

Liquidity, Debt Denomination, and Currency Dominance

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May 2023

Currency dominance: world features US dollar dominance

- Historical precedents: Dutch florin (17th–18th c.), British pound sterling (19th–20th c.)

This paper: liquidity-based theory for currency dominance in debt issuance

- Debt obligations are denominated in the unit required to be delivered at settlement
- Obtaining unit for settlement is less costly in more liquid money markets

US \$ is attractive for issuance because of a large, liquid \$ stock of instruments for settlement

Key mechanism: complementarity in liquidity supply (issuance) & demand (settlement)

⇒ Endogenous positive feedback: \$ issuance begets more debt market liquidity for settlement

Related Literature

International monetary system:

- *Dollar world*: Matsuyama Kiyotaki Matsui (1993), Obstfeld Dornbusch McKinnon (1995), Tirole (2002), Gourinchas Rey (2007a,b), Eichengreen Mehl Chitu (2017), Maggiori (2017), Farhi Maggiori (2018), He Krishnamurthy Milbradt (2019), Ilizetzi Reinhart Rogoff (2019), Gopinath Stein (2021), Chahrour Valchev (2021)
- *Historical precedents*: Keynes (1923), Nurske (1944), Dickson (1967), Despres Kindleberger Salant (1969), Lindert (1969), King (1972), Flandreau Jobst (2006), Eichengreen Flandreau (2008), Eichengreen (2008, 2012, 2017), Quinn Roberds (2014a,b), Kynaston (2015a,b), Roberds Velde (2016), Payne Szoke Hall Sargent (2022), Bolt Frost Shin Wierds (2023)

Safe asset shortages:

- Holmstrom Tirole (1998), Caballero Farhi Gourinchas (2008), Caballero Krishnamurthy (2009), Farhi Gourinchas Rey (2011), Krishnamurthy Vissing-Jorgensen (2012), Gorton Lewellen Metrick (2012), Obstfeld (2012), Greenwood Hanson Stein (2015)

US dollar dominance:

- *Trade invoicing*: Engel (2006), Goldberg Tille (2008), Gopinath Itskhoki Rigobon (2010), Gopinath Boz Casas Díez Gourinchas Plagborg-Møller (2020), Amiti Itskhoki Konings (2022), Mukhin (2022)
- *Global finance*: Krugman (1984), Frankel (1992), Cetorelli Goldberg (2012), Bruno Shin (2015a,b), Ivashina Scharfstein Stein (2015), McCauley McGuire Sushko (2015), Du Tepper Verdelhan (2018), Bahaj Reis (2020, 2021), Koijen Yogo (2020), Maggiori Neiman Schreger (2020), Bianchi Bigio Engel (2021), Jiang Krishnamurthy Lustig (2021), Kekre Lenel (2021), Jiang Richmond Zhang (2022), Correa Du Liao (2022), Eren Malamud (2022), Arslanalp Eichengreen Simpson-Bell (2022), Du Huber (2023)

Search frictions in financial markets:

- Kiyotaki Wright (1989, 1993), Pagano (1989), Trejos Wright (1995), Freeman (1996), Duffie Garleanu Pedersen (2005, 2007), Lagos Wright (2005), Garleanu Pedersen (2007), Vayanos Wang (2007), Vayanos Weill (2008), Weill (2008, 2020), Lagos Rocheteau (2009), Doepke Schneider (2017), Copeland Duffie Yang (2021), Passadore Xu (2022)

**Historical Example:
The First Global Currency**

The First Global Currency: Dutch Florin (17th – 18th c.)

International payments made in **illiquid metallic coin** for much of history

- Hundreds of types; costly to verify, insure, and transport; **uncertain supply** at any given time/place

Bank of Amsterdam (1609) overcame fractions with **florin (ledger currency)**

- Standardized **unit of account**: obtainable with coin deposits for payments via account transfers

Florin was liquid \implies florin-denominated “bill on Amsterdam” used internationally

- At any given time, florins available in Amsterdam; **yield premium** for florin-denominated assets

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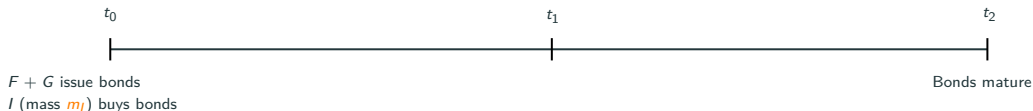
- At any given time, florins available in Amsterdam; **yield premium** for florin-denominated assets

Contrast with **illiquid Spanish “pieces of eight”** as a potential alternative global currency

- Spain bigger, wealthier, 6 \times trade volumes, but serial defaulter

Model: Within-Country Setup

Debt Market: Firms and Investors



Debt suppliers & demanders at t_0 :

- Entrepreneur-owned **Firms** (mass F) and **Government** (mass G) issue bonds at t_0
 - Entrepreneurs borrow to finance project which costs β^2 , and generates profits $\pi = 1$
- **Investors** (mass I) buy bonds, have endowments w ; each investor can invest in 1 bond

Preferences (risk neutral):

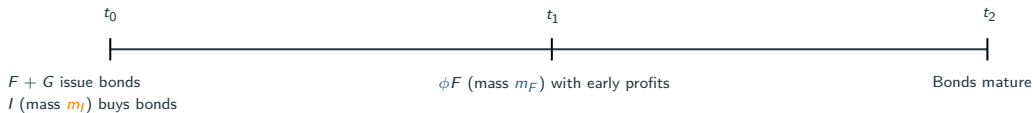
$$u_i^{F,I} = c_0 + \beta c_1 + \beta^2 c_2, \quad c_t \geq 0$$

Bonds:

- Face value 1, mature at t_2 , indivisible
- Zero default risk, perfect substitutes \implies same endogenous price P_0

Total bonds mass: $m_I = F + G \leq I$

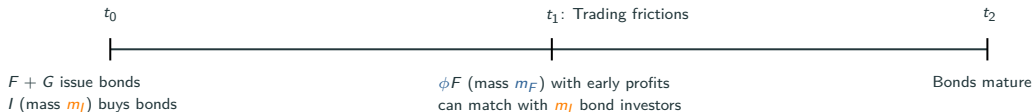
Timing Mismatch Generates Liquidity Demand at t_1



Central element: potential for timing mismatch generates liquidity demand

- Firms receive profits $\pi = 1$ at either t_1 or t_2
- Probability of early profits $\phi \rightarrow$ mass $m_F = \phi F$ of mismatched firms

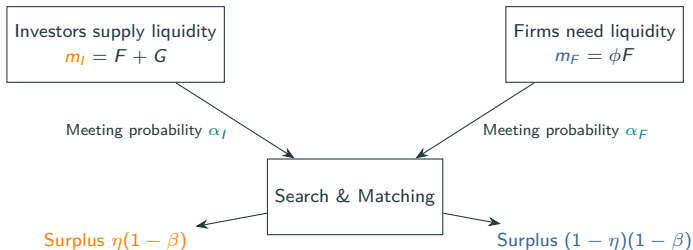
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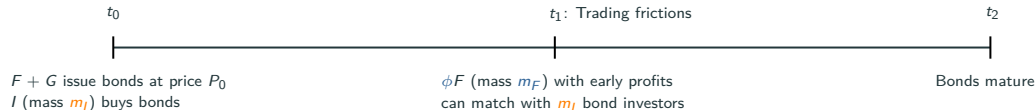
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Gains from asset trade $(1 - \beta)$ possible in the market at t_1 if firm is early:



Asset Market Equilibrium and Issuance Benefits



Solving for P_0 : market at t_0 is Walrasian, so investor bids result in price

$$P_0 = \underbrace{\alpha_I \beta (\beta + (1 - \eta)(1 - \beta))}_{P(\text{Matched}) \times \text{PV of Sale Price}} + \underbrace{(1 - \alpha_I) \beta^2}_{P(\text{Not Matched}) \times \text{PV of 1}}$$

Convenience yield at t_0 captured by $P_0 - \beta^2 = \beta(1 - \beta)(1 - \eta) \times \alpha_I$

- A fully illiquid bond ($\alpha_I = 0$) would be priced at β^2

Expected utility from debt issuance for firm i is increasing α_I and α_F :

$$\mathbb{E}[u_i^F] = \beta(1 - \beta) \times \left[\underbrace{(1 - \eta) \alpha_I}_{\text{Convenience yield at } t_0} + \underbrace{\eta \phi \alpha_F}_{\text{Benefit of liquidity at } t_1} \right]$$

Closing the Model With Search Specification, Complementary Issuance Benefits

Matching function at t_1 : number of meetings between firms (demanders) and investors (suppliers) is

$$n = \lambda m_F^\theta m_I^\theta, \quad \lambda > 0, \quad \underbrace{\theta > 1/2}_{\text{Increasing returns}}$$

- Duffie Garleanu Pedersen (2005) case: $\theta = 1$, micro-foundations in Duffie Qiao Sun (2018)

Meeting probabilities:

$$\underbrace{\alpha_F = \frac{n}{m_F} = \lambda m_I^\theta m_F^{\theta-1}}_{P(\text{Firm finds a bond seller})}, \quad \underbrace{\alpha_I = \frac{n}{m_I} = \lambda m_F^\theta m_I^{\theta-1}}_{P(\text{Bond seller finds a firm})}$$

Expected firm utility given equilibrium prices and probabilities (taking $\theta = 1$ case):

$$\mathbb{E}[u_i^F] = \lambda\beta(1 - \beta) \times \left[\underbrace{(1 - \eta)m_F}_{\substack{\text{Convenience yield at } t_0, \\ \text{increasing in liquidity demand } m_F}} + \underbrace{\eta\phi m_I}_{\substack{\text{Benefit of liquidity at } t_1, \\ \text{increasing in liquidity supply } m_I}} \right]$$

Model: Two-Country Environment

Debt Denomination Choice

Two countries $j = A, B$ with fundamentals $\{G_j, F_j, \lambda_j\}$

Currency denomination choice for firms i in each country

- Fixed cost $\propto K_i$ of foreign issuance
 - Ex: expected costs of balance sheet currency mismatch, underwriting, risk aversion (hedging), ...

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Endogenous masses $\mathcal{M} = (m_{F,A}, m_{I,A}, m_{F,B}, m_{I,B})$

Four denomination possibilities with expected utility denoted:

$$U_{A \rightarrow A}(\mathcal{M}) \qquad U_{A \rightarrow B}(\mathcal{M}, K_i)$$

$$U_{B \rightarrow B}(\mathcal{M}) \qquad U_{B \rightarrow A}(\mathcal{M}, K_i)$$

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Firm optimality requires threshold strategy: firms issue in foreign currency iff $K_i \leq \bar{K}$

- $H(K_i)$ is the (Pareto) CDF of $K_i \in [\underline{K}, \infty) \rightarrow$ share $H(\bar{K})$ issues in foreign currency

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- **Class BA (focus today)** and **class AB (symmetric analysis)** equilibria can arise

International Equilibrium Conditions

Define \hat{K} as the equilibrium value of \bar{K} , equilibrium characterized by:

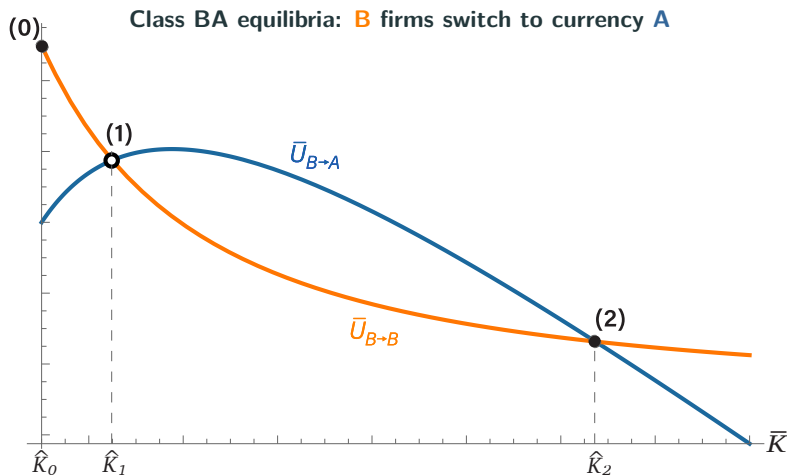
1. **Firm optimality:** the *marginal firm* ($K_i = \bar{K}$) has $K_i = \hat{K}$ in equilibrium and satisfies

$$\bar{U}_{j' \rightarrow j}(\hat{K}) = \bar{U}_{j' \rightarrow j'}(\hat{K})$$

2. **Market clearing:** given \hat{K} , masses \mathcal{M} satisfy

$$\begin{aligned} m_{I,j} &= G_j + F_j + H(\hat{K})F_{j'} & m_{I,j'} &= G_{j'} + [1 - H(\hat{K})] F_{j'} \\ m_{F,j} &= \phi [F_j + H(\hat{K})F_{j'}] & m_{F,j'} &= \phi [1 - H(\hat{K})] F_{j'} \end{aligned}$$

Multiple Equilibria With Symmetric Fundamentals



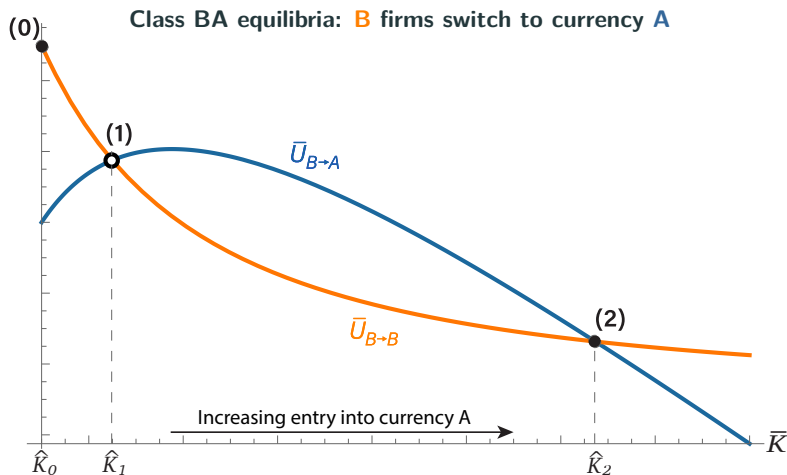
$$\bar{U}_{B \rightarrow A} = \lambda_A [m_{F,A}(\bar{K}) + \phi m_{I,A}(\bar{K})] - \bar{K}$$

Expected utility of foreign denomination

$$\bar{U}_{B \rightarrow B} = \lambda_B [m_{F,B}(\bar{K}) + \phi m_{I,B}(\bar{K})]$$

Expected utility of home denomination

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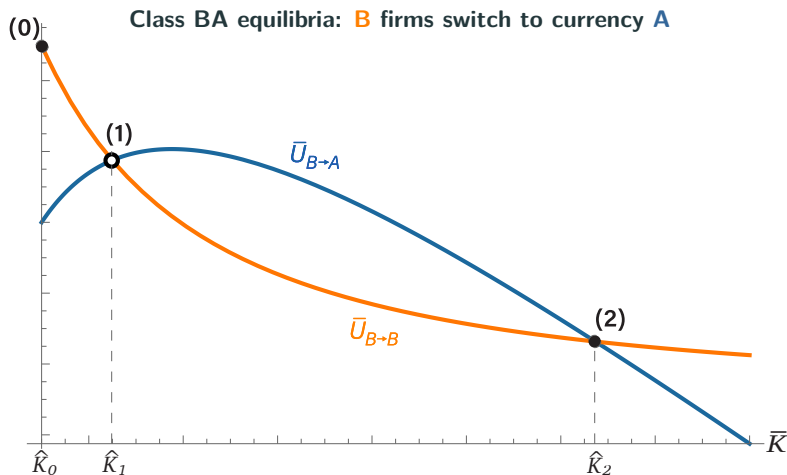
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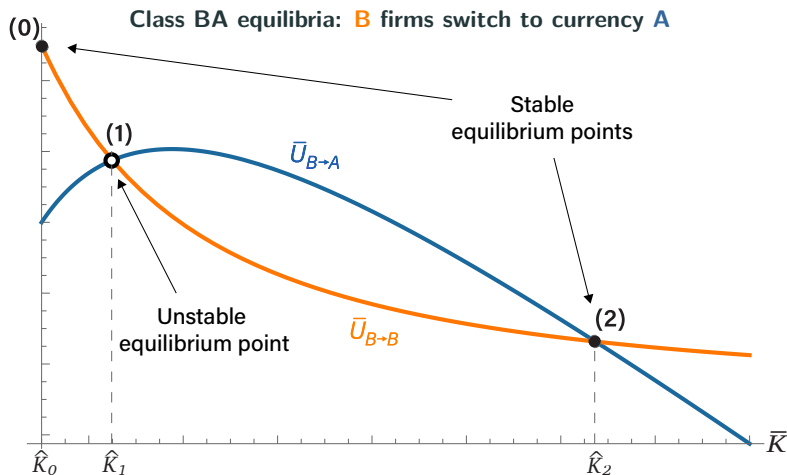
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Liquidity and Dominance Throughout History

Result 1: Understanding Historical Transitions - Fundamental Asymmetries Generate Dominance

Italian city-states (15th – 16th c.) also prominent in trade and finance, but no dominant currency:

- Symmetry → stable multipolar arrangement

Amsterdam disrupted multipolarity:

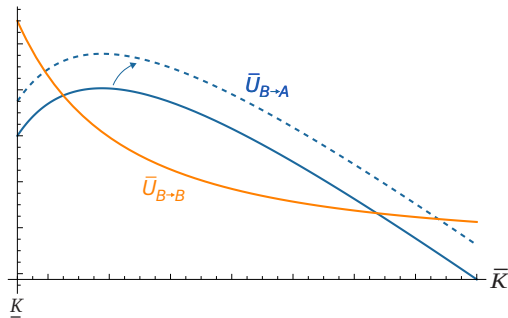
- Govt commitment and financial technology generated asymmetrically large G

Transition to British pound had similar features:

- Bank of Amsterdam collapses in 1791 ($\downarrow G_A$)
- Britain wins Napoleonic Wars ($\uparrow G_B$)

⇒ In paper: $\uparrow F$ not sufficient for eq. transition

Increasing G_A sufficiently leads to unique equilibrium selection:



$$\bar{U}_{B \rightarrow A} = \phi \lambda_A [G_A + 2F_A + 2H(\bar{K})F_B] - \bar{K}$$

Result 2: Complementarities Between Dominance and Sovereign Liquidity Provision Incentives

Specify the **government's objective** as

$$W_j = \underbrace{F_j \int u_{i,j}^F(K_i) dH(K_i)}_{\text{Domestic firm utility}} + \underbrace{G_j (P_{0,j} - \beta^2)}_{\text{Seignorage conv. yield}}$$

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Consider: $B \rightarrow A$ equilibrium with $G_A > G_B$, $\lambda_A = \lambda_B$, $F_A = F_B$

1. **Bigger incentive to create liquidity** (G) for the leader (A): $\frac{\partial W_A}{\partial G_A} > \frac{\partial W_B}{\partial G_B}$
2. **Complementarity**: investment incentive reinforced by endogenous rise in entry (\hat{K}):

$$\frac{\partial^2 W_A}{\partial G_A \partial \hat{K}} > 0, \quad \frac{\partial \hat{K}}{\partial G_A} > 0$$

Incentives manifested in history of **Bank of England**: LoLR, backstopping of private credit market

⇒ More in paper: analogous complementarity in incentives to facilitate private liquidity creation

Result 3: Additional Complementarity Arises from International Trade Invoicing

International trade and finance are highly related

- Ex: bills of exchange in Amsterdam both **settlement instruments** for trade and source of **credit**

Trade invoicing is complementary to currency dominance in debt denomination

- If revenues in dominant currency, lower FX mismatch reduces K_i (as in Gopinath Stein 2021)
- Shifting $H(K)$ to the left \rightarrow **more entry** with $\hat{K}_1 > \hat{K}_0$:

$$\underbrace{\lambda_A \phi [2F_A + G_A + F_B H(\hat{K}_0)] - \hat{K}_0}_{\bar{U}_{B \rightarrow A}} = \underbrace{\lambda_B \phi [2F_B + G_B + F_B (1 - H(\hat{K}_0))]}_{\bar{U}_{B \rightarrow B}}$$

- If firms *choose* invoicing currency, generate trade dominance as by-product of financial dominance

\implies Additional complementarity that reinforces dominant equilibrium

Welfare, Aggregate Risk, and International Cooperation

Result 4: Welfare and International Cooperation

Global planner has objective:

$$W = W_A + W_B$$

Socially optimal entry > **competitive equilibrium** because entry carries positive *liquidity externality*

$$\underbrace{K^*}_{\text{Socially optimal entry}} > \underbrace{\hat{K}_{\max}}_{\text{Competitive Equilibrium}}$$

- First best (K^*) is a Pareto improvement over competitive equilibrium (with transfers)

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Country A underprovisions G_A relative to global planner if $\frac{\partial W}{\partial G_A} > \frac{\partial W_A}{\partial G_A}$

- In this case, there are **gains from international cooperation in liquidity supply**
 - Historical analog: **Bretton Woods** → major economies coordinated on US-provided liquidity
 - This case occurs in the model if F_B is sufficiently larger than G_B

Result 5: Aggregate Risk and State-Contingent Liquidity, Role of Swap Lines

Aggregate risk:

- State at t_1 is $\omega \in \Omega$ with probability $q_\omega \rightarrow$ aggregate liquidity demand shock: ϕ_ω
- State-contingent liquidity supply G_ω^A chosen in advance at t_0

Equilibrium indifference condition now features **moments** of the $(\phi_\omega, G_\omega^A)$ distribution:

$$\lambda_A \left(\mathbb{E}[\phi_\omega] \left(2(F_A + H(\hat{K})F_B) + \mathbb{E}[G_\omega^A] \right) + \text{Cov}[\phi_\omega, G_\omega^A] \right) - \hat{K} = \lambda_B \mathbb{E}[\phi_\omega] \left(2(1 - H(\hat{K}))F_B + G_B \right)$$

- **State-contingent liquidity provision** (positive covariance) induces entry

Policy tool: Central bank swap lines that provide liquidity when it is most demanded

Conclusion: Dollar Dominance Today

Sources of dominance we highlight appear in many features of the dollar:

- Base for USD-denominated money markets is T-Bills (large, liquid, safe stock)
- Financial technologies make private assets liquid (repo, securitization, banking)
- Fed swap lines: contingent expansion of US \$-denominated liquidity
- Complementarities in dollar issuance by wide spectrum of entities:
 - Safe liquidity suppliers taking advantage of US \$ convenience yields (e.g., KFW)
 - Other lower-rated global corporates also issue US \$ drawn in by liquidity benefit

Renminbi dominance question: current Chinese financial system lacks these elements

Model equilibrium:

- Equilibrium lemmas [Go](#)
- Formal firm problem [Go](#)
- Class AB equilibria [Go](#)
- Increasing F_A [Go](#)

Theoretical extensions:

- Issuance complementarities [Go](#)
- The $\theta < 1$ case [Go](#)
- Limited pledgeability [Go](#)
- Sovereign denomination choice [Go](#)

History:

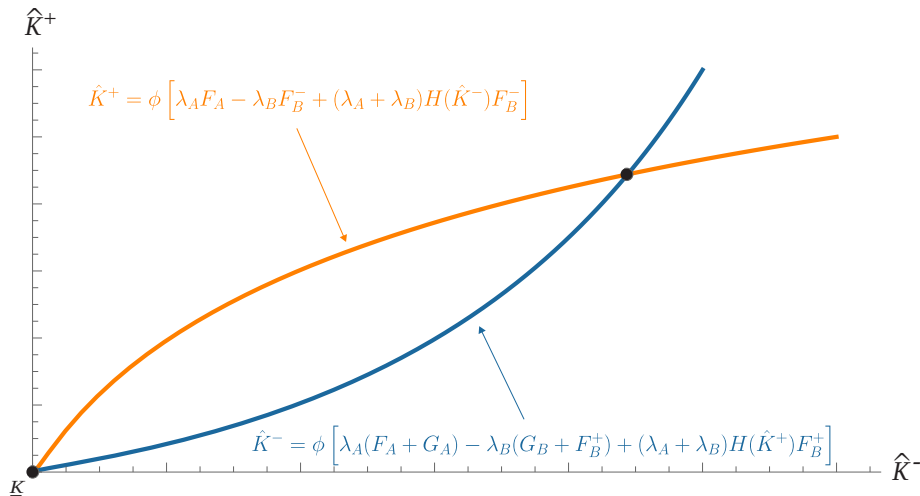
- Bank of Amsterdam mandate [Go](#)
- Florin quantities [Go](#)
- The florin *agio* [Go](#)
- Bank of England evolution [Go](#)

Empirics:

- Debt quantities [Go](#)
- British dominance [Go](#)
- Finance and trade [Go](#)

Extra Slides

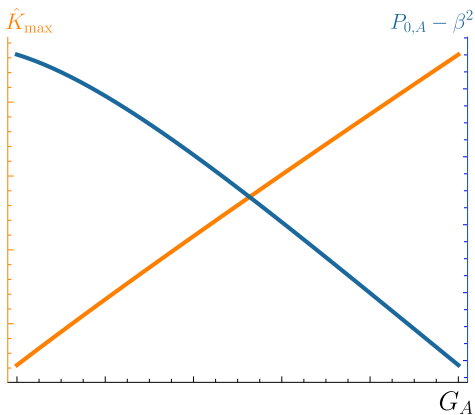
The Denomination Choices of Safe and Risky Private Borrowers Are Complementary



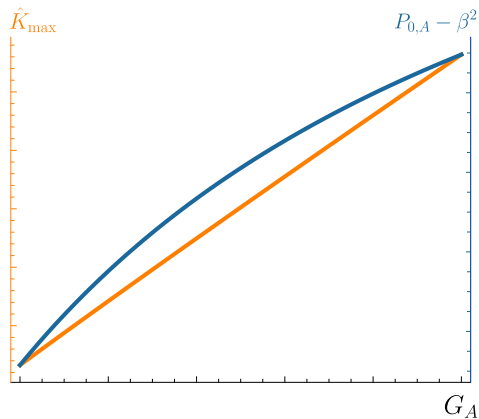
Convenience Yields and Sovereign Debt Supply

$$P_{0,j} - \beta^2 = \frac{\lambda_j \beta(1 - \beta)}{2} m_{F,j}^\theta m_{I,j}^{\theta-1}$$

(a) Case 1: Convenience yield decreasing in G_A

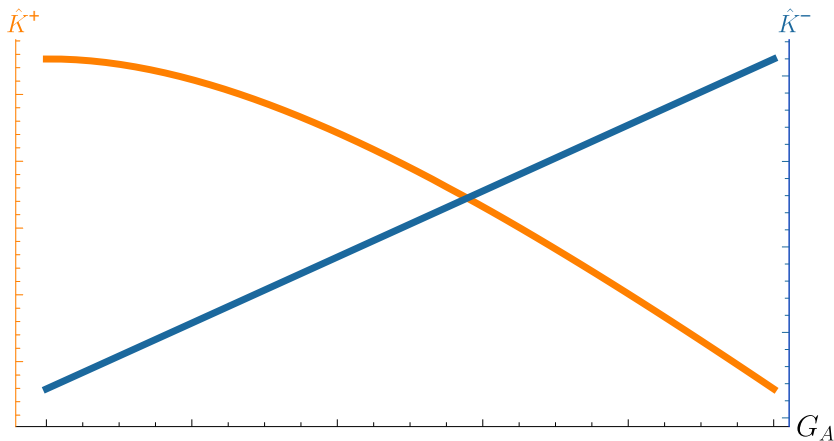


(b) Case 2: Convenience yield increasing in G_A



Crowding In and Crowding Out of Heterogeneous Private Borrowers

- In general case ($\theta < 1$), can generate negative impact of sovereign debt supply on conv. yields
- As a result, more government debt **crowds out** safe borrowers while **crowding in** risky borrowers



Sovereign Incentives to Facilitate Private Sector Liquidity Creation Increasing in Dominance

Improving capacity of **private sector** to issue safe money-like assets also part of financial development

Extend model to include country-specific **pledgeability** parameter ρ_j

- After currency choice, firms find out if revenues are fully pledgeable (probability ρ_j) or not

Ex ante expectation of pledgeability is ρ_j , so equilibrium condition becomes:

$$\rho_A [\lambda_A(m_{F,A} + \phi m_{I,A}) - \hat{K}] = \rho_B [\lambda_B(m_{F,B} + \phi m_{I,B})]$$

As in previous case, sovereign incentives to invest in firm pledgeability **complementary** to dominance:

$$\frac{\partial W_A}{\partial \rho_A} > \frac{\partial W_B}{\partial \rho_B}, \quad \frac{\partial^2 W_A}{\partial \rho_A \partial \hat{K}} > 0, \quad \frac{\partial \hat{K}}{\partial \rho_A} > 0$$

Mandate from the Bank's founding decree:

"To check all agio of the current money and confusion of coin, and to be of use to all persons who are in need of any kind of coin in business."

The Entrepreneur's Decision Problem

Entrepreneur chooses whether to issue (D_i) at t_0 and whether to trade (T_i) at t_1 :

$$\max_{D_i, T_i} E[c_0 + \beta c_1 + \beta^2 c_2]$$

subject to

$$c_0 = D_i(P_0 - \beta^2),$$

$$c_1 = \begin{cases} 0, & \text{late;} \\ 0, & \text{early, but not matched;} \\ D_i T_i \eta(1 - \beta), & \text{early, and matched} \end{cases}$$

$$c_2 = 0.$$

Since $P_0 \geq \beta^2$ and $\beta < 1$, solution is to set $D_i = 1$ and $T_i = 1$

[\[Back\]](#)

Lemmas: Necessary Conditions for Firm Optimality

Lemma 1

Consider firms \hat{i} and i in country j , where $K_i < K_{\hat{i}}$. If it is optimal for firm \hat{i} to issue in foreign currency $j' \neq j$, then it is optimal for firm i to issue in foreign currency j' .

Lemma 2

Suppose that there is a positive mass of firms in j that find it optimal to issue in currency j' . Then, no firms in j' will issue in currency j .

Lemma 3

A necessary condition for a collection of firm denominations choices $\mathcal{D}_{i,j}$ to be consistent with firm optimality is that it must take the following threshold form:

$$\mathcal{D}_{i,j'} = \begin{cases} 1 & \text{if } K_i < \bar{K}, \\ 0 & \text{if } K_i \geq \bar{K}, \end{cases} \quad \mathcal{D}_{i,j} = 0.$$

Consider the choice for firms in A and define \hat{K} as the equilibrium value of \bar{K}

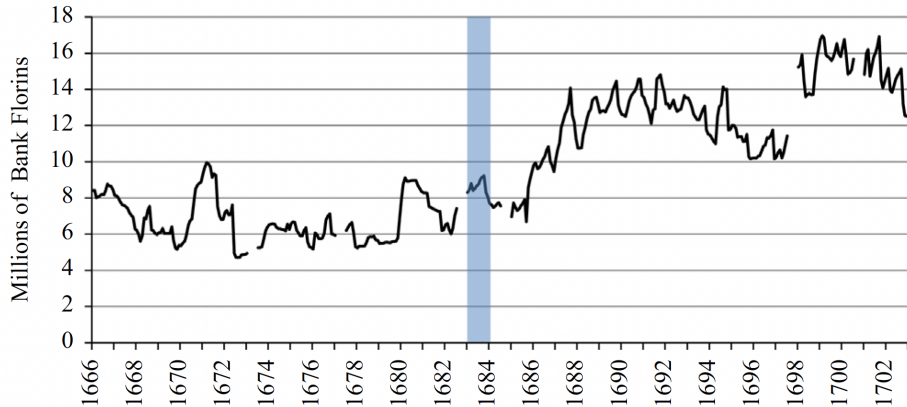
The **threshold firm** ($K_i = \bar{K}$) has $K_i = \hat{K}$ in equilibrium and satisfies:

$$\underbrace{\lambda_A [m_{F,A} + \phi m_{I,A}]}_{\bar{U}_{A \rightarrow A}(\bar{\mathcal{M}}): \text{Utility from issuing in home currency}} = \underbrace{\lambda_B [m_{F,B} + \phi m_{I,B}] - \hat{K}}_{\bar{U}_{A \rightarrow B}(\bar{\mathcal{M}}, \hat{K}): \text{Utility from issuing in foreign currency}}$$

Given \hat{K} , masses are:

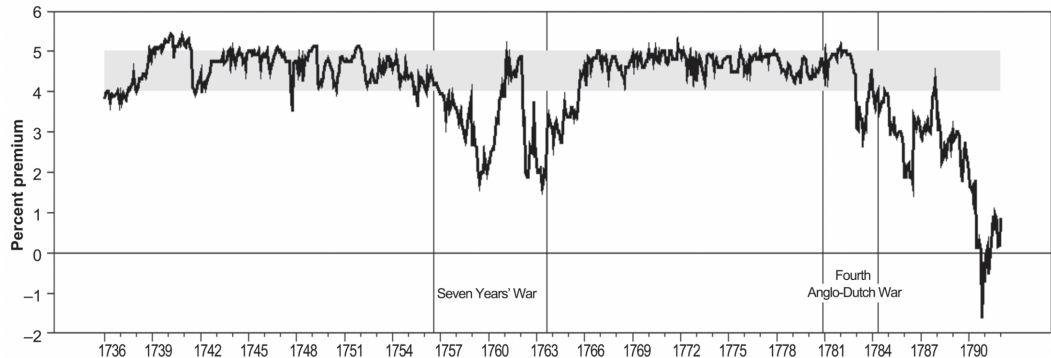
$$\begin{aligned} m_{I,A} &= G_A + [1 - H(\hat{K})] F_A & m_{I,B} &= G_B + F_B + H(\hat{K}) F_A \\ m_{F,A} &= \phi [1 - H(\hat{K})] F_A & m_{F,B} &= \phi [F_B + H(\hat{K}) F_A] \end{aligned}$$

Financial Innovation Driving Florin Success



Monthly bank balances (1666 – 1703); Source: Quinn and Roberds (2014)

End of Dutch Dominance

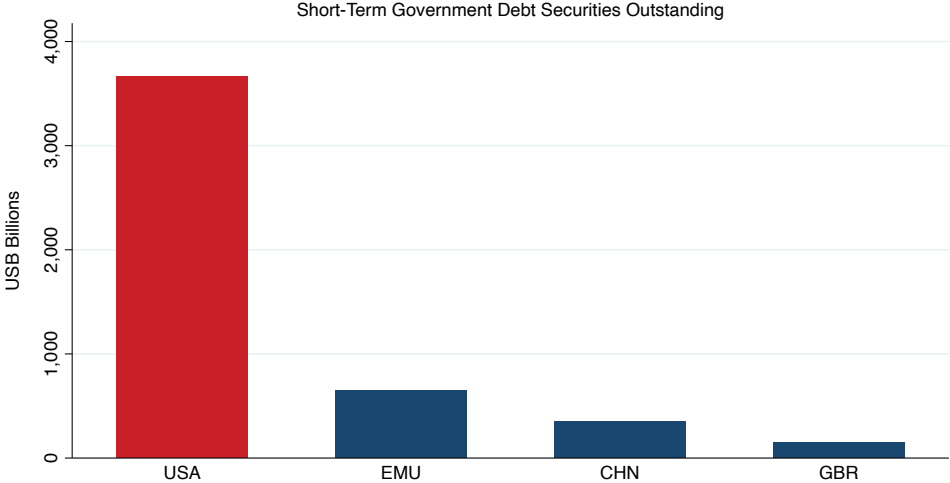


Agio: percent premium of bank florin over current guilders (1736 – 1792)

Source: Quinn and Roberds (2019)

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Short-Term Government Debt Supply Vastly Higher in the United States



Allowing Denomination Choice for G Entrenches Dominance

Allow government in B to choose amount of denomination in currency A : $G_B^* \in [0, G_B]$

Government's objective:

$$W_j = \underbrace{G_j (P_{0,j} - \beta^2)}_{\text{Seignorage conv. yield}} + F_j \underbrace{\int u_{i,j}^F(K_i) dH(K_i)}_{\text{Domestic firm utility}}$$

In equilibrium in the baseline model, the follower's objective (B) is

$$W_B = G_B \times \underbrace{\lambda_B m_{F,B}}_{\substack{\text{Conv. Yield in B} \\ \text{for govt debt}}} + F_B(1 - H(\hat{K})) \times \underbrace{\lambda_B (m_{F,B} + \phi m_{I,B})}_{\substack{\text{Conv. Yield +} \\ \text{Liquidity Benefit in B} \\ \text{for firm debt}}} + \underbrace{U_{B \rightarrow A}}_{\text{Switchers}}$$

With the choice, B trades off **better convenience yields** in govt debt with **lower liquidity benefit** to private firms

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$$W_B = \underbrace{G_B^* \times \lambda_A m_{F,A}}_{\substack{\text{Higher conv. yield} \\ \text{for own debt}}} + (1 - G_B^*) \lambda_B m_{F,B} + F_B(1 - H(\hat{K})) \times \lambda_B(m_{F,B} + \phi \underbrace{((1 - G_B^*)}_{\substack{\text{Lower liquidity benefits} \\ \text{for domestic firm debt}}}) + F_B H(\hat{K})) + \dots$$

$\implies G_B^*$ will issue more in A if **convenience yields** are much better and **private firms are small**

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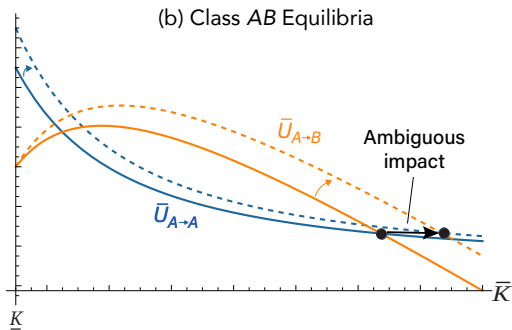
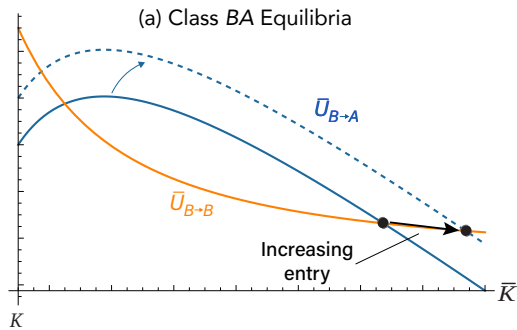
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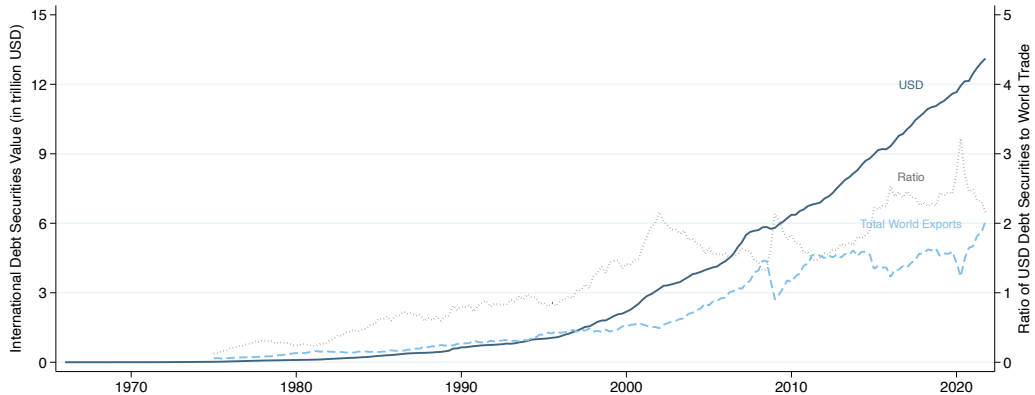
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Increasing Private Sector Size Has Ambiguous Equilibrium Impact



Trade Volumes and Financial Quantities Today



The Evolution of British Pound Dominance

