln \Pi^w_{t+1}$$

where κ denotes the slope of the Phillips curve, $\mathcal{M}$ denotes the wage markup and $v'(n_t)/\mathrm{e}^{-c_t}$ is the marginal rate of substitution between consumption and labor for the average household.

3 Self-insurance and income-risk channels in general equilibrium

Fisher parity (UIP) Since our framework abstracts from aggregate risk and all unconstrained households can take unrestricted positions in the Home and Foreign bonds, in equilibrium the returns on Home and Foreign bonds are equalized in real terms:

$$1 + r_t = (1 + r_t^*) \frac{\mathcal{Q}_{t+1}}{\mathcal{Q}_t} \tag{7}$$

Unconstrained Home households' decisions The optimal decisions of an unconstrained household can be characterized in closed form and are described in the Proposition below.

Proposition 1 (Unconstrained Households' saving decisions)**.** *Given real interest interest rates $\{r_t\}_{t=0}^{\infty}$, RER $\{\mathcal{Q}_t\}_{t=0}^{\infty}$ and aggregate output $\{y_t\}_{t=0}^{\infty}$, the date t consumption of an unconstrained Home household j born at date $\tau \leq t$ with real financial wealth $a_{j,t}^{\tau} = A_{j,t}^{\tau}/P_t$ and idiosyncratic productivity $\xi_{j,t}^{\tau}$ is:*

$$c_{j,uc,t}^{\tau} = c_{uc,t} + \mu_t \left(a_{j,t}^{\tau} - a_t + h_{j,t}^{\tau} \right), \tag{8}$$

where $h_{j,t}^{\tau}$ (human wealth) denotes the demeaned expected discounted lifetime labor income of the household:

$$h_{j,t}^{\tau} = \mathbb{E}_t \sum_{s=0}^{\infty} \frac{\vartheta^s}{\prod_{k=0}^{s-1}(1 + r_{t+k})} \left(y_{j,t}^{\tau} - p_H(\mathcal{Q}_t) y_t \right) = \left[\sum_{s=0}^{\infty} \frac{(\vartheta\lambda)^s}{\prod_{k=0}^{s-1}(1 + r_{t+k})} \sigma_{y,t+s} \right] \xi_{j,t}^{\tau},$$

and a_t denotes the per-capita average financial wealth of Home households. μ_t denotes the marginal propensity to consume of unconstrained households and evolves according to:

$$\mu_t^{-1} = 1 + \frac{\vartheta}{1 + r_t} \mu_{t+1}^{-1}, \tag{9}$$

$c_{uc,t}$ is the average consumption of unconstrained Home households and satisfies the aggregate Euler equation:

$$\Delta c_{uc,t+1} = \ln \beta \left(\frac{1 + i_t}{\Pi_{t+1}} \right) + \frac{\varphi}{2} \sigma_{c,t+1}^2 - \frac{1 - \vartheta}{\vartheta} \mu_{t+1} a_{t+1} \tag{10}$$

where $\sigma_{c,t+1}$ denotes the consumption risk that uc Home households expect to face at date $t+1$ and is given by:

$$\sigma_{c,t} = \mu_t \sigma_{y,t} + \lambda \left(1 - \mu_t \right) \sigma_{c,t+1} \tag{11}$$

Proof. See Appendix B. □

Equation (8) shows that the consumption function of unconstrained households is linear in both financial and human wealth. Since unconstrained households can access asset markets, their consumption at date t depends on lifetime income $h_{j,t}^{\tau}$, rather than just current income. μ_t denotes the household's marginal propensity to consume (MPC): an increase in lifetime income by one dollar at date t induces an increase in consumption by μ_t.[8] Importantly, due to the assumption of CARA preferences, all Home

[8] μ_t still denotes the MPC out of a one time change in current income but it also denotes the change in consumption induced by a unit change in the expected discounted lifetime labor income of the household.

households have the same MPC μ_t, and all Foreign households have MPC μ_t^*, permitting linear aggregation in both economies. Thus, despite there being a non-degenerate distribution of wealth and income in both economies, the dynamics of aggregate consumption can be characterized without tracking the dynamics of the distributions of financial and human wealth.

The aggregate Euler equation for unconstrained Home households (10) describes the evolution of their average per-capita consumption growth. As in the case with complete markets, the first term on the RHS of (10) shows that consumption growth of unconstrained Home households depends positively on the real interest rate: higher real interest rates increase consumption growth since households choose to postpone consumption. However, since unconstrained households are unable to fully insure themselves against idiosyncratic income risk, higher consumption risk in the future also increases the desired level of precautionary savings and lowers the demand for current consumption. This is captured by the second term in (10), which shows that the higher the coefficient of risk aversion φ, the greater is the decline in current consumption for any increase in consumption risk $\sigma^2_{c,t+1}$.

Equation (11) describes how consumption risk evolves. To understand the forces which govern how much consumption risk households face, it is easiest to consider the special case in which individual income risk is i.i.d. ($\lambda = 0$). In this case, (11) simplifies to $\sigma_{c,t} = \mu_t \sigma_{y,t}$, showing that for any given level of income risk $\sigma^2_{y,t}$, a higher μ_t increases the pass-through from income to consumption risk. More generally, when income risk is not i.i.d. ($\lambda > 0$), (11) can be solved forwards to get $\sigma^2_{c,t} = \mu_t \sigma_{h,t}$, where $\sigma_{h,t}$ denotes the lifetime income risk that households face, and is given by:

$$\sigma_{h,t} = \sum_{s=0}^{\infty} \frac{(\lambda \vartheta)^s}{\prod_{k=0}^{s-1} (1 + r_{t+k})} \sigma_{y,t+s}, \tag{12}$$

which shows that, for unconstrained households, the entire path of future income risk determines consumption risk and thus affects the desired level of date t precautionary savings.

Equation (11) shows that monetary policy can affect consumption risk in two ways: by affecting income risk $\sigma_{y,t}$, and the pass-through μ_t. Equation (9) shows that monetary policy can affect consumption risk through the *self-insurance* channel by affecting μ_t. Solving (9) forwards yields:

$$\mu_t = \left(\sum_{s=0}^{\infty} \frac{\vartheta^s}{\prod_{k=0}^{s-1} (1 + r_{t+k})} \right)^{-1}, \tag{13}$$

which shows that μ_t positively depends on the future path of real interest rates. A lower path of rates allows a household with a one-off fall in current income to prevent a decline in consumption by borrowing, since the cost of doing so is relatively low. However, if interest rates are expected to be high in the future, borrowing costs are higher, discouraging the household from borrowing as much and resulting in a larger decline in consumption following the same reduction in current income. Thus, for a given sequence of current and future income risk, lower interest rates tend to reduce consumption risk.

In addition to the self-insurance channel, monetary policy also affects consumption risk by directly affecting the amount of lifetime income risk $\sigma_{h,t}$ that households face. We refer to this as the *income-risk* channel. To see this, we can use (2), (12) and (13) to express consumption risk as:

$$\sigma_{c,t} = \sum_{s=0}^{\infty} \frac{\vartheta^s \lambda^s}{\prod_{k=0}^{s-1}(1+r_{t+k})} \sigma_y \mathrm{e}^{-\Theta y_{t+s}} \Bigg/ \sum_{s=0}^{\infty} \frac{\vartheta^s}{\prod_{k=0}^{s-1}(1+r_{t+k})}$$

When monetary policy commits to a lower future path of real interest rates, it engineers an economic expansion and results in higher output in the future. This higher expected aggregate output lowers the income risk faced by households. When income risk is countercyclical, $\Theta > 0$ (see (2)). Beyond reducing the level of income risk, a lower path of rates also mitigates consumption risk by shrinking the amount of pass-through from income shocks. Thus, lower real interest rates unambiguously lower consumption risk, except in the special case in which individual income follows a random walk $\lambda = 1$ and income risk is acyclical $\Theta = 0$. In this special case, then real interest rates do not affect the level of consumption risk. This special case, which we term as the *acyclical consumption risk* case, will serve as a useful benchmark later.

Finally, the last term on the right hand side of (10) reflects the Blanchard-Yaari correction: in our OLG economy, the consumption growth of households born at date t differs from aggregate consumption growth because they are born with zero wealth and accumulate/decumulate wealth over their lifetime. This term ensures that all economic variables return to their initial steady state levels after a temporary shock dissipates.[9] In the limit with $\vartheta = 1$, this term vanishes and the world economy is non-stationary.

HtM households' decisions HtM households cannot borrow or save and are unable to use asset markets to smooth the effects of income shocks across time. Consequently, their consumption depends on their current income rather than lifetime income. In particular, their MPC out of changes in current income is equal to 1. Consequently, while an increase in future consumption risk causes unconstrained households to increase their desired level of precautionary savings, it does not affect the consumption decisions of HtM households since they simply cannot save for precautionary reasons. Proposition 2 describes their consumption decisions.

Proposition 2 (HtM Households' decisions)**.** *The date t consumption of a HtM Home household j born at date $\tau \leq t$ is given by $c_{htm,t}(j,\tau) = y_{j,t}^{\tau}$. Furthermore, averaging across all HtM households, $c_{HtM,t} = p_H(\mathcal{Q}_t)y_t$ denotes the average per-capita consumption of Home HtM households.*

Propositions 1 and 2 also highlight that consumption of an unconstrained household vs that of a HtM household responds differently to changes in exchange rates. To see this, consider the effect of a one-time temporary RER depreciation on the consumption of an unconstrained and a HtM Home household. Holding all else constant, the change in exchange rate by $d\mathcal{Q}_t > 0$ lowers the current income of both types of Home households by $p'_H(\mathcal{Q}_t)y_t d\mathcal{Q}_t < 0$. This change in income causes the HtM households to reduce their consumption demand one-for-one, while the unconstrained households reduce their consumption by only $\mu_t \times p'_H(\mathcal{Q}_t)y_t d\mathcal{Q}_t < p'_H(\mathcal{Q}_t)y_t d\mathcal{Q}_t$, which is smaller than the change in income since the pass-through $\mu_t < 1$. Since unconstrained households can access asset markets, they can reduce the pass-through from changes in current income to consumption by borrowing or saving. Thus, holding all else constant, the consumption of HtM households is more sensitive to changes in the RER than that of

[9] Mechanically, the term involving a_{t+1} in (10) shows that if the net foreign asset position of Home is above its steady state level ($a_{t+1} > 0$), households save less other things being equal, which manifests in a lower path for consumption growth.

unconstrained households in both Home and Foreign. Auclert et al. (2021) label this excessive sensitivity of HtM consumption as the *real-income* channel.

Aggregate Home consumption The date t aggregate consumption of Home households is given by

$$c_t = (1-\mathcal{O})c_{\text{uc},t} + \mathcal{O}c_{\text{htm},t} \tag{14}$$

Similarly, the date t consumption of Foreign households is given by $c_t^* = (1-\mathcal{O}^*)c_{\text{uc},t}^* + \mathcal{O}^* c_{\text{htm},t}^*$.

Net foreign asset position The last ingredient to close the model is the asset market clearing condition. Home's net foreign asset (nfa) position evolves according to

$$\frac{1-\mathcal{O}}{1+r_t}a_{t+1} = (1-\mathcal{O})a_t + p_H(\mathcal{Q}_t)y_t - c_t, \tag{15}$$

and using world asset market clearing, the nfa position of Foreign can be written as

$$\mathcal{Q}_t(1-\mathcal{O}^*)a_t^* + (1-\mathcal{O})a_t = 0 \tag{16}$$

Overall, given the path of monetary policy rates in the two countries $\{r_{H,t}, r_{F,t}^*\}_{t=0}^{\infty}$, the dynamics of the RER $\mathcal{Q}_t$ are described by (7). (10)-(11) characterize the dynamics of average per-capita consumption of unconstrained Home households, while the average per-capita consumption by HtM households is equal to their income $c_{\text{htm},t} = p_H(\mathcal{Q}_t)y_t$, and (14) describes aggregate consumption by all Home households. An analogous set of equations describe the dynamics of aggregate consumption in Foreign. The goods-market clearing conditions (6a) and (6b) characterize the evolution of Home and Foreign output for a given path of the RER and aggregate consumption in Home and Foreign. Finally, (15) and (16) describe the evolution of nfa in Home and Foreign respectively. The date 0 net foreign asset position of Home and Foreign is assumed to be 0, $a_0 = a_0^* = 0$ and the economy is in the zero-inflation steady state prior to date 0. In the next section, we characterize the first-order dynamics of the world economy following a monetary shock in Foreign, around a zero inflation symmetric steady state, which we describe next.

It is useful to reiterate that, as in Ghironi (2006), the presence of the Blanchard-Yaari demographics ($\vartheta < 1$) implies that our world economy is stationary and returns to the initial steady state following any temporary shock. In contrast, if $\vartheta = 1$, the economy in general does not revert to the initial steady state even following a temporary shock.[10] However, even in the $\vartheta = 1$ limit, Appendix C.5 shows that output returns to its level in the initial steady state in both countries. We use this property to derive analytical insights in the $\vartheta = 1$ limit and numerically show that these insights remain valid in the stationary case.

[10] Unlike in Auclert et al. (2021), our HANK economy is not "*automatically*" stationary despite the presence of uninsurable idiosyncratic risk and incomplete markets. This is because the consumption function of households in our economy is linear in wealth due to the assumption of CARA utility and no hard borrowing limits. In contrast, in a large number of HANK models with CRRA utility and hard-borrowing limits, the consumption function is typically concave, implying that households tend to have *target* levels of wealth that they want to accumulate for precautionary savings purposes (see, for example, Carroll and Kimball (1996, 2001); Carroll (2004); Toche (2005) and Carroll and Toche (2009)). While shocks may move households away from this target level, they return to this target level of assets once the shocks have dissipated. In contrast, introducing the overlapping generations structure stationarizes our economy, as in Ghironi (2006).

3.1 The zero-inflation symmetric steady state

We assume that both countries are symmetric in steady state at date 0, i.e. $a = a^* = 0$ and that (net) PPI inflation in both Home and Foreign is zero and that the nominal and real exchange rates are equal to 1, $\mathcal{E} = \mathcal{Q} = 1$. We set $v'(1), v'(1) = \mathrm{e}/\mathcal{M}$ to normalize output and consumption in both countries to unity, $y = y^* = c = c^* = 1$. Steady state income risk is also the same in Home and Foreign: $\sigma_y = \sigma_{y^*}$. Consequently, the steady state real interest rates (r and r^*), the MPC out of transfers (μ and μ^*), and consumption risk σ_c and σ_{h^*} are identical in Home and Foreign:

$$r = r^* = \beta^{-1} e^{-\frac{\varphi\sigma_c^2}{2}} - 1, \qquad \mu = \mu^* = 1 - \widetilde{\beta} \qquad \text{and} \qquad \sigma_c = \sigma_c^* = \left(\frac{1 - \widetilde{\beta}}{1 - \widetilde{\beta}\lambda} \right) \sigma_y$$

where $\widetilde{\beta} = \vartheta/(1 + r)$. Since unconstrained households in each country face consumption risk in steady state, the desired precautionary savings pushes down the real interest rates: $r = r^* < \beta^{-1} - 1$. Higher consumption risk σ_c^2 or risk aversion φ increase the gap between $\beta^{-1} - 1$ and r.

3.2 Log-linearized dynamics

In what follows, all hatted variables denote log-deviations from their steady state values, except for nfa where $\widehat{a}_t, \widehat{a}_t^*$ denote level-deviations. Next, we summarize the set of equations which characterize the first-order dynamics of the world economy for a given path of monetary policy $\{\widehat{r}_{H,t}, \widehat{r}_{F,t}^*\}_{t=0}^{\infty}$.[11]

Interest rate parity Given the sequence $\{\widehat{r}_{H,t}, \widehat{r}_{F,t}^*\}_{t=0}^{\infty}$, the real exchange rate satisfies the UIP condition

$$\Delta \widehat{q}_{t+1} = (1 - 2\alpha) \left(\widehat{r}_{H,t} - \widehat{r}_{F,t}^* \right), \tag{17}$$

which is simply the log-linearized version of (7) where we have made use of the relationship between PPI- and CPI-based real interest rates in Home and Foreign: $\widehat{r}_t = \widehat{r}_{H,t} - \frac{\alpha}{1-2\alpha} \Delta \widehat{q}_{t+1}$ and $\widehat{r}_t^* = \widehat{r}_{F,t}^* + \frac{\alpha}{1-2\alpha} \Delta q_{t+1}$. (17) shows that holding Home monetary policy fixed $\widehat{r}_{H,t} = 0$, a one-off temporary Foreign interest rate increase $\widehat{r}_{F,t}^* > 0$ generates an *expected* RER appreciation between dates t and $t + 1$: the RER depreciates at date t, $\widehat{q}_t < 0$, following which it appreciates to return to steady state.

Aggregate Euler equations and consumption risk The aggregate consumption growth in Home and Foreign can be written as the average of the consumption growth of uc and HtM households:

$$\Delta \widehat{c}_{t+1} = (1 - \mathcal{O}) \left\{ \widehat{r}_t + \varphi \sigma_c^2 \widehat{\sigma}_{c,t+1} - \frac{(1-\vartheta)(1-\widetilde{\beta})}{\vartheta} \widehat{a}_{t+1} \right\} + \mathcal{O} \left\{ \Delta \widehat{y}_{t+1} - \frac{\alpha}{1 - 2\alpha} \Delta \widehat{q}_{t+1} \right\} \tag{18a}$$

$$\Delta \widehat{c}_{t+1}^* = (1 - \mathcal{O}^*) \left\{ \widehat{r}_t^* + \varphi \sigma_c^2 \widehat{\sigma}_{c,t+1}^* - \frac{(1-\vartheta)(1-\widetilde{\beta})}{\vartheta} \widehat{a}_{t+1}^* \right\} + \mathcal{O}^* \left\{ \Delta \widehat{y}_{t+1}^* + \frac{\alpha}{1 - 2\alpha} \Delta \widehat{q}_{t+1} \right\} \tag{18b}$$

where $\widehat{r}_t = \widehat{r}_{H,t} - \frac{\alpha}{1-2\alpha} \Delta \widehat{q}_{t+1}$ and $\widehat{r}_t^* = \widehat{r}_{F,t}^* + \frac{\alpha}{1-2\alpha} \Delta q_{t+1}$ denote the CPI-based Home and Foreign real interest rates respectively. The terms in the first parenthesis of the two equations denote the consumption growth of unconstrained households in Home and Foreign respectively, while the terms in the second

[11] See Appendix C for a derivation.

parenthesis denote the consumption growth of HtMs in the two countries. The terms involving $\widehat{a}_{t+1}$ and $\widehat{a}^*_{t+1}$ denote the Blanchard-Yaari correction which renders the economy stationary. $\widehat{\sigma}_{c,t+1}$ and $\widehat{\sigma}^*_{c,t+1}$ denote the log-deviation of consumption risk in the two countries, and are given by:

$$\widehat{\sigma}_{c,t} = \underbrace{(1-\lambda)\,\widehat{\mu}_t}_{\text{self-insurance}} - \underbrace{(1-\widetilde{\beta}\lambda)\Theta\widehat{y}_t}_{\text{cyclical income risk}} + \widetilde{\beta}\lambda\widehat{\sigma}_{c,t+1} \qquad \text{where} \qquad \widehat{\mu}_t = \widetilde{\beta}\left(\widehat{r}_t + \widehat{\mu}_{t+1}\right) \tag{19a}$$

$$\widehat{\sigma}^*_{c,t} = \overbrace{(1-\lambda)\,\widehat{\mu}^*_t} - \overbrace{\left(1-\widetilde{\beta}\lambda\right)\Theta^*\widehat{y}^*_t} + \widetilde{\beta}\lambda\widehat{\sigma}^*_{c,t+1} \qquad \text{where} \qquad \widehat{\mu}^*_t = \widetilde{\beta}\left(\widehat{r}^*_t + \widehat{\mu}^*_{t+1}\right) \tag{19b}$$

The first term on the RHS captures the self-insurance channel: a lower passthrough $\widehat{\mu}_t$ reduces the passthrough from a given level of income risk to consumption risk. The second term on the RHS of (19a) represents the income risk channel: if income risk is countercyclical ($\Theta > 0$), higher output $\widehat{y}_t > 0$ reduces the income risk and consumption risk, holding passthrough fixed.

Goods market clearing The market-clearing conditions (6a)-(6b), up to first-order, can be written as

$$\widehat{y}_t = (1-\alpha)\,\widehat{c}_t + \alpha\widehat{c}^*_t + \frac{2\alpha\,(1-\alpha)}{1-2\alpha}\eta\widehat{q}_t \qquad \text{and} \qquad \widehat{y}^*_t = (1-\alpha)\,\widehat{c}^*_t + \alpha\widehat{c}_t - \frac{2\alpha\,(1-\alpha)}{1-2\alpha}\eta\widehat{q}_t \tag{20}$$

Evolution of nfa and asset market clearing Finally, the evolution of nfa can be written as

$$\frac{1-\mathcal{O}}{1+r}\widehat{a}_{t+1} = (1-\mathcal{O})\widehat{a}_t + \widehat{y}_t - \widehat{c}_t - \frac{\alpha}{1-2\alpha}\widehat{q}_t, \tag{21}$$

and global asset market clearing implies that $(1-\mathcal{O}^*)\,\widehat{a}^*_t = -\,(1-\mathcal{O})\,\widehat{a}_t$.

4 Spillovers and spillbacks

In this section we use our model to inspect in some detail the policy transmission mechanism among interdependent economies. Rather than considering an arbitrary path of Foreign and Home interest rates, we focus on the effect of an unanticipated contractionary Foreign monetary policy shock at date 0, which then mean-reverts at some rate $\rho \in (0,1)$, while the Home real interest rate remains fixed at its steady state level: $\widehat{r}^*_{F,t} = \rho^t \widehat{r}^*_{F,0} > 0$ and $\widehat{r}_{H,t} = 0$ for all $t \geq 0$.

Effect on RER Consequently, in all our experiments, the Foreign policy rate is higher than the Home policy rate. Holding all else constant, this increases the demand for Foreign bonds relative to Home bonds and results in a depreciation of the RER.[12] Imposing $\widehat{r}_{H,t} = 0$ in (17), we obtain:

$$\Delta\widehat{q}_{t+1} = (1-2\alpha)\left(\widehat{r}_{H,t} - \widehat{r}^*_{F,t}\right) = (1-2\alpha)\left(-\widehat{r}^*_{F,t}\right) < 0,$$

which implies that, following the Foreign interest rate increase, the RER depreciates on impact ($\widehat{q}_t > 0$) and appreciates in the future ($\widehat{q}_{t+1} < \widehat{q}_t$).

[12] The fact that we choose to shock the Foreign rate, rather than Home monetary policy is without loss of generality because the two countries are symmetric in our baseline model.

Effect on CPI-based interest rates While the policy rate in Home is unchanged $\widehat{r}_{H,t} = 0$, the CPI-based real interest rates (which determine the consumption growth of unconstrained households) are not unchanged. In fact, the expected RER appreciation implies that the Home CPI-based real interest rate increases even though the Home policy rate remains unchanged: $\widehat{r}_t = \widehat{r}_{H,t} - \frac{\alpha}{1-2\alpha}\Delta\widehat{q}_{t+1} > 0$. To assess the effects on the Foreign CPI-based real interest rate, observe that $\widehat{r}^*_t = \widehat{r}^*_{F,t} + \frac{\alpha}{1-2\alpha}\Delta\widehat{q}_{t+1}$. If the RER were fixed ($\Delta\widehat{q}_{t+1} = 0$), a contractionary Foreign monetary policy shock would imply the same increase in the CPI-based Foreign real interest rate, or $\widehat{r}^*_t = \widehat{r}^*_{F,t} > 0$. However, the expected depreciation due to the change in policy counteracts some of the increase in the Foreign CPI-based real interest rate and $\widehat{r}^*_t$ moves less than one-for-one with the policy rate: $\widehat{r}^*_t = (1-\alpha)\widehat{r}^*_{F,t} > 0$. Overall, a higher policy rate in Foreign leads to an increase in both Home and Foreign CPI-based real interest rates:

$$\frac{d\widehat{r}_t}{d\widehat{r}^*_{F,t}} = \alpha \qquad \text{and} \qquad \frac{d\widehat{r}^*_t}{d\widehat{r}^*_{F,t}} = 1-\alpha \tag{22}$$

Home and Foreign "IS" curves To characterize output *spillovers* and *spillbacks* due to changes in monetary policy, it is useful to combine (18a), (18b) and (20) in order to derive Home and Foreign IS equations, which describe the effects of a change in Home or Foreign monetary policy on Home and Foreign output. Appendix C.4 shows that for a given path of policy rates $\{\widehat{r}_{H,t}, \widehat{r}^*_{F,t}\}_{t=0}^{\infty}$, the Home and Foreign IS equations can be written as:

$$\begin{aligned}\begin{bmatrix}\Delta\widehat{y}_{t+1}\\ \Delta\widehat{y}^*_{t+1}\end{bmatrix} = {} & \underbrace{\mathbb{X}\left(\mathbb{I}-\mathbb{O}\right)\begin{bmatrix}\widehat{r}_t\\ \widehat{r}^*_t\end{bmatrix}}_{\substack{\text{intertemporal}\\ \text{substitution}}} + \underbrace{\begin{bmatrix}\frac{2(1-\alpha)\alpha}{1-2\alpha}\eta\\ -\frac{2(1-\alpha)\alpha}{1-2\alpha}\eta\end{bmatrix}\Delta\widehat{q}_{t+1}}_{\text{expenditure switching}} + \underbrace{\mathbb{X}\left(\mathbb{I}-\mathbb{O}\right)\varphi\sigma_c^2\begin{bmatrix}\widehat{\sigma}_{c,t+1}\\ \widehat{\sigma}^*_{c,t+1}\end{bmatrix}}_{\text{precautionary savings}} \\ & + \underbrace{\mathbb{X}\mathbb{O}\begin{bmatrix}-\frac{\alpha}{1-2\alpha}\\ \frac{\alpha}{1-2\alpha}\end{bmatrix}\Delta\widehat{q}_{t+1}}_{\text{real-income channel}} + \underbrace{\mathbb{X}\mathbb{O}\begin{bmatrix}\Delta\widehat{y}_{t+1}\\ \Delta\widehat{y}^*_{t+1}\end{bmatrix}}_{\substack{\text{Keynesian}\\ \text{multiplier}}} + \underbrace{\mathbb{X}\left(\mathbb{I}-\mathbb{O}\right)\begin{bmatrix}-\frac{(1-\vartheta)(1-\widetilde{\beta})}{\vartheta}\\ \frac{(1-\vartheta)(1-\widetilde{\beta})}{\vartheta}\end{bmatrix}\widehat{a}_{t+1}}_{\text{Blanchard-Yaari correction}},\end{aligned} \tag{23}$$

where $\mathbb{O} = \begin{bmatrix}\mathcal{O} & 0\\ 0 & \mathcal{O}^*\end{bmatrix}$ accounts for the fraction of HtMs in each economy, and $\mathbb{X} = \begin{bmatrix}1-\alpha & \alpha\\ \alpha & 1-\alpha\end{bmatrix}$ describes the allocation of expenditures by Home and Foreign households to Home and Foreign goods, holding the RER fixed at steady state ($\mathcal{Q} = 1$). When a Home household spends 1 additional unit of Home currency on consumption, $1-\alpha$ is spent on Home goods and α on Foreign goods, as shown in the first column. Similarly, the second column shows that a Foreign household who spends 1 unit of Foreign currency on consumption spends a fraction α and $1-\alpha$ on Home and Foreign goods respectively.

Equation (23) shows that the effect of a change in monetary policy in one country affects output in the two countries through the following channels: (i) *intertemporal substitution*, which captures the direct effect of a higher CPI-based real interest rates in Home or Foreign holding all else constant; (ii) the *expenditure switching* channel, which captures the effect of a change in the RER $\widehat{q}_t$ on the demand for Home and Foreign goods, holding all else constant; (iii) the *precautionary savings channel*, which captures the effect of a change in consumption risk faced by Home and Foreign households following the Foreign interest rate increase; (iv) the *real income channel*, which captures the effect of a change in the purchasing power of current income of Home and Foreign HtMs induced by the RER depreciation following the Foreign monetary tightening. In addition to these forces, the presence of HtM households also induces a *Keynesian multiplier* effect, which is represented by the fifth term on the RHS of (23). Because HtM households have a marginal propensity to consume equal to one, any change in their income induced by the four channels mentioned above translates into a one-for-one change in their consumption. This works to amplify the direct effect of the four channels on Home and Foreign output.

Finally, the last term on the RHS of (23) reflects the Blanchard-Yaari correction discussed earlier. This term stationarizes the model when $\vartheta < 1$. With $\vartheta = 1$, following a temporary shock, nfa and most macroeconomic quantities and prices do not return to their respective values in the initial steady state. However, as Appendix C.5 shows, even in the $\vartheta = 1$ limit, both Home and Foreign output return to their respective values in the original steady state once the temporary shock dissipates. This feature allows us to derive analytical results with more ease than in the $\vartheta < 1$ case.[13] Thus, while our analytical results relate to the $\vartheta = 1$ limit, we also numerically solve the stationarized model with $\vartheta < 1$ and show that the insights from the $\vartheta = 1$ limit are still valid.[14]

When we solve the model with $\vartheta < 1$, we set $\vartheta = 1 - 62^{-1}$ following Benhabib and Bisin (2006). Following Galí and Monacelli (2005), we set the degree of openness at $\alpha = 0.4$. As is common in the literature, we set the coefficient of relative risk-aversion for the median household at $\varphi = 2$. Following Itskhoki and Mukhin (2021) we set the intratemporal elasticity of substitution at $\eta = 1.5$. We set β to target a steady state interest rate of $r = 4\%$ in both economies. For the individual income processes, we set the autocorrelation of income at $\lambda = 0.973$ and the steady-state standard deviation of income at $\sigma_y = 0.35$, which is broadly consistent with Heathcote et al. (2010). While there is no consensus in the literature regarding empirical estimates of the cyclicality of income risk, we use the findings of Nakajima and Smirnyagin (2019) to calibrate our baseline and set $\Theta = \Theta^* = 3.5$. In support of this parameterization, Nakajima and Smirnyagin (2019) find the difference in income risk (after accounting for taxes and transfers) between expansions and recessions to be around 0.05. Assuming that the difference between GDP growth in expansions and contractions is about 0.04, we obtain $-d \ln \sigma_y / d \ln y \approx (\Delta\sigma_y/\sigma_y)/\Delta \ln y = (0.05/0.35)/0.04 \approx 3.5$. In line with the estimates of Kaplan et al. (2014), in our baseline we set the fraction of HtM households in both countries at $\mathcal{O} = \mathcal{O}^* = 0.3$. Finally, we set the persistence of the Foreign monetary policy shock at $\rho = 0.8$.

Next, we discuss in detail how each of the forces above shape the magnitude and sign of international monetary policy spillovers and spillbacks.

4.1 Perfect-insurance benchmark

The perfect-insurance benchmark (which we will refer to as the RANK benchmark for brevity) is defined as one in which all households in both economies are unconstrained: $\mathcal{O} = \mathcal{O}^* = 0$. In addition, households in both countries do not face idiosyncratic risk ($\sigma_y = \sigma_y^* = 0$). Consequently, all households within

[13]As shown in Appendix C.5, the fact that output returns to its pre-shock level in both countries is due to the presence of cyclical income risk. Absent cyclical income, this argument need not hold. In order to maintain comparability with the full model, when we shut off cyclical income risk in sections 4.1 and 4.2, we assume that monetary policy picks the equilibrium in which output in both countries return to their pre-shock levels in the long-run.

[14]We solve for a minimum-state variable solution to the model with $\vartheta < 1$. Details are available in Appendix H.

a country have the same level of consumption.[15] In RANK, (23) can be further simplified to

$$\begin{bmatrix}\Delta\widehat{y}_{t+1}\\ \Delta\widehat{y}^*_{t+1}\end{bmatrix} = \underbrace{\mathbb{X}\begin{bmatrix}\widehat{r}_t\\ \widehat{r}^*_t\end{bmatrix}}_{\substack{\text{intertemporal}\\ \text{substitution}}} + \underbrace{\begin{bmatrix}\frac{2\alpha(1-\alpha)\eta}{1-2\alpha}\\ -\frac{2\alpha(1-\alpha)\eta}{1-2\alpha}\end{bmatrix}\Delta\widehat{q}_{t+1}}_{\text{expenditure switching}} = \underbrace{\mathbb{X}\begin{bmatrix}\alpha\\ 1-\alpha\end{bmatrix}\widehat{r}^*_{F,t}}_{\substack{\text{intertemporal}\\ \text{substitution}}} + \underbrace{\begin{bmatrix}\frac{2\alpha(1-\alpha)\eta}{1-2\alpha}\\ -\frac{2\alpha(1-\alpha)\eta}{1-2\alpha}\end{bmatrix}\Delta\widehat{q}_{t+1}}_{\text{expenditure switching}}, \quad (24)$$

showing that only the intertemporal substitution and expenditure switching channels are at work.

The intertemporal-substitution channel Given our specification of monetary policy, (22) shows that a higher Foreign policy rate $\widehat{r}^*_{F,t} > 0$ implies that the CPI-based real interest rates in both Home and Foreign rise following the monetary policy shock: $\widehat{r}_t = \alpha\widehat{r}^*_{F,t} > 0$ and $\widehat{r}^*_t = (1-\alpha)\widehat{r}^*_{F,t} > 0$. Holding all else constant, the higher CPI-based real interest rates incentivize households in both Home and Foreign to postpone their date t consumption of both Home and Foreign goods. Because of nominal rigidities, this lower demand translates into lower Home and Foreign output at date t compared to date $t+1$ via the intertemporal substitution channel. Equivalently, the intertemporal substitution channel leads to higher growth rates of Home and Foreign output $\Delta\widehat{y}_{t+1}, \Delta\widehat{y}^*_{t+1} > 0$ following a Foreign monetary tightening.

The expenditure-switching channel However, a different force also contributes to shaping the total effect of the Foreign tightening in RANK. As mentioned above, the higher Foreign policy rate implies that the RER at date t depreciates ($\widehat{q}_t > 0$), making Home goods cheaper relative to Foreign goods thus leading both Home and Foreign households to increase their demand for Home goods and reduce their demand for Foreign goods. Thus, the expenditure-switching channel tends to increase Home output at date t relative to date $t+1$ and lowers Foreign output relative to date $t+1$. Equivalently, the Foreign monetary tightening leads to negative Home output growth $\Delta\widehat{y}_{t+1} < 0$ and positive Foreign output growth $\Delta\widehat{y}^*_{t+1} > 0$ following the RER depreciation.

Since the two channels work in opposite directions, the sign and magnitude of *spillovers* and *spillbacks* is determined by which channel dominates, which in turn depends on the magnitude of the intratemporal elasticity of substitution η relative to the IES (which is equal to 1). Proposition 3 formalizes this.

Proposition 3 (RANK)**.** *The effect of a Foreign monetary contraction $\widehat{r}^*_{F,t} > 0$ in RANK is given by*

$$\frac{d\widehat{y}_t}{d\widehat{r}^*_{F,t}} = \underbrace{\chi\left(\eta-1\right)}_{\textit{spillover}} \qquad \textit{and} \qquad \frac{d\widehat{y}^*_t}{d\widehat{r}^*_{F,t}} = \underbrace{-\frac{1}{1-\rho}}_{\substack{\textit{closed-economy}\\ \textit{effect}}} \underbrace{-\chi\left(\eta-1\right)}_{\textit{spillback}}, \quad (25)$$

where $\chi = \frac{2\alpha(1-\alpha)}{1-\rho}$ is a positive constant.

Proof. See Appendix D. □

In what follows, we formally use the term "spillback" to refer to the effect of a Foreign policy tightening on Foreign output minus the effect of the same policy in the closed economy limit, i.e., when all

[15] Markets are still incomplete in this benchmark since only two risk-free bonds are traded.

households only consume goods produced in their countries, which is the same as setting $\alpha = 0$. In other words, we define spillbacks as:

$$\text{spillback} = \left.\frac{d\widehat{y}_t^*}{d\widehat{r}_{F,t}^*}\right|_{\alpha>0} - \left.\frac{d\widehat{y}_t^*}{d\widehat{r}_{F,t}^*}\right|_{\alpha=0}$$

Thus, in (25), the Foreign spillback is the last term on the right hand side and is equal in absolute value to the Home spillover: $-\chi(\eta - 1)$.

Proposition 3 shows that under the Cole and Obstfeld (1991) parameterization, i.e. when the intertemporal and intratemporal elasticities are equal ($\eta = 1$), there are *zero spillovers* from Foreign monetary policy on Home output. This case corresponds to the *insularity case* in Corsetti and Pesenti (2001). The RER depreciation resulting from the Foreign monetary tightening increases the demand for the relatively cheaper Home goods via the expenditure switching channel, which exactly offsets the reduction in demand for Home goods via the intertemporal substitution channel, leaving Home output unchanged. Moreover, even the domestic effect of a Foreign monetary contraction is the same as would arise in the closed economy limit, which is defined as the case in which households in each country have complete home-bias in consumption, $\alpha = 0$: $\frac{\partial \widehat{y}_t^*}{\partial \widehat{r}_{F,t}^*} = -\frac{1}{1-\rho} < 0$, i.e., there are *zero spillbacks*. While Proposition 3 is derived under $\vartheta = 1$, the top row of Figure 1 shows that this characterization is also accurate with $\vartheta < 1$.

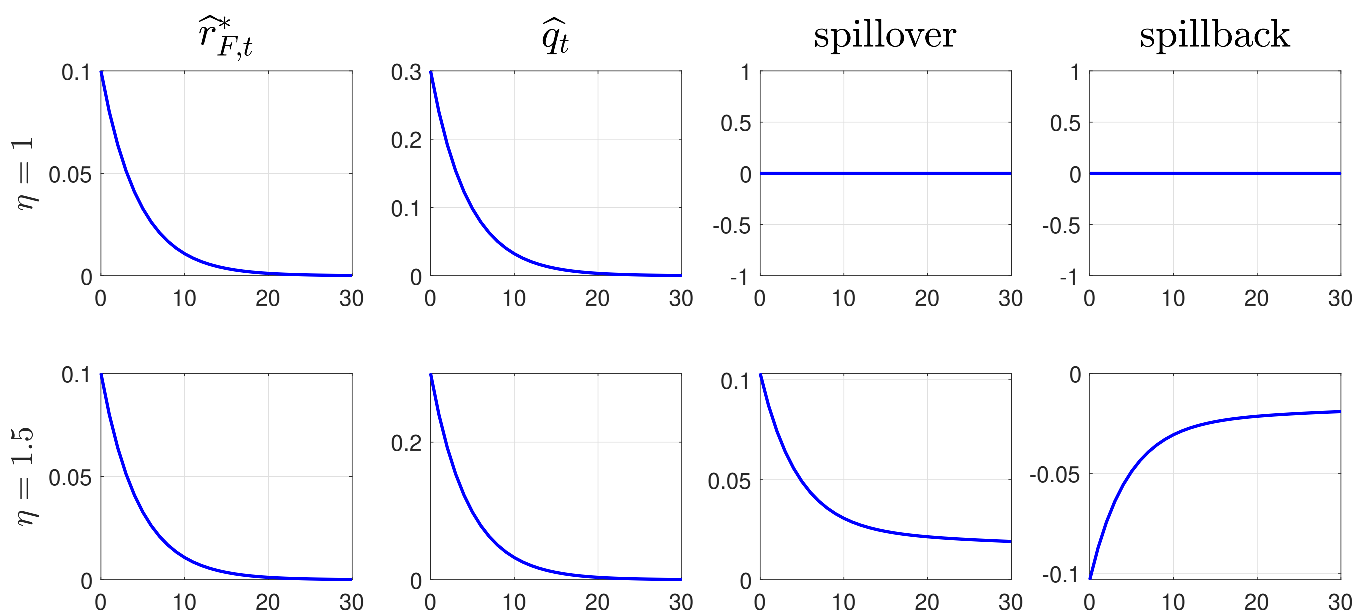

Figure 1: Impulse Responses (IRF) to a Foreign monetary tightening with $\vartheta < 1$ in RANK. The top row corresponds to IRFs with $\eta = 1$, while the bottom row has $\eta = 1.5$. The first column in each row depicts the path of the the Foreign policy rate, the second column depicts the resulting path of the RER, the third column depicts the response of Home output (spillover) and the fourth column depicts the spillback.

However, with $\eta > 1$, spillovers and spillbacks are not zero: a Foreign policy rate hike results in an expansion of Home output due to the RER depreciation. A larger η implies that even a small RER depreciation triggers a relatively larger expenditure switching by Home and Foreign households towards the now cheaper Home goods. This increase in demand for Home goods is large enough to offset the decline in demand for Home goods via the intertemporal substitution channel. Thus, the net effect of a Foreign tightening is to increase Home output: a *positive spillover*. In contrast, the expenditure-switching channel works to reduce the demand for Foreign goods and, alongside intertemporal substitution, results in a decline in Foreign output which is larger than in the closed economy case: a *negative spillback*. This prediction is also true with $\vartheta < 1$, as can be seen from the second row of Figure 1. Home output rises and Foreign output declines on impact, and then slowly revert back to their steady state values as the shock dissipates.

In contrast, $\eta < 1$ implies that even a large change in the RER triggers a relatively small expenditure switching towards Home goods. Consequently, the expenditure-switching channel is not strong enough to offset the decline in demand for Home goods via the intertemporal substitution channel and the net effect of a Foreign tightening is to lower Home output: a *negative spillover*. Foreign output still declines in this case, even though it declines by less than if the Foreign economy were a closed economy: a positive *spillback*. As explained earlier, this cushioning of Foreign output relative to the closed economy limit ($\alpha = 0$) happens because a Foreign monetary tightening by $\widehat{r}^*_{F,t} > 0$ in the open economy ($\alpha > 0$) increases the Foreign CPI-based interest rate (which is the real interest rate relevant for households' consumption decisions) by less than one-for-one: $\widehat{r}^*_t = (1-\alpha)\widehat{r}^*_{F,t}$. This smaller increase in CPI-based real interest rates due to the RER depreciation causes Foreign households to lower their demand by less then in the closed economy limit. Furthermore, since a small η implies only a small shift away from Foreign goods via expenditure switching, Foreign output declines by less than in the closed economy limit.

A final point is worth clarifying in some detail. While Auclert et al. (2021) argue that the RANK benchmark is unable to deliver a situation in which Home output declines alongside a RER depreciation, the discussion above highlights that with $\eta < 1$, RANK *can* deliver a *contractionary depreciation*.[16] The discrepancy between our finding and theirs is due to two important differences in the specification of monetary policy in the two models. First, Auclert et al. (2021) specify monetary policy in Home as keeping the CPI-based real interest rate unchanged in response to the Foreign monetary shock, while we specify monetary policy in Home as keeping PPI-based real interest rates unchanged. As Appendix I.2 shows, the specification in Auclert et al. (2021) implies that Home monetary policy actually *cuts* nominal rates in response to the rate hike in Foreign, thus stimulating demand for Home goods even more than is caused by the depreciation. Second, Auclert et al. (2021) model the change in Foreign interest rates as a response to a discount factor shock, which keeps Foreign consumption unchanged. Instead, as is standard in the literature on international monetary spillovers, we model the change in Foreign monetary policy as not a response to any other shock, but rather a true exogenous monetary policy shock. Consequently, the Foreign tightening that we study leads to a decline in current consumption demand by Foreign households via the intertemporal substitution channel. Thus, as Appendix I.2 shows, the differences in their findings relative to ours reflect the differences in the specification of monetary policy, which in their case nullifies the intertemporal substitution channel. Departing from either one of their assumptions makes the intertemporal substitution channel active once again, so that even RANK can deliver a contractionary depreciation. In fact, as shown above, under our specification of monetary policy, the intertemporal substitution is active and overwhelms the expenditure-switching channel when $\eta < 1$, leading to a decline in Home output despite a RER depreciation.

[16] $\eta < 1$ in our model corresponds to a case with a small trade elasticity, which is exactly the condition that Auclert et al. (2021) require for the real-income channel to deliver a contractionary depreciation.

4.2 MPC heterogeneity: the role of the real-income channel

The HANK literature in general, and in particular, Auclert et al. (2021) argue that accounting for households with high MPCs can affect monetary transmission in the open economy via the *real-income* channel. To study the contribution of this facet of incomplete market economies to spillovers and spillbacks, we now introduce some high MPC households to both economies, and assume $\mathcal{O}, \mathcal{O}^* > 0$. For clarity of exposition, we maintain the assumption that households do not face idiosyncratic income risk $\sigma_y^2 = \sigma_{y^*}^2 = 0$. Even though households do not face idiosyncratic shocks to their productivity, the consumption of unconstrained and constrained households within a country can diverge in response to a RER depreciation following a Foreign monetary tightening.

Recall that the consumption of unconstrained households and HtM households is differentially exposed to changes in RER. A one-time temporary RER depreciation $d\mathcal{Q}_t > 0$, holding all else constant, lowers the date t income of both types of Home households by $p_H'(\mathcal{Q}_t)y_t d\mathcal{Q}_t < 0$. But since HtMs cannot access credit markets, this translates into a one-for-one decline in their consumption demand. This is referred to as the *real-income* channel: an RER depreciation lowers the *real* purchasing power of HtM households' income, forcing them to lower their consumption. In contrast, the consumption of unconstrained households is more insulated against the decline in their real current income, since their consumption depends on lifetime income. This is captured by the fact that their date t consumption only declines by less than the decline in their current income $dc_{u,t} = \mu_t \times p_H'(\mathcal{Q}_t)y_t d\mathcal{Q}_t$ where $\mu_t < 1$ is the pass-through from income to consumption, and reflects the fact that these households can borrow and save to smooth consumption in response to transitory changes in income. This shows that incorporating HtM households can lead to larger fluctuations in consumption demand and thus can affect spillovers and spillbacks. We explore this next.

Since HtMs in both Home and Foreign have an MPC of 1 and consume their entire income at each date, the average consumption of Home and Foreign HtMs can be expressed as:

$$\widehat{c}_{\text{htm},t} = -\frac{\alpha}{1-2\alpha}\widehat{q}_t + \widehat{y}_t \qquad \text{and} \qquad \widehat{c}^*_{\text{htm},t} = \frac{\alpha}{1-2\alpha}\widehat{q}_t + \widehat{y}^*_t \tag{26}$$

Thus, holding Home and Foreign output $\widehat{y}_t$ and $\widehat{y}^*_t$ constant, a RER depreciation $\widehat{q}_t > 0$ lowers the *real income* of Home HtMs but increases the real income of Foreign HtMs, resulting in a one-for-one decline in Home HtMs' consumption and a one-for-one increase in Foreign HtMs' consumption. However, to translate these declines in aggregate consumption demand onto changes in Home and Foreign output, we have to account for the fact that the RER depreciation also triggers expenditure switching towards the cheaper Home good. Consequently, holding all else unchanged, the change in demand for Home goods by Home HtMs and Foreign HtMs is given by:

$$\frac{\partial \widehat{c}_{H,\text{htm},t}}{\partial \widehat{q}_t} = \underbrace{-\frac{\alpha(1-\alpha)}{1-2\alpha}}_{\substack{\text{real income}\\\text{channel}}} + \underbrace{\frac{\alpha(1-\alpha)}{1-2\alpha}}_{\substack{\text{expenditure}\\\text{switching}}}\eta \lesseqgtr 0 \qquad \Leftrightarrow \qquad \eta \lesseqgtr 1 \tag{27a}$$

$$\frac{\partial \widehat{c}^*_{H,\text{htm},t}}{\partial \widehat{q}_t} = \underbrace{\frac{\alpha^2}{1-2\alpha}}_{\substack{\text{real income}\\\text{channel}}} + \underbrace{\frac{\alpha(1-\alpha)\eta}{1-2\alpha}}_{\substack{\text{expenditure}\\\text{switching}}} > 0 \qquad \text{for} \qquad \eta > 0 \tag{27b}$$

Equation (27a) shows that the Home HtM demand for Home goods can be split into two parts: (i) the first term on the right hand side of (27a) represents the effect of the real-income channel, insofar as a reduction in the purchasing power of Home HtMs' income causes them to reduce their demand for Home goods; (ii) at the same time, because Home goods are now cheaper, HtM households switch away from Foreign to Home goods. How much they switch their expenditure towards Home goods depends on η. In fact, if $\eta > 1$, their demand for Home goods increases on net, as the expenditure switching effect dominates the decline in their real income. The opposite is true if $\eta < 1$, while their demand for Home goods remains unchanged if $\eta = 1$. In contrast, (27b) shows that — regardless of η — both the real-income channel and the expenditure-switching channel cause Foreign HtMs (who now have a higher real income) to increase their demand for Home goods. Overall, as has also been pointed out by Auclert et al. (2021), (27a)-(27b) show that for η sufficiently smaller than 1, the real income channel can result in a decline for Home goods despite a reduction in their relative price.

Appendix E.1 shows that the exact opposite is true for the demand for Foreign goods by Home and Foreign HtMs. Since Foreign goods are now more expensive and the real income of Home HtMs is lower, their demand for Foreign goods is unambiguously lower. However, Foreign HtMs now have a higher real income, which tends to increase their demand for Foreign goods. Yet, the fact that these goods are more expensive causes them to switch their expenditure away from them. Again, the magnitude of η relative to 1 determines which force dominates. When $\eta > 1$, Foreign HtMs' demand for Foreign goods declines despite them being richer, as the expenditure switching channel dominates. The opposite is true for $\eta < 1$, while Foreign HtM demand for Foreign goods remains unchanged if $\eta = 1$.

Moving from HtM to unconstrained households, whether their demand for Home goods increases or decreases following a Foreign monetary tightening depends on the strength of the intertemporal-substitution vs the expenditure-switching channels, precisely as in RANK. For $\eta > 1$ the demand for Home goods by unconstrained households increases following a Foreign tightening, while it declines if $\eta < 1$ and remains unchanged if $\eta = 1$.

While the discussion above described the *direct* effect of the RER depreciation induced by a Foreign monetary tightening, it is not the total effect, as we were implicitly holding output fixed in both countries. However, in general equilibrium the change in demand for Home and Foreign goods also triggers a change in output because of the presence of nominal rigidities. These changes in output, in turn, further modify the current income of households. Because HtMs have an MPC equal to 1, the change in income translates into a one-to-one change in their consumption, leading to a *Keynesian multiplier* effect. Proposition 4 below summarizes the general-equilibrium effect of a RER depreciation induced by a Foreign monetary policy rate hike.

Proposition 4. *Let $\mathcal{O}, \mathcal{O}^* > 0$. Then, the effect of an increase in the Foreign monetary policy rate $\widehat{r}^*_{F,t}$, holding Home monetary policy fixed $\widehat{r}_{H,t} = 0$ on Home and Foreign output is given by:*

$$\frac{d\widehat{y}_t}{d\widehat{r}^*_{F,t}} = \underbrace{\mathbb{M}\left(\mathcal{O}, \mathcal{O}^*\right)}_{\substack{\text{spillover-}\\\text{multiplier}}} \times \underbrace{\chi\Big(\eta - 1\Big)}_{\substack{\text{spillover in}\\\text{RANK}}} \qquad \textit{and} \qquad \frac{d\widehat{y}^*_t}{d\widehat{r}^*_{F,t}} = \underbrace{-\frac{1}{1-\rho}}_{\substack{\text{closed-economy}\\\text{effect}}} - \underbrace{\mathbb{M}\left(\mathcal{O}^*, \mathcal{O}\right)}_{\substack{\text{spillback-}\\\text{multiplier}}} \times \underbrace{\chi\Big(\eta - 1\Big)}_{\substack{\text{spillback in}\\\text{RANK}}}, \tag{28}$$

$$\textit{where} \qquad \mathbb{M}(x, y) = \frac{1 - y}{1 - (1-\alpha)(x + y) + (1 - 2\alpha)\, xy} > 0 \qquad \forall\, (x, y) \in [0, 1) \times [0, 1) \tag{29}$$

captures the Keynesian multiplier effect.

Proof. See Appendix E.2. □

Equation (28) makes clear that the *sign* of the spillover and spillback is independent of $\mathcal{O}$ and $\mathcal{O}^*$, and only depends on the magnitude of η relative to 1, as in RANK. However, the presence of high MPC households alters the *magnitude* of spillovers and spillbacks relative to RANK via the Keynesian multiplier $\mathbb{M}(\bullet,\bullet)$. Appendix E.2 shows that for two countries with the same fraction of HtMs, i.e., $\mathcal{O}=\mathcal{O}^*$, the spillover and spillback multipliers are larger than 1 for any $\mathcal{O}\in(0,1)$:

$$\mathbb{M}(\mathcal{O},\mathcal{O})=\frac{1}{1-(1-2\alpha)\,\mathcal{O}}>1 \qquad \text{for any } \mathcal{O}\in(0,1)$$

Thus, a higher fraction of HtMs tends to amplify spillovers and spillbacks when this fraction is similar in both countries.

To see why, consider the case $\eta>1$ for ease of exposition, and since it is the empirically relevant case. As discussed above, with $\eta>1$, the direct effect of the RER depreciation caused by a Foreign monetary tightening is to increase the demand for Home goods. This increased spending on Home goods leads to an increase in the income accruing to Home HtMs. Since they have an MPC of 1, the higher income translates into a one-to-one increase in consumption of both Home and Foreign goods. This additional expenditure on Home goods results in even more income accruing to Home HtMs through the Keynesian multiplier effect. Furthermore, Foreign households also spend more on Home goods since they are now cheaper. This additional spending on Home goods also increases the income of Home HtMs, which reinforces the increase in Home output. The net effect of these multiple rounds of spending increases is that Home output increases by more than in the perfect-insurance benchmark, and is captured by the spillover multiplier, which is larger than 1. In contrast, the direct effect of the Foreign monetary tightening is to lower the demand for Foreign goods. This reduction in spending on Foreign goods decreases the income accruing to Foreign HtMs. As the latter reduce their spending on Foreign goods, this further reduces their income and consumption.[17] The net effect, along with Home households switching their expenditures towards the cheaper Home goods, is that Foreign output falls more than in RANK and is represented by a spillback multiplier, which is greater than 1.

Thus, for two countries with a similar fraction of HtM households — one may think of two advanced economies such as U.S and the Euro area — spillovers and spillbacks are larger than in RANK. Furthermore, since $\mathbb{M}_1(\Theta,\Theta)>0$ and $\mathbb{M}_2(\Theta,\Theta)<0$, increasing the fraction of Home HtMs increases the magnitude of spillovers and reduces the magnitude of spillbacks. In fact, holding the fraction of Home HtMs constant, a smaller fraction of HtMs in Foreign make the spillback become smaller. In the limit, as $\mathcal{O}^*=0$ the spillback multiplier becomes less than 1: $\mathbb{M}(0,\mathcal{O})=\frac{1-\mathcal{O}}{1-(1-\alpha)\mathcal{O}}<1$, while the spillover multiplier is still larger than 1: $\mathbb{M}(\mathcal{O},0)=\frac{1}{1-(1-\alpha)\mathcal{O}}>1$. If one were willing to interpret Home as an emerging economy with a relatively higher share of HtM households and Foreign as an advanced economy, the spillover of a monetary tightening in the advanced economy onto the emerging economy would be relatively larger, while the spillbacks onto the advanced economy itself would be relatively smaller. Intuitively, our model highlights a mechanism by which the same U.S. monetary policy shock can lead to a larger effect on output in emerging markets, than, say, in the Euro area.

[17] The net effect on the real income of Foreign HtMs is ambiguous because the depreciation also increases their real income.

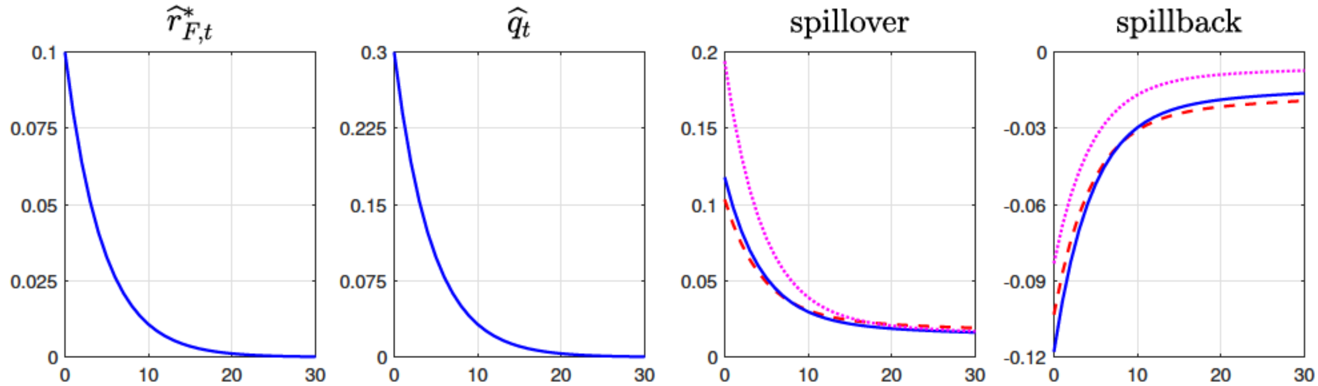

Figure 2: Spillover and Spillbacks with HtMs: The solid-blue lines depict dynamics in a world economy with $\mathcal{O} = \mathcal{O}^* = 0.3$, the dashed-red lines depict the dynamics in the RANK benchmark, and the dotted-magenta lines depict the asymmetric case in which Home is an "emerging market" and has a larger fraction of HtMs.

While the above analysis holds in the context of the $\vartheta = 1$ limit, Figure 2 shows that the basic intuition carries through in the case with $\vartheta < 1$. The solid-blue lines depict the dynamics in the economy with an equal fraction of HtM households in Home and Foreign ($\mathcal{O} = \mathcal{O}^* = 0.3$). Since $\eta > 1$ in our calibration, as in Proposition 4, Home output increases following a Foreign monetary tightening and the magnitude of this increase is larger than in RANK, which is depicted by the dashed-red line. However, importantly, the sign of the spillovers and spillbacks is identical to RANK. The dotted-magenta line depicts the dynamics in the case in which Home is calibrated to be an "emerging" economy with a higher fraction of HtM households, while Foreign is an "advanced" economy with a smaller fraction of HtMs. Following Zhou (2022), we set $\mathcal{O} = 0.7$ for the emerging economy, while we keep $\mathcal{O}^* = 0.3$ for the advanced Foreign economy. While this increases the magnitude of spillovers and reduces spillbacks relative to the symmetric case, the sign of the spillover and spillback are still the same as in RANK.

It is also worth pointing out a corollary of Proposition 4. Under the Cole and Obstfeld (1991) parameterization $\eta = 1$, the fraction of HtMs $\mathcal{O}, \mathcal{O}^*$ do not matter in the assessing the magnitude of spillovers and spillbacks. This is simply because in this special case the demand for Home goods by Home HtMs and unconstrained households remains unchanged after the depreciation, since the expenditure switching channel cancels out both intertemporal substitution and the decline in real income of Home HtMs. In contrast, Foreign HtM households increase their demand for Home goods by the same amount as the unconstrained Foreign households, implying that the actual fraction of HtM households is irrelevant for the general-equilibrium response of Home and Foreign output. This result is summarized below.

Corollary 1 (An as-if result)**.** *If $\eta = 1$, the effect of an increase in the Foreign monetary policy rate, holding Home monetary policy fixed on Home and Foreign output is the same as in RANK.*

Finally, Proposition 4 offers valuable insights that complement the arguments presented by Auclert et al. (2021). The latter suggests that, while the RANK benchmark cannot deliver a contractionary depreciation in Home, incorporating the real-income channel can do so, provided the trade elasticity is sufficiently small. Our analysis aligns with part of this assertion, confirming that, for $\eta < 1$ (which implies a small trade elasticity), a Foreign monetary tightening can indeed lead to a simultaneous reduction in Home output and a RER depreciation. However, as was discussed at the end of Section 4.1, when

$\eta < 1$ even RANK is capable of delivering a contractionary depreciation. Thus, our framework clarifies that the real-income channel induces a contractionary depreciation in Home *if and only if* RANK also features a contractionary depreciation. As mentioned earlier, the only reason that RANK cannot deliver a contractionary depreciation at Home in Auclert et al. (2021) is because their specification of monetary policy nullifies the intertemporal substitution channel.

4.3 Precautionary-savings channel

Next, we study how the precautionary-savings channel affects spillovers and spillbacks. To do so, we depart from RANK by introducing idiosyncratic income risk but, for simplicity, assume a zero measure of HtM households in both countries ($\mathcal{O}, \mathcal{O}^* = 0$). In this case, (23) under $\vartheta = 1$, simplifies to:

$$\begin{bmatrix} \Delta\widehat{y}_{t+1} \\ \Delta\widehat{y}^*_{t+1} \end{bmatrix} = \underbrace{\mathbb{X}\begin{bmatrix} \alpha \\ 1-\alpha \end{bmatrix}\widehat{r}^*_{F,t}}_{\text{intertemporal substitution}} + \underbrace{\begin{bmatrix} \frac{2\alpha(1-\alpha)\eta}{1-2\alpha} \\ -\frac{2\alpha(1-\alpha)\eta}{1-2\alpha} \end{bmatrix}\Delta\widehat{q}_{t+1}}_{\text{expenditure switching}} + \underbrace{\varphi\sigma_c^2\mathbb{X}\begin{bmatrix} \widehat{\sigma}_{c,t+1} \\ \widehat{\sigma}^*_{c,t+1} \end{bmatrix}}_{\text{precautionary saving due to consumption risk}}$$

While intertemporal-substitution and expenditure-switching effects are still at work, as in RANK, the precautionary-savings channel now also affects the sign and magnitude of monetary spillovers. The strength of this channel depends on how expected consumption risk in Home and Foreign is affected by the policy shock. Appendix F shows, using (19a) and (19b), that monetary policy affects expected consumption through the *self-insurance* and cyclical *income-risk* channels:

$$\widehat{\sigma}_{c,t+1} = \underbrace{\frac{\alpha\widetilde{\beta}}{1-\widetilde{\beta}\rho}\left(\frac{1-\lambda}{1-\widetilde{\beta}\lambda\rho}\right)\widehat{r}^*_{F,t+1}}_{\text{self-insurance}} - \underbrace{\left(1-\widetilde{\beta}\lambda\right)\Theta\sum_{s=0}^{\infty}(\widetilde{\beta}\lambda)^s\widehat{y}_{t+1+s}}_{\text{cyclicality of income-risk}} \tag{30a}$$

$$\widehat{\sigma}^*_{c,t+1} = \overbrace{\frac{(1-\alpha)\widetilde{\beta}}{1-\widetilde{\beta}\rho}\left(\frac{1-\lambda}{1-\widetilde{\beta}\lambda\rho}\right)\widehat{r}^*_{F,t+1}} - \overbrace{\left(1-\widetilde{\beta}\lambda\right)\Theta^*\sum_{s=0}^{\infty}(\widetilde{\beta}\lambda)^s\widehat{y}^*_{t+1+s}} \tag{30b}$$

The first term on the RHS of (30a) and (30b) shows that higher interest rates increase the pass-through from a given level of income risk to consumption risk. Thus, a contractionary Foreign monetary policy shock increases consumption risk in both Home and Foreign via the *self-insurance channel*, except if individual income follows a random walk ($\lambda = 1$).[18] Furthermore, if a rate hike in Foreign lowers output in both countries, and if income risk is countercyclical in both Home and Foreign $\Theta, \Theta^* > 0$, then the lower output in both countries also increases consumption risk in both countries via the income-risk channel. However, if a monetary tightening in Foreign raises Home output, then the income-risk channel works to lower consumption risk in Home and the net effect is thus ambiguous.

A special, yet informative case of (30a) and (30b) is when $\lambda = 1$ and $\Theta = \Theta^* = 0$. In this case with random walk income and acyclical income risk, (30a) and (30b) collapse to $\widehat{\sigma}_{c,t+1} = \widehat{\sigma}^*_{c,t+1} = 0$, i.e., a change in Foreign monetary policy has no effect on consumption risk in Home and Foreign: consumption

[18]As discussed previously, the self-insurance channel is not operative if $\lambda = 1$, since all changes in income are expected to be permanent and households do not wish to borrow or save in response to these changes. Consequently, interest rate changes do not matter for self-insurance.

risk is *acyclical* and is constant at its steady state level. Consequently, as the corollary below summarizes, the size and magnitude of international spillovers is the same as in RANK.

Proposition 5 (Acyclical consumption risk). *If $\lambda = 1$ and $\Theta = \Theta^* = 0$, then consumption risk does not change the sign or magnitude of spillovers and spillbacks relative to RANK, which are still given by (25) in Proposition 3.*

Proof. See Appendix F.1. □

Away from this special case with acyclical consumption risk, the precautionary savings channel does affect spillovers and spillbacks. Since the the precautionary savings channel itself depends on the self-insurance and income risk channels, we first study the role of the self-insurance channel by considering the case in which both Home and Foreign have acyclical income risk $\Theta = \Theta^* = 0$, but $\lambda < 1$.

Proposition 6 (Self-insurance channel). *Suppose households face acyclical income risk, i.e., $\Theta = \Theta^* = 0$ but $\sigma_y^2 > 0$, then the effect of a Foreign monetary tightening, $\widehat{r}^*_{F,t} > 0$, holding Home monetary policy fixed $\widehat{r}_{H,t} = 0$, on Home and Foreign output is given by:*

$$\frac{d\widehat{y}_t}{d\widehat{r}^*_{F,t}} = \underbrace{\chi\Big(\eta - 1 - \gamma\Big)}_{\text{spillover}} \qquad \text{and} \qquad \frac{d\widehat{y}^*_t}{d\widehat{r}^*_{F,t}} = \underbrace{-\frac{1+\gamma}{1-\rho}}_{\text{closed-economy effect}} \underbrace{-\chi\Big(\eta - 1 - \gamma\Big)}_{\text{spillback}}, \tag{31}$$

$$\text{where} \qquad \gamma = \left(\frac{\widetilde{\beta}\rho}{1-\widetilde{\beta}\rho}\right)\left(\frac{1-\lambda}{1-\widetilde{\beta}\lambda\rho}\right)\varphi\sigma_c^2 \geq 1, \qquad \text{with strict inequality for } \lambda \in [0,1) \tag{32}$$

Proof. See Appendix F.2. □

Recall from (25) that in RANK, the sign of the spillover and spillback depends on the magnitude of η relative to 1. In contrast, (31) shows that in the case with *acyclical* income risk, the sign depends on the magnitude of η relative to $1 + \gamma > 1$. The term γ captures the additional decline in current demand due to the self-insurance channel. The intuition underlying this expression is straightforward in the closed-economy limiting case ($\alpha = 0$ and $\chi = 0$), in which there are trivially no spillovers and spillbacks. Even in the closed economy limit, accounting for the self-insurance channel, the effect of a given Foreign rate hike on Foreign output is larger than in RANK: it is given by $-(1+\gamma)/(1-\rho)$, compared to $-1/(1-\rho)$ in (25). This larger decline is due to the fact that, in addition to the intertemporal substitution channel which lowers Foreign output by $-1/(1-\rho)$, the self-insurance channel also lowers demand and hence output. (19b) shows that a higher path of Foreign rates increases the pass-through from income to consumption risk, i.e. raises μ^*_t. Even though income risk is acyclical, Foreign households face greater consumption risk, causing them to increase their desired level of precautionary savings and lower their consumption demand. This lowers Foreign output more than in RANK.

In the open economy case ($\alpha > 0$) the monetary tightening in Foreign raises CPI-based real interest rates in the global economy, thus resulting in an increase in pass-through from income to consumption risk in both Home and Foreign: $\widehat{\mu}_t, \widehat{\mu}^*_t > 0$. Thus, both Home and Foreign unconstrained households reduce their current demand for both Home and Foreign goods and increase their precautionary savings. Consequently, Home and Foreign output falls by more than in RANK in response to a Foreign policy hike. In fact, spillovers and spillbacks under acyclical income risk are equivalent to those in a RANK economy with a larger IES (equal to $1 + \gamma$), instead of an IES of 1.

The bottom line is that, different from the RANK benchmark which features *positive* spillovers when $\eta > 1$, the self-insurance channel can, at least theoretically, *flip the sign* of spillovers and spillbacks if γ is large enough, i.e., $\gamma > \eta - 1$. However, if $\gamma < \eta - 1$, then the self-insurance channel still results in positive spillovers but the magnitude of the spillover is actually smaller than in RANK. In contrast, if $\eta < 1$ RANK features *negative* spillovers. Accounting for the self-insurance channel amplifies this *negative* spillover. (31) shows that same is true for the properties of spillbacks.

How large γ is relative to $\eta - 1$ crucially depends on the persistence of individual income. This parameter is key in determining the strength of the self-insurance channel. As described earlier, if $\lambda = 1$, the self-insurance channel is not operative and $\gamma = 0$, even though households face risk. However, with $\lambda < 1$, $\gamma > 0$. In fact, holding steady-state income risk and real interest rates fixed, the change in γ due to a change in λ can be written as:

$$\frac{\partial \gamma(\lambda)}{\partial \lambda} = \underbrace{-\frac{\widetilde{\beta}\rho}{\left(1-\widetilde{\beta}\lambda\rho\right)^2}\varphi\sigma_c^2}_{<0} + \left(\frac{\widetilde{\beta}\rho}{1-\widetilde{\beta}\rho}\right)\left(\frac{1-\lambda}{1-\widetilde{\beta}\lambda\rho}\right)\varphi \underbrace{\frac{2\widetilde{\beta}}{1-\widetilde{\beta}\lambda}\sigma_c^2}_{=\frac{\partial\sigma_c^2}{\partial\lambda}>0},$$

implying that a higher λ has two opposing effects on the strength of the self-insurance channel. First, holding consumption risk fixed, a higher λ makes the self-insurance channel less relevant. If λ is large, a decline in current income also implies that expected lifetime income declines by almost the same amount. Consequently, households do not borrow as much (in the limit as $\lambda = 1$, they don't borrow at all because the fall in income is expected to be permanent). This effect is captured by the first term on the right hand side of the expression above. On the other hand, holding income risk σ_y^2 fixed, a higher λ implies that households face greater consumption risk (the last term on the RHS of the expression above). Thus, the effect of a higher λ is in general ambiguous.

For λ close to 0, the second term dominates and increasing λ from 0 results in a higher γ, while for λ closer to 1, the first term dominates and a further increase in λ tends to lower γ. Given our calibration, the strength of the self-insurance channel initially increases, peaks at about $\lambda = 0.98$ and then declines, vanishing at $\lambda = 1$. Now, in our calibration — which features $\lambda = 0.973$ — the implied level of γ is 0.04, which is not large enough to flip the sign of spillovers since $\eta - 1 = 0.5$. However, our model with CARA preferences and no hard-borrowing constraints for unconstrained households features a small MPC μ relative to empirical estimates. For any given level of income risk, the model likely understates the average consumption risk σ_c^2 that households face, and hence implies a smaller γ. Thus, is it possible that in quantitatively realistic HANK models which can generate a higher level of consumption risk, the implied γ could be large enough to flip the sign of the spillover and spillback relative to RANK.

The other component which affects the strength of the precautionary savings channel is the cyclical *income risk* channel. Proposition 7 presents the effects of a Foreign rate hike on Home output when income risk in Home and Foreign is countercyclical ($\Theta, \Theta^* > 0$).

Proposition 7 (Precautionary-savings channel)**.** *If income risk in Home and Foreign is countercyclical* $\Theta, \Theta^* > 0$*, but not too countercyclical* $\Theta, \Theta^* < \overline{\Theta}$*,*[19] *then the effect of a Foreign monetary tightening* ($\widehat{r}^*_{F,t} > 0$)*, holding*

[19] The exact expression for $\overline{\Theta} > 0$ is available in Appendix F.

Home monetary policy fixed $\widehat{r}_{H,t}=0$, on Home and Foreign output is given by:

$$\frac{d\widehat{y}_t}{d\widehat{r}^*_{F,t}} = \underbrace{\mathbb{M}\left(\theta,\theta^*\right)}_{\substack{\text{spillover-}\\\text{multiplier}}} \times \chi\Big(\eta-(1+\gamma)\Omega(\theta^*)\Big) \tag{33a}$$

$$\frac{d\widehat{y}^*_t}{d\widehat{r}^*_{F,t}} = -\underbrace{\frac{1}{1-\theta^*}\frac{1+\gamma}{1-\rho}}_{\substack{\text{closed-economy}\\\text{effect}}} - \underbrace{\mathbb{M}\left(\theta^*,\theta\right)}_{\substack{\text{spillback-}\\\text{multiplier}}} \times \chi\Big(\eta-(1+\gamma)\Omega(\theta^*)\Big), \tag{33b}$$

where $\theta=\Theta/\overline{\Theta},\theta^*=\Theta^*/\overline{\Theta}$ and $(\theta,\theta^*)\in[0,1)\times[0,1)$ and the function $\mathbb{M}(x,y)$ is the same as in (29). Ω captures the effect of cyclical income risk and can be written as

$$\Omega(\theta^*)=1+\frac{\theta^*}{2\left(1-\alpha\right)\left(1-\theta^*\right)}>1 \qquad \forall\theta^*\in(0,1). \tag{34}$$

Furthermore, $\exists\widetilde{\theta}^*\in(0,1)$, such that $(1+\gamma)\times\Omega(\widetilde{\theta}^*)=\eta$ and for $\theta^*>\widetilde{\theta}^*$, $(1+\gamma)\times\Omega(\widetilde{\theta}^*)>\eta$, i.e., for large enough θ^*, the sign of the spillover and spillback flips relative to RANK.

Proof. See Appendix F. □

Ω in (33a)-(33b) captures the contribution of cyclical income risk to spillovers and spillbacks. Recall that, even when $\eta>1$, in response to a Foreign monetary tightening Home output can (i) stay unchanged relative to RANK if $\lambda=1$, which implies $\gamma=0$, or (ii) increase less relative to RANK if γ is positive but not too large, or (iii) decline if the self-insurance channel is strong enough, i.e., if γ is large. In contrast, the self-insurance channel causes Foreign output to decline by at least as much as in RANK.

When income risk is countercyclical in Foreign, the decline in Foreign output following a monetary tightening implies that Foreign households face higher income risk. Furthermore, since interest rates are expected to be higher in the future as well, lower Foreign output in the future means that Foreign households expect to face higher income risk in the future. This implies that they face even higher consumption risk in the future relative to the acyclical income risk case. Consequently, their desired level of precautionary savings further increases, causing them to cut back on consumption even more.

Even in the closed economy limit ($\alpha=0$), this lower demand leads to a larger decline in Foreign output: the first term on the RHS of (33b) shows that the decline in Foreign output is larger by a factor $1/(1-\theta^*)>1$ relative to the acyclical income risk case. Here $\theta^*=\Theta^*/\overline{\Theta}$ captures the cyclicality of income risk; the larger the cyclicality, the larger the amplification. In fact as $\Theta^*\to\overline{\Theta}$ or $\theta^*\to1$, we have: $\frac{1}{1-\theta^*}\to\infty$. Notice that this effect of countercyclical income risk is also present if the self-insurance channel is not operational ($\lambda=1$), in which case $\gamma=0$. Even in this case, countercyclical income risk results in a larger drop in Foreign output relative to RANK: $-\frac{1}{1-\theta^*}\frac{1}{1-\rho}<-\frac{1}{1-\rho}$.

In the open economy ($\alpha>0$), countercyclical income risk in Foreign also affects the response of Home output to Foreign monetary policy. As in the closed economy limit, countercyclical income risk causes Foreign households to further reduce consumption of both Foreign and Home goods, relative to the acyclical income case. This decline in demand for Home goods tends to lower Home output. Whether this effect can result in an actual decline in Home output depends on the strength of the income risk channel relative to the other channels at work. Proposition 7 shows that, when risk is countercyclical enough in Foreign, i.e., when $\theta^* \in (0,1)$ is sufficiently high, then the implied Ω is large enough so that $(1+\gamma)\Omega > \eta$ and the combination of intertemporal substitution, self-insurance and income risk effects overwhelm the expenditure switching channel. In this case Home output declines in response to a Foreign monetary contraction even though the RER depreciates making Home goods cheaper.

In addition to this direct effect, changes in Home output also affect the level of income risk that Home households face. If income risk in Foreign is countercyclical enough, $(1+\gamma)\Omega > \eta$, then the decline in Home output implies that Home households face higher income risk, which causes them to reduce their consumption of both Home and Foreign goods. This further amplifies the decline in both Home and Foreign output. In contrast, if θ^* is not large enough to cause a decline in Home output so that Home output still increases (but to a lesser extend than in RANK), then the higher level of Home output implies that Home households face *lower* income risk, causing them to increase their consumption demand. Overall, the increase in Home output is amplified. This amplification effect on Home and Foreign output is captured by the spillover and spillback multipliers respectively.

Finally, it is worth noting that countercyclical income risk at Home only affects the magnitude (not the sign) of spillovers and spillbacks relative to acyclical risk or RANK. Instead, countercyclical risk in Foreign can cause a decline in Home output following a Foreign monetary contraction even if under the acyclical income risk and the RANK benchmarks we would obtain positive spillovers. In fact, given our calibration, $(1+\gamma)\Omega = 1.658$, which is larger than $\eta = 1.5$, implying that even though the self-insurance channel is not strong enough to flip the sign of the spillover, the full strength of the precautionary savings channel can generate negative spillovers in Home output alongside a RER depreciation. This is true even with $\eta > 1$, i.e., when RANK features positive spillovers.

Interestingly, even in the countercyclical risk case, the spillover and spillback multipliers are still given by (29), as in the case with HtMs. The difference is that, rather than the fractions of HtM households, the relevant arguments in the function $\mathbb{M}(\bullet,\bullet)$ are the cyclicality of income risk in both countries. Thus, rather than an *intratemporal* Keynesian multiplier that is generated by the presence of high MPC households, the equilibrium effect of countercyclical income risk can instead be thought of as an *intertemporal* Keynesian multiplier. This multiplier-like effect stems from the fact that persistently lower output in the future increases the income risk faced by households, causing them to reduce their demand, which further reduces output leading to a further increase in income risk and so on.

By the same logic as in Section 4.2, the spillover and spillback multipliers due to countercyclical risk are larger than one for two countries with comparable countercyclicalities of income risk:

$$\mathbb{M}(\theta,\theta) = \frac{1}{1-(1-\alpha)\theta} > 1, \qquad \forall \theta \in (0,1)$$

In contrast, in the case where Home is – say – an emerging economy where income risk is more countercyclical than – say – the advanced Foreign economy ($\theta > \theta^*$), the spillovers of a Foreign monetary shock on Home output are larger relative to the case in which two similar economies are compared. Also, as in the Section 4.2, the spillback on the advanced economy is smaller and can even become smaller than in RANK if the countercyclicality in Home is much larger than in Foreign.

Figure 3 plots spillovers and spillbacks for our calibrated economy with $\vartheta < 1$. As in the RANK case, a date 0 increase in $\widehat{r}^*_{F,t}$ after which it slowly returns to steady state (panel a), results in a date 0 RER depreciation, after which it gradually appreciates and returns to its steady state value (panel b).

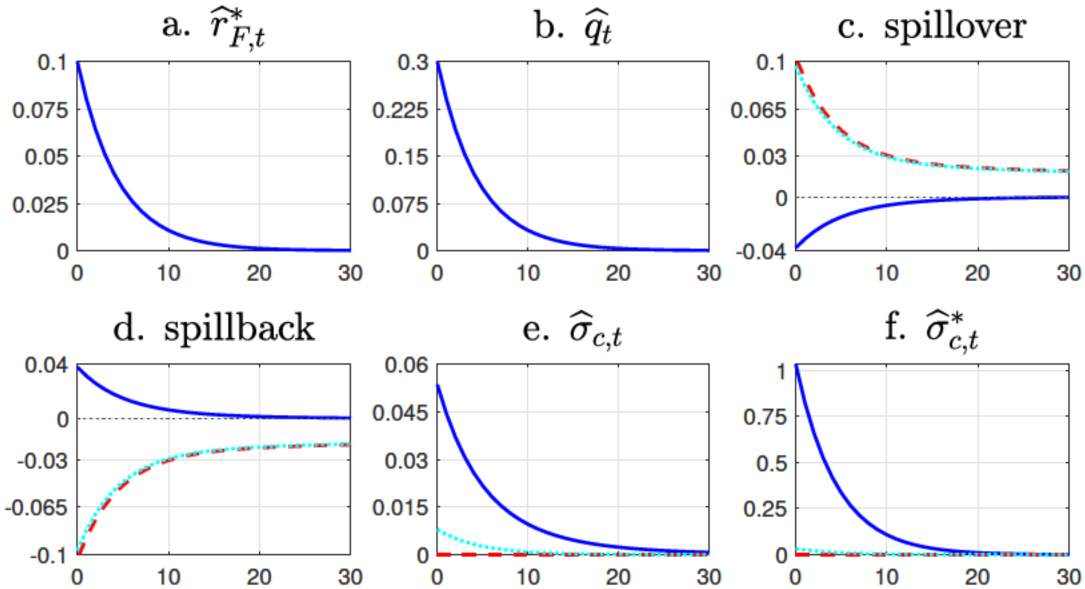

Figure 3: Precautinary-savings channel. The solid blue lines depict dynamics in a world economy in which households face countercyclical risk, while the dashed-red curves depict dynamics in the RANK limit. The dashed-cyan lines depict the contribution of the self-insurance channel.

As in the $\vartheta = 1$ case, while Home output increases on impact in RANK (red-dashed line in panel c), the solid-blue curve shows that Home output declines following the monetary tightening because of the precautionary savings channel. The dotted-cyan curve depicts the contribution of the self-insurance channel, showing that — given our calibration — most of the precautionary-savings channel effects are quantitatively swamped by the income-risk channel.

In consumption-risk space, the difference between the dotted-cyan and solid-blue curves (panels e and f) shows how much consumption risk increases because of the income risk channel. Even though consumption risk increases in both Home and Foreign in the acyclical income risk case, the income-risk channel (depicted by the solid-blue lines) contributes most of the increase in consumption risk in both countries. Finally, panel d shows that with countercyclical income risk, Foreign output falls by more due to the precautionary-savings channel (solid-blue line) relative to both the acyclical income risk case (dotted-cyan line) and RANK (dashed-red line).

Finally, recall that under the Cole and Obstfeld (1991) parameterization spillovers and spillbacks were zero in not just RANK, but also in the benchmark with the real-income channel studied in section 4.2. However, accounting for the precautionary-savings channel implies that even in the Cole and Obstfeld (1991) limit, spillovers are *not zero*; in fact, there are *negative* spillovers: Home output falls despite the RER depreciation. Moreover, Foreign output declines more than in RANK.

Corollary 2. *As long as $\Theta^* > 0$ or $\lambda < 1$, under the Cole and Obstfeld (1991) parameterization ($\eta = 1$), then Home output declines following a Foreign monetary tightening ($\widehat{r}^*_{F,t} > 0$), holding Home monetary policy fixed $\widehat{r}_{H,t} = 0$. Furthermore, spillbacks onto Foreign output are also positive.*

Proof. With countercyclical risk, $(1+\gamma)\Omega > 1$ unless consumption risk is acyclical in Foreign. Otherwise, the result follows from using this information in (33a) and (33b) and imposing $\eta = 1$. □

4.4 Bringing it all together

In summary, the discussion in Sections 4.2 and 4.3 showed how accounting for uninsurable idiosyncratic income risk shapes the profile of monetary policy spillover and spillbacks, which depend on the role played by the real-income channel, the presence of some high MPC households, and the precautionary-savings channel. Table 1 summarizes how each of these channels affect sign and magnitude of spillovers and spillbacks following a Foreign monetary tightening which induces a RER depreciation.

	SPILLOVER			SPILLBACK		
	RANK	MPC heterogeneity	Precautionary savings	RANK	MPC heterogeneity	Precautionary savings
$\eta > 1$	+	+	+/−	−	−	−/+
$\eta = 1$	0	0	−	0	0	+
$\eta < 1$	−	−	−	+	+	+

Table 1: Sign of spillover and spillback in response to a Foreign monetary tightening

Table 1 reiterates the lessons from sections 4.2 and 4.3. It shows that, depending on the magnitude of η relative to 1, even the RANK benchmark can generate positive, negative or zero spillovers. Adding MPC heterogeneity can amplify the magnitude of spillovers and spillbacks via the real-income channel, but the real-income channel *cannot* switch the sign of spillovers and spillbacks relative to RANK. However, the precautionary-savings channel can not just amplify the spillovers relative to RANK, it can also flip the sign if income risk is countercyclical enough. Furthermore, in the empirically relevant case of $\eta > 1$, only the precautionary savings channel can deliver negative spillovers.

Leaving aside considerations about their sign, we can focus purely on the amplification of the spillover and spillback multipliers associated with the real-income channel when compared to the precautionary-savings channel. In our calibration in which both countries are symmetric, the real-income channel induces a spillover and spillback multiplier of $\mathbb{M}(0.3, 0.3) = 1.32$, i.e., spillovers and spillbacks are about 30% larger due to the real-income channel compared to a RANK benchmark with the same preference parameters. In contrast, under our calibration the spillover and spillback multiplier with precautionary savings is equal to $\mathbb{M} = 1.62$, which is about 20% larger. These results suggest that the precautionary-savings channel is able to affect the magnitude of spillovers and deliver a level of amplification in a comparable way as the real-income channel.

5 Dominant Currency Pricing

Our baseline model above featured full exchange rate pass-through (EPRT), as it implicitly assumed a paradigm of Producer Currency Pricing (PCP) in international invoicing. However, there is growing empirical support for the Dominant Currency Pricing (DCP) paradigm as a better description of the global pricing system (Gopinath, 2015; Gopinath et al., 2020). As these papers analyze, in contrast to the symmetric PCP, DCP features not just incomplete ERPT, but also asymmetric ERPT. While we relegate the details of the full model under a DCP paradigm to Appendix G, we present a brief description of the differences relative to our baseline model.

We model the DCP paradigm by assuming that the currency of Foreign is the dominant currency and ERPT in Foreign is less than full. Then, under DCP, exports from Home to Foreign are invoiced in

Foreign currency. In contrast, the import price at Home still satisfies the law of one-price and features full ERPT. Algebraically:

$$\frac{P_{F,t}}{P_t} = \mathcal{Q}_t \frac{P^*_{F,t}}{P^*_t} \qquad \text{and} \qquad \frac{P^*_{H,t}}{P^*_t} = \left(\frac{P_{H,t}}{P_t}\right)^{\iota} \mathcal{Q}_t^{-\iota}, \tag{35}$$

where $\iota \in [0,1]$ controls the degree of exchange rate pass-through. As (35) shows, a 1% RER depreciation (higher $\mathcal{Q}_t$) results in a 1% increase in the relative price of Foreign goods for Home households (higher $P_{F,t}/P_t$), holding Foreign prices fixed. However, in Foreign a 1% RER depreciation only translates into a lower import price for Home goods by a fraction $\iota \leq 1\%$. Thus, holding all else constant, under DCP Foreign households face smaller incentives to shift their demand towards Home goods following a depreciation, i.e., DCP makes the expenditure-switching channel weaker. Consequently, Proposition 8 shows that – when ERPT is sufficiently low – RANK can deliver a contraction in Home output alongside a RER depreciation, even when $\eta > 1$.

Proposition 8 (Perfect-Insurance under DCP)**.** *In a DCP regime ($\iota < 1$), the effect of an increase in the Foreign monetary policy rate $\widehat{r}^*_{F,t}$, holding Home monetary policy fixed $\widehat{r}_{H,t} = 0$, on Home and Foreign output is given by:*

$$\frac{d\widehat{y}_t}{d\widehat{r}^*_{F,t}} = \underbrace{\chi_d\Big(\delta(\iota)\eta - 1\Big)}_{\text{spillover}} \qquad \text{and} \qquad \frac{d\widehat{y}^*_t}{d\widehat{r}^*_{F,t}} = \underbrace{-\frac{1}{1-\rho}}_{\substack{\text{closed-economy}\\ \text{effect}}} \underbrace{-\chi_d\delta(\iota)\Big(\eta - 1\Big)}_{\text{spillback}}, \tag{36}$$

where χ_d is a positive constant defined in Appendix G.1 and

$$\delta(\iota) = 1 - \frac{1-\iota}{2-\alpha\,(1-\iota)} < 1, \qquad \forall \iota \in [0,1), \tag{37}$$

Furthermore, even with $\eta > 1$, as long as $\iota \in [0, \underline{\iota})$, Home output contracts despite a RER deprecation in RANK with DCP, i.e. $\frac{d\widehat{y}_t}{d\widehat{r}^*_{F,t}} < 0$. However, for $\iota \in [\underline{\iota}, 1]$, spillovers are still positive.

Proof. See Appendix G.1. ☐

In the baseline model the sign of the RANK spillovers depended on the magnitude of η relative to 1. In contrast, Proposition 8 shows that, in a DCP regime, the sign of spillovers depends on the relative magnitude of $\delta(\iota) \times \eta$ and 1. For $0 \leq \iota < 1$, the parameter $\delta(\iota)$ is less than one so that the strength of the expenditure-switching channel is weakened (as $\delta(\iota) \times \eta < \eta$). In fact, for low enough ι (small ERPT), (36) shows that even with $\eta > 1$ the RANK benchmark under DCP can generate negative spillovers: Home output can fall in response to a Foreign monetary tightening despite the RER depreciation.[20]

The solid-red line in Figure 4 depicts this graphically by plotting $d\widehat{y}_t/d\widehat{r}^*_{F,t}$ as a function of ERPT ι:[21] the solid-red line is negative for small ι. When ι is small, the export price of Home goods is not very responsive to changes in the RER: following a RER depreciation , the relative price of Home goods does not change appreciably for Foreign households and they are less willing to substitute towards Home goods. Even though Home households still switch towards Home goods, the lack of "extra" demand from Foreign households causes Home output to decline. In contrast, for higher ERPT, Foreign demand for

[20] With $\eta < 1$, even the baseline featured negative spillovers in RANK. Spillovers become larger if one includes $\iota < 1$.

[21] The rest of the parameters are set to the same values as in our calibration for the baseline model.

Home goods increases enough and Home output still increases following a Foreign monetary tightening, as in the baseline. Finally, it is worth noting that unlike in the baseline model, due to the asymmetric nature of DCP, the sign of spillovers is no longer the same as that of spillbacks. Instead, (36) shows that the sign of the spillback still depends on the sign of η relative to 1 because DCP implies low ERPT into Foreign import prices but not for Foreign export prices.

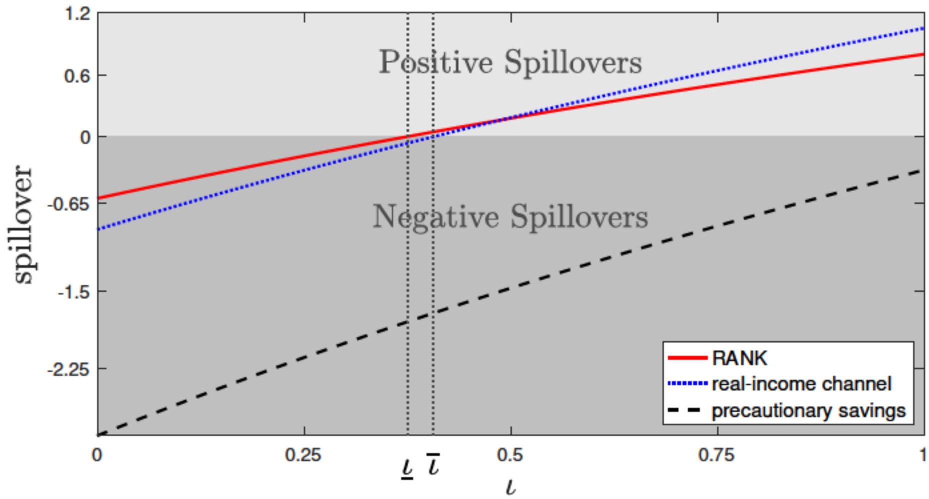

Figure 4: Spillover in response to a Foreign monetary tightening under DCP as a function of ι. The three curves plot the spillovers $\left(d\widehat{y}_t / d\widehat{r}^*_{F,t}\right)$ in the $\vartheta = 1$ limit, in three different cases. The solid red line corresponds to the RANK case under DCP. The dotted-blue line depicts the case which incorporates the effect of the real-income channel, but households face no idiosyncratic risk. The dashed-black line depicts the spillover due to the precautionary savings channel and features no HtM households.

Despite negative spillovers being more likely in a DCP regime than with full ERPT, the key insights from the baseline model broadly still hold. First consider the case in which households face no idiosyncratic risk but there are $\mathcal{O}, \mathcal{O}^*$ HtM households in Home and Foreign respectively. As in RANK, the expenditure-switching channel is weaker in this case because the RER depreciation does not lower the relative price of Home goods for Foreign households. Moreover, while the depreciation lowers the real income of Home HtMs as in the baseline (holding Home output fixed), the real income of Foreign HtMs increases by less because the Foreign price of Home goods does not respond as much (in the limit with $\iota = 0$, the real-income of Foreign HtMs remains unchanged). Consequently, Foreign HtM demand for Home goods increases by less than in the baseline. As such, Appendix G.2 shows that if ERPT ι is small enough ($\iota < \bar{\iota}$), then the real-income channel in conjunction with DCP can result in negative spillovers, but for $\iota > \bar{\iota}$, the real-income channel still delivers positive spillovers.

Figure 4 shows that unlike in the baseline, if $\iota \in [\underline{\iota}, \bar{\iota}]$, then the real-income channel *can* flip the sign of the spillover relative to RANK. Given our calibration, $\underline{\iota}$ is around 38%, while $\bar{\iota}$ is about 41%, suggesting if the estimates of ERPT lie in this range, then the real-income channel in a DCP regime can generate negative spillovers when RANK features positive spillovers. Outside this interval, if $\iota < \underline{\iota}$, then there are negative spillovers in RANK, as well as with the real-income channel; if $\iota > \bar{\iota}$, then there are positive spillovers in RANK, as well as with the real-income channel. Empirically, Gopinath et al. (2020) find ι to lie in the range $50 - 80\%$, which is comfortably larger than $\iota \in [0.38, 0.41]$, suggesting that it is unlikely that the real-income channel can flip the sign of spillovers relative to RANK in a model calibrated to capture a realistic level of ERPT. Proposition 9 formalizes this discussion and presents the equilibrium spillovers accounting for the real-income channel in DCP. The term $\digamma(\iota, \mathcal{O}^*) \geq 1$ captures the fact that due to limited ERPT, the real-income of Foreign HtMs does not increase as much as in the baseline, implying that their demand for Home goods increases by less.

Proposition 9 (The real-income channel under DCP)**.** *As long as ERPT ι is not too small $\iota > \bar{\iota}$ or the fraction of HtMs in Foreign $\mathcal{O}^*$ is not too large, then even in a DCP regime the real income channel does not flip the sign of spillovers relative to RANK. However, as in the baseline model, the real-income channel still amplifies the spillovers relative to RANK. Mathematically, the effect of an increase in the Foreign monetary policy rate $\widehat{r}^*_{F,t}$, holding Home monetary policy fixed $\widehat{r}_{H,t} = 0$, on Home output under a DCP regime is given by:*

$$\frac{d\widehat{y}_t}{d\widehat{r}^*_{F,t}} = \underbrace{\mathbb{M}\left(\mathcal{O}, \mathcal{O}^*\right)}_{\substack{\text{spillover-}\\ \text{multiplier}}} \times \chi_d \Big(\delta\eta - \digamma\left(\iota, \mathcal{O}^*\right)\Big), \tag{38}$$

where the spillover multiplier $\mathbb{M}\left(\mathcal{O}, \mathcal{O}^*\right)$ is given by (29), and $\digamma\left(\iota, \mathcal{O}^*\right) = \delta(\iota) + \Big(1 - \delta(\iota)\Big)\left(1 + \frac{\alpha\mathcal{O}^*}{1-\mathcal{O}^*}\right) \geq 1$ captures the effect of the real-income channel.

Proof. See Appendix G.2. □

Finally, Proposition 10 shows that DCP makes it more likely that the precautionary savings channel changes the sign of spillovers relative to RANK, because the DCP weakens the expenditure switching channel as explained earlier.

Proposition 10 (The precautionary-savings channel under DCP)**.** *Suppose that Home and Foreign have no HtM households ($\mathcal{O} = \mathcal{O}^* = 0$), but all households face idiosyncratic income risk ($\sigma_y, \sigma^*_y > 0$). If income risk is acyclical or countercyclical $\Theta, \Theta^* \geq 0$, but not too countercyclical $\Theta, \Theta^* < \overline{\Theta}$, then the effect of an increase in the Foreign monetary policy rate $\widehat{r}^*_{F,t}$, holding Home monetary policy fixed $\widehat{r}_{H,t} = 0$, in a DCP regime, on Home output is given by*

$$\frac{d\widehat{y}_t}{d\widehat{r}^*_{F,t}} = \underbrace{\mathbb{M}\left(\theta, \theta^*\right)}_{\substack{\text{spillover-}\\ \text{multiplier}}} \times \chi_d \Big(\delta(\iota)\eta - (1+\gamma) \times \Omega(\theta^*, \iota)\Big), \tag{39}$$

where θ and θ^* are defined in Proposition 7 and γ, which captures the effect of the self-insurance channel, is defined in Proposition 6, while $\Omega(\theta^*, \iota)$ captures the effect of countercyclical income risk and is given by:[22]

$$\Omega\left(\theta^*, \iota\right) = 1 + \left[\frac{1 - \alpha^2\left(1-\iota\right)}{2 - \alpha\left(1-\iota\right)}\right] \frac{\theta^*}{\left(1-\alpha\right)\left(1-\theta^*\right)} \geq 1 \qquad \textit{with strict inequality for } \theta^* > 0, \tag{40}$$

As in the baseline, a high enough θ^* can switch the sign of the spillover relative to RANK.

Proof. See Appendix G.3. □

As in the benchmark case, the effects of the precautionary-savings channel can be split into two components: γ, which captures the contribution of the self-insurance channel, and Ω, which captures the effect of the income-risk channel. The Proposition above shows that the expression for γ is the same

[22](40) nests the PCP case with $\iota = 1$, in which case it simplifies and is identical to (34).

as in the baseline model with full ERPT, implying that the self-insurance channel is unaffected by DCP. However, for $\iota < 1$, the contribution of cyclical income risk increases and $\Omega(\theta^*, \iota) > \Omega(\theta^*, 1)$.

To see why, recall that when Foreign households face countercyclical risk, a decline in Foreign output raises the level of income risk (and hence consumption risk) faced by Foreign households following a Foreign monetary tightening. This higher income risk causes Foreign households to increase their desired level of precautionary savings and to reduce current consumption demand for both Home and Foreign goods. If income risk is countercyclical enough, their demand for Home goods declines sufficiently (despite the fall in the relative price of Home goods) and results in a decline in Home output, thus flipping the sign of the spillover relative to RANK.

Under DCP ($\iota < 1$), the RER depreciation does not lower the relative price of Home goods as much as in the baseline. Thus, Foreign demand for Home goods falls by more when $\iota < 1$, making it more likely that a Foreign monetary tightening has negative spillovers onto Home output. This is depicted by the dashed-black line in Figure 4. Recall that in our calibration, even with full ERPT ($\iota = 1$) there were negative spillovers. The black line shows that lowering ERPT ι makes the spillovers even more negative and Home output declines more following a Foreign monetary tightening. Importantly though, even for $\iota > \underline{\iota}$, when RANK under DCP features positive spillovers, the precautionary-savings channel can cause spillovers to become negative.

6 Conclusion

We have presented a simple analytical HANK model which allows us to identify the precise roles played by the precautionary-savings channel and the real-income channel in shaping the sign and magnitude of international monetary policy spillovers. We showed that while both channels can amplify spillovers and spillbacks relative to RANK, only the precautionary savings channel can cause the sign of spillovers to flip relative to RANK. This conclusion also holds for an empirically realistic level of exchange rate pass-through. Finally, while we have focused on *output* spillovers and spillbacks, the analysis can easily be extended to study *inflation* interdependencies. We leave this for future work.

References

Acharya, Sushant and Edouard Challe, "Inequality and Optimal Monetary Policy in the Open Economy," Technical Report 2023. 1

_ **and Keshav Dogra**, "Understanding HANK: Insights from a PRANK," *Econometrica*, May 2020, *88* (3), 1113–1158. 1

Auclert, Adrien, Matthew Rognlie, Martin Souchier, and Ludwig Straub, "Exchange Rates and Monetary Policy with Heterogeneous Agents: Sizing up the Real Income Channel," Working Paper 28872, National Bureau of Economic Research May 2021. 1, 1, 2.4, 3, 10, 4.1, 16, 4.2, 4.2, 4.2, I.2

Bayer, Christian, Gernot Müller, Alexander Kriwoluzky, and Fabian Seyrich, "A HANK2 model of monetary unions," Technical Report, CEPR Discussion Papers 2023. 4

Benhabib, Jess and Alberto Bisin, "The Distribution of Wealth and Redistributive Policies," Technical Report 2006. 4

Benigno, Gianluca and Pierpaolo Benigno, "Price stability in open economies," *The Review of Economic Studies*, 2003, *70* (4), 743–764. 1

Bernanke, Ben S, "Federal reserve policy in an international context," *IMF Economic Review*, 2017, *65*, 1–32. 1

Betts, Caroline and Michael B Devereux, "Exchange rate dynamics in a model of pricing-to-market," *Journal of international Economics*, 2000, *50* (1), 215–244. 1

Bianchi, Javier and Louphou Coulibaly, "Liquidity traps, prudential policies, and international spillovers," Technical Report, National Bureau of Economic Research 2022. 1

__ **and** __ , "Financial Integration and Monetary Policy Coordination," Technical Report, National Bureau of Economic Research 2024. 1

Bodenstein, Martin, Giancarlo Corsetti, and Luca Guerrieri, "The Elusive Gains from Nationally Oriented Monetary Policy," Technical Report 2023. 1

Carroll, Christopher, "Theoretical Foundations of Buffer Stock Saving," Working Paper 10867, National Bureau of Economic Research November 2004. 10

Carroll, Christopher D. and Miles S. Kimball, "On the Concavity of the Consumption Function," *Econometrica*, 1996, *64* (4), 981–992. 10

__ **and** __ , "Liquidity Constraints and Precautionary Saving," Working Paper 8496, National Bureau of Economic Research October 2001. 10

Carroll, Christopher D and Patrick Toche, "A Tractable Model of Buffer Stock Saving," Working Paper 15265, National Bureau of Economic Research August 2009. 10

Carstens, A., "Monetary Policy Options and Tools," in "Global Dimensions of Unconventional Monetary Policy, Proceedings - Economic Policy Symposium - Jackson Hole," Federal Reserve Bank of Kansas City, 2013, pp. 135–148. 1

Clarida, Richard, Jordi Galı, and Mark Gertler, "A Simple Framework for International Monetary Policy Analysis," *Journal of monetary economics*, 2002, *49* (5), 879–904. 1

Cole, Harold L. and Maurice Obstfeld, "Commodity trade and international risk sharing : How much do financial markets matter?," *Journal of Monetary Economics*, August 1991, *28* (1), 3–24. 1, 2.2, 4.1, 4.2, 4.3, 2, I.1

Corsetti, Giancarlo and Paolo Pesenti, "Welfare and macroeconomic interdependence," *Quarterly Journal of Economics*, May 2001, *116* (2), 421–445. 1, 1, 2.2, 4.1, I.1

Cugat, Gabriela, "Emerging Markets, Household Heterogeneity, and Exchange Rate Policy," Working Paper 2019. 3

Devereux, Michael B and Charles Engel, "Monetary Policy in the Open Economy Revisited: Price Setting and Exchange-Rate Flexibility," *The Review of Economic Studies*, 2003, *70* (4), 765–783. 1

Dornbusch, Rudiger, "Expectations and exchange rate dynamics," *Journal of political Economy*, 1976, *84* (6), 1161–1176. 1

Druedahl, Jeppe, Søren Hove Ravn, Laura Sunder-Plassmann, Jacob Marott Sundram, and Nicolai Waldstrøm, "The Transmission of Foreign Demand Shocks," Technical Report, Working Paper 2022. 3

Ferra, Sergio De, Kurt Mitman, and Federica Romei, "Household heterogeneity and the transmission of foreign shocks," *Journal of International Economics*, 2020, *124*, 103303. 1

Ferrante, Francesco and Nils Gornemann, "Devaluations, deposit dollarization, and household heterogeneity," International Finance Discussion Paper 1336 2022. 1

Fleming, J Marcus, "Domestic Financial Policies under Fixed and under Floating Exchange Rates (Politiques finacierieures interieures avec un systeme de taux de change fixe et avec un systeme de taux de change fluctuant)(Politica financiera interna bajo sistemas de tipos de cambio fijos o de tipos de cambio fluctuantes)," *Staff Papers-International Monetary Fund*, 1962, pp. 369–380. 1

Fornaro, Luca and Federica Romei, "Monetary policy in an unbalanced global economy," Technical Report 2023. 1

Gabaix, Xavier and Matteo Maggiori, " International Liquidity and Exchange Rate Dynamics *," *The Quarterly Journal of Economics*, 03 2015, *130* (3), 1369–1420. 1

Galí, Jordi and Tommaso Monacelli, "Monetary Policy and Exchange Rate Volatility in a Small Open Economy," *The Review of Economic Studies*, 07 2005, *72* (3), 707–734. 2.3, 4

Ghironi, Fabio, "Macroeconomic interdependence under incomplete markets," *Journal of International Economics*, 2006, *70* (2), 428–450. 3, 10

Giagheddu, Marta, "The distributional implications of fiscal devaluations," *Available at SSRN 3651258*, 2020. 3

Gopinath, Gita, "The international price system," Technical Report, National Bureau of Economic Research 2015. 1, 5

_ , Emine Boz, Camila Casas, Federico J. Díez, Pierre-Olivier Gourinchas, and Mikkel Plagborg-Møller, "Dominant Currency Paradigm," *American Economic Review*, March 2020, *110* (3), 677–719. 1, 5, 5

Guo, Xing, Pablo Ottonello, and Diego J Perez, "Monetary policy and redistribution in open economies," *Journal of Political Economy Macroeconomics*, 2023, *1* (1), 191–241. 1

Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante, "The Macroeconomic Implications of Rising Wage Inequality in the United States," *Journal of Political Economy*, 2010, *118* (4), 681–722. 4

Hong, Seungki, "Emerging market business cycles with heterogeneous agents," Technical Report, Working Paper 2020. 3

Itskhoki, Oleg and Dmitry Mukhin, "Exchange Rate Disconnect in General Equilibrium," *Journal of Political Economy*, 2021, *129* (8), 2183–2232. 4

Iyer, Tara, "Optimal monetary policy in an open emerging market economy," Technical Report 2016. 3

Kalemli-Özcan, Sebnem, "U.S. Monetary Policy and International Risk Spillovers," Working Paper 26297, National Bureau of Economic Research September 2019. 1

Kaplan, Greg, Giovanni L Violante, and Justin Weidner, "The wealthy hand-to-mouth," Technical Report, National Bureau of Economic Research 2014. 4

McCallum, Bennett T, "On non-uniqueness in rational expectations models: An attempt at perspective," *Journal of monetary Economics*, 1983, *11* (2), 139–168. H

McKay, Alisdair, Emi Nakamura, and Jon Steinsson, "The Power of Forward Guidance Revisited," *American Economic Review*, October 2016, *106* (10), 3133–3158. 2.2

Mundell, Robert A, "Capital mobility and stabilization policy under fixed and flexible exchange rates," *Canadian Journal of Economics and Political Science/Revue canadienne de economiques et science politique*, 1963, *29* (4), 475–485. 1

Nakajima, Makoto and Vladimir Smirnyagin, "Cyclical Labor Income Risk," Working Paper 2019. 2.1, 4

Obstfeld, Maurice and Kenneth Rogoff, "Exchange rate dynamics redux," *Journal of political economy*, 1995, *103* (3), 624–660. 1

_ and _ , *Foundations of international macroeconomics*, MIT press, 1996. 1

_ and _ , "Global implications of self-oriented national monetary rules," *The Quarterly journal of economics*, 2002, *117* (2), 503–535. 1

Oskolkov, Aleksei, "Exchange rate policy and heterogeneity in small open economies," *Journal of International Economics*, 2023, *142*, 103750. 1

Rajan, Raghuram, "Competitive monetary easing: is it yesterday once more?," *Macroeconomics and Finance in Emerging Market Economies*, 2015, *8* (1-2), 5–16. 1

Rey, Hélène, "Dilemma not trilemma: the global financial cycle and monetary policy independence," Technical Report, National Bureau of Economic Research 2015. 1

Rotemberg, Julio J., "Monopolistic Price Adjustment and Aggregate Output," *Review of Economic Studies*, 1982, *49* (4), 517–531. 2.4

Storesletten, Kjetil, Chris Telmer, and Amir Yaron, "Cyclical Dynamics in Idiosyncratic Labor Market Risk," *Journal of Political Economy*, 2004, *112* (3), 695–717. 2.1

Sunel, Enes, "Welfare consequences of gradual disinflation in emerging economies," *Journal of Money, Credit and Banking*, 2018, *50* (4), 705–755. 3

Tille, Cedric, "The role of consumption substitutability in the international transmission of monetary shocks," *Journal of International Economics*, 2001, *53* (2), 421–444. 1

Toche, Patrick, "A tractable model of precautionary saving in continuous time," *Economics letters*, 2005, *87* (2), 267–272. 10

Vicondoa, Alejandro, "Monetary news in the United States and business cycles in emerging economies," *Journal of International Economics*, 2019, *117*, 79–90. 1

Zhou, Haonan, "Open economy, redistribution, and the aggregate impact of external shocks," Working Paper 2022. 1, 4.2

Özge Akinci and Albert Queralto, "Exchange Rate Dynamics and Monetary Spillovers with Imperfect Financial Markets," *The Review of Financial Studies*, 10 2023, *37* (2), 309–355. 1

Appendix

A Supply side and pricing behavior

A.1 Final goods producers

A perfectly competitive wholesale firm combines the Home good and final good into the Home consumption good according to the aggregator

$$c_t = \left[\alpha^{\frac{1}{\eta}} \left(c_{F,t}\right)^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} \left(c_{H,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

The firm's profit maximizing problem can be written as

$$\max_{c_{H,t}, c_{F,t}} P_t \left[\alpha^{\frac{1}{\eta}} \left(c_{F,t}\right)^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} \left(c_{H,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} - P_{H,t}c_{H,t} - P_{F,t}c_{F,t}$$

This yields the standard demand functions

$$c_{H,t}\left(j,\tau\right) = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} c_t\left(j,\tau\right) \qquad \text{and} \qquad c_{F,t}\left(j,\tau\right) = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} c_t\left(j,\tau\right)$$

where $P_t = \left[\eta P_{F,t}^{1-\eta} + (1-\eta)\, P_{H,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}$ is the price of the Home consumption basket.

A.2 Relative prices

Using the definitions of the CPI price indices in the two countries along with the definition of the RER, we have:

$$
\begin{aligned}
1 &= \alpha\left(\mathcal{Q}_t \frac{P^*_{F,t}}{P^*_t}\right)^{1-\eta} + (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{1-\eta} \\
\mathcal{Q}_t^{1-\eta} &= \alpha\left(\frac{P_{H,t}}{P_t}\right)^{1-\eta} + (1-\alpha)\left(\mathcal{Q}_t \frac{P^*_{F,t}}{P^*_t}\right)^{1-\eta}
\end{aligned}
$$

Combine these to solve for $P_{H,t}/P_t$ and $P^*_{F,t}/P^*_t$ to get:

$$
\frac{P_{H,t}}{P_t} = p_H(\mathcal{Q}_t) \equiv \left[\frac{1-\alpha-\alpha\mathcal{Q}_t^{1-\eta}}{1-2\alpha}\right]^{\frac{1}{1-\eta}} \tag{A.1}
$$

$$
\frac{P^*_{F,t}}{P^*_t} = p^*_F(\mathcal{Q}_t) \equiv \left[\frac{1-\alpha-\alpha\mathcal{Q}_t^{\eta-1}}{1-2\alpha}\right]^{\frac{1}{1-\eta}} \tag{A.2}
$$

Also, taking log-linear approximations of (A.1) and (A.2) around steady state yields

$$
\widehat{p}_{H,t} = p'_H(1)\widehat{q}_t = -\frac{\alpha}{1-2\alpha}\widehat{q}_t \qquad \text{and} \qquad \widehat{p}^*_{F,t} = p^{*\prime}_F(1)\widehat{q}_t = \frac{\alpha}{1-2\alpha}\widehat{q}_t \tag{A.3}
$$

B Optimal consumption-savings decisions

B.1 Unconstrained households

In this section, we characterize the solution to an unconstrained Home household's problem for a given sequence $\{y_t, w_t, r_t\}_{t=0}^{\infty}$. In what follows, to reduce notational clutter, we exclude the k subscript, since this section characterizes the decisions of unconstrained households. Using the functional forms of $u(\cdot)$ and $\mathcal{R}(\cdot)$, an unconstrained Home household (j,τ)'s problem can be written as:

$$
\mathbb{W}^\tau_{j,t} = -\ln\left\{(1-\beta\vartheta)\, e^{-\gamma^{-1}c^\tau_{j,t}} + \beta\vartheta\left[\mathbb{E}e^{-\varphi\mathbb{W}^\tau_{j,t+1}}\right]^{\frac{1}{\varphi}}\right\} \tag{B.1}
$$

subject to

$$
\frac{\vartheta}{1+r_t}a^\tau_{j,t+1} = y_{\mathrm{uc},t} + \sigma_{y,t}\xi^\tau_{j,t} + a^\tau_{j,t} - c^\tau_{j,t} \tag{B.2}
$$

where

$$
\begin{aligned}
y_{\mathrm{uc},t} &= w_t n_t \bar{z} + D_{\mathrm{uc},t} + T_{\mathrm{uc},t} \qquad \text{where} \qquad w_t = \frac{W_t}{P_t} && \text{(B.3)}\\
\xi^\tau_{j,t} &= \lambda\xi^\tau_{j,t-1} + v^\tau_{j,t}\,, \qquad \xi^\tau_{j,\tau-1} = 0, \qquad v^\tau_{j,t} \sim N(0,1) && \text{(B.4)}\\
P_t a^\tau_{j,t} &= A^\tau_{j,t} = B^\tau_{H,j,t} + \mathcal{E}_t B^\tau_{F,j,t} && \text{(B.5)}\\
\sigma_{y,t} &= w_t n_t \sigma_t && \text{(B.6)}\\
a^\tau_{j,\tau} &= 0 && \text{(B.7)}
\end{aligned}
$$

Since new households have 0 wealth, aggregating (B.2) across all unconstrained households, we get:

$$\frac{1}{1+r_t}a_{t+1} = y_{\mathrm{uc},t} + a_t - c_{\mathrm{uc},t} \tag{B.8}$$

where $c_{\mathrm{uc},t}$ denotes the average per-capita consumption of unconstrained households. Subtract (B.8) from (B.2) to get:

$$a^{\tau}_{j,t+1} - a_{t+1} = \left(\frac{1+r_t}{\vartheta}\right)\left[\sigma_{y,t}\xi^{\tau}_{j,t} + \left(a^{\tau}_{j,t} - a_t\right) - \left(c^{\tau}_{j,t} - c_{\mathrm{uc},t}\right)\right] + \frac{1-\vartheta}{\vartheta}a_{t+1}$$

$$a^{\tau}_{j,t+1} - a_{t+1} = \left(\frac{1+r_t}{\vartheta}\right)\left[\sigma_{y,t}\xi^{\tau}_{j,t} + \left(a^{\tau}_{j,t} - a_t\right) - \left(c^{\tau}_{j,t} - c_{\mathrm{uc},t}\right)\right] + \frac{1-\vartheta}{\vartheta}a_{t+1} \tag{B.9}$$

Next, guess that the value-function $\mathbb{W}^{\tau}_{j,t}$ is given by:

$$\mathbb{W}^{\tau}_{j,t} = \overline{\mathbb{W}}_t + \mu_t\left(a^{\tau}_{j,t} - a_t + h^{\tau}_{j,t}\right) \tag{B.10}$$

where h^{τ} denotes the human wealth of the household relative to the average unconstrained household:

$$\begin{aligned} h^{\tau}_{j,t} &= \mathbb{E}_t\sum_{s=0}^{\infty}\left(\frac{\vartheta^s}{\prod_{k=0}^{s-1}(1+r_{t+k})}\right)w_{t+s}n_{t+s}\left(z^{\tau}_{j,t+\tau} - \bar{z}\right) \\ &= \underbrace{\left[\sum_{s=0}^{\infty}\frac{\vartheta^s\lambda^s}{\prod_{k=0}^{s-1}(1+r_{t+k})}\sigma_{y,t+s}\right]}_{\sigma_{h,t}}\xi^{\tau}_{j,t} \end{aligned} \tag{B.11}$$

Using (B.11), we can rewrite (B.10) as

$$\mathbb{W}^{\tau}_{j,t} = \overline{\mathbb{W}}_t + \mu_t\left(a^{\tau}_{j,t} - a_t + \sigma_{h,t}\xi^{\tau}_{j,t}\right) = \overline{\mathbb{W}}_t + \mu_t\left(a^{\tau}_{j,t} - a_t\right) + \sigma_{c,t}\xi^{\tau}_{j,t}, \tag{B.12}$$

where $\sigma_{c,t} = \mu_t\sigma_{h,t}$. Next, taking the derivative of (B.1) w.r.t. $c^{\tau}_{j,t}$ and setting it equal to zero yields the first-order optimality condition:

$$(1-\beta\vartheta)\,e^{-c^{\tau}_{j,t}} = \beta\vartheta\frac{\mu_{t+1}(1+r_t)}{\vartheta}\left[\mathbb{E}e^{-\varphi\mathbb{W}^{s}_{t+1}}\right]^{\frac{1}{\varphi}} \tag{B.13}$$

Using (B.12) and since $\xi^{\tau}_{j,t}$ is normally distributed, we can rewrite the above as:

$$(1-\beta\vartheta)\,e^{-c^{\tau}_{j,t}} = \beta\vartheta\frac{\mu_{t+1}(1+r_t)}{\vartheta}e^{-\left\{\overline{\mathbb{W}}_{t+1}+\mu_{t+1}\left(a^{\tau}_{j,t+1}-a_{t+1}\right)+\lambda\sigma_{c,t+1}\xi^{\tau}_{j,t}\right\}+\frac{\varphi}{2}\sigma^2_{c,t+1}} \tag{B.14}$$

Taking logs on both sides of (B.14), using (B.9) and further simplifying yields the consumption-decision rule for the household:

$$c^{\tau}_{j,t} = c_{\mathrm{uc},t} + \frac{1}{1+\frac{(1+r_t)\mu_{t+1}}{\vartheta}}\left[\frac{(1+r_t)\,\mu_{t+1}}{\vartheta}\sigma_{y,t} + \lambda\sigma_{c,t+1}\right]\xi^{\tau}_{j,t} + \frac{\frac{(1+r_t)\mu_{t+1}}{\vartheta}}{1+\frac{(1+r_t)\mu_{t+1}}{\vartheta}}\left(a^{\tau}_{j,t} - a_t\right), \tag{B.15}$$

where $c_{\mathrm{uc},t}$ is given by:

$$c_{\mathrm{uc},t} \;=\; -\ln\left[\frac{\beta\vartheta}{1-\beta\vartheta}\frac{\mu_{t+1}\left(1+r_t\right)}{\vartheta}\right]+\overline{\mathrm{W}}_{t+1}+\frac{1-\vartheta}{\vartheta}\mu_{t+1}a_{t+1}-\frac{\varphi}{2}\sigma_{c,t+1}^2 \tag{B.16}$$

Next, we need to now verify our guess (B.10). Using (B.13), (B.15) and (B.16), we can re-write (B.1) as:

$$\begin{aligned}\mathrm{W}_{j,t}^{\tau} \;=\; & -\ln\left\{\beta\left(1+\frac{\vartheta}{1+r_t}\mu_{t+1}^{-1}\right)\mu_{t+1}\left(1+r_t\right)\right\}+\overline{\mathrm{W}}_{t+1}+\frac{1-\vartheta}{\vartheta}\mu_{t+1}a_{t+1}-\frac{\varphi}{2}\sigma_{c,t+1}^2\\ &+\frac{1}{1+\frac{(1+r_t)\mu_{t+1}}{\vartheta}}\left[\frac{\left(1+r_t\right)\mu_{t+1}}{\vartheta}\sigma_{y,t}+\lambda\sigma_{c,t+1}\right]\xi_{j,t}^{\tau}+\frac{\frac{(1+r_t)\mu_{t+1}}{\vartheta}}{1+\frac{(1+r_t)\mu_{t+1}}{\vartheta}}\left(a_{j,t}^{\tau}-a_t\right)\end{aligned}$$

Equating this expression to the guess (B.12):

$$\begin{aligned}\overline{\mathrm{W}}_t+\mu_t\left(a_{j,t}^{\tau}-a_t\right)+\sigma_{c,t}\xi_{j,t}^{\tau} \;=\; & -\ln\left\{\beta\left(1+\frac{\vartheta}{1+r_t}\mu_{t+1}^{-1}\right)\mu_{t+1}\left(1+r_t\right)\right\}+\overline{\mathrm{W}}_{t+1}\\ &+\frac{1-\vartheta}{\vartheta}\mu_{t+1}a_{t+1}-\frac{\varphi}{2}\sigma_{c,t}^2\\ &+\frac{\frac{(1+r_t)\mu_{t+1}}{\vartheta}}{1+\frac{(1+r_t)\mu_{t+1}}{\vartheta}}\left(a_{j,t}^{\tau}-a_t\right)\\ &+\frac{1}{1+\frac{(1+r_t)\mu_{t+1}}{\vartheta}}\left[\frac{\left(1+r_t\right)\mu_{t+1}}{\vartheta}\sigma_{y,t}+\lambda\sigma_{c,t+1}\right]\xi_{j,t}^{\tau}\end{aligned}$$

For this equation to be satisfied for any sequence of $\xi_{j,t}^{\tau}$, r_t and $a_{j,t}^{\tau}-a_t$, we need:

$$\mu_t \;=\; \frac{\mu_{t+1}\frac{1+r_t}{\vartheta}}{1+\mu_{t+1}\frac{1+r_t}{\vartheta}} \qquad \Leftrightarrow \qquad \mu_t^{-1}=1+\frac{\vartheta}{1+r_t}\mu_{t+1}^{-1} \tag{B.17}$$

$$\sigma_{c,t} \;=\; \mu_t\sigma_{y,t}+\lambda\left(1-\mu_t\right)\sigma_{c,t+1} \tag{B.18}$$

$$\overline{\mathrm{W}}_t \;=\; -\ln\left\{\beta\mu_t^{-1}\mu_{t+1}\left(1+r_t\right)\right\}+\overline{\mathrm{W}}_{t+1}+\frac{1-\vartheta}{\vartheta}\mu_{t+1}a_{t+1}-\frac{\varphi}{2}\sigma_{c,t+1}^2 \tag{B.19}$$

Next, combining (B.16), (B.17) and (B.19), we have

$$\overline{\mathrm{W}}_t=\ln\mu_t+c_{\mathrm{uc},t}-\ln\left(1-\beta\vartheta\right)$$

Using this in (B.19), we can derive the aggregate Euler equation for all unconstrained Home households:

$$\Delta c_{\mathrm{uc},t+1}=\ln\beta\left(1+r_t\right)+\frac{\varphi}{2}\sigma_{c,t+1}^2-\frac{1-\vartheta}{\vartheta}\mu_{t+1}a_{t+1}$$

and the value function is given by:

$$\mathrm{W}_{j,t}^{\tau}=\ln\mu_t+c_{\mathrm{uc},t}-\ln\left(1-\beta\vartheta\right)+\mu_t\left(a_{j,t}^{\tau}-a_t\right)+\sigma_{c,t}\xi_{j,t}^{\tau}$$

B.2 HtM households and aggregate Home consumption

Since HtM households are excluded from asset markets, each of them consume their respective income:

$$c^{\tau}_{j,\text{htm},t} = y^{\tau}_{j,\text{htm},t} \tag{B.20}$$

Aggregating across all HtM households yields:

$$c_{\text{htm},t} = y_{\text{htm},t} \tag{B.21}$$

Subtracting (B.21) from (B.20) yields:

$$c_{\text{htm},t}(j,\tau) = y_{\text{htm},t} + \sigma_{y,t}\xi^{\tau}_{j,t}, \tag{B.22}$$

which is the same as in Proposition 1. Finally, total consumption demand of Home households can then be written as

$$c_t = \mathcal{O} c_{\text{htm},t} + (1-\mathcal{O})\, c_{\text{uc},t} \tag{B.23}$$

C Linearized baseline model

C.1 Market clearing conditions

Using the expressions for $p_H(\mathcal{Q}_t)$ and $p^*_F(\mathcal{Q}_t)$ in (A.1) and (A.2) in the market clearing equations (6a) and (6b), we have

$$y_t = \left\{(1-\alpha)\, c_t + \alpha \mathcal{Q}_t^{\eta} c_t^*\right\} \left[\frac{1-\alpha-\alpha\mathcal{Q}_t^{1-\eta}}{1-2\alpha}\right]^{\frac{-\eta}{1-\eta}} \tag{C.1}$$

$$y^*_t = \left\{(1-\alpha)\, c_t^* + \alpha \mathcal{Q}_t^{-\eta} c_t\right\} \left[\frac{1-\alpha-\alpha\mathcal{Q}_t^{\eta-1}}{1-2\alpha}\right]^{\frac{-\eta}{1-\eta}} \tag{C.2}$$

Log-linearizing, we have

$$\begin{bmatrix}\Delta\widehat{y}_{t+1}\\ \Delta\widehat{y}^*_{t+1}\end{bmatrix} = \mathbb{X}\begin{bmatrix}\Delta\widehat{c}_{t+1}\\ \Delta\widehat{c}^*_{t+1}\end{bmatrix} + \begin{bmatrix}\frac{2(1-\alpha)\alpha}{1-2\alpha}\eta\\ -\frac{2(1-\alpha)\alpha}{1-2\alpha}\eta\end{bmatrix}\Delta\widehat{q}_{t+1}, \tag{C.3}$$

where

$$\mathbb{X} = \begin{bmatrix}1-\alpha & \alpha\\ \alpha & 1-\alpha\end{bmatrix}$$

C.2 Uncovered Interest Rate Parity (UIP)

Log-linearizing the UIP equation (7) in real terms, we get

$$\Delta\widehat{q}_{t+1} = (1-2\alpha)(\widehat{r}_{H,t} - \widehat{r}^*_{F,t})$$

which is the same as (17) in the main text.

C.3 Net Foreign Asset positions (NFA)

$$\frac{1}{1+r}\widehat{a}_{t+1} = \widehat{a}_t + \widehat{y}_t - \widehat{c}_t - \frac{\alpha}{1-2\alpha}\widehat{q}_t$$

C.4 Deriving the IS curves

We start by taking a first-order approximation of the aggregated Euler equation of unconstrained Home households (10) around the zero inflation symmetric steady state:

$$\Delta\widehat{c}_{\text{uc},t+1} = \widehat{r}_t + \varphi\sigma_c^2\widehat{\sigma}_{c,t+1} - \frac{(1-\widetilde{\beta})(1-\vartheta)}{\vartheta}\widehat{a}_{t+1} \tag{C.4}$$

where $\widehat{\sigma}_{c,t}$ denotes the log-deviation of the unconstrained Home households' consumption risk from its steady state level and $\widehat{a}_{t+1}$ denotes the level deviation of Home nfa from its steady state value of 0. $\widehat{r}_t = \widehat{r}_{H,t} - \frac{\alpha}{1-2\alpha}\Delta\widehat{q}_{t+1}$, where $\widehat{r}_t$ denotes the log deviation of the CPI based gross Home real interest rate, $\widehat{r}_{H,t}$ denotes the log-deviation of the PPI based gross Home real interest rate and $\widehat{q}_t$ denotes the log-deviation of the RER from their respective steady state values. Similarly, the aggregated Euler equation for unconstrained Foreign households can be written as

$$\Delta\widehat{c}^*_{\text{uc},t+1} = \widehat{r}^*_t + \varphi\sigma_c^2\widehat{\sigma}^*_{c,t+1} + \frac{(1-\widetilde{\beta})(1-\vartheta)}{\vartheta}\widehat{a}_{t+1}, \tag{C.5}$$

where we have used the first-order approximation of the asset market clearing condition $\widehat{a}_t = -\widehat{a}^*_t = 0$. $\widehat{\sigma}^*_{c,t}$ denotes the log-deviation of consumption risk faced by unconstrained Foreign households from its steady state level. $\widehat{r}^*_t = \widehat{r}^*_{F,t} + \frac{\alpha}{1-2\alpha}\Delta\widehat{q}_{t+1}$, where $\widehat{r}^*_t$ denotes the log deviation of the CPI based gross Foreign real interest rate and $\widehat{r}^*_{F,t}$ denotes the log-deviation of the PPI based gross Foreign real interest rate from their respective steady state values.

Next, recall that the since average consumption of Home HtM households is equal to their average income, (in log-deviations) this can be written as:

$$\widehat{c}_{\text{htm},t} = \widehat{y}_t - \frac{\alpha}{1-2\alpha}\widehat{q}_t \qquad \Rightarrow \qquad \Delta\widehat{c}_{\text{htm},t+1} = \Delta\widehat{y}_{t+1} - \frac{\alpha}{1-2\alpha}\Delta\widehat{q}_{t+1} \tag{C.6}$$

Since aggregate Home consumption (in log-deviations) is given by $\widehat{c}_t = \mathcal{O}\widehat{c}_{\text{htm},t} + (1-\mathcal{O})\widehat{c}_{\text{uc},t}$, using (C.6), we have

$$\begin{aligned}\Delta\widehat{c}_{t+1} \quad &= \quad \mathcal{O}\left(\Delta\widehat{y}_{t+1} - \frac{\alpha}{1-2\alpha}\Delta\widehat{q}_{t+1}\right) + (1-\mathcal{O})\left(\widehat{r}_{H,t} - \frac{\alpha}{1-2\alpha}\Delta\widehat{q}_{t+1}\right) \\ &\quad + (1-\mathcal{O})\,\varphi\sigma_c^2\widehat{\sigma}_{c,t+1} - (1-\mathcal{O})\,\frac{\widetilde{\beta}(1-\vartheta)}{\vartheta}\widehat{a}_{t+1}\end{aligned} \tag{C.7}$$

Similarly, since $\widehat{c}^*_{\text{htm},t} = \widehat{y}^*_t + \frac{\alpha}{1-2\alpha}\widehat{q}_t$ and $\widehat{c}^*_t = \mathcal{O}^*\widehat{c}^*_{\text{htm},t} + (1-\mathcal{O}^*)\widehat{c}^*_{\text{uc},t}$, we have:

$$\begin{aligned}\Delta\widehat{c}^*_{t+1} &= \mathcal{O}^*\left(\Delta\widehat{y}^*_{t+1} + \frac{\alpha}{1-2\alpha}\Delta\widehat{q}_{t+1}\right) + (1-\mathcal{O}^*)\left(\widehat{r}^*_{F,t} + \frac{\alpha}{1-2\alpha}\Delta\widehat{q}_{t+1}\right) \\ &\quad + (1-\mathcal{O}^*)\,\varphi\sigma_c^2\widehat{\sigma}^*_{c,t+1} + (1-\mathcal{O}^*)\,\frac{\widetilde{\beta}(1-\vartheta)}{\vartheta}\widehat{a}_{t+1}\end{aligned} \tag{C.8}$$

Writing (C.7) and (C.8) in matrix-form:

$$\begin{aligned}\begin{bmatrix}\Delta\widehat{c}_{t+1}\\ \Delta\widehat{c}^*_{t+1}\end{bmatrix} &= \underbrace{(\mathbb{I}-\mathbb{O})\left\{\begin{bmatrix}\widehat{r}_t\\ \widehat{r}^*_t\end{bmatrix} + \varphi\sigma_c^2\begin{bmatrix}\widehat{\sigma}_{c,t+1}\\ \widehat{\sigma}^*_{c,t+1}\end{bmatrix} + \left(\frac{1-\vartheta}{\vartheta}\right)\begin{bmatrix}-\left(1-\widetilde{\beta}\right)\\ \left(1-\widetilde{\beta}\right)\end{bmatrix}\widehat{a}_{t+1}\right\}}_{\text{consumption growth of unconstrained households}} \\ &\quad + \underbrace{\mathbb{O}\begin{bmatrix}\Delta\widehat{y}_{t+1} - \frac{\alpha}{1-2\alpha}\Delta\widehat{q}_{t+1}\\ \Delta\widehat{y}^*_{t+1} + \frac{\alpha}{1-2\alpha}\Delta\widehat{q}_{t+1}\end{bmatrix}}_{\text{consumption growth of HtM households}},\end{aligned} \tag{C.9}$$

where

$$\mathbb{O} = \begin{bmatrix}\mathcal{O} & 0\\ 0 & \mathcal{O}^*\end{bmatrix}$$

To transform this into output growth terms, we can substitute (C.9) in to (C.3) to get:

$$\begin{aligned}\begin{bmatrix}\Delta\widehat{y}_{t+1}\\ \Delta\widehat{y}^*_{t+1}\end{bmatrix} &= \underbrace{\mathbb{X}\,(\mathbb{I}-\mathbb{O})\begin{bmatrix}\widehat{r}_t\\ \widehat{r}^*_t\end{bmatrix}}_{\substack{\text{intertemporal}\\ \text{substitution}}} + \underbrace{\begin{bmatrix}\frac{2(1-\alpha)\alpha}{1-2\alpha}\eta\\ -\frac{2(1-\alpha)\alpha}{1-2\alpha}\eta\end{bmatrix}\Delta\widehat{q}_{t+1}}_{\text{expenditure switching}} + \underbrace{\mathbb{X}\,(\mathbb{I}-\mathbb{O})\,\varphi\sigma_c^2\begin{bmatrix}\widehat{\sigma}_{c,t+1}\\ \widehat{\sigma}^*_{c,t+1}\end{bmatrix}}_{\text{precautionary savings}} \\ &\quad \underbrace{+\,\mathbb{X}\mathbb{O}\begin{bmatrix}-\frac{\alpha}{1-2\alpha}\\ \frac{\alpha}{1-2\alpha}\end{bmatrix}\Delta\widehat{q}_{t+1}}_{\text{real-income channel}} \underbrace{+\,\mathbb{X}\mathbb{O}\begin{bmatrix}\Delta\widehat{y}_{t+1}\\ \Delta\widehat{y}^*_{t+1}\end{bmatrix}}_{\substack{\text{Keynesian}\\ \text{multiplier}}} + \underbrace{\frac{(1-\vartheta)}{\vartheta}\mathbb{X}\,(\mathbb{I}-\mathbb{O})\begin{bmatrix}-\left(1-\widetilde{\beta}\right)\\ \left(1-\widetilde{\beta}\right)\end{bmatrix}\widehat{a}_{t+1}}_{\text{Blanchard-Yaari correction}},\end{aligned} \tag{C.10}$$

which is the same as (23) in the main text.

C.5 Output returns to its original steady state value with $\vartheta = 1$

Setting $\vartheta = 1$, (23) can be simplified to:

$$\begin{aligned}\begin{bmatrix}\Delta\widehat{y}_{t+1}\\ \Delta\widehat{y}^*_{t+1}\end{bmatrix} &= \underbrace{\mathbb{X}\,(\mathbb{I}-\mathbb{O})\begin{bmatrix}\widehat{r}_t\\ \widehat{r}^*_t\end{bmatrix}}_{\substack{\text{intertemporal}\\ \text{substitution}}} + \underbrace{\begin{bmatrix}\frac{2(1-\alpha)\alpha}{1-2\alpha}\eta\\ -\frac{2(1-\alpha)\alpha}{1-2\alpha}\eta\end{bmatrix}\Delta\widehat{q}_{t+1}}_{\text{expenditure switching}} + \underbrace{\mathbb{X}\,(\mathbb{I}-\mathbb{O})\,\varphi\sigma_c^2\begin{bmatrix}\widehat{\sigma}_{c,t+1}\\ \widehat{\sigma}^*_{c,t+1}\end{bmatrix}}_{\text{precautionary savings}} \\ &\quad \underbrace{+\,\mathbb{X}\mathbb{O}\begin{bmatrix}-\frac{\alpha}{1-2\alpha}\\ \frac{\alpha}{1-2\alpha}\end{bmatrix}\Delta\widehat{q}_{t+1}}_{\text{real-income channel}} \underbrace{+\,\mathbb{X}\mathbb{O}\begin{bmatrix}\Delta\widehat{y}_{t+1}\\ \Delta\widehat{y}^*_{t+1}\end{bmatrix}}_{\substack{\text{Keynesian}\\ \text{multiplier}}}\end{aligned}$$

By definition, in any steady state, since there are no monetary policy shocks, we have $\widehat{r}^*_{F,t} = 0$. In addition, since in any steady state, all macroeconomic variables are constant, i.e., $\widehat{y}_t, \widehat{y}^*_t, \widehat{q}_t, \widehat{\sigma}_{c,t}$ and $\widehat{\sigma}^*_{c,t}$ converge to some constant values. Together with $\widehat{r}^*_{F,t} = 0$, $\Delta\widehat{q}_t = 0$ imply that $\widehat{r}_t = \widehat{r}^*_t = 0$ in steady state. It is clear from inspection that the equation above is satisfied in steady state. However while the argument above guarantees that it does not $\widehat{y}_t$ and $\widehat{y}^*_t$ are constant in steady state, it does not guarantee that $\widehat{y}_t = \widehat{y}^*_t = 0$ in steady state. To see that this must also be the case, recall that consumption risk in Home and Foreign can be written as

$$\widehat{\sigma}_{c,t} = (1-\lambda)\,\widehat{\mu}_t - (1-\widetilde{\beta}\lambda)\Theta\widehat{y}_t + \widetilde{\beta}\lambda\widehat{\sigma}_{c,t+1} \qquad \text{and} \qquad \widehat{\mu}_t = \widetilde{\beta}\,(\widehat{r}_t + \widehat{\mu}_{t+1})$$
$$\widehat{\sigma}^*_{c,t} = (1-\lambda)\,\widehat{\mu}^*_t - (1-\widetilde{\beta}\lambda)\Theta^*\widehat{y}^*_t + \widetilde{\beta}\lambda\widehat{\sigma}^*_{c,t+1} \qquad \text{and} \qquad \widehat{\mu}_t = \widetilde{\beta}\,(\widehat{r}^*_t + \widehat{\mu}^*_{t+1})$$

Since $\widehat{r}_t = \widehat{r}^*_t = 0$ in steady state, the expression above imply that $\widehat{\mu}_t = \widehat{\mu}^*_t = 0$ in steady state. Then, as long as income risk is not acyclical $\Theta, \Theta^* \neq 0$, it must be that $\widehat{y}_t = \widehat{y}^*_t = 0$ in steady state. □

D Analytical results in RANK

In RANK, we have $\mathcal{O}, \mathcal{O}^* = 0$ and $\sigma_c^2 = 0$. Imposing this along with $\vartheta = 0$, we can simplify (23) to:

$$\begin{bmatrix} \Delta\widehat{y}_{t+1} \\ \Delta\widehat{y}^*_{t+1} \end{bmatrix} = \underbrace{\mathbb{X}\begin{bmatrix} \alpha \\ 1-\alpha \end{bmatrix}\widehat{r}^*_{F,t}}_{\substack{\text{intertemporal} \\ \text{substitution}}} + \underbrace{\begin{bmatrix} -2\,(1-\alpha)\,\alpha\eta \\ 2\,(1-\alpha)\,\alpha\eta \end{bmatrix}\widehat{r}^*_{F,t}}_{\substack{\text{expenditure-} \\ \text{switching}}} \tag{D.1}$$

Next, as shown in Appendix C.5, in the limit with $\vartheta = 1$, since output in both economies returns to their respective levels in the initial steady state, the first-order equilibrium response of Home and Foreign output can be written as linear functions of the Foreign monetary policy shock:

$$\widehat{y}_t = \mathbf{a}_y \times \widehat{r}^*_{F,t} \qquad \text{and} \qquad \widehat{y}^*_t = \mathbf{a}^*_y \times \widehat{r}^*_{F,t},$$

where $\mathbf{a}_y = d\widehat{y}_t / d\widehat{r}^*_{F,t}$ and $\mathbf{a}^*_y = d\widehat{y}^*_t / d\widehat{r}^*_{F,t}$. Using this and the fact that $\widehat{r}^*_{F,t+s} = \rho^s \widehat{r}^*_{F,t}$, we can rewrite (D.1) as:

$$\begin{bmatrix} \mathbf{a}_y \\ \mathbf{a}^*_y \end{bmatrix} = \begin{bmatrix} \frac{2\alpha(1-\alpha)}{1-\rho}\,(\eta - 1) \\ -\frac{1}{1-\rho} - \frac{2\alpha(1-\alpha)}{1-\rho}\,(\eta-1) \end{bmatrix}, \tag{D.2}$$

which are the same expressions as in Proposition 3.

E Analytical results with real-income channel

E.1 Demand for Foreign goods by Home and Foreign HtMs

The main text depicts how incorporating the real-income channel affects the demand for Home goods by Home and Foreign HtMs. To save space, we present expressions here which show the real-income channel affects the demand for Foreign goods by Home and Foreign HtMs. In particular, the demand for

Foreign goods by Home and Foreign HtMs, up to first-order, can be written as:

$$\frac{\partial \widehat{c}_{F,\text{htm},t}}{\partial \widehat{q}_t} = -\underbrace{\frac{\alpha^2}{1-2\alpha}}_{\substack{\text{real income}\\ \text{channel}}} - \underbrace{\frac{\alpha(1-\alpha)}{1-2\alpha}\eta}_{\substack{\text{expenditure}\\ \text{switching}}} < 0 \qquad \text{for} \qquad \eta > 0 \tag{E.1a}$$

$$\frac{\partial \widehat{c}^*_{F,\text{htm},t}}{\partial \widehat{q}_t} = \underbrace{\frac{\alpha(1-\alpha)}{1-2\alpha}}_{\substack{\text{real income}\\ \text{channel}}} - \underbrace{\frac{\alpha(1-\alpha)\eta}{1-2\alpha}}_{\substack{\text{expenditure}\\ \text{switching}}} \lesseqgtr 0 \qquad \Leftrightarrow \qquad 1 \lesseqgtr \eta \tag{E.1b}$$

As mentioned in the main text, following a RER depreciation ($\widehat{q}_t > 0$), the demand for Foreign goods by Home HtMs unambiguously declines for any $\eta > 0$. This is because the depreciation lowers their real income, which works to lower their demand for both Home and Foreign goods. In addition the depreciation renders Foreign goods relatively more expensive, which causes Home HtMs to switch their demand away from Foreign goods to Home goods. Thus, both the real-income channel and expenditure-switching channel work towards reducing the Home HtM demand for Foreign goods. In contrast the RER depreciation increases the real-income of Foreign HtMs, which causes them to raise their demand for both Home and Foreign goods. But since Foreign goods are now relatively more expensive, even Foreign HtMs reduce their demand for these via the expenditure-switching channel, and the net effect on the demand for Foreign goods by Foreign HtMs depends on the magnitude of η relative to 1.

E.2 General equilibrium effects incorporating the real-income channel

In the case with $\mathcal{O}, \mathcal{O}^* \neq 0$ but no risk $\sigma_c^2 = 0$ and $\vartheta = 0$, (23) can be simplified to:

$$\begin{bmatrix} \Delta\widehat{y}_{t+1} \\ \Delta\widehat{y}^*_{t+1} \end{bmatrix} = \underbrace{\mathbb{X}\left(\mathbb{I}-\mathbb{O}\right)\begin{bmatrix} \alpha \\ 1-\alpha \end{bmatrix}\widehat{r}^*_{F,t}}_{\substack{\text{intertemporal}\\ \text{substitution}}} + \underbrace{\begin{bmatrix} -2\left(1-\alpha\right)\alpha\eta \\ 2\left(1-\alpha\right)\alpha\eta \end{bmatrix}\widehat{r}^*_{F,t}}_{\substack{\text{expenditure-}\\ \text{switching}}} + \underbrace{\mathbb{X}\mathbb{O}\begin{bmatrix} \alpha \\ -\alpha \end{bmatrix}\widehat{r}^*_{F,t}}_{\substack{\text{real-income}\\ \text{channel}}} + \underbrace{\mathbb{X}\mathbb{O}\begin{bmatrix} \Delta\widehat{y}_{t+1} \\ \Delta\widehat{y}^*_{t+1} \end{bmatrix}}_{\text{HtM multiplier}} \tag{E.2}$$

Subtracting the last term on the RHS from both sides, we can rewrite the above as:

$$\left(\mathbb{I}-\mathbb{X}\mathbb{O}\right)\begin{bmatrix} \Delta\widehat{y}_{t+1} \\ \Delta\widehat{y}^*_{t+1} \end{bmatrix} = \begin{bmatrix} 2\alpha\left(1-\alpha\right)\left(1-\eta\right)-\alpha\mathcal{O}^* \\ 1-2\alpha\left(1-\alpha\right)+2\left(1-\alpha\right)\alpha\eta-\left(1-\alpha\right)\mathcal{O}^* \end{bmatrix}\widehat{r}^*_{F,t}$$

Then, since output in Home and Foreign is stationary in the $\vartheta = 1$ limit, we know that in equilibrium, it must be that $\widehat{y}_t = \mathbf{a}_y \widehat{r}^*_{F,t}$ and $\widehat{y}^*_t = \mathbf{a}^*_y \widehat{r}^*_{F,t}$, where $\mathbf{a}_y \equiv d\widehat{y}_t / d\widehat{r}^*_{F,t}$ and $\mathbf{a}^*_y \equiv d\widehat{y}^*_t / d\widehat{r}^*_{F,t}$. Using this in the expression above, we can rewrite it as:

$$\begin{bmatrix} \mathbf{a}_y \\ \mathbf{a}^*_y \end{bmatrix} = \frac{1}{\Gamma}\begin{bmatrix} 1-\left(1-\alpha\right)\mathcal{O}^* & \alpha\mathcal{O}^* \\ \alpha\mathcal{O} & 1-\left(1-\alpha\right)\mathcal{O} \end{bmatrix}\begin{bmatrix} \frac{2\alpha(1-\alpha)}{1-\rho}\left(\eta-1\right)+\frac{\alpha\mathcal{O}^*}{1-\rho} \\ -\frac{1}{1-\rho}-\frac{2\alpha(1-\alpha)}{1-\rho}\left(\eta-1\right)+\frac{(1-\alpha)\mathcal{O}^*}{1-\rho}, \end{bmatrix} \tag{E.3}$$

where

$$\Gamma = 1-\left(1-\alpha\right)\left(\mathcal{O}+\mathcal{O}^*\right)+\left(1-2\alpha\right)\mathcal{O}\mathcal{O}^* > 0 \tag{E.4}$$

Unpacking (E.3), we can write:

$$\frac{d\widehat{y}_t}{d\widehat{r}^*_{F,t}} = \mathbb{M}(\mathcal{O},\mathcal{O}^*) \times \frac{2(1-\alpha)\alpha}{1-\rho}(\eta-1) \tag{E.5}$$

$$\frac{d\widehat{y}^*_t}{d\widehat{r}^*_{F,t}} = -\frac{1}{1-\rho} - \left(\frac{1-\mathcal{O}}{1-\mathcal{O}^*}\right)\mathbb{M}(\mathcal{O}^*,\mathcal{O}) \times \frac{2\alpha(1-\alpha)}{1-\rho}(\eta-1), \tag{E.6}$$

where

$$\mathbb{M}(x,y) = \frac{1-y}{1-(1-\alpha)(x+y)+(1-2\alpha)xy}$$

These are the same expressions as those presented in Proposition 4. Also, it is clear by observation that for any $x \in (0,1)$, the following properties of $\mathbb{M}$ are true:

$$\mathbb{M}(x,x) = \frac{1}{1-(1-2\alpha)x} > 1,$$

$$\mathbb{M}_1(x,x) = \frac{\alpha+(1-2\alpha)(1-x)}{(1-x)[1-(1-2\alpha)x]^2} > 0,$$

and

$$\mathbb{M}_2(x,x) = \frac{-\alpha(1-x)}{(1-x)[1-(1-2\alpha)x]^2} < 0$$

F Analytical results with precautionary-savings channel

In the case with $\vartheta = 1$ and $\mathcal{O} = \mathcal{O}^* = 0$, equation (23) can be specialized to

$$\begin{bmatrix}\Delta\widehat{y}_{t+1}\\ \Delta\widehat{y}^*_{t+1}\end{bmatrix} = \underbrace{\mathbb{X}\begin{bmatrix}\widehat{r}_t\\ \widehat{r}^*_t\end{bmatrix}}_{\substack{\text{intertemporal}\\ \text{substitution}}} + \underbrace{\begin{bmatrix}\frac{2(1-\alpha)\alpha}{1-2\alpha}\eta\\ -\frac{2(1-\alpha)\alpha}{1-2\alpha}\eta\end{bmatrix}\Delta\widehat{q}_{t+1}}_{\text{expenditure switching}} + \underbrace{\mathbb{X}\varphi\sigma_c^2\begin{bmatrix}\widehat{\sigma}_{c,t+1}\\ \widehat{\sigma}^*_{c,t+1}\end{bmatrix}}_{\text{precautionary savings}} \tag{F.1}$$

Next, using equations (19a) and (19b), we have

$$\widehat{\mu}_t = \widetilde{\beta}\sum_{s=0}^{\infty}\widetilde{\beta}^s\widehat{r}_{t+s} = \alpha\widetilde{\beta}\sum_{s=0}^{\infty}\widetilde{\beta}^s\widehat{r}^*_{F,t+s} = \frac{\alpha\widetilde{\beta}}{1-\widetilde{\beta}\rho}\widehat{r}^*_{F,t}$$

$$\widehat{\mu}^*_t = \widetilde{\beta}\sum_{s=0}^{\infty}\widetilde{\beta}^s\widehat{r}^*_{t+s} = (1-\alpha)\widetilde{\beta}\sum_{s=0}^{\infty}\widetilde{\beta}^s\widehat{r}^*_{F,t+s} = \frac{(1-\alpha)\widetilde{\beta}}{1-\widetilde{\beta}\rho}\widehat{r}^*_{F,t}$$

Consequently, using these expressions and solving equations (19a) and (19b) forwards to get date t consumption risk in Home and Foreign, we get:

$$\widehat{\sigma}_{c,t+1} = \frac{\alpha\widetilde{\beta}}{1-\widetilde{\beta}\rho}\left(\frac{1-\lambda}{1-\widetilde{\beta}\lambda\rho}\right)\widehat{r}^*_{F,t+1} - \left(1-\widetilde{\beta}\lambda\right)\Theta\sum_{s=0}^{\infty}(\widetilde{\beta}\lambda)^s\widehat{y}_{t+1+s} \tag{F.2}$$

$$\widehat{\sigma}^*_{c,t+1} = \frac{(1-\alpha)\widetilde{\beta}}{1-\widetilde{\beta}\rho}\left(\frac{1-\lambda}{1-\widetilde{\beta}\lambda\rho}\right)\widehat{r}^*_{F,t+1} - \left(1-\widetilde{\beta}\lambda\right)\Theta^*\sum_{s=0}^{\infty}(\widetilde{\beta}\lambda)^s\widehat{y}^*_{t+1+s} \tag{F.3}$$

Next, as shown in Appendix C.5, in the limit with $\vartheta = 1$, since output in both economies returns to their respective levels in the initial steady state, the first-order equilibrium response of Home and Foreign output can be written as linear functions of the Foreign monetary policy shock:

$$\widehat{y}_t = \mathbf{a}_y \times \widehat{r}^*_{F,t} \qquad \text{and} \qquad \widehat{y}^*_t = \mathbf{a}^*_y \times \widehat{r}^*_{F,t}$$

Using this and the fact that $\widehat{r}^*_{F,t+s} = \rho^s \widehat{r}^*_{F,t}$, we can rewrite (F.2) and (F.3) as:

$$\widehat{\sigma}_{c,t+1} = \rho \left[\frac{\alpha\widetilde{\beta}}{1-\widetilde{\beta}\rho} \left(\frac{1-\lambda}{1-\widetilde{\beta}\lambda\rho} \right) - \left(\frac{1-\widetilde{\beta}\lambda}{1-\widetilde{\beta}\lambda\rho} \right) \Theta \mathbf{a}_y \right] \widehat{r}^*_{F,t} \tag{F.4}$$

$$\widehat{\sigma}^*_{c,t+1} = \rho \left[\frac{(1-\alpha)\widetilde{\beta}}{1-\widetilde{\beta}\rho} \left(\frac{1-\lambda}{1-\widetilde{\beta}\lambda\rho} \right) - \left(\frac{1-\widetilde{\beta}\lambda}{1-\widetilde{\beta}\lambda\rho} \right) \Theta^* \mathbf{a}^*_y \right] \widehat{r}^*_{F,t} \tag{F.5}$$

Plugging (F.4) and (F.5) back into the IS equations (F.1) yields:

$$[1-(1-\alpha)\theta]\,\mathbf{a}_y - \alpha\theta^*\mathbf{a}^*_y = \frac{2\alpha(1-\alpha)}{1-\rho}(\eta - 1 - \gamma) \tag{F.6}$$

$$-\alpha\theta\mathbf{a}_y + [1-(1-\alpha)\theta^*]\,\mathbf{a}^*_y = -\frac{1+\gamma}{1-\rho} - \frac{2\alpha(1-\alpha)}{1-\rho}(\eta - 1 - \gamma), \tag{F.7}$$

where $\theta = \Theta/\overline{\Theta}, \theta^* = \Theta^*/\overline{\Theta}$ and

$$\overline{\Theta} = \left[\frac{\rho}{1-\rho} \varphi\sigma_c^2 \left(\frac{1-\widetilde{\beta}\lambda}{1-\widetilde{\beta}\lambda\rho} \right) \right]^{-1} \tag{F.8}$$

$$\gamma = \varphi\sigma_c^2 \frac{\widetilde{\beta}\rho}{1-\widetilde{\beta}\rho} \left(\frac{1-\lambda}{1-\widetilde{\beta}\lambda\rho} \right) \tag{F.9}$$

Solving (F.6) and (F.7) for $\mathbf{a}_y, \mathbf{a}^*_y$ yields

$$\mathbf{a}_y = \mathbb{M}(\theta,\theta^*) \times \frac{2\alpha(1-\alpha)}{1-\rho}(\eta - (1+\gamma)\Omega) \tag{F.10}$$

$$\mathbf{a}^*_y = -\frac{1}{1-\theta^*}\frac{1+\gamma}{1-\rho} - \mathbb{M}(\theta^*,\theta) \times \frac{2\alpha(1-\alpha)}{1-\rho}(\eta - (1+\gamma)\Omega) \tag{F.11}$$

where

$$\mathbb{M}(x,y) = \frac{1-y}{1-(1-\alpha)(x+y)+(1-2\alpha)xy} > 0 \qquad \forall (x,y) \in [0,1)\times[0,1)$$

$$\Omega(\theta^*) = 1 + \frac{\theta^*}{2(1-\alpha)(1-\theta^*)} \geq 1 \qquad \forall \theta^* \in [0,1),$$

which are the same as the expressions in Proposition 7 in the main text.

Finally, to see that a high enough θ^* results in the sign of spillovers to flip relative to RANK, notice that for $\theta^* = 0$, $\Omega(0) = 1$, $\Omega(1) = \infty$ and $\frac{\partial\Omega}{\partial\theta^*} = \frac{2-\theta}{2(1-\alpha)(1-\theta)^2} > 0$ for all $\theta^* \in [0,1]$. Thus, since $\Omega(\theta^*)$ is continuous and strictly increasing $[0,1]$ and has range $[1,\infty)$, there always exists a large enough $\theta^* = \widetilde{\theta}^*$ such that $(1+\gamma)\Omega(\widetilde{\theta}^*) = \eta$, and for all $\theta^* \in (\widetilde{\theta}^*, 1)$, $(1+\gamma)\Omega(\widetilde{\theta}^*) > \eta$ for any given η. Consequently, given $\eta > 1$, a high enough θ^* implies that the income risk channel flips the sign of the spillovers relative to RANK, if the self-insurance channel is too weak to do so. □

F.1 Acyclical consumption risk

In the case with acyclical consumption risk $\theta = \theta^* = 0$ and $\lambda = 1$. In this case (F.10) and (F.11) simplify to

$$\mathbf{a}_y = \frac{2\alpha(1-\alpha)}{1-\rho}(\eta - 1) \tag{F.12}$$

$$\mathbf{a}_y^* = -\frac{1}{1-\rho} - \frac{2\alpha(1-\alpha)}{1-\rho}(\eta-1), \tag{F.13}$$

which are the same as the expressions in Proposition 3 in the main text.

F.2 Only self-insurance channel, no cyclical risk

In the case with acyclical income risk $\theta = \theta^* = 0$, (F.10) and (F.11) simplify to

$$\mathbf{a}_y = \frac{2\alpha(1-\alpha)}{1-\rho}(\eta - \gamma) \tag{F.14}$$

$$\mathbf{a}_y^* = -\frac{\gamma}{1-\rho} - \frac{2\alpha(1-\alpha)}{1-\rho}(\eta-\gamma), \tag{F.15}$$

which are the same expression as in Proposition 6 in the main text.

G Dominant Currency Pricing

Introducing DCP does not change the household's consumption decision rules relative to the baseline model. As mentioned in the main text, what does change is the relationship between the Foreign price of Home goods, which is now given by:

$$\frac{P^*_{H,t}}{P^*_t} = \left(\frac{P_{H,t}}{P_t}\frac{1}{\mathcal{Q}_t}\right)^{\iota}$$

where $\iota \in [0,1)$; $\iota = 1$ corresponds to the baseline case. Then, as in the baseline, using the definitions of the CPI price indices in the two countries along with the definition of the RER, we have:

$$1 = \alpha\Big(p_F^*(\mathcal{Q}_t)\Big)^{1-\eta}\mathcal{Q}_t^{1-\eta} + (1-\alpha)\Big(p_H(\mathcal{Q}_t)\Big)^{1-\eta}$$

$$\mathcal{Q}_t^{\iota(1-\eta)} = \alpha\Big(p_H(\mathcal{Q}_t)\Big)^{\iota(1-\eta)} + (1-\alpha)\Big(p_F^*(\mathcal{Q}_t)\Big)^{1-\eta}\mathcal{Q}_t^{(1-\iota)(1-\eta)},$$

where

$$p_F^*(\mathcal{Q}_t) = \frac{P^*_{F,t}}{P^*_t} \qquad \text{and} \qquad p_H(\mathcal{Q}_t) = \frac{P_{H,t}}{P_t}$$

Up to first-order this implies that the passthrough from changes in the RER $\widehat{q}_t$ into Foreign prices of Home goods are smaller than in the baseline model:

$$\widehat{p}_{H,t} = -\frac{\alpha\left[1-\alpha\left(1-\iota\right)\right]}{1-2\alpha+\alpha^2\left(1-\iota\right)}\widehat{q}_t \tag{G.1}$$

$$\widehat{p}^*_{F,t} = \frac{\alpha\iota}{1-2\alpha-\alpha^2\left(\iota-1\right)}\widehat{q}_t \tag{G.2}$$

Since the definitions of $p_H(\mathcal{Q}_t)$ and $p^*_F(\mathcal{Q}_t)$ are different in the DCP limit, the relation between CPI and PPI based real interest rates is also altered. Up to first-order, for a given level of Home and Foreign PPI bsaed real interest rates $\widehat{r}_{H,t}$ and $\widehat{r}^*_{F,t}$, the interest parity condition in PPI terms can now be written as:

$$\widehat{r}_{H,t} - \widehat{r}^*_{F,t} = \frac{1-\alpha\left(1-\iota\right)}{1-2\alpha-\alpha^2\left(\iota-1\right)}\Delta\widehat{q}_{t+1},$$

Consequently, the CPI based real interest rates at Home and Foreign are given by:

$$\widehat{r}_t = \widehat{r}_{H,t} - \frac{\alpha\left[1-\alpha\left(1-\iota\right)\right]}{1-2\alpha+\alpha^2\left(1-\iota\right)}\Delta\widehat{q}_{t+1} \tag{G.3}$$

$$\widehat{r}^*_t = \widehat{r}^*_{F,t} + \frac{\alpha\iota}{1-2\alpha+\alpha^2\left(1-\iota\right)}\Delta\widehat{q}_{t+1}, \tag{G.4}$$

In all our experiments, we hold $\widehat{r}_{H,t} = 0$, so the interest parity condition can be simplified to:

$$\Delta\widehat{q}_{t+1} = -\frac{1-2\alpha+\alpha^2\left(1-\iota\right)}{1-\alpha\left(1-\iota\right)}\widehat{r}^*_{F,t}, \tag{G.5}$$

and the CPI based real interest rates in the two countries are given by:

$$\widehat{r}_t = \alpha\widehat{r}^*_{F,t} \qquad \text{and} \qquad \widehat{r}^*_t = \frac{1-\alpha}{1-\alpha\left(1-\iota\right)}\widehat{r}^*_{F,t} \tag{G.6}$$

The expression show that, as in the baseline, a Foreign monetary tightening $\widehat{r}^*_{F,t} > 0$ results in a higher CPI based real interest rate in Home and Foreign. However, the Foreign CPI based real interest rate increase by more, the smaller the exchange rate passthrough (ERPT) ι; e.g., for $\iota = 0$, an increase in $r^*_{F,t}$ translates into a one-for-one increase in the CPI based real interest rate. In contrast, with full ERPT, $\iota = 1$, $\widehat{r}^*_t = (1-\alpha)\widehat{r}^*_{F,t}$, which implies an increase which is less than one-for-one. This can be explained by the fact that ι controls how much cheaper Home goods are for Foreign households after a RER depreciation. When ι is high, the relative price of Home goods for Foreign households falls following a depreciation (and are expected to be higher tomorrow since the depreciation is temporary). Consequently, this profile of the Foreign price of Home goods offsets some of the effects of monetary tightening. In contrast, when ι is small, say when $\iota = 0$, the Foreign price of Home goods is unaffected by changes in the RER and there is no force offsetting the monetary tightening.

Then, the aggregate Euler equations for Home and Foreign in the limit with $\vartheta = 1$, under DCP, up to first-order can be written as:

$$\Delta\widehat{c}_{t+1} = (1-\mho)\left\{\alpha\widehat{r}^{*}_{F,t} + \varphi\sigma^2_c\widehat{\sigma}_{c,t+1}\right\} + \mho\left\{\Delta\widehat{y}_{t+1} - \frac{\alpha\left[1-\alpha\left(1-\iota\right)\right]}{1-2\alpha+\alpha^2\left(1-\iota\right)}\Delta\widehat{q}_{t+1}\right\} \quad \text{(G.7a)}$$

$$\Delta\widehat{c}^{*}_{t+1} = (1-\mho^{*})\left\{\frac{1-\alpha}{1-\alpha\left(1-\iota\right)}\widehat{r}^{*}_{F,t} + \varphi\sigma^2_c\widehat{\sigma}^{*}_{c,t+1}\right\} + \mho^{*}\left\{\Delta\widehat{y}^{*}_{t+1} + \frac{\alpha\iota}{1-2\alpha-\alpha^2\left(\iota-1\right)}\Delta\widehat{q}_{t+1},\right\} \quad \text{(G.7b)}$$

where we have used (G.6) to substitute out the CPI based real interest rates in terms of the Foreign monetary policy rate $\widehat{r}^{*}_{F,t}$, and also used (G.1)-(G.2) in accounting for changes in the real value of HtM income in Home and Foreign.

Finally, log-linearizing the market clearing conditions (6a)-(6b) and using (G.1)-(G.2), we can express the market clearing conditions in terms of the RER, up to first-order, as:

$$\widehat{y}_t = (1-\alpha)\,\widehat{c}_t + \alpha\widehat{c}^{*}_t + \alpha\left(1-\alpha\right)\left[\frac{2-\left(1+\alpha\right)\left(1-\iota\right)}{1-2\alpha+\alpha^2\left(1-\iota\right)}\right]\eta\widehat{q}_t \quad \text{(G.8)}$$

$$\widehat{y}^{*}_t = \alpha\widehat{c}_t + (1-\alpha)\,\widehat{c}^{*}_t - \alpha\left(1-\alpha\right)\left[\frac{2-\left(1+\alpha\right)\left(1-\iota\right)}{1-2\alpha+\alpha^2\left(1-\iota\right)}\right]\eta\widehat{q}_t \quad \text{(G.9)}$$

These expressions highlight that in DCP, for $\iota < 1$, changes in the RER have a smaller effect on $\widehat{y}_t$ and $\widehat{y}^{*}_t$ than with $\iota = 1$, holding all else constant.

G.1 Perfect-insurance benchmark

In the prefect-insurance benchmark $\mho = \mho^{*} = 0$ and $\sigma^2_c = 0$, combining (G.7a), (G.7b), (G.8) and (G.9) yields the Home and Foreign IS curves:

$$\Delta\widehat{y}_{t+1} = -\alpha\left(1-\alpha\right)\frac{2-\alpha\left(1-\iota\right)}{1-\alpha\left(1-\iota\right)}\Big(\delta(\iota)\eta-1\Big)\widehat{r}^{*}_{F,t} \quad \text{(G.10)}$$

$$\Delta\widehat{y}^{*}_t = \left[1+\alpha\left(1-\alpha\right)\frac{2-\left(1+\alpha\right)\left(1+\iota\right)}{1-\alpha\left(1-\iota\right)}\left(\eta-1\right)\right]\widehat{r}^{*}_{F,t}, \quad \text{(G.11)}$$

where

$$\delta(\iota) = 1 - \frac{1-\iota}{2-\alpha\left(1-\iota\right)} \quad \text{(G.12)}$$

Since, in the $\vartheta = 1$ limit, Home and Foreign output are simply linear functions of $\widehat{r}^{*}_{F,t}$, it is clear by inspection that the following solve (G.10)-(G.11):

$$\frac{d\widehat{y}_t}{d\widehat{r}^{*}_{F,t}} = \chi_d\Big(\delta(\iota)\eta-1\Big) \quad \text{(G.13)}$$

$$\frac{d\widehat{y}^{*}_t}{d\widehat{r}^{*}_{F,t}} = -\frac{1}{1-\rho} - \chi_d\delta(\iota)\left(\eta-1\right), \quad \text{(G.14)}$$

where

$$\chi_d = \frac{\alpha\left(1-\alpha\right)}{1-\rho}\frac{2-\alpha\left(1-\iota\right)}{1-\alpha\left(1-\iota\right)} > 0 \quad \text{(G.15)}$$

G.2 Real-income channel

Specializing (G.7a)-(G.7b) to the case with $\mathcal{O}, \mathcal{O}^* > 0$ and no idiosyncratic risk yields:

$$\Delta \widehat{c}_{t+1} = (1-\mathcal{O})\,\alpha \widehat{r}^*_{F,t} + \mathcal{O}\left(\Delta \widehat{y}_{t+1} + \alpha \widehat{r}^*_{F,t}\right) \tag{G.16}$$

$$\Delta \widehat{c}^*_{t+1} = \frac{(1-\alpha)(1-\mathcal{O}^*)}{1-\alpha(1-\iota)}\widehat{r}^*_{F,t} + \mathcal{O}^*\left(\Delta \widehat{y}^*_{t+1} - \frac{\alpha\iota}{1-\alpha(1-\iota)}\widehat{r}^*_{F,t}\right) \tag{G.17}$$

Combining these with the market clearing conditions (G.8)-(G.9), we can derive the Home and Foreign IS equations, which can be written in matrix form as:

$$\begin{bmatrix} 1-(1-\alpha)\mathcal{O} & -\alpha\mathcal{O}^* \\ -\alpha\mathcal{O} & 1-(1-\alpha)\mathcal{O}^* \end{bmatrix} \begin{bmatrix} \Delta \widehat{y}_{t+1} \\ \Delta \widehat{y}^*_{t+1} \end{bmatrix} = \begin{bmatrix} -\chi_d\left(1-\delta(\iota)\eta\right) + \frac{\alpha\mathcal{O}^*}{1-\rho} \\ -\frac{1}{1-\rho} - \chi^*_d(\eta-1) + \frac{(1-\alpha)\mathcal{O}^*}{1-\rho} \end{bmatrix},$$

where $\delta(\iota)$ and χ_d are defined in (G.12) and (G.15) respectively. Then, following the same steps as in Appendix E.2, the equilibrium spillover can be written as:

$$\frac{d\widehat{y}_t}{d\widehat{r}^*_{F,t}} = \mathbb{M}(\mathcal{O}, \mathcal{O}^*)\,\chi_d\left(\delta\eta - \digamma\right), \tag{G.18}$$

where

$$\digamma = \delta + (1-\delta)\left(1 + \frac{\alpha\mathcal{O}^*}{1-\mathcal{O}^*}\right) \in \left[1, 1 + \frac{\alpha\mathcal{O}^*}{1-\mathcal{O}^*}\right]$$

These are the same expressions as those presented in Proposition 9. We want to find the conditions which cause the real-income channel to flip the sign relative to RANK. When $\delta\eta < 1$, then RANK already has negative spillovers and so does the economy with HtM households. So we know that for $\iota < \underline{\iota}$, where $\bar{\iota}$ is implicitly defined by the condition $\delta(\underline{\iota}) = \eta^{-1}$, both RANK and the real income channel result in negative spillovers. Next, for the real income channel to result in positive spillovers, we need $\iota > \bar{\iota}$, where $\bar{\iota}$ is defined by the condition

$$\delta(\bar{\iota}) = \eta^{-1}\digamma(\bar{\iota})$$

Since (G.12) shows that δ is an increasing function of ι, and $\digamma$ is a decreasing function of ι, it follows that $\bar{\iota} > \underline{\iota}$. Thus, in the interval $\underline{\iota} \leq \iota \leq \bar{\iota}$, the real income channel can flip the sign relative to RANK in the DCP regime.

G.3 Precautionary-savings channel

In order to characterize the equilibrium spillover due to the precautionary-savings channel in a DCP regime, we first have to characterize how consumption risk in Home and Foreign responds to a Foreign tightening under DCP. To see this, we start with how the Foreign tightening affects the passthrough from income to consumption, $(\widehat{\mu}_t, \widehat{\mu}^*_t)$ is affected by the introduction of DCP. As in the baseline model, $\widehat{\mu}_t$ and $\widehat{\mu}^*_t$ can be written as the discounted sum of expected CPI based real interest rates in Home and Foreign respectively: $\widehat{\mu}_t = \widetilde{\beta}\sum_{s=0}^{\infty}\widetilde{\beta}^s \widehat{r}_{t+s}$, $\widehat{\mu}^*_t = \widetilde{\beta}\sum_{s=0}^{\infty}\widetilde{\beta}^s \widehat{r}^*_{t+s}$. Using, (G.6), this can be written in terms of the

Foreign monetary policy shock:

$$\widehat{\mu}_t = \widetilde{\beta}\sum_{s=0}^{\infty}\alpha\widetilde{\beta}^s\widehat{r}^*_{F,t+s} = \frac{\alpha\widetilde{\beta}}{1-\widetilde{\beta}\rho}\widehat{r}^*_{F,t} \tag{G.19}$$

$$\widehat{\mu}^*_t = \frac{\widetilde{\beta}\left(1-\alpha\right)}{1-\alpha\left(1-\iota\right)}\frac{1}{1-\widetilde{\beta}\rho}\widehat{r}^*_{F,t} \tag{G.20}$$

Next, as shown in Appendix C.5, in the limit with $\vartheta = 1$, since output in both economies returns to their respective levels in the initial steady state, the first-order equilibrium response of Home and Foreign output can be written as linear functions of the Foreign monetary policy shock. Following similar steps as in Appendix F, we can express consumption risk in Home and Foreign as

$$\widehat{\sigma}_{c,t+1} = \left[\frac{\alpha\widetilde{\beta}\rho}{1-\widetilde{\beta}\rho}\left(\frac{1-\lambda}{1-\widetilde{\beta}\lambda\rho}\right) - \left(\frac{1-\widetilde{\beta}\lambda}{1-\widetilde{\beta}\lambda\rho}\right)\Theta\mathbf{a}_y\right]\widehat{r}^*_{F,t} \tag{G.21}$$

$$\widehat{\sigma}^*_{c,t+1} = \left[\frac{\left(1-\alpha\right)\widetilde{\beta}\rho}{1-\widetilde{\beta}\rho}\left(\frac{1-\lambda}{1-\widetilde{\beta}\lambda\rho}\right)\frac{1}{1-\alpha\left(1-\iota\right)} - \left(\frac{1-\widetilde{\beta}\lambda}{1-\widetilde{\beta}\lambda\rho}\right)\Theta^*\mathbf{a}^*_y\right]\widehat{r}^*_{F,t}, \tag{G.22}$$

where $\mathbf{a}_y = d\widehat{y}_t/d\widehat{r}^*_{F,t}$ and $\mathbf{a}^*_y = d\widehat{y}^*_t/d\widehat{r}^*_{F,t}$. Next, specializing (G.7a)-(G.7b) to the case with $\mathcal{O} = \mathcal{O}^* = 0$ but $\sigma_c^2 > 0$, and using the expressions above, we can express the aggregate consumption growth in Home and Foreign as:

$$\Delta\widehat{c}_{t+1} = \left\{\alpha + \alpha\frac{\widetilde{\beta}\rho}{1-\widetilde{\beta}\rho}\varphi\sigma_c^2\left(\frac{1-\lambda}{1-\widetilde{\beta}\lambda\rho}\right) - \varphi\sigma_c^2\rho\left(\frac{1-\widetilde{\beta}\lambda}{1-\widetilde{\beta}\lambda\rho}\right)\Theta\mathbf{a}_y\right\}\widehat{r}^*_{F,t} \tag{G.23}$$

$$\Delta\widehat{c}^*_{t+1} = \left\{\frac{1-\alpha}{1-\alpha\left(1-\iota\right)} + \varphi\sigma_c^2\frac{\widetilde{\beta}\rho\left(1-\alpha\right)}{\left(1-\widetilde{\beta}\rho\right)\left[1-\alpha\left(1-\iota\right)\right]}\left(\frac{1-\lambda}{1-\widetilde{\beta}\lambda\rho}\right) - \varphi\sigma_c^2\rho\left(\frac{1-\widetilde{\beta}\lambda}{1-\widetilde{\beta}\lambda\rho}\right)\Theta^*\mathbf{a}^*_y\right\}\widehat{r}^*_{F,t} \tag{G.24}$$

Finally, combining this with the market clearing conditions (G.8)-(G.9) and using the fact that output is stationary in both economies, this yields two equations in two unknowns $\mathbf{a}_y$ and $\mathbf{a}^*_y$:

$$\left[1-\left(1-\alpha\right)\theta\right]\mathbf{a}_y - \alpha\theta^*\mathbf{a}^*_y = \chi_d\left(\iota\right)\left(\delta\left(\iota\right)\eta - 1 - \gamma\right) \tag{G.25}$$

$$-\alpha\theta\mathbf{a}_y + \left[1-\left(1-\alpha\right)\theta^*\right]\mathbf{a}^*_y = -\frac{\gamma}{1-\rho} - \chi_d\left(\iota\right)\delta\left(\iota\right)\left(\eta - 1 - \gamma\right), \tag{G.26}$$

where θ and θ^* are defined the same way as in Appendix F, and δ and χ_d are defined in (G.12) and (G.15) respectively. Solving these out, we get the following expression for equilibrium spillovers:

$$\mathbf{a}_y = \frac{d\widehat{y}_t}{d\widehat{r}^*_{F,t}} = \underbrace{\mathbb{M}\left(\theta,\theta^*\right)}_{\substack{\text{spillover-}\\\text{multiplier}}}\times\chi_d\Big(\delta(\iota)\eta - (1+\gamma)\times\Omega(\theta^*,\iota)\Big), \tag{G.27}$$

where

$$\Omega(\theta^*,\iota) = 1 + \left[\frac{1-\alpha^2\left(1-\iota\right)}{2-\alpha\left(1-\iota\right)}\right]\frac{\theta^*}{\left(1-\alpha\right)\left(1-\theta^*\right)}$$

H Minimum-state-variable rational expectations solution

When we consider the model with $\vartheta < 1$, we compute the minimum-state-variable(MSV) equilibrium (McCallum, 1983). For our economy, the minimum state-variable representiaton involves two state variables at any date t: nfa $\widehat{a}_t$ and the monetary policy shock $\widehat{r}^*_{F,t}$. Consequently, in the unique MSV equilibirum in the linearized economy, all endogenous variables at any date t are time-invariant linear functions of $\widehat{a}_t$ and $\widehat{r}^*_{F,t}$. In particular, we can guess that:

$$\widehat{a}_{t+1} = h_a\widehat{a}_t + h_r\widehat{r}^*_{F,t} \tag{H.1}$$

$$Y_t = \mathbf{g}_a\widehat{a}_t + \mathbf{g}_r\widehat{r}^*_{F,t} \tag{H.2}$$

where Y_t denotes the vector of $n-2$ jump variables that we want to characterize the equilibrium dynamics for. $\mathbf{g}_a$ and $\mathbf{g}_r$ are two $n-2\times 1$ time and state-invariant vectors.

The linearized model can be written in matrix form as:

$$\begin{bmatrix} a_{11} & 0 & \mathbf{0} \\ 0 & 1 & \mathbf{0} \\ \underbrace{\mathbf{a}_{31}}_{n-2\times 1} & \underbrace{\mathbf{a}_{32}}_{n-2\times 1} & \underbrace{\mathbf{a}_{33}}_{n-2\times n-2} \end{bmatrix} \begin{bmatrix} \widehat{a}_{t+1} \\ \widehat{r}^*_{F,t+1} \\ Y_{t+1} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & \underbrace{\mathbf{b}_{13}}_{1\times n-2} \\ 0 & \rho & \mathbf{0} \\ \mathbf{b}_{31} & \mathbf{b}_{32} & \mathbf{b}_{33} \end{bmatrix} \begin{bmatrix} \widehat{a}_t \\ \widehat{r}^*_{F,t} \\ Y_t \end{bmatrix}, \tag{H.3}$$

where $\{a_{i,j}, b_{i,j}\}$ are constants which depend on model parameters. The first equation in (H.3) is the nfa equation, which is represented by (21) in the main text. The second equation describes the evolution of the monetary policy shock $\widehat{r}^*_{F,t}$, and the rest of the equations are the other model equations.

To compute the MSV equilibrium, we can substitute (H.1) and (H.2) into (H.3) to get:

$$\begin{bmatrix} a_{11} & 0 & 0 \\ 0 & 1 & 0 \\ \mathbf{a}_{31} & \mathbf{a}_{32} & \mathbf{a}_{33} \end{bmatrix} \begin{bmatrix} h_a\widehat{a}_t + h_r\widehat{r}^*_{F,t} \\ \widehat{r}^*_{F,t+1} \\ \mathbf{g}_a h_a\widehat{a}_t + (\mathbf{g}_a h_r + \rho\mathbf{g}_r)\,\widehat{r}^*_{F,t} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & \mathbf{b}_{13} \\ 0 & \rho & 0 \\ \mathbf{b}_{31} & \mathbf{b}_{32} & \mathbf{b}_{33} \end{bmatrix} \begin{bmatrix} \widehat{a}_t \\ \widehat{r}^*_{F,t} \\ g_a\widehat{a}_t + g_r\widehat{r}^*_{F,t} \end{bmatrix} \tag{H.4}$$

Then, for (H.1) and (H.2) to constitute a valid rational expectations equilibrium, (H.4) must be satisfied for any sequence of $\{\widehat{a}_t, \widehat{r}^*_{F,t}\}$, i.e., the following for equations must hold:

$$h_a = \frac{b_{11}}{a_{11}} + \frac{1}{a_{11}}\mathbf{b}_{13}\mathbf{g}_a \tag{H.5}$$

$$(h_a\mathbf{a}_{33} - \mathbf{b}_{33})\,\mathbf{g}_a = (\mathbf{b}_{31} - \mathbf{a}_{31}h_a) \tag{H.6}$$

$$h_r = \frac{b_{12}}{a_{11}} + \frac{1}{a_{11}}\mathbf{b}_{13}\mathbf{g}_r \tag{H.7}$$

$$(\mathbf{b}_{33} - \rho\mathbf{a}_{33})\,\mathbf{g}_r = h_r\,(\mathbf{a}_{31} + \mathbf{a}_{33}\mathbf{g}_a) + (\rho\mathbf{a}_{32} - \mathbf{b}_{32}) \tag{H.8}$$

Combining (H.5) and (H.6), we get a quadratic equation in h_a:

$$b_{11} + \mathbf{b}_{13}\,(h_a\mathbf{a}_{33} - \mathbf{b}_{33})^{-1}\,(\mathbf{b}_{31} - \mathbf{a}_{31}h_a) - a_{11}h_a = 0 \tag{H.9}$$

If a unique bounded MSV equilibrium exists, then one of the roots of the quadratic equation satisfies $|h_a| < 1$ and the other satisfies $|h_a| > 1$. As is standard, we set h_a to equal the root which satisfies

$|h_a| < 1$.[23] Given h_a, (H.6), (H.7) and (H.8) are linear in $\mathbf{g}_a, \mathbf{g}_r$ and h_r and can be solved to yield:

$$\begin{aligned}
\mathbf{g}_a &= (h_a \mathbf{a}_{33} - \mathbf{b}_{33})^{-1} (\mathbf{b}_{31} - \mathbf{a}_{31} h_a) \\
\mathbf{g}_r &= \left[a_{11} (\mathbf{b}_{33} - \rho \mathbf{a}_{33}) - (\mathbf{a}_{31} + \mathbf{a}_{33} \mathbf{g}_a) \mathbf{b}_{13}\right]^{-1} \left[b_{12} \mathbf{a}_{31} + b_{12} \mathbf{a}_{33} \mathbf{g}_a + \rho a_{11} \mathbf{a}_{32} - a_{11} \mathbf{b}_{32}\right] \\
h_r &= \frac{b_{12}}{a_{11}} + \frac{1}{a_{11}} \mathbf{b}_{13} \left[a_{11} (\mathbf{b}_{33} - \rho \mathbf{a}_{33}) - (\mathbf{a}_{31} + \mathbf{a}_{33} \mathbf{g}_a) \mathbf{b}_{13}\right]^{-1} \left[b_{12} \mathbf{a}_{31} + b_{12} \mathbf{a}_{33} \mathbf{g}_a + \rho a_{11} \mathbf{a}_{32} - a_{11} \mathbf{b}_{32}\right]
\end{aligned}$$

□

I PPI- vs CPI-based real interest rates

The standard assumption in the NK literature is that central banks set the nominal rate of interest. However, as explained in Section 2.3, it is sometimes convenient to work with the assumption that the central bank directly chooses the path of real interest rates. While this is innocuous in closed economy models, a choice needs to be made in the context of open economy models since there are two real interest rates that the central bank could choose: the PPI based and the CPI based real interest rates. This Appendix shows that choosing the PPI based real interest rate is equivalent to the standard assumption of central banks choosing the nominal rate, while choosing the CPI based real interest rate understates the importance of the intertemporal substitution channel. This illustration will be clearest in the limit in which nominal wages in both Home and Foreign are rigid ($\kappa = 0$ in the wage Phillips curve). In this case, PPI inflation in both countries is constant $\Pi_{H,t} = \Pi^*_{F,t} = 1$. However, CPI inflation in both countries can still change over time due to movements in the RER. To see this, recall from Appendix A.2, that the relative prices $P_{H,t}/P_t$ and $P^*_{F,t}/P^*_t$ can be written as:

$$\frac{P_{H,t}}{P_t} = \left[\frac{1 - \alpha - \alpha \mathcal{Q}_t^{1-\eta}}{1 - 2\alpha}\right]^{\frac{1}{1-\eta}} \qquad \text{and} \qquad \frac{P^*_{F,t}}{P^*_t} = \left[\frac{1 - \alpha - \alpha \mathcal{Q}_t^{\eta-1}}{1 - 2\alpha}\right]^{\frac{1}{1-\eta}}$$

Imposing $P_{H,t} = 1$ and $P^*_{F,t} = 1$, Home and Foreign CPI can be written as:

$$P_t = \left[\frac{1 - \alpha - \alpha \mathcal{Q}_t^{1-\eta}}{1 - 2\alpha}\right]^{\frac{1}{\eta-1}} \qquad \text{and} \qquad P^*_t = \left[\frac{1 - \alpha - \alpha \mathcal{Q}_t^{\eta-1}}{1 - 2\alpha}\right]^{\frac{1}{\eta-1}}$$

Up to first-order, this implies that Home and Foreign CPI inflation can be expressed as:

$$\widehat{\pi}_t = \frac{\alpha}{1 - 2\alpha} \Delta \widehat{q}_t \qquad \text{and} \qquad \widehat{\pi}^*_t = -\frac{\alpha}{1 - 2\alpha} \Delta \widehat{q}_t,$$

even though PPI inflation in both Home and Foreign is zero: $\widehat{\pi}_{H,t} = \widehat{\pi}^*_{F,t} = 0$. Consequently, specifying monetary policy in terms of a path of nominal interest rate is identical to specifying it in terms of the PPI

[23] If there are multiple bounded MSV equilibria, then both the roots of the quadratic equation satisfy $|h_a| < 1$, in which case we pick the h_a which has smaller absolute value.

based real interest rates since:

$$\begin{aligned}\widehat{r}_{H,t} &= \widehat{i}_t - \pi_{H,t+1} = \widehat{i}_t \\ \widehat{r}^*_{F,t} &= \widehat{i}^*_t - \pi^*_{F,t+1} = \widehat{i}^*_t\end{aligned}$$

Consequently, if we were to specify monetary policy in terms of a path of nominal rates, the analysis in the main text would remain unchanged as long as we assumed that nominal wages were rigid in both countries.

Next, consider the situation where we specify monetary policy as specifying a path of CPI based real interest rates. In this case, our experiment would correspond to $\widehat{r}^*_t > 0$ and $\widehat{r}_t = 0$. Using this in the UIP, $\widehat{r}_t = \widehat{r}^*_t + \Delta\widehat{q}_{t+1}$, we get:

$$\Delta\widehat{q}_{t+1} = -\widehat{r}^*_t < 0$$

To see what this means in terms of nominal rates at Home, recall that

$$\widehat{i}_t = \widehat{r}_t + \widehat{\pi}_{t+1} = 0 + \frac{\alpha}{1-2\alpha}\Delta\widehat{q}_{t+1} = -\frac{\alpha}{1-2\alpha}\widehat{r}^*_t < 0,$$

which implies that the Home central bank *cuts* nominal rates in response to the Foreign monetary tightening. Thus, the net effect on Home and Foreign output cannot be interpreted as just the effect of a Foreign tightening, it is the combined effect of a hike in Foreign nominal rates and a cut in Home nominal rates.

I.1 Deriving the insularity benchmark of Corsetti and Pesenti (2001)

In this section, we consider the Cole and Obstfeld (1991) parameterization ($\eta = 1$). Then, in the RANK benchmark, our model can be expressed as the two IS curves:

$$\begin{aligned}\Delta\widehat{y}_{t+1} &= (1-\alpha)\widehat{r}_t + \alpha\widehat{r}^*_t + \frac{2\alpha(1-\alpha)}{1-2\alpha}\Delta\widehat{q}_{t+1} \\ \Delta\widehat{y}^*_{t+1} &= \alpha\widehat{r}_t + (1-\alpha)\widehat{r}^*_t - \frac{2\alpha(1-\alpha)}{1-2\alpha}\Delta\widehat{q}_{t+1},\end{aligned}$$

and the UIP:

$$\widehat{r}_t = \widehat{r}^*_t + \Delta\widehat{q}_{t+1}$$

If we specify monetary policy in terms of CPI based real interest rates and set $\widehat{r}^*_t = \rho^t\widehat{r}^*_0 \geq 0$ and $\widehat{r}_t = 0$ for all $t \geq 0$, the UIP implies that $\Delta\widehat{q}_{t+1} = -\widehat{r}^*_t$. Plugging this into the IS equations yields:

$$\Delta\widehat{y}_{t+1} = -\frac{\alpha}{1-2\alpha}\widehat{r}^*_t \neq 0 \qquad \text{and} \qquad \Delta\widehat{y}^*_{t+1} = \frac{1-\alpha}{1-2\alpha}\widehat{r}^*_t$$

Thus, Home output is affected by a change in monetary policy even under the Cole and Obstfeld (1991) parameterization if we specify monetary policy as a path of CPI based real rates. Thus, this specification of monetary policy does not allow us to recover the insularity benchmark from Corsetti and Pesenti (2001). Instead, Appendix D shows that we recover the insularity result if we specify monetary policy as a path of PPI based real interest rates.

I.2 Specification of monetary policy as in Auclert et al. (2021)

In this section, we show that if we impose the monetary policy specification of Auclert et al. (2021) in our RANK benchmark, like them, we would also conclude that in response to a Foreign monetary tightening Home output must increase.

In the RANK benchmark and the specification of Auclert et al. (2021), the FOreign Euler equation can be written as:

$$\Delta\widehat{c}^*_{t+1} = \widehat{r}^*_t - \widehat{\beta}_t,$$

where $\widehat{\beta}_t > 0$ denotes a discount factor shock in Foreign. Auclert et al. (2021) assume that Foreign monetary policy acts to nullify the effect of the discount factor shock on Foreign consumption, i.e., $\widehat{r}^*_t = \widehat{\beta}_t > 0$. This specification nullifies the effect of the intertemporal substitution channel on Foreign consumption: the effect of higher Foreign interest rates on consumption is nullified because of the discount factor shock, which makes agents less patient.

Next, the Home aggregate Euler equations (up to first-order) can be written as:

$$\Delta\widehat{c}_{t+1} = \widehat{r}_t,$$

since there is no discount factor shock at Home. Auclert et al. (2021) set $\widehat{r}_t = 0$, which implies that the intertemporal substitution channel is also shut down at Home. Together, both these assumptions imply that holding the RER unchanged, consumption in Home and Foreign remain unchanged following the increase in Foreign monetary policy. Finally, we can forward and first-difference the market clearing conditions (20):

$$\begin{aligned}
\Delta\widehat{y}_{t+1} &= (1-\alpha)\,\Delta\widehat{c}_{t+1} + \alpha\Delta\widehat{c}^*_{t+1} + \frac{2\alpha\,(1-\alpha)}{1-2\alpha}\eta\Delta\widehat{q}_{t+1} \\
\Delta\widehat{y}^*_{t+1} &= (1-\alpha)\,\Delta\widehat{c}^*_{t+1} + \alpha\Delta\widehat{c}_{t+1} - \frac{2\alpha\,(1-\alpha)}{1-2\alpha}\eta\Delta\widehat{q}_{t+1}
\end{aligned}$$

Since the monetary policy specification implies that $\Delta\widehat{c}_{t+1} = \widehat{c}^*_{t+1}$; imposing this in the expressions above reveals that only the expenditure switching channel is left operational:

$$\begin{aligned}
\Delta\widehat{y}_{t+1} &= \frac{2\alpha\,(1-\alpha)}{1-2\alpha}\eta\Delta\widehat{q}_{t+1} \\
\Delta\widehat{y}^*_{t+1} &= -\frac{2\alpha\,(1-\alpha)}{1-2\alpha}\eta\Delta\widehat{q}_{t+1}
\end{aligned}$$

Using the fact that $\widehat{r}_t = 0$ in the UIP $\widehat{r}_t = \widehat{r}^*_t + \Delta\widehat{q}_{t+1}$, we get $\Delta\widehat{q}_{t+1} = -\widehat{r}^*_t$. Plugging this in the previous expressions:

$$\begin{aligned}
\Delta\widehat{y}_{t+1} &= -\frac{2\alpha\,(1-\alpha)}{1-2\alpha}\eta\widehat{r}^*_t \\
\Delta\widehat{y}^*_{t+1} &= \frac{2\alpha\,(1-\alpha)}{1-2\alpha}\eta\widehat{r}^*_t,
\end{aligned}$$

which imply that in equilibrium

$$\frac{d\widehat{y}_t}{d\widehat{r}_t^*} = \frac{2\alpha(1-\alpha)}{(1-2\alpha)(1-\rho)}\eta > 0$$
$$\frac{d\widehat{y}_t^*}{d\widehat{r}_t^*} = \frac{2\alpha(1-\alpha)}{(1-2\alpha)(1-\rho)}\eta,$$

which show that, as in Auclert et al. (2021), Home output must rise following the RER depreciation following a Foreign monetary contraction, regardless of the magnitude of $\eta > 0$. This clarifies that the findings of Auclert et al. (2021) are due to their specific assumptions on the conduct of monetary policy. With more general specifications of monetary policy, this property that RANK cannot generate a contractionary depreciation does not hold. As shown in Appendix D, given our specification of monetary policy, which relaxes both assumptions, Home output can decline following a RER depreciation, even in RANK, if $\eta < 1$. Even relaxing one of the assumptions of Auclert et al. (2021) on monetary policy would imply that the intertemporal channel is not nullified, in which case even RANK can deliver a contractionary depreciation at Home. This can easily be seen by assuming that the Foreign monetary shock is not the response to a discount factor shock. This is simply accomplished by setting $\widehat{\beta}_t = 0$ and $\widehat{r}_t^* > 0$. Going through the same calculations as above shows that Home output can decline in RANK alongside a RER depreciation as long as η is small enough.