Money and Capital*

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Abstract

We analyze a microfounded model of monetary exchange with periodic centralized and decentralized markets, extended to include capital as a factor of production. Different from earlier attempts to integrate capital and monetary theory, our framework does not dichotomize: one cannot solve independently for equilibrium in the centralized and decentralized markets. Because of feedback across markets, money has interesting implications for investment, consumption, and employment. We calibrate and use the model to study quantitatively the effects of monetary and fiscal policy, taking into account long-run transitions. As an example, we find that the cost of 10% inflation can be between 1% and 4% of consumption, and that replacing inflation with distortionary taxes may be beneficial. We also find the two holdup problem present in our model are quite important in generating these results.

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1 Introduction

The goal of this paper is to extend, analyze, and study policy in, some models from the new monetary economics literature, both qualitatively and also quantitatively, as people have been routinely doing in, say, real business cycle theory for a long time (see Cooley 1995 for a representative sample). Our approach builds on the model with periodic meetings of centralized and decentralized markets in Lagos and Wright (2005), hereafter LW. However, for our purposes it is critical to extend that simple framework to include neoclassical firms employing labor and capital, as in the standard growth model. There are two main ways to motivate what we do and how we do it.

First, a previous attempt to integrate monetary and growth theory in Aruoba and Wright (2003) was at best partially successful, because that specification displays a strong *dichotomy*: one can solve independently for equilibrium allocations in the centralized and decentralized markets. This has some implications that seem undesirable, including the prediction that monetary policy can have no impact on investment, employment or consumption in the centralized market.[1] Although we discuss several ways to break the dichotomy, our preferred specification has some capital produced in the centralized market used as an input to decentralized market production. Then, since inflation is a tax on decentralized market activity, it influences the demand for capital, implying potentially rich feedback across markets. Now monetary policy can have interesting implications for investment, employment and consumption.

Second, there have been very few previous attempts to take microfounded monetary theories to the data, especially versions with capital.[2] It is because we are interested in quantitative analysis that we think it is important to include capital, which is an staple in mainstream macroeconomics. For the same reason, we include some other ingredients,

[1] Moreover, one might say that it implies money and growth theory are really not integrated at all. See Howitt (2003) and Waller (2003) for additional discussion.

[2] One exception is Shi (1999), who uses a very different approach. A few attempts to quantify simple search-based monetary models without capital are surveyed in Craig and Rocheteau (2005).

including government spending and taxation – which are not only important for calibration purposes, they also allow us to analyze fiscal policy and combined monetary-fiscal policy experiments. In discussing policy, the analysis here is more challenging and certainly more interesting than in models without capital, because we need to take into account transition paths. Therefore we need to solve numerically for the equilibrium decision rules, as opposed to merely comparing steady states.

In terms of theory, our contribution is to construct a novel monetary model with capital used as a factor of production in both centralized and decentralized markets (as we discuss in detail below, capital is only a factor of production and not a medium of exchange). Again, this generates interesting feedback across markets and from money to all variables. We consider both the case where prices in the decentralized market are determined via bargaining, and the case where they are determined competitively. This allows us to highlight certain effects due to bargaining, referred to as *holdup problems*, on the demand for money and capital.[3] In addition to these effects, we have the usual monetary distortion from nominal interest rates, and distortions due to taxation. Even with all these effects, the model is tractable, and we show how to solve it explicitly in an example with common functional forms.

In terms of quantitative economics, we calibrate to mostly standard observations. We do this for several specifications, including one that dichotomizes, and versions with bargaining as well as competitive pricing. Different specifications do a more or less reasonable job of capturing the key observations. We use the model to perform several policy experiments. For example, we measure the welfare gain of going from 10% inflation to the Friedman rule, with or without adjusting taxes to keep revenue constant, taking into account long-run transitions. Although naturally the answer depends on the version of the model and several

[3] The holdup problem with money demand was discussed in LW. Holdup problems with investment are often thought to be an important factor influencing aggregate capital accumulation, but are difficult to incorporate into the standard growth model (Caballero and Hamour 1998; Caballero 1999). In our framework they rise naturally, due to the decentralized nature of some trades, and interact with policy in interesting ways. By comparing equilibrium with bargaining and with price taking, we can evaluate the importance of holdup problems qualitatively and quantitatively.

other details, this gain can be between 1% to 4% of consumption. We also find that it may be desirable to reduce inflation even if this entails an increase in taxation.

The paper is organized as follows. In Section 2 we describe our baseline model and compare equilibria under bargaining and competitive pricing. In Section 3 we work through an explicit example, and analyze several extensions. In Section 4 we discuss calibration. In Section 5 we report the results, and discuss policy and robustness issues. In Section 6 we conclude.[4]

2 The Basic Model

2.1 General Assumptions

There is a $[0, 1]$ continuum of infinitely-lived agents. Time is discrete, and each period is divided into two subperiods. In one subperiod there is a frictionless or centralized market, referred to as the CM; in the other there is a decentralized market, referred to as the DM, with two main frictions. These frictions are: (i) a double-coincidence problem, generated here by taste and technology shocks; and (ii) anonymity, which precludes credit. This means that some medium of exchange is essential, as is standard in modern monetary theory (Kocherlakota 1998; Wallace 2001). The main issue in much of this literature (e.g. Kiyotaki and Wright 1989) is to determine endogenously which object will play this role. In order to focus on other questions, however, other papers avoid this issue by assuming there is a unique storable asset – perhaps fiat money, perhaps commodity money, perhaps something

[4] A few words are perhaps in order as to why we use the LW framework. First, having some decentralized trade is what makes a medium of exchange essential. Then, having a periodic centralized market generates a huge gain in tractability over similar models without it such as Green and Zhou (1998, 2002), Molico (1999), Zhou (1999), or Zhu (2003, 2005). This is because, with a centralized market, combined with quasi-linear preferences, we do not have to keep track of trading histories via the distribution of money holdings as a state variable. While models that do have to keep track of this distribution are clearly interesting, it is nice to have a benchmark that is easy to analyze and understand, the way e.g. the complete-market, representative-agent, neoclassical growth model serves as a benchmark in business cycle theory. Previous generations of search-based monetary theory, including Kiyotaki and Wright (1993), Shi (1995), or Trejos and Wright (1995), are also easy to analyze and understand, but mainly because of assumptions that would seem to preclude quantitative work as it is normally practiced.

else – that qualifies as a potential medium of exchange.[5]

For the current project, we want to follow the latter approach and avoid the interesting but difficult problem of determining the medium of exchange endogenously. We cannot, however, assume there is a unique storable asset that qualifies for this role in a paper called "Money and Capital." Our approach is to assume that physical capital is fixed in place in the CM, and thus cannot be traded in the DM. Then, to address the issue of why claims to (rather than physical units of) capital do not circulate in the DM, we assume that agents can costlessly counterfeit such claims, but cannot so easily counterfeit currency. Given this, sellers will never accept claims to capital from anonymous buyers in the DM, any more than they would accept personal IOU's, but they could accept money.[6]

So money is the only object that can serve as a medium of exchange in this environment, while capital is simply a productive input. We emphasize that we do not regard this as a particularly interesting or elegant solution to the rate-of-return-dominance puzzle – how can money and other assets paying higher rates of return coexist? It is rather a device that allows us to study interactions between money and capital when one serves as medium of exchange and the other as a factor of production. Our position is that, even if we do not have a prize-winning answer to the rate-of-return-dominance question, for now, it is interesting to study other issues in models that include many of the ingredients from micro-based monetary economics, including double-coincidence problems, bargaining problems, and so on.

While we acknowledge that our assumptions about capital are crude, at the same time we insist that they are logically consistent assumptions about the physical environment, and not direct assumptions about agents' behavior. As a general principle, it should be

[5] For example, in Trejos and Wright (1995), it is assumed that "Agents consume *services* (or, equivalently, nonstorable goods)" to rule out commodity money, which is studied elsewhere, and concentrate on the role of fiat money in models with search and bargaining. Note, however, that there is no constraint saying that agents have to use money in order to consume – they are free to try direct barter if they like, e.g., and there typically is some barter in equilibrium.

[6] One need not interpret money literally as cash. He, Huang and Wright (2005) study a related model, where agents can deposit money in bank accounts in the CM, and pay with either cash or checks or maybe debit cards in the DM (see also Berentsen Camera and Waller 2005). It would seem feasible to do something similar here, but for the current paper we thought that adding banking might take us too far afield.

clear that it is better to be explicit about the assumptions leading to an outcome, rather than assuming the outcome as a "reduced-form" for something left implicit. This is not (or at least, not only) because some people may doubt that there exist logically consistent assumptions generating the outcome in question, but because one ought to want to know what *other* implications these assumptions may have. The only way to know this is to be explicit about the environment.[7]

To continue, in the CM there is a general good that can be use for consumption or investment. It is produced using labor H and capital K, hired by firms in perfectly competitive markets. As usual, profit maximization implies $r = F_K(K, H)$ and $w = F_H(K, H)$, where F is the technology, r is the rental rate and w the real wage, and by constant returns equilibrium profits will be 0. In the DM these firms do not operate, but an agent's own effort e and capital k may be used with technology $f(e, k)$. Note that k appears as an input in DM production, even though it cannot be produced or traded in the DM in our baseline model (it can be in some extensions in Section 3). So while perhaps k cannot be physically moved to the location where the DM convenes, it still may increase productivity at that location; as an example, think about logging on to a computer from a remote site.

To generate a double-coincidence problem we adopt the following specification in the DM: with probability σ each agent wants to consume but cannot produce; with probability σ each agent can produce but does not want to consume; and with probability $1 - 2\sigma$ he can neither produce nor consume. This is equivalent for our purposes to the standard bilateral matching specification in the literature, where there is a probability σ of wanting to consume a good produced by a random partner. We frame things here in terms of taste and technology shocks, rather than matching, because it facilitates some parts of the discussion,

[7] We pause here to say that it is of course interesting to think about the coexistence of currency and other assets at a deeper level. Lagos and Rocheteau (2005) discuss this in a related model (see also Waller 2003). Devices that could potentially be used to capture why capital does not drive out money if we allowed it to circulate include government policies like those in Aiyagari et al. (1994), Shi (2005), and Lagos (2005), or maybe private information as in Williamson and Wright (1994), Trejos (1997), and Berentsen and Rocheteau (2004). We leave for future work the exploration of these ideas in other models with money and capital.

but otherwise very little hinges on this part of the specification.[8]

Instantaneous utility in the CM is $U(x) - Ah$, where x is consumption and h hours. In the DM, with probability σ an agent is a consumer and has utility $u(q)$, and with probability σ he is a producer and has utility $-\ell(e)$, where q is consumption and e effort. Assume $U(x)$, $u(q)$ and $\ell(e)$ have the usual properties. Linearity in h is not important in principle, but yields a big gain in tractability; alternatively one can assume general utility and indivisible labor as in Rogerson (1988), implying a quasi-linear reduced form (see Rocheteau et al. 2005 for details). In any case, it is convenient to write disutility in the DM as a production cost: given k, solve $q = f(e,k)$ for $e = \xi(q,k)$ and let $c(q,k) = \ell[\xi(q,k)]$. The Appendix verifies that $c_q > 0$, $c_k < 0$, $c_{qq} > 0$, and $c_{kk} > 0$ under the usual monotonicity and convexity assumptions on f and ℓ, and $c_{qk} < 0$ if $f_k f_{ee} < f_e f_{ek}$, which holds if k is a normal input.

The government sets the money growth rate τ so that $M_{+1} = (1+\tau)M$, where $+1$ denotes next period; as Fisher equation holds, an equivalent policy is to set the nominal interest rate, or the inflation rate, in this economy. Government also consumes G, levies a lump-sum tax T, a labor income tax t_h, and a capital income tax t_k in the CM. In principle it also levies sales taxes in both the CM and DM, t_x and t_q; however we set $t_q = 0$ because it reduces the notation, streamlines the presentation, avoids a discussion about taxing anonymous trades, and does not matter for the quantitative results. Letting δ be depreciation on capital, which is tax deductible here, and p the CM price level, the government budget constraint is

$$G = T + t_h wH + t_k rK - \delta t_k K + t_x X + \tau M/p,$$

Agents discount between the CM and DM at rate β, but to reduce notation, not between the DM and CM (think about the DM meeting first within each period). If $W(m,k)$ and $V(m,k)$ are the value functions of agents in the CM and DM, then

$$\begin{aligned} W(m,k) &= \max_{x,h,m_{+1},k_{+1}} \left\{ U(x) - Ah + \beta V_{+1}(m_{+1}, k_{+1}) \right\} \qquad (1) \\ \text{s.t. } (1+t_x)\,x &= w\,(1-t_h)\,h + \left[1 + (r-\delta)\,(1-t_k)\right] k - k_{+1} - T + \frac{m - m_{+1}}{p} \end{aligned}$$

[8] As discussed in fn. 5, random matching models allow some barter trades; we can as well, by having agents sometimes produce and also want to consume something other than what they produce.

After eliminating h using the budget equation, the FOC are

$$\begin{aligned}
x &: U'(x) = \frac{A(1+t_x)}{w(1-t_h)} \\
m_{+1} &: \frac{A}{pw(1-t_h)} = \beta V_{+1,m}(m_{+1},k_{+1}) \\
k_{+1} &: \frac{A}{w(1-t_h)} = \beta V_{+1,k}(m_{+1},k_{+1}),
\end{aligned} \tag{2}$$

assuming an interior solution; see LW for assumptions to guarantee interiority in these kinds of models.[9] Because (m,k) does not appear in (2), for any distribution of (m,k) across agents entering the CM the distribution of (m_{+1},k_{+1}) across agents leaving is degenerate. Also, from the envelope conditions W is linear in (m,k):

$$W_m(m,k) = \frac{A}{pw(1-t_h)} \tag{3}$$

$$W_k(m,k) = \frac{A[1+(r-\delta)(1-t_k)]}{w(1-t_h)} \tag{4}$$

Moving back within the period to the DM market, we have

$$V(m,k) = \sigma V^b(m,k) + \sigma V^s(m,k) + (1-2\sigma)W(m,k), \tag{5}$$

where

$$V^b(m,k) = u(q_b) + W(m-d_b,k) \tag{6}$$

$$V^s(m,k) = -c(q_s,k) + W(m+d_s,k), \tag{7}$$

while q_b and d_b are output and money exchanged when buying, and q_s and d_s when selling. Using the linearity implied by (3), we have

$$V(m,k) = W(m,k) + \sigma\left[u(q_b) - \frac{d_b A}{pw(1-t_h)}\right] + \sigma\left[\frac{d_s A}{pw(1-t_h)} - c(q_s,k)\right]. \tag{8}$$

[9] The second order conditions can be complicated, since they can involve third derivatives of u and c, in models with bargaining. We simply assume V is strictly concave for now (but see LW for assumptions to guarantee this); we check it numerically in the calibrated models below. With price taking, on the other hand, this is a non-issue: V is always concave.

Differentiation yields

$$\begin{aligned} V_m(m,k) &= \frac{A}{pw\,(1-t_h)} + \sigma\left[u'\frac{\partial q_b}{\partial m} - \frac{A}{pw\,(1-t_h)}\frac{\partial d_b}{\partial m}\right] \\ &+\sigma\left[\frac{A}{pw\,(1-t_h)}\frac{\partial d_s}{\partial m} - c_q\frac{\partial q_s}{\partial m}\right] \end{aligned} \tag{9}$$

$$\begin{aligned} V_k(m,k) &= \frac{A\left[1+(r-\delta)\,(1-t_k)\right]}{w\,(1-t_h)} + \sigma\left[u'\frac{\partial q_b}{\partial k} - \frac{A+A\,(r-\delta)\,(1-t_k)}{w\,(1-t_h)}\frac{\partial d_b}{\partial k}\right] \\ &+\sigma\left[\frac{A+A\,(r-\delta)\,(1-t_k)}{w\,(1-t_h)}\frac{\partial d_s}{\partial k} - c_q\frac{\partial q_s}{\partial k} - c_k\right]. \end{aligned} \tag{10}$$

It remains to specify how the terms of trade (q,d) are determined, so that we can substitute for the derivatives in (9) and (10); this will differ across versions of the model considered below. Before pursuing equilibrium, however, consider the planner's problem in an economy *without* anonymity, so that money is not essential:

$$\begin{aligned} J(K) &= \max_{q,X,H,K_{+1}} \left\{U(X) - AH + \sigma\left[u(q) - c(q,K)\right] + \beta J_{+1}(K_{+1})\right\} \\ s.t.\ X &= F(K,H) + (1-\delta)K - K_{+1} - G \end{aligned} \tag{11}$$

Eliminating X, and again assuming interiority, we have the FOC:

$$\begin{aligned} q &: u'(q) = c_q(q,K) \\ H &: A = U'(X)F_H(K,H) \\ K_{+1} &: U'(X) = \beta J'_{+1}(K_{+1}) \end{aligned} \tag{12}$$

From the envelope condition $J'(K) = U'(X)[F_K(K,H) + 1 - \delta] - \sigma c_k(q,K)$, we get

$$U'(X) = \beta U'(X_{+1})[F_K(K_{+1},H_{+1}) + 1 - \delta] - \sigma\beta c_k(q_{+1},K_{+1}). \tag{13}$$

Using the first condition in (12), given K we have $q = q^*(K)$ where $q^*(K)$ solves $u'(q) = c_q(q,K)$. Then the paths for (K_{+1},H,X) satisfy the the Euler equation (13), the second equation in (12), and the constraint in (11). These are all fairly standard, except for the presence of the term $-\beta\sigma c_k(q_{+1},K_{+1}) > 0$ in (13), which reflects the fact that in general investment not only affects CM but also DM productivity. If K did not appear in $c(q)$, this term would vanish and the system would dichotomize: we can first set $q = q^*$, where q^* solves $u'(q) = c'(q)$, and then solve the other conditions independently for (K_{+1}, H, X). In general, however, we need to solve the conditions simultaneously.

2.2 Bargaining

Suppose each agent with a desire to consume in the DM is paired with one who can produce. Since buyers are anonymous, trade must be quid pro quo, and here this means they must pay with money. Let the buyer's and seller's states be (m_b, k_b) and (m_s, k_s). Then the terms of trade (q, d) solve the generalized Nash solution, with bargaining power for the buyer given by θ and threat points given by continuation values. The buyer's payoff from the trade is $u(q) + W(m_b - d, k_b)$ and his threat point $W(m_b, k_b)$, so (3) implies his surplus is $u(q) - Ad/pw\,(1 - t_h)$. Similarly, the seller's surplus is $Ad/pw\,(1 - t_h) - c(q, k_s)$. Hence our bargaining solution is

$$\max_{q,d} \left[u(q) - \frac{Ad}{pw\,(1 - t_h)}\right]^{\theta} \left[\frac{Ad}{pw\,(1 - t_h)} - c(q, k_s)\right]^{1-\theta} \quad \text{s.t. } d \leq m_b.$$

As in LW, one can show that in any equilibrium $d = m_b$. This implies $q \leq q^*(k_s)$ where $q^*(k_s)$ is the solution to $u'(q) = c_q(q, k_s)$, and typically the inequality is strict.[10] In any case, inserting $d = m_b$ and taking the FOC with respect to q, we get

$$\frac{m_b}{p} = \frac{g(q, k_s)w\,(1 - t_h)}{A}, \tag{14}$$

where

$$g(q, k_s) \equiv \frac{\theta c(q, k_s)u'(q) + (1 - \theta)u(q)c_q(q, k_s)}{\theta u'(q) + (1 - \theta)c_q(q, k_s)}. \tag{15}$$

We write $q = q(m_b, k_s)$, where $q(\cdot)$ is given by solving (14) for q as a function of (m_b, k_s) (the dependence on prices and θ is implicit). Now one can compute $\partial d/\partial m_b = 1$, $\partial q/\partial m_b = A/pw\,(1 - t_h)\,g_q > 0$ and $\partial q/\partial k_s = -g_k/g_q > 0$ where

$$g_q = \frac{u'c_q[\theta u' + (1 - \theta)c_q] + \theta(1 - \theta)(u - c)[u'c_{qq} - c_q u'']}{[\theta u' + (1 - \theta)c_q]^2} > 0 \tag{16}$$

$$g_k = \frac{\theta u'c_k\,[\theta u' + (1 - \theta)c_q] + \theta(1 - \theta)(u - c)u'c_{qk}}{[\theta u' + (1 - \theta)c_q]^2} < 0, \tag{17}$$

[10] In models without capital, e.g., $q < q^*$ unless $\theta = 1$ and the nominal interest rate is 0.

while the other derivatives in (9) and (10) are 0.

Inserting these derivatives and imposing $(m,k) = (M,K)$, we can reduce (9) and (10) to

$$V_m(M,K) = \frac{(1-\sigma)A}{pw\,(1-t_h)} + \frac{\sigma A u'(q)}{pw\,(1-t_h)\,g_q(q,K)} \tag{18}$$

$$V_k(M,K) = \frac{A + A\,(r-\delta)\,(1-t_k)}{w\,(1-t_h)} - \sigma\gamma(q,K), \tag{19}$$

where it is understood that $q = q(M,K)$, and[11]

$$\gamma(q,K) \equiv \frac{c_k(q,K)g_q(q,K) - c_q\,(q,K)\,g_k(q,K)}{g_q(q,K)} < 0.$$

Substituting (18) and (19), as well as prices $p = AM/w\,(1-t_h)\,g(q,K)$, $r = F_K(K,H)$, and $w = F_H(K,H)$, into the FOC for m_{+1} and k_{+1} in (2), we get the equilibrium conditions

$$\frac{g(q,K)}{M} = \frac{\beta g(q_{+1},K_{+1})}{M_{+1}}\left[1-\sigma+\sigma\frac{u'(q_{+1})}{g_q(q_{+1},K_{+1})}\right] \tag{20}$$

$$\begin{aligned} U'(X) &= \beta U'(X_{+1})\left\{1 + \left[F_K(K_{+1},H_{+1}) - \delta\right](1-t_k)\right\} \\ &\quad -\sigma\beta\,(1+t_x)\,\gamma(q_{+1},K_{+1}). \end{aligned} \tag{21}$$

The other equilibrium conditions come from the FOC for X and the resource constraint,

$$U'(X) = \frac{A\,(1+t_x)}{(1-t_h)\,F_H(K,H)} \tag{22}$$

$$X + G = F(K,H) + (1-\delta)K - K_{+1}. \tag{23}$$

An *equilibrium* can now be defined as (positive, bounded) paths for (q, K_{+1}, H, X) satisfying (20)-(23), given policy and initial K_0.[12] When $M_{+1} = (1+\tau)\,M$ for fixed τ, a *steady state* is a constant solution (q,K,H,X) to (20)-(23). This means inflation equals τ, and if we

[11] The term $-\sigma\gamma(q,K)$ in (19) is the marginal value of capital in the DM, which in general depends on the bargaining solution, and in particular on θ. Note that $\gamma = c_k + c_q \partial q/\partial K$, where the first term is the cost saving from having more capital and the second is the cost increase from having to produce more when you have more capital. It is this second term that captures the capital holdup problem.

[12] We focus on monetary equilibria, where $q > 0$. A nonmonetary equilibrium satisfies $q = 0$ instead of (20), (21) with $\gamma = 0$, and (22)-(23), which are *exactly* the equilibrium conditions for the standard nonmonetary model (Hansen 1985). Also, as in LW, one can give a more comprehensive definition of equilibrium – including generalized descriptions of decision rules, payoffs, and distributions – but we see little gain from such pedantics here.

define ρ by $\beta = \frac{1}{1+\rho}$ and the nominal interest rate by the Fisher equation $i = (1+\rho)(1+\tau)-1$, in steady state (20)-(21) simplify to

$$\frac{i}{\sigma} = \frac{u'(q)}{g_q(q,K)} - 1 \tag{24}$$

$$\rho = [F_K(K,H) - \delta]\,(1-t_k) - \sigma\,(1+t_x)\,\frac{\gamma(q,K)}{U'(x)}. \tag{25}$$

Note for future reference that the price level in the DM is defined implicitly by $\tilde{p} = M/q$, which is to be contrasted with the price level p in the CM.

A very special case of this model is the specification in Aruoba and Wright (2003), where capital is not used in the DM, so $c(q,K) = c(q)$ and $\gamma(q,K) = 0$. That version dichotomizes: (20) determines a path for q, while (21)-(23) determine paths for (K_{+1}, H, X), independently. Hence, the path for M affects q but not (K_{+1}, H, X).[13] When the dichotomy prevails, many properties of this model are similar to one without capital, like LW. Thus, assuming a unique steady state, $\partial q/\partial i < 0$. Since $q < q^*$ for $i > 0$, welfare is maximized at the Friedman Rule $i = 0$. However, if $\theta < 1$, then $q < q^*$ even at $i = 0$. LW interpret this as a holdup problem with money demand: the buyer bears the cost of acquiring cash in the CM, but if $\theta < 1$ he must share the surplus that this cash generates in trade with the seller, which lowers the demand for money and hence q (see Rocheteau and Waller 2005 for more discussion).

The dichotomy does not hold when capital enters the DM cost function, since then K and q both appear in (20) and (21), so there is no way in general to solve for q independently of the other variables. In this case private investors, like the planner, not only take into account the fact that K affects productivity in the CM, but also in the DM (as we will see, however, this does not necessarily mean equilibrium investment is efficient). A change in monetary policy, by affecting q, thus affects investment. Intuitively, inflation reduces the return to trading in the DM, which affects the value of K in that market. Since K is also

[13] Although money does not affect CM consumption or investment at the individual level, it does affect individual employment. However, the effects cancel, so that it does not affect aggregate employment, and agents do not care about the effects on individual employment because utility is linear in h (see Aruoba and Wright 2003 for details). Money does affect welfare, however, since it affects q – i.e., dichotomy does *not* mean (super)neutrality.

used in the CM, this will impact on productivity, employment, output, and consumption in that market.

Notice, however, that even when K enters the DM production function, if $\theta = 1$ then $\gamma(q, K) = 0$. In this case the model is recursive, if not dichotomous: (21)-(23) can be solved for (K_{+1}, H, X); then, given K (20) determines q. So when $\theta = 1$, anything like fiscal policy that influences K will affect q, but there is no feedback in the other direction, and monetary policy still cannot influence investment, employment or consumption in the CM. Intuitively, when $\theta = 1$ sellers get none of the DM surplus, so they realize no cost savings from bringing extra capital to the DM and hence the investment decision is based solely on returns in the CM. This holdup problem in the demand for capital does not require $\theta = 1$; it applies whenever $\theta > 0$, as sellers underinvest unless they get the *full* return.[14]

The distortion described above is in addition to the usual inefficiencies that arise when $i > 0$ in monetary economies, the problem in money demand that arises with bargaining when $\theta < 1$, and the obvious problems associated with distorting taxes. If we ran the Friedman Rule ($i = 0$) and used lump sum taxes exclusively, we would be left with only the holdup problems. In some models all such problems can be resolved simultaneously if one simply sets θ correctly; see Hosios (1990) or, for a recent update, Rogerson et al. (2005). This is impossible here: $\theta = 1$ resolves the problem in the demand for money, but this is the worst case for investment; and $\theta = 0$ resolves the problem in the demand for capital, but this this is the worst case for money (it implies $q = 0$). With Nash bargaining, there is no θ that eliminates both problems.

[14]Holdup problems in investment in general are standard fare in microeconomics, but perhaps need more attention in macro. As Caballero and Hamour (1998) put it, "From a macroeconomic perspective, the prevalence of unprotected specific rents makes it a potentially central factor in determining the functioning of the aggregate economy." See also Caballero (1999), who says "the quintessential problem of investment is that is almost always sunk ... opening a vulnerable flank ... The problem is far more serious ... when the exposed flanks are largely controlled by economic agents with the will and freedom to behave opportunistically." Holdup problems are usually attributed to a lack of complete contracting, which makes perfect sense in search-based models, to the extent that it is not possible to contract before you contact someone.

2.3 Price Taking

With care, competitive price taking can be used in models like this, instead of bargaining (Rocheteau and Wright 2005). The CM is unchanged. The value function for the DM has the same form as (5), but (6) and (7) change. For a buyer,

$$V^b(m,k) = \max_q \ \{u(q) + W(m - \tilde{p}q, k)\} \text{ s.t. } \tilde{p}q \leq m, \tag{26}$$

where $\tilde{p}$ is the price level in the DM, now taken parametrically; and for a seller

$$V^s(m,k) = \max_q \{-c(q,k) + W(m + \tilde{p}q, k)\}. \tag{27}$$

Market clearing implies buyers and sellers choose the same q. Also, as in the bargaining version, $\tilde{p}q = m = M$, and so $q = M/\tilde{p}$ in equilibrium. The FOC from (27) is $c_q(q,k) = \tilde{p}W_m = \tilde{p}A/pw(1-t_h)$. Notice $\partial q/\partial k = -c_{qk}/c_{qq} > 0$, since $c_{qk} < 0$, as long as k is a normal input. Inserting $\tilde{p} = M/q$ into the FOC, we get the analog to (14) from the bargaining model:

$$\frac{M}{p} = \frac{qc_q(q,k)w(1-t_h)}{A} \tag{28}$$

Given this, the analogs to (18) and (19) are:[15]

$$\begin{aligned} V_m(m,k) &= \frac{(1-\sigma)A}{pw(1-t_h)} + \frac{\sigma u'(q)}{\tilde{p}} \\ V_k(m,k) &= \frac{A + A(r-\delta)(1-t_k)}{w(1-t_h)} - \sigma c_k(q,k) \end{aligned}$$

Inserting these into (2) yields the analogs to (20) and (21):

$$\frac{c_q(q,K)q}{M} = \frac{\beta c_q(q_{+1},K_{+1})q_{+1}}{M_{+1}}\left[1 - \sigma + \sigma\frac{u'(q_{+1})}{c_q(q_{+1},K_{+1})}\right] \tag{29}$$

$$\begin{aligned} U'(X) = {} & \beta U'(X_{+1})\{1 + [F_K(K_{+1},H_{+1}) - \delta](1-t_k)\} \\ & -\sigma\beta(1+t_x)c_k(q_{+1},K_{+1}) \end{aligned} \tag{30}$$

The other equilibrium conditions do not change, and are repeated here for convenience:

$$U'(X) = \frac{A(1+t_x)}{F_H(K,H)(1-t_h)} \tag{31}$$

$$X + G = F(K,H) + (1-\delta)K - K_{+1}. \tag{32}$$

[15] Notice that with price taking $V_{mm} = \sigma u''/\tilde{p}^2$, $V_{mk} = 0$ and $V_{kk} = -\sigma\left(c_{qq}c_{kk} - c_{qk}^2\right)/c_{qq}$.

Equilibrium is now given by (positive, bounded) paths for (q, K_{+1}, H, X) satisfying (29)-(32), given policy and K_0.

The difference between the bargaining and price-taking models is in the difference between (20)-(21) and (29)-(30). The first pair of equations differ because, in general, we do not have $g(q,K) = c_q(q,K)q$ and $g_q(q,K) = c_q(q,K)$, except in the special case where $\theta = 1$, which implies $g(q,K) = c(q,K)$, and $c(q,K)$ is linear in q. The second pair of equations differ because, in general, we do not have $\gamma(q,K) = c_k(q,K)$, unless $\theta = 0$. This suggests that the price-taking model avoids *both* holdup problems, as we now verify.

First, set $t_k = t_h = t_x = 0$. Then (30)-(32) are exactly the conditions for (K_{+1}, H, X) from the planner's problem in Section 2.1. If $q = q^*(K)$ also solves (29), then the first best is an equilibrium, where we recall that $q^*(K)$ solves $u'(q) = c_q(q,K)$. Substituting this into (29) yields

$$\frac{c_q(q,K)q}{M} = \frac{\beta c_q(q_{+1}, K_{+1})q_{+1}}{M_{+1}}.$$

Using (28) to eliminate $c_q(q,K)q$, this reduces to $1/pw = \beta/p_{+1}w_{+1}$. By virtue of $w = F_H(K,H) = A/U'(X)$, this can also be written $U'(X)/p = \beta U'(X_{+1})/p_{+1}$. Therefore, we arrive at

$$\frac{p_{+1}}{p} = \frac{\beta U'(X_{+1})}{U'(X)} = \frac{1}{1+r}, \tag{33}$$

where r is the equilibrium real interest rate between t and $t+1$.

We conclude from (33) that $q = q^*(K)$ solves (29) iff $1 = (1+\pi)(1+r) = 1+i$, where π is the inflation rate and i the nominal interest rate along the equilibrium path (and not only in steady state). Hence, if we run the Friedman rule $i = 0$, given we use only lump sum taxes, the monetary equilibrium under price taking coincides with the solution to the planner's problem. In particular, there are no holdup problems under price taking.[16]

[16] The argument in the text applies to the equilibrium path; less ambitiously but more easily, one could focus on steady states. The steady state conditions for q and K under price taking are:

$$\text{(a) } \frac{i}{\sigma} = \frac{u'(q)}{c_q(q,K)} - 1 \text{ and (b) } \rho = [F_K(K,H) - \delta](1-t_k) - \sigma(1+t_x)\frac{c_k(q,K)}{U'(X)}$$

Comparing this to bargaining, (a) is the same as (24) iff $c_q = g_q$, which holds iff $\theta = 1$, while (b) is the same

3 Extensions and Examples

Consider the following functional forms, which are the ones we ultimately calibrate below:

$$
\begin{aligned}
U(x) &= B\frac{x^{1-\varepsilon}-1}{1-\varepsilon} \\
u(q) &= \frac{(q+b)^{1-\eta}-1}{1-\eta} \\
F(K,H) &= K^{\alpha}H^{1-\alpha} \\
c(q,k) &= q^{\psi}k^{1-\psi}
\end{aligned}
\tag{34}
$$

The cost function comes from $\ell(e)=e$ and $q=e^{\chi}k^{1-\chi}$ where $0<\chi\leq 1$, so $\psi=1/\chi\geq 1$; when $\psi=1$ the model dichotomizes. The other parameters satisfy B, ε, η, $b>0$ and $0<\alpha<1$. The only nonstandard parameter is b, which guarantees $u(0)=0$ for all η; if $b\approx 0$ then u is approximately CRRA.

For ease of presentation, in this subsection we focus on pricing taking, and briefly mention bargaining at the end. With these functional forms (29)-(32) can be written:

$$
\frac{K^{1-\psi}}{q^{-\psi}} = \frac{\beta}{1+\tau}\left[(1-\sigma)\frac{K_{+1}^{1-\psi}}{q_{+1}^{-\psi}}+\sigma\psi(q_{+1}+b)^{-\eta}q_{+1}\right] \tag{35}
$$

$$
\frac{X_{+1}^{\varepsilon}}{X^{\varepsilon}} = \beta(1-t_k)\left[\alpha\left(\frac{K_{+1}}{H_{+1}}\right)^{\alpha-1}+1-\delta\right]-\frac{\sigma\beta(1+\tau_x)(1-\psi)}{B}\frac{X_{+1}^{\varepsilon}K_{+1}^{-\psi}}{q_{+1}^{-\psi}} \tag{36}
$$

$$
X = \left[\frac{B(1-\alpha)(1-t_h)}{A(1+t_x)}\frac{K^{\alpha}}{H^{\alpha}}\right]^{1/\varepsilon} \tag{37}
$$

$$
X = K^{\alpha}H^{1-\alpha}+(1-\delta)K-K_{+1}-G \tag{38}
$$

Let $\mathbb{k}=K/H$, and combine (38) and (37) to get

$$
\frac{\mathbb{k}}{K}\left[\frac{(1-\alpha)(1-t_h)}{A(1+t_x)}\mathbb{k}^{\alpha}\right]^{1/\varepsilon} = \mathbb{k}^{\alpha}+(1-\delta)\mathbb{k}+\mathbb{k}_{+1}-\frac{G}{K}\mathbb{k}.
$$

Hence, in steady state,

$$
K = \frac{\mathbb{k}^{1-\alpha}\left[\frac{(1-\alpha)(1-t_h)}{A(1+t_x)}B\mathbb{k}^{\alpha}\right]^{1/\varepsilon}}{1-(\delta+\frac{G}{K})\mathbb{k}^{1-\alpha}}. \tag{39}
$$

as (25) iff $c_k=\gamma$, which holds iff $\theta=0$ (the linearity of c in q is not relevant for steady state comparisons). If $i=0$ then (a) implies $q=q^*(K)$, and if $t_k=t_x=0$ then (b) implies K solves the steady state version of (13) from the planner's problem. Finally, if we also have $t_h=0$ then (30) reduces to the FOC for H from the planner's problem. Hence, the Friedman rule and lump sum taxation together imply the first best is the unique monetary steady state under price taking.

Given $b \approx 0$, (35)-(37) reduce to:

$$
\begin{aligned}
q &= \left[\frac{\sigma}{\psi(i+\sigma)}\right]^{\frac{1}{\psi+\eta-1}} K^{\frac{\psi-1}{\psi+\eta-1}} && (40)\\
X &= \left[\frac{(1-\alpha)(1-t_h)B}{A(1+t_x)}\mathbb{k}^{\alpha}\right]^{1/\varepsilon} && (41)\\
1 &= \beta\left[1+(\alpha\mathbb{k}^{\alpha-1}-\delta)(1-t_k)\right] && (42)\\
&\quad + \tfrac{(\psi-1)\sigma\beta(1-\alpha)(1-t_h)}{A}\left[\tfrac{\sigma}{\psi(i+\sigma)}\right]^{\frac{\psi}{\psi+\eta-1}}\mathbb{k}^{\frac{\alpha(\psi+\eta-1)-(1-\alpha)\psi\eta}{\psi+\eta-1}}\left\{\frac{1-(\delta+G/K)\mathbb{k}^{1-\alpha}}{\left[\frac{(1-\alpha)(1-t_h)B}{A(1+t_x)}\mathbb{k}\right]^{1/\varepsilon}}\right\}^{\frac{\psi\eta}{\psi+\eta-1}}
\end{aligned}
$$

Notice (42) is one equation in $\mathbb{k}$. The RHS approaches ∞ as $\mathbb{k} \to 0$ and approaches a value less than 1 as $\mathbb{k} \to [1-(\delta+G/K)]^{1/(1-\alpha)}$. Hence it has a solution. The solution is unique if we assume $\alpha(\psi+\eta-1) < (1-\alpha)\,\psi\eta$, since then the RHS is strictly decreasing. Given $\mathbb{k}$, (39) yields K, (40) yields q, (41) yields X, and $H = \mathbb{k}/K$. So we have existence, uniqueness under a simple condition, and an easy solution method, for steady state.[17]

3.1 Extension: Two Capital Goods

We now consider some extensions to show that there are several other ways to break the dichotomy.[18] First, we generalize the assumption that the same stock of capital k is used in both markets. Suppose there are two types of capital, k used in the CM and z used in the DM, which depreciate at rates δ and ω. Although they are used as inputs in different markets, production of both k and z occurs in the CM here, and following the approach in the baseline model neither k nor z can be used as a medium of exchange. Also, for the sake of illustration, there is no tax on z, and we only consider the bargaining version (price-taking is similar).

[17] Things are slightly harder when we use bargaining instead of price taking, since we cannot get a closed form for q as a function of $\mathbb{k}$, but it is still easy to solve numerically.

[18] We think it is important to know that, although we mainly pursue one baseline model, the general ideas are robust – but readers who are more interested in quantitative results in the baseline version than in theoretical generalizations could skip to the next section without loss of continuity.

The problem in the CM is now

$$\begin{aligned}
W(m,k,z) &= \max_{x,h,m_{+1},k_{+1},z_{+1}} \{U(x) - Ah + \beta V(m_{+1},k_{+1},z_{+1})\} \\
\text{s.t. } (1+t_x)\,x &= w\,(1-t_h)\,h + [1+(r-\delta)\,(1-t_k)]\,k - k_{+1} - T + \frac{m-m_{+1}}{p} \\
&\quad + (1-\omega)\,z - z_{+1}.
\end{aligned}$$

Eliminating h using the budget equation, we have the FOC:

$$\begin{aligned}
x &: U'(x) = \frac{A\,(1+t_x)}{w\,(1-t_h)} \\
m_{+1} &: \frac{A\,(1+t_x)}{pw\,(1-t_h)} = \beta V_m(m_{+1},k_{+1},z_{+1}) \\
k_{+1} &: \frac{A}{w\,(1-t_h)} = \beta V_k(m_{+1},k_{+1},z_{+1}) \\
z_{+1} &: \frac{A}{w\,(1-t_h)} = \beta V_z(m_{+1},k_{+1},z_{+1}).
\end{aligned}$$

The envelope conditions for W_m, W_k and W_z are derived in the obvious way. The usual logic implies the distribution of (m,k,z) is degenerate for agents leaving the CM.

The DM is as before, except we replace $c(q,k)$ with $c(q,z)$ and $g(q,k)$ with $g(q,z)$. The value function in the DM and the envelope conditions for V_m, V_k and V_z are derived in the obvious way. This leads to:

$$g(q,Z) = \frac{\beta g(q_{+1},Z_{+1})}{1+\tau}\left[1-\sigma+\sigma\frac{u'(q_{+1})}{g_q(q_{+1},Z_{+1})}\right] \tag{43}$$

$$U'(X) = \beta U'(X_{+1})\,\{1+[F_K(K_{+1},H_{+1})-\delta]\,(1-t_k)\} \tag{44}$$

$$U'(X) = \beta U'(X_{+1})\left[1-\omega-\frac{(1+t_x)\,\sigma\gamma(q_{+1},Z_{+1})}{U'(x_{+1})}\right] \tag{45}$$

$$U'(X) = \frac{A\,(1+t_x)}{F_H(K,H)\,(1-t_h)} \tag{46}$$

$$X+G = F(K,H)+(1-\delta)\,K-K_{+1}+(1-\omega)\,Z-Z_{+1} \tag{47}$$

An equilibrium is now given by (positive, bounded) paths for (q,K_{+1},Z_{+1},H,X) satisfying (43)-(47).

Notice (43) is equivalent to (20), except Z replaces K. Also, (44) is the standard condition for K from the one-sector growth model: in contrast to (21), γ is not in (44), but now it shows up in (45). Still, the model does not dichotomize because Z is used in the DM and produced in the CM. An increase in i affects q and Z, and this generally must affect something in the CM. At $\theta = 1$ and $i = 0$, we get the efficient q conditional on Z, but when $\theta = 1$ we actually have $Z = 0$ because sellers get no surplus in the DM.[19] In any case, it is clear that this model is similar to the version with a single capital good, and since the latter is simpler we revert to it in what follows.

3.2 Extension: Capital Produced in DM

So far all investment occurs in the CM. Since it has been known since Stockman (1981) that it can make a difference if cash is needed to buy k, we now consider the alternative where k is acquired in the DM. For the sake of illustration, as in Shi (1999), we assume agents do not consume the output of the DM, but use it as an intermediate input that is transformed one-for-one into k, which is then an input to CM production: each period a fraction σ of agents can produce this intermediate input, the same fraction can transform it into capital, and $1 - 2\sigma$ can do neither.

The CM problem is now

$$\begin{aligned} W(m,k) &= \max_{x,h,m_{+1}} \{U(x) - Ah + \beta V(m_{+1},k)\} \\ \text{s.t. } (1+t_x)\,x &= w\,(1-t_h)\,h + [1 + (r-\delta)\,(1-t_k)]\,k - T + \frac{m - m_{+1}}{p}. \end{aligned}$$

The FOC are

$$x \quad : \quad U'(x) = \frac{A\,(1+t_x)}{w\,(1-t_h)} \tag{48}$$

$$m_{+1} \quad : \quad \frac{A}{pw\,(1-t_h)} = \beta V_m(m_{+1},k). \tag{49}$$

The envelope conditions are still given by (3) and (4). Since k is obtained in the DM, there is a distribution of k across agents, say $\Phi_k(k)$. Since the FOC for m_{+1} is not independent

[19] This extreme form of the holdup problem does not arise in the benchmark model because the same K is used in both markets.

of k it is not obvious that the distribution of m_{+1}, call it $\Phi_m(m_{+1})$, is degenerate. We now show that Φ_m is degenerate.

Assuming bargaining, the buyer gives up d units of money and acquires q units of intermediate goods which yields $k - k_{-1} = q$ additional units of capital for the CM. The usual methods imply (q, d) is independent of (m_s, k_b, k_s), $d = m_b$, and

$$\frac{m_b}{p} = g(q) \equiv \frac{[\theta c(q) + (1-\theta)qc'(q)]\left[1 + (r-\delta)(1-t_k)\right]}{\theta\left[1 + (r-\delta)(1-t_k)\right]A/w + (1-\theta)c'(q)}.$$

Also,

$$\begin{aligned} V(m,k) &= W(m,k) + \sigma\left[\frac{A + A(r-\delta)(1-t_k)}{w(1-t_h)}q(m) - \frac{Am}{pw(1-t_h)}\right] \\ &\quad +\sigma\int\left\{\frac{\tilde{d}(\tilde{m})A}{pw(1-t_h)} - c\left[q(\tilde{m})\right]\right\}d\Phi_m(\tilde{m}), \end{aligned}$$

where we integrate with respect to to Φ_m. Hence

$$V_m(m,k) = \frac{A}{pw(1-t_h)}\left[1 - \sigma + \sigma\frac{1 + (r-\delta)(1-t_k)}{g'(q)}\right].$$

Since $V_m(m,k)$ is independent of the buyer's k, the choice of m_{+1} in the CM is the same for everyone, by (49). Again, Φ_m is degenerate, whether or not Φ_k is.[20]

Following the usual procedure, we arrive at:

$$\frac{g(q)}{F_H(K,H)} = \frac{\beta g(q_{+1})}{(1+\tau)F_H(K_{+1},H_{+1})}\left\{1 - \sigma + \sigma\frac{1 + [F_K(K_{+1},H_{+1}) - \delta](1-t_k)}{g_q(q_{+1},r_{+1},w_{+1})}\right\} \quad (50)$$

$$K_{+1} = (K + \sigma q)(1-\delta) \quad (51)$$

$$U'(X) = \frac{A(1+t_x)}{F_H(K,H)(1-t_h)} \quad (52)$$

$$X + G = F(K,H) + (K+\sigma q)(1-\delta) \quad (53)$$

An equilibrium is now given by (positive, bounded) paths for (q, K_{+1}, H, X) satisfying (43)-(47). This system does not dichotomize, since H and K appear in (50). Intuitively, changing i affects the amount of intermediate goods traded in the DM, and hence K, not unlike the results in Stockman (1981). Although it may be interesting to pursue this line, for simplicity we revert to the baseline model for the rest of this paper.[21]

[20] As always, we do require interior solutions.

[21] Implicitly we did not allow existing k to trade in the CM in this version of the model, but we now argue

3.3 Extension: Nonseparable Utility

Finally, we show how to break the dichotomy with a more general but still quasi-linear utility function, $\hat{U}(x,q,e) - Ah$. Although one can do it in a variety of ways, suppose that x interacts with the (q,e) brought in from the *previous* DM, so the latter are state variables in the current CM. To isolate the effects of nonseparable utility, assume k does not appear in the DM technology; then we can write $e = \xi(q) \equiv f^{-1}(q)$.

In this case, the CM problem is

$$\begin{aligned} W(m,k,q,e) &= \max_{x,h,m_{+1},k_{+1}} \left\{ \hat{U}(x,q,e) - Ah + \beta V(m_{+1},k_{+1}) \right\} \\ \text{s.t. } x &= wh + (1+r-\delta)k - k_{+1} - T + \frac{m - m_{+1}}{p}, \end{aligned}$$

where we shut off distorting taxes, merely to reduce notation. The FOC are:

$$\begin{aligned} x &: \hat{U}_x(x,q,e) = \frac{A}{w} \\ m_{+1} &: \frac{A}{pw} = \beta V_m(m_{+1},k_{+1}) \\ k_{+1} &: \frac{A}{w} = \beta V_k(m_{+1},k_{+1}) \end{aligned} \tag{54}$$

We again get a degenerate distribution of (m,k), but now there is a distribution of x in the CM, since this choice for an agent is affected by what happened in the DM. Let $x_s = x_s(q,w)$, $x_b = x_b(q,w)$ and $x_0 = x_0(w)$ be the choices of agents who were sellers, buyers and non-traders in the previous DM, where from the first condition in (54) these solve

$$\hat{U}_x[x_s(q,w), 0, \xi(q)] = \hat{U}_x[x_b(q,w), q, 0] = \hat{U}_x[x_0(w), 0, 0] = \frac{A}{w}.$$

that this is without loss of generality. Suppose agents can trade existing k, and let λ be the price. The FOC for k_{+1} is $A\lambda/w\,(1-t_h) = \beta V_k(m_{+1},k_{+1})$. Inserting $V_k = W_k$ leads to

$$\frac{\lambda}{F_H(K,H)} = \frac{\beta\lambda_{+1}\left\{1 + \left[F_k(K_{+1},H_{+1}) - \delta\right](1-t_k)\right\}}{F_H(K_{+1},H_{+1})}.$$

This is independent of individual k, and merely pins down the path for λ in the secondary market so that no arbitrage opportunities exist. Agents are indifferent to trading capital at this price, so the distribution Φ_k is not pinned down. Intuitively, this is because payoffs are linear in wealth (again we require interiority for the result).

Assume Nash bargaining. Then, by the usual logic, $d = m_b$ and q solves a version of (14) with $g(q, w)$ replacing $g(q, k)$, where $g(q, w)$ satisfies

$$\begin{aligned} \Upsilon(q,w)g(q,w) &= (1-\theta)\left\{\hat{U}\left[x_0(w),0,0\right]-\hat{U}\left[x_b(q,w),q,0\right]\right\}\hat{U}_e\left[x_s(q,w),0,\xi(q)\right]\xi'(q) \\ &\quad +\theta\left\{\hat{U}\left[x_0(w),0,0\right]-\hat{U}\left[x_s(q,w),0,\xi(q)\right]\right\}\hat{U}_q\left[x_b(q,w),q,0\right] \\ &\quad +(1-\theta)\frac{A}{w}\left[x_b(q,w)-x_0(w)\right]\hat{U}_e\left[x_s(q,w),0,\xi(q)\right]\xi'(q) \\ &\quad +\theta\frac{A}{w}\left[x_s(q,w)-x_0(w)\right]\hat{U}_q\left[x_b(q,w),q,0\right], \end{aligned} \tag{55}$$

with $\Upsilon(q, w) \equiv \theta U_q [x_b(q, w), q, 0] - (1 - \theta) U_e [x_s(q, w), 0, \xi(q)] \xi'(q) > 0$. This appears onerous, but it simplifies a lot in some cases. Of course, if $\hat{U} = U(x) + u(q) - \ell(e)$ is separable, then $g(q, w) = g(q)$, and we are back to a model that dichotomizes. In the intermediate case $\hat{U} = \tilde{U}(x, q) - \ell(e)$, where we can write $c(q) = \ell[\xi(q)]$ because e and q enter separably, the RHS of (55) reduces to

$$\theta c(q)\tilde{U}_q\left[x_b(q,w),q\right]+(1-\theta)\left\{\tilde{U}\left[x_b(q,w),q\right]-\tilde{U}\left[x_0(w),0\right]+\frac{A}{w}\left[x_0(w)-x_b(q,w)\right]\right\}$$

and $\Upsilon(q, w) = \theta\tilde{U}_q [x_b(q, w), q] - (1 - \theta) c'(q)$. Alternatively, for any $\hat{U}$, if $\theta = 1$,

$$g(q,w)=U\left[x_0(w),0,0\right]-U\left[x_s(q,w),0,\xi(q)\right]+\frac{A}{w}\left[x_s(q,w)-x_0(w)\right]. \tag{56}$$

In any case, the usual methods lead to the equilibrium condition

$$g(q,w)=\frac{\beta g(q_{+1},w_{+1})}{1+\tau}\left[1-\sigma+\sigma\frac{U_q\left[x_b(q_{+1},w_{+1}),q_{+1},0\right]}{g_q(q_{+1},w_{+1})}\right].$$

It is clear that q cannot be determined independently of $w = F_H(K, H)$, unless $\hat{U}$ is separable. This version can be interesting in some applications (e.g. Rocheteau et al. 2005). However, for most of what follows we return to the case of $\hat{U} = U(x) + u(q) - \ell(e)$, and pursue the impact of breaking the dichotomy by having k enter the DM technology in the baseline model.[22]

[22] There is one detail to mention in the nonseparable-utility model, which is the distribution of x and h

4 Quantitative Analysis

4.1 Accounting

In order to calibrate the model, we first need to do some simple accounting. The price levels in the CM and DM are p and $\tilde{p} = M/q$, respectively, where p satisfies

$$p = \frac{AM}{(1-t_h)\, g(q,K)\, F_H(K,H)} \tag{57}$$

in the bargaining version of the model, by (14), and

$$p = \frac{AM}{(1-t_h)\, qc_q(q,K)\, F_H(K,H)} \tag{58}$$

in the price-taking version, by (28). Nominal outputs of the two markets are σM and $pF(K,H)$. We use p as the unit of account by which we convert all nominal variables into real terms. Hence, real output is $Y = \sigma M/p + F(K,H)$, and the share of output produced in the DM is $s_D = \sigma M/pY$.

Define the markup μ by equating $1+\mu$ to the ratio of price to marginal cost. The markup in the CM market is always 0, since it is competitive. The markup in the DM under price taking is also 0. With bargaining, however, the markup in the DM is derived as follows. Marginal cost in terms of utility is $c_q(q,K)$. Due to quasi-linearity, a dollar is always worth $A/p(1-t_h)w$ utils, so marginal cost in dollars is $c_q(q,K)\,p(1-t_h)\,w/A$. Since $\tilde{p} = M/q$, the DC markup μ_D is given by

$$1+\mu_D = \frac{M/q}{c_q(q,K)\,p(1-t_h)\,w/A} = \frac{g(q,K)}{qc_q(q,K)}, \tag{59}$$

after eliminating M using (57). The aggregate markup is $\mu = s_D\mu_D$.

across individuals. Consider a steady-state, which implies

$$\begin{aligned} \frac{i}{\sigma} &= \frac{U_q\{x_b[q,F_H(K,H)],q,0\}}{g_q[q,F_H(K,H)]} - 1 \\ \rho &= F_K(K,H) - \delta \\ X &= F(K,H) - \delta K. \end{aligned}$$

Here aggregate CM consumption is given by $X = \sigma x_b(q,w) + \sigma x_s(q,w) + (1-2\sigma)\,x_0(w)$ with individual x_j given by (56). Then aggregate CM employment is $H = \sigma h_s + \sigma h_b + (1-2\sigma)\,h_0$ with individual h_j given by the budget equation.

We will also discuss certain elasticities in the quantitative analysis, which are derived in the model in standard fashion. For example, what is referred to as the interest elasticity of money demand is given by $\xi = \frac{\partial(M/p)}{\partial i}\frac{i}{M/p}$. Consider the bargaining model (the price-taking version is similar). Inserting $M/p = (1 - t_h)\, g\,(q, K)\, F_H(K, H)/A$ from (57) and differentiating, we get

$$\xi = \frac{\partial\,(M/p)}{\partial i}\frac{i}{M/p} = \left(g_q\frac{\partial q}{\partial i} + g_k\frac{\partial K}{\partial i}\right)\frac{i}{g} + \left(F_{HH}\frac{\partial H}{\partial i} + F_{HK}\frac{\partial K}{\partial i}\right)\frac{i}{F_H}. \tag{60}$$

It is now a matter of substituting $\partial q/\partial i$, $\partial K/\partial i$ and $\partial H/\partial i$, which we derive in the Appendix, to yield ξ as a function of the allocation and parameters.

4.2 Steady State Calibration

We now describe our calibration strategy, using the functional forms from Section 3.1. Beginning with preferences, we first set β to match a real interest of $r = 0.035$.[23] Now recall

$$U(x) = B\frac{x^{1-\varepsilon} - 1}{1-\varepsilon} \text{ and } u(q) = \frac{(q+b)^{1-\eta} - 1}{1-\eta}.$$

We set $b = 0.0001$ so that the utility of consumption in the DM is approximately CRRA, as it is in the CM. As a benchmark we set $\varepsilon = \eta = 1$, mainly to facilitate comparison with previous studies, but we check robustness with respect to this choice below.[24] The remaining preference parameters are the weight on CM consumption B, and the weight on CM hours A, which are determined as described below.

Moving to policy parameters, we can directly observe the average inflation rate $\tau = 0.036$ as the average annual change in GDP deflator. We can also directly observe taxes. We use $t_h = 0.242$ and $t_k = 0.548$, the average effective marginal rates in McGrattan et al. (1997). We then set t_x to the average of excise plus sales tax over consumption expenditure, which

[23] When we refer to the data, we mean annual U.S. data from 1951-2004. Our real interest rate is computed from an average nominal rate on Aaa-rated corportate bonds of 7.2% and an average inflation rate (changes in the GDP deflator) of 3.6% over this period. We choose the period length to be a year, for now, but change this below.

[24] Also, $\varepsilon = \eta = 1$ is a good benchmark because this is what we require for balanced growth in a generalized version of the model with technical change (details available upon request).

we measure to be 0.069. The other policy parameter is G; although we can observe G/Y directly, since Y is endogenous, we solve for G as described below.

Moving to technology, we set depreciation to $\delta = 0.07$, matching observed I/K, where capital is measured as residential and nonresidential structures, plus producer equipment and software, but not consumer durables or inventories.[25] We then set the coefficient α in the CM production function to match labor's share in the data, which we measure as $LS = 1 - \alpha = 0.712$ using the method in Prescott (1986); note that we will use this same method to compute LS in the model (basically, we treat DM income in the model like proprietor's income in the data, which is split into labor income and capital income according to α, although this does not matter much for the results). This leaves us with the coefficient ψ in the DM cost function, the probability σ of being a consumer or a producer in the DM, and (in bargaining models) the buyer's share θ.

Table 1 partitions parameters into two groups: ones we have already set based on 'obvious' observations, as discussed above; and six yet to be determined, with six other observations that we now discuss. First is average hours worked, as a function of discretionary time, which as is standard we set to $H = 1/3$ (Juster and Stafford 1991). Second is average velocity, which is $v = M/pY = 5.76$ when we measure M by $M1$.[26] Third is $G/Y = 0.25$. Fourth is $K/Y = 2.32$. Fifth is the elasticity of M/p with respect to i, which we estimate to be $\xi = -0.226$, as discussed below. Last is the markup, which we set to $\mu = 0.10$ (Basu and Fernald 1997). Our method is to set the six remaining parameters simultaneously to minimize the distance between these targets in the data and in the model.

Table 1 - Calibration Parameters and Targets

[25] As is common practice we remove durable goods and net exports from all measurements. As such, our measure of consumption is defined as private nondurable consumption and services expenditures, investment is private fixed investment and output is the sum of the two plus government consumption expenditures and gross investment.

[26] As mentioned in Section 2, it would seem that one can rewrite the model by introducing banks and having agents in the DM pay with either cash or checks, following He et al. (2005). This suggests that it is reasonable to use $M1$ as an empirical notion of money. In any case, we check robustness to using other measures.

(i) 'Obvious' Parameters

Parameters	β	b	ε	η	t_h	t_k	t_x	δ	α
Targets	0.966	0.0001	1	1	0.242	0.548	0.069	0.070	0.288

(ii) Remaining Parameters

Parameters	A	B	G	ψ	σ	θ
Targets	H	v	G/Y	K/Y	$-\xi$	μ
Target Values	0.33	5.76	0.25	2.32	0.226	0.10

Before presenting results, we point out that our approach is fairly standard, with perhaps two exceptions. For one thing, we target K/Y, even though we have already used LS to pin down α. In the one-sector growth model, given β, δ and t_k, α and K/Y are tied together through the steady-state condition $\alpha Y/K = \delta + (1-\beta)/\beta(1-t_k)$. This is not true given the DM, as long as $\psi > 1$. The bigger is ψ, the greater are the returns to investing, and the idea is that ψ is determined by K/Y. This works well because we have taxes in the model, and as is well known, given α matches LS and t_k is set realistically, K/Y tends to be too low (see e.g. Greenwood et al. 1995). In principle, the extra return on K generated by the DM can offset the impact of t_k, for the right value of ψ.

The other issue is the interest elasticity of money demand, ξ. Following a common specification in the literature (see e.g. Goldfeld and Sichel 1990), we specify log real money demand ($\tilde{m}_t$) as a linear function of log nominal interest ($\tilde{\imath}_t$) and log real output ($\tilde{y}_t$), allowing for first-order autocorrelation in the residuals. Due to nonstationarity, we estimate this in first differences.[27] This leads to

$$\Delta\tilde{m}_t = \beta_y \Delta\tilde{y}_t + \beta_i \Delta\tilde{\imath}_t - \rho\beta_y\Delta\tilde{y}_{t-1} - \rho\beta_i\Delta\tilde{\imath}_{t-1} + \rho\Delta\tilde{m}_{t-1} + \zeta_t \tag{61}$$

$$\beta_y = 0.369 \;\; (0.124)\,,\;\; \beta_i = -0.226 \;\; (0.045)\,,\; \rho = 0.347 \;\; (0.131) \;\; R^2 = 0.423$$

where ρ is the AR(1) coefficient for the residuals in the original equation in levels and the numbers in parentheses are standard errors. The estimated long-run interest elasticity is $\xi = -0.226$, with a relatively small standard error of 0.05. We match this to the theoretical

[27] Since we first difference logs our estimating equation is in growth rates, but one can still characterize the relevant parameter restrictions from the original specification.

long-run elasticity, given e.g. by (60) in the bargaining model. The fit of this money demand regression is shown in Figure 1.

4.3 Solving for Decision Rules

The above discussion about calibration concerns steady states. For much of what we want to do, we need to go beyond this and solve for equilibrium decision rules, because we want to be able to analyze transitions between steady states after a policy change. This subsection briefly describes our method for computing the decision rules.

As is standard, we begin by scaling all nominal variables by the aggregate money stock, so that $\hat{m} = m/M$, $\hat{p} = p/M$, etc. Then the individual state variable becomes $(\hat{m}, k, K)$. In equilibrium, $\hat{m} = 1$ and $k = K$. A recursive equilibrium is then described by time-invariant functions $[q(K), K_{+1}(K), H(K), X(K)]$, solving (20)-(23) for the bargaining version or (29)-(32) for the price-taking version, plus value functions $[W(K), V(K)]$ solving versions of (1) or (8). We solve these equations numerically, using a nonlinear global approximation, which is especially important for accurate welfare computations.[28] Figure 3 plots decision rules and value function for a typical parametrization. Notice they are quite nonlinear, and again this can be important for the results.[29]

For expositional purposes, let $[q^{\pi}(K), K^{\pi}_{+1}(K), H^{\pi}(K), X^{\pi}(K)]$ and $[W^{\pi}(K), V^{\pi}(K)]$ describe equilibrium, given government policy π. A steady state solves $K^{\pi} = K^{\pi}_{+1}(K^{\pi})$. Generally, there is a non-trivial transition path after a change in π. When we change policy from π_1 and π_2, our welfare comparison is between $W^{\pi_1}(K^{\pi_1})$ and $W^{\pi_2}(K^{\pi_1})$; i.e. we look at utility under the new policy starting at the old steady state. For computing welfare, we use a standard consumption-equivalent measure: we compute the number Λ such that going from policy π_1 to policy π_2 makes agents just as well off as staying at π_1 and changing (both CM and DM) consumption by a factor Λ.

[28] Specifically, we use the Weighted Residual Method with Chebyshev Polynomials and Orthogonal Collocation. See Judd (1992) for details; see Aruoba et al. (2005) for a recent discussion of different solution methods including this method.

[29] We plot the decision rules for $\pm 99\%$ of the steady state of K, which is around 0.85 for his parameterization. In simulations, the economy remains within roughly $\pm 15 - 20\%$ of the steady state.

5 Results

5.1 Main Results

5.1.1 Calibration

The results of our calibration are reported in Table 2. The first column repeats the values of various moments that we use in the calibration. As a benchmark, the next two columns report results when we simply fix $\psi = 1$ and pick the remaining parameters to match all moments, which is the special case in which the model dichotomizes discussed in Aruoba and Wright (2003). The first column gives up on the markup μ and fixes $\theta = 1$, while the second includes μ as a target and calibrates θ. In the first column, with $\theta = 1$, because there is no holdup problem with money demand and the model dichotomizes, the results are identical with price taking (but only because $\psi = 1$). The other columns include ψ as a calibrated parameter. The next column fixes $\theta = 1$, the next one calibrates θ and targets μ, and the final column uses price taking, again giving up on μ. In each of these cases the model matches most of the targets well, with the exception of K/Y.

The bargaining models are not as good on K/Y as price-taking. The former gets a value of 1.87 while the latter matches the target exactly. As we explained above, we are able to include both LS and K/Y as targets due to the $-\sigma\,(1 + t_x)\,\gamma(q, K)/U'(X)$ term in (25) for the bargaining version and the $-\sigma\,(1 + t_x)\,c_k(q, K)/U'\,(X)$ term we would get imposing steady state in (30) for the price-taking version. When we fix $\psi = 1$, these terms are equal to zero. Since we fix α to match LS directly, this pins down the value for K/Y, which turns out to be lower than what is in the data. This is why the first two versions of the model is unable to match K/Y. In the next three versions, ψ is calibrated and all calibrated values are greater than unity. For the bargaining versions (columns 3 and 4), having $\psi > 1$ helps increase K/Y slightly but this change is smaller than 0.01 and is not reflected in the table. In these versions the extra term in the right-hand side, which is the marginal benefit of capital in the DM, is very small, and almost invariant to ψ due to the investment hold up problem. This is because the benefit of holding extra capital is split between the buyer and the seller (the agent who holds the capital) and the calibrated value of θ gives a fairly large share to the former and the seller chooses not to respond to changes in inflation as much as he would in the absence of the holdup problem. However, in the price-taking version, we are able to match K/Y exactly since changing ψ effects the benefit of capital in the DM to a much greater extent as there is no holdup problem.[30] Of course, the price-taking version misses the markup μ completely.

When we do calibrate to the markup μ in the bargaining versions, θ comes out to be around 3/4. Notice that in the third column we actually get a negative markup, which would be strange in competitive model but is no problem in a bargaining model; because $\theta = 1$ in this column, the consumers are making take it or leave it offers, so price equals average cost which exceeds marginal cost.[31] When ψ is calibrated, the value is around 1.8 in the bargaining models and 2.8 under price-taking; the data want a model that does not dichotomize.

In order to assess our calibration we also compute some statistics that we do not calibrate to in any of the cases. In particular, we look at the elasticity of investment and output with respect to inflation. Using quarterly data, we estimate them as -0.023 and -0.004, respectively where the former is statistically significant while the latter is not.[32] As such, we focus primarily on the former. This number may appear small, but if one thinks about it correctly it is not: doubling inflation rate from our benchmark value of 0.035 to 0.7 could cause aggregate investment to fall by 2%, which is nothing to scoff at. The elasticity of investment with respect to inflation implied by the first three columns is identically 0, since the first two dichotomize, and while in the third case there is feedback from the CM to q there is no feedback from the DM or monetary policy to the CM. In the versions with no dichotomy, there is a negative elasticity implied by the model. The effect in the bargaining model is weak (an elasticity of -0.001) and in the later it is too strong (an elasticity of

[30] Mathematically, we can show that $-\gamma(q, K) < -c_k(q, K)$ which means that K/Y will always be smaller in the bargaining version.

[31] This is not true in the first column because when $\psi = 1$ average cost equals marginal cost.

[32] These estimates are obtained in a similar way as (61).

-0.060).[33] This difference between the two versions is due to, as was the case for matching K/Y, the holdup problems. While inflation, which is a tax on DM activity, does reduce the incentive to invest, the effect is small. Of course, the effect depends on the calibrated curvature of the DM cost function, ψ. But this is the best the model can do on this dimension: If we choose ψ to make the effect as big as possible holding the other parameters fixed, or if we replace K/Y as a target with this elasticity, we end up at basically the same ψ and implied elasticity. The holdup problem simply chokes of this effect quantitatively.

In the price-taking version, there is no holdup problem, and the effect of inflation on investment is quite big. But this is not a serious issue: if we want to do better on this dimension, we can replace K/Y or LS as a target by the relevant elasticity and get it spot on at -0.023 with relatively little sacrifice in terms of other targets. So the price-taking version can do very well on this dimension but, of course, it cannot match the markup. What is certainly true, however, is that to the extent that one takes seriously a negative relation between I and τ, this is inconsistent with any model that dichotomizes, such as either version with $\psi = 1$.[34]

We also report the share of the DM in output s_D, which varies between 4.2% and 4.7%. We think these numbers are very reasonable, in the sense that we would be uncomfortable if the model predicted that anonymous bilateral trade was too big a share of GDP. Because s_D is relatively small, we need a big markup in the DM to match the average markup. One reason s_D is small is that the probability σ is relatively small, around 1/4.

We can also see how well the model matches up with the money demand data, which is a common, if somewhat informal way to proceed, used by Lucas (2000). Figure 2 shows the relationship on which Lucas (2000) focuses, $M/pY = 1/v$, versus i, in the data and in the model, for a typical parameterization.[35] Notice that, as is typical, it is not easy to fit

[33] Similar qualitative conclusions hold for the elasticity of output.

[34] In the baseline model, it is not possible to generate a *positive* relation between τ and output, investment, consumption or employment in the CM, but one could do so by considering the extension in the previous section to nonseparable utility.

[35] There are only very minor differences across different versions of the model in this dimension.

the low interest rate observations in the upper left part of the scatter plot (shown in a box) which are all from 1951-1960. Ignoring this decade, we think our money demand curve looks pretty good, about as good as those used by Lucas.[36]

5.1.2 Allocations and Welfare

Having reasonably calibrated the model, we turn to comparing allocations and welfare at different inflation levels.[37] Table 3 reports ratios of (q, y, Y, K, H, X) for each version of the model at two values of τ, 10% and the value corresponding to the Friedman rule, where $i = 0$.[38] In the versions of the model with $\psi = 1$, we see the dichotomy as DM variables are not affected. Moreover, the ratio for q is around 0.64 for these version, but when capital is used in the DM the ratio is closer to 0.78 in the bargaining models and 0.8 in the competitive model. Thus, inflation does not reduce DM output as much when capital is used to produce it. GDP goes down by around 2% with 10% in all columns but the last, where it actually goes down by 6%. This is because in this model the calibrated ψ is big – almost 3 – which means capital is very important in DM production. Since there is no holdup problem in the competitive economy, agents take this into account when they invest; with inflation the decline in q reduces K by over 10%. This has a big effect on CM output, while it barely changes for all the other versions.

The table also reports for each column ratios of (q, y, Y, K, H, X) at the equilibrium at 10% inflation and at the first best. Naturally, these numbers are relatively small, due to the large effect of distorting taxation in the model. Thus, in equilibrium Y and X are only 2/3 of their first best values, K and q are around 1/2 of the first best, and H is 3/4 of the first best. Again, these effects are mainly due to distorting taxation – one can calculate how much is due to taxation and how much to inflation by combining the results discussed in

[36] Furthermore, this specification assume a unit interest elasticity of money demand, which is often rejected by the data. Unfortunately, it is not possible to generalize this plot to allow for any elasticity since this requires matching the *level* of output in the model to that in the data which would be very difficult. Our current calibration only requires mathing the ratio of money balances to output.

[37] With our functional forms, the value function $V(.)$ is always concave and there is a unique equilibrium. We verify concavity for the bargaining case numerically everytime we compute an equilibrium.

[38] Here y is the output in the CM and Y is the real GDP.

this and the previous paragraphs.

In Figures 6 and 7 we plot the key allocations as we vary the interest rate from 0% to 100%, for the bargaining and the price-taking versions, respectively. We find that all our qualitative conclusions regarding 10% inflation are also valid for any interest rate in this range.

Next we turn to the welfare implications of reducing inflation from 10% to the Friedman rule.[39] In this section, we do the *usual* experiment where the lost revenue due to a lower seigniorage revenue is replaced by higher lumpsum taxes. We consider more complicated experiments – ones where the government is not able to change lump sum taxes – in the next section. In the first column, which means a model that dichotomizes and $\theta = 1$ (or equivalently with $\psi = 1$, competitive pricing), this is worth 0.7% of consumption. This is also true in the third column. These results are commensurate with what Lucas (2000) finds, presumably because these models do not have holdup problems. They are also directly comparable to the results in Cooley and Hansen (1991) who obtains a loss of 1% of output (which comes to almost exactly 0.7% of consumption) in a model with distortionary taxes where the only other inefficiency is the Friedman inefficiency.

In columns 2 and 4, in bargaining models calibrated to match the markup μ, we find that reducing inflation from 10% to the Friedman rule is worth around 3% of consumption. In column 5, which is competitive pricing and $\psi = 2.8$, it is worth closer to 2%, which is still big even though there is no holdup problems because of the way inflation affects K as discussed above.

Only in the last two columns is there an effect of τ on K, and hence only in these cases is there a nontrivial transition after a reduction in inflation. In particular, a reduction of inflation leads to a higher steady state level of capital, which is the source of the increased welfare. During the transition, however, welfare goes down because agents have to work and save more to build up K. Our results suggest that the benefit in the long run outweighs the

[39] As noted earlier, this is not a steady state comparison but we take the welfare during the transition in to account.

loss during the transition, which is quite sizable in the price-taking version.[40] To understand the source of the welfare loss during the transition, consider Figure 4 and Figure 5, where we plot the transition path of the variables of interest following a reduction of the inflation rate from 10% to the Friedman rule in period 1 for the parametrization in column 4 and 5, respectively. Unity on the y-axis corresponds to the initial steady state and 1.01 would show an increase of 1% compared to the steady state. All lines converge to the new steady state in about 35 periods for the bargaining version and 70 periods for the price taking version. We see that DM output immediately jumps up by a large amount and quickly converges to its new steady state value. Hours in the CM, on the other hand jump up initially and slowly go down to the new steady state value, just above the original one. Similarly, consumption in the CM jumps down by a small amount on impact before slowly converging to a new level. In order to accumulate the extra capital, agents work more and consume less initially. This is the source of the welfare loss during the transition. Comparing the two figures, it becomes clear that the welfare loss during the transition is larger in the price taking version because the amount of capital that needs to be accumulated in order to reach the new steady state is bigger.

Table 3 also reports the welfare gain of switching from a equilibrium at $\tau = 0.1$ to the first best, but only in terms of steady states (we do not compute the transition path to the first best because it cannot be supported as an equilibrium except for the price-taking version). These numbers are very big, between 21% and 46% but again this is mainly due to the effects of eliminating distortionary taxation. Similar results appear in nonmonetary models when distortionary taxation is eliminated – e.g. McGrattan et al. (1997) find it is worth around 30% of consumption.

It is important to note is that the presence of the holdup problems affect welfare very significantly. For example, the welfare loss of Friedman rule in the price taking version is 17.1%, which is simply the welfare loss due to taxes since in the absence of taxes, this

[40] To make this absolutely clear, note that the 1.8% figure we report is the difference between the long-run benefit (comparing steady states) of 3.5% and the short-run loss of 1.7%.

specification achieves the first best. When we add the investment holdup problem, i.e. when we look at column 3, the loss increases to 28.4%. We can interpret the difference between the two numbers, around 11% as the loss due to the investment holdup problem. Similarly, when we compare column 4 with column 3, the only thing that is added is the money demand holdup problem, and that creates a loss of about 13% of consumption. So, even at the Friedman rule, the holdup problems create an inefficiency which is close to twice that created by distortionary taxes.

Figures 8 and 9 plot these welfare measures for a wide grid between 0% and 100% for the bargaining and the price-taking versions, respectively. Once again, we find that all our qualitative conclusions are valid for these interest rates. In particular, we see that the loss during the transition is much smaller in the bargaining version than the price-taking version. Moreover, the welfare loss of inflation levels of at a level below 10% of consumption for the price-taking version and 20% for the bargaining version.

5.1.3 Fiscal Policy

The usual practice for computing welfare cost of inflation in the literature is to look at the case where the lost revenue due to less seigniorage is compensated by an increase in lumpsum taxes. Cooley and Hansen (1991) consider the case where lumpsum taxation is not feasible for the government and hence one or more proportional taxes should be used to make up for the lost revenue. They find that, at least for the values they consider, such a policy – replacing inflation with a distortionary tax – is not welfare improving. We repeat this exercise here. A priori, it seems that this conclusion may not be true in our model since the welfare gain of reducing inflation will be bigger than what Cooley and Hansen (1991) find in the context of their model and as such, it may outweigh the loss due to increased taxes.

Before turning to results, let us explain the experiment in some detail. For this experiment, we endow the government with the ability to issue bonds, denoted by B_t, that pay a real interest rate equal to the discount rate of the agents, ρ, which is the real interest rate of the economy. The new budget constraint for the government is given by

$$G + (1+\rho) B_t = \mathcal{T}_t + B_{t+1} \tag{62}$$

where $\mathcal{T}_t = T + t_h w_t H_t + t_k r_t K_t - \delta t_k K_t + t_x X_t + \tau M_t / p_t$ denotes total revenues. We assume that this borrowing (or lending if $B_t < 0$) is done with an entity outside the model, i.e. the agents do not hold these bonds. Just like a model of a consumer, we assume the government faces the borrowing constraint[41]

$$\lim_{t \to \infty} \frac{B_{t+1}}{(1+\rho)^t} = \lim_{t \to \infty} \beta^t B_{t+1} \leq 0 \tag{63}$$

which states that the present value of government borrowing in the limit is nonpositive.

We will conduct our experiments as follows. At time $t = -1$, the economy is at the steady state with $\tau = 10\%$ and all taxes as initially calibrated with $B_0 = 0$ and $K_0 = K_1^{SS}$. At time $t = 0$, the government announces that $\tau = 0\%$ and one of the tax rates, for example $t_h = t_h^2$, forever. We look for the new tax rate which brings the economy back to the steady state. In this new steady state we have

$$G + (1+\rho) B_2^{SS} = \mathcal{T}_2^{SS} + B_2^{SS} \tag{64}$$

which implies

$$B_2^{SS} = \frac{1}{\rho} \left(\mathcal{T}_2^{SS} - G \right) \tag{65}$$

This means that the government holds a constant amount of debt whose interest payments are just covered by the budget surplus every period. This will satisfy the borrowing constraint. Note that the government will accumulate some debt (or potentially accumulate some assets) during the transition and when it reaches the new steady state (in finite or infinite time) it will keep a constant amount of debt in perpetuity.[42]

[41] Note that this constraint is equivalent to the restriction $B_{t+1} \leq \mathcal{B} < \infty$.

[42] Note that in this setup we cannot ask the government to pay down the debt eventually since in the new steady state the economy will not have primary budget balance. Therefore, if the new steady state gives a primary budget surplus, the government will keep a constant amount of debt and if itgives a deficit, it will keep a constant amount of assets whose interest receipts are used to balance the budget.

The results of these experiments for changes in lumpsum taxes, labor income tax and consumption tax are reported on Table 4.[43] The first panel reports the results from the experiment where the lost revenue is replaced by an increase in lumpsum taxes. These results correspond to the welfare numbers reported in Table 3. The conclusion is, as discussed above, this policy always generates a welfare gain, and this gain is larger in the versions of the model with the holdup problems. Also note that this policy requires a smaller increase in the lumpsum taxes for the price-taking version compared to all other versions. [44]

When we look at the other two panels of the table, the numbers reflect the net effect of two changes: the change in inflation and the change in taxes. We can isolate the latter by looking at the difference between this net effect and the numbers in the panel for lumpsum taxes since they correspond to the effect of the change in inflation.[45] The conclusions for the labor income and consumption taxes are qualitatively identical so we focus on the labor income tax. We find that the introduction of increased labor income taxes reduce capital by about $3-4\%$ which lead to similar declines in labor supply and CM consumption. As a result is GDP is lower in all version of the model by about 2%, except for price-taking which increases by 1%. This is due to the fact that the reduction in inflation increases GDP by 4% and this outweighs the reduction due to increase taxes. As was the case above, the price-taking version requires a smaller increase in taxes: the labor income tax increase by 4 percentage points in all version except the price-taking version which has an increase of 3 percentage points. Finally, looking at the welfare results we see that for the version of the model where the welfare gain of reducing inflation was relatively small (columns 1 and 3)

[43]We could not solve for a new tax rate when we used capital income tax to make up for the lost revenue. In a nutshell, an increase in the capital income tax would lead to a significantly smaller level of capital and the tax revenue from that smaller economy would not cover the government's expenditures. The government would keep increasing taxes to increase revenue but this would have a very big effect on the size of the economy and that would outweigh the increase in revenue. Cooley and Hansen (1991) also report that this experiment is not feasible with capital income taxes and 10% inflation.

[44]The table shows that T/Y is negative for all models indicating a lumpsum subsidy. This subsidy is the smallest (2.56% of GDP) for the price-taking version since capital is much higher and therefore all other taxes provide enough revenue.

[45]For example, q_2/q_1 for the price-taking version with change in labor income tax reflects a 15% which is comprised of a 17% increase due to the change in inflation a 2% reduction due to increased taxes.

the net effect of this new policy is a welfare loss. In fact the numbers we find are very similar to what Cooley and Hansen (1991) find. Of course, these two cases are those without the money demand holdup problem (column 1 does not have investment hold up problem either) which maps exactly in to their setup. However, for all other cases, we find that this new policy is indeed welfare improving. We can get a net benefit of up to 0.5% of consumption.

The transition works in opposite directions for the bargaining and price-taking versions. Figures 9 and 10 report the transition paths for the bargaining and price-taking versions, respectively, for this combined policy. We see that in the bargaining version the new steady state for capital is lower than the initial level and as a result the agents work less and consume more initially to deaccumulate the extra capital. This leads to a welfare gain during the transition. Even though GDP falls overall, the benefit of increased DM output along with the transition effect lead to a net increase in welfare. We see just the opposite for the price-taking case where the agents work more and consume less initially to accumulate the extra capital.

5.2 Robustness

To be added.

6 Conclusions

To be added.

A Appendix

A.1 The Cost Function

Here we verify the properties of the DM (utility) cost function $c(q,k)$ that we stated in Section 2. This cost function comes from a DM production function $q = f(k,e)$ that is strictly increasing and concave, and a disutility of effort function $\ell(e)$ that is is strictly increasing and convex. Also, by definition, saying k is a normal input into f means that in the problem $\min\{we + rk\}$ s.t. $f(k,e) \geq q$, the solution satisfies $\partial k/\partial q = f_e f_{ek} - f_k f_{ee} > 0$.

To proceed, first rewrite $q = f(k,e)$ as $e = \xi(q,k)$. Then $\partial e/\partial q = \xi_q = 1/f_e > 0$ and $\partial e/\partial k = \xi_k = -f_k/f_e < 0$. Also $\xi_{qq} = -f_{ee}/f_e^3 > 0$, $\xi_{kk} = -\left(f_e^2 f_{kk} - 2f_e f_k f_{ke} + f_k^2 f_{ee}\right)/f_e^3 > 0$, and $\xi_{kq} = -\left(f_{ek} f_e - f_{ee} f_k\right)/f_e^3$. Hence, $c_q = \ell'/f_e > 0$, $c_k = -\ell' f_k/f_e < 0$, $c_{qq} = \left[\ell'' \ell'^2 f_e - \ell' f_{ee}\right]/f_e^3 > 0$, $c_{kk} = -\left[\ell'\left(f_e f_{kk} - 2f_e f_k f_{ke} + f_k^2 f_{ee}\right) - f_e f_k^2 \ell''\right]/f_e^3 > 0$ and $c_{qk} = -\left[\ell'' f_e f_k - \ell'\left(f_k f_{ee} - f_e f_{ek}\right)\right]/f_e^3$. These results establish that c is increasing and convex in q and decreasing and convex in k, and that $c_{qk} < 0$ if k is a normal input, as asserted in the text.

A.2 Money Demand Elasticity

The interest elasticity of money demand is $\xi = \frac{\partial(M/P)}{\partial i}\frac{i}{M/P}$. We show how to compute this in the bargaining model (the price-taking version is similar). First, using (57),

$$\xi = \left[g_q \frac{\partial q}{\partial i} + g_k \frac{\partial K}{\partial i}\right]\frac{i}{g} + \left[F_{HH}\frac{\partial H}{\partial i} + F_{HK}\frac{\partial K}{\partial i}\right]\frac{i}{F_H},$$

where we drop the arguments to save space. It is now a matter of substituting $\partial q/\partial i$, $\partial K/\partial i$ and $\partial H/\partial i$.

Eliminating X, we can write the steady state as 3 equations in (q,K,H):

$$\frac{i}{\sigma} = \frac{u'(q)}{g_q(q,K)} - 1 \tag{66}$$

$$\rho = \left[F_K(K,H) - \delta\right](1 - t_k) - \frac{\sigma(1+t_x)\gamma(q,K)}{U'\left[F(K,H) - \delta K - G\right]} \tag{67}$$

$$U'\left[F(K,H) - \delta K - G\right] F_H(K,H) = \frac{A(1+t_x)}{(1-t_h)} \tag{68}$$

We take the total derivative of this system to obtain

$$B \left[\begin{array}{c} dq \\ dK \\ qH \end{array} \right] = \left[\begin{array}{c} di \\ 0 \\ 0 \end{array} \right]$$

where

$$B = \left[\begin{array}{ccc} \frac{\sigma(g_q u'' - u' g_{qq})}{g_q^2} & -\frac{\sigma u' g_{qk}}{g_q^2} & 0 \\ -\frac{\sigma(1+t_x)\gamma_q U'}{U'^2} & \Theta & \frac{(1-t_k)U'^2 F_{KH} + \sigma(1+t_x)\gamma U'' F_H}{U'^2} \\ 0 & (F_K - \delta) F_H U'' + F_{KH} U' & F_H^2 U'' + F_{HH} U' \end{array} \right] \tag{69}$$

and $\Theta = (1 - t_k) F_{KK} - \frac{\sigma(1+t_x)}{(U')^2} [\gamma_k U' - (F_K - \delta) \gamma U'']$. We can now compute the partials as

$$\frac{\partial q}{\partial i} = B_{11}^{-1} \; \frac{\partial K}{\partial i} = B_{21}^{-1} \; \frac{\partial H}{\partial i} = B_{31}^{-1}$$

where B_{ij}^{-1} refers to the (i, j) element of B^{-1}.

References

[1] Aiyagari S.R., N. Wallace and R. Wright (1996) "Coexistence of Money and Interest-Bearing Securities," *Journal of Monetary Economics* 37, 397-420.

[2] Aruoba, S.B., J. Fernandez-Villaverde and J.R. Ramirez (2005) "Comparing Solution Methods for Dynamic Equilibrium Economies," *Journal of Economic Dynamics and Control*, forthcoming.

[3] Aruoba, S. B. and R. Wright (2003) "Search, Money and Capital: A Neoclassical Dichotomy," *Journal of Money, Credit and Banking* 35, 1086-1105.

[4] Basu, S. and Fernald, (1997) "Returns to scale in U.S. production: Estimates and implications," *Journal of Political Economy* 105, 249-283.

[5] Berentsen, A. and G. Rocheteau (2004) "Money and Information," *Review of Economic Studies* 71, 915-944.

[6] Caballero, R. J. (1999) "Aggregate Investment," in *Handbook of Macroeconomics*, ch. 12, vol. 1B, ed. by J. Taylor and M. Woodford. Amsterdam: North Holland, 814-862.

[7] Caballero R. J. and M. Hammour (1998) "The Macroeconomics of Specificity," *Journal of Political Economy* 106, 724-767.

[8] Craig. B. and G. Rocheteau (2005) "Search for the Welfare Costs of In‡ation," *mimeo*, Federal Reserve Bank of Cleveland.

[9] Cooley, T. (1995) *Frontiers of Business Cycle Research.* Princeton: Princeton University Press.

[10] Cooley, T. and G.D. Hansen (1989) "The Inflation Tax in a Real Business Cycle Model," *American Economic Review,* 79, 733-748.

[11] Cooley, T. and G.D. Hansen (1991) "The Welfare Cost of Moderate Inflations," *Journal of Money Credit and Banking,* 23, 483-503.

[12] Goldfeld, S. and D. Sichel (1990) "The Demand for Money," in *The Handbook of Monetary Economics*, vol. 1, ed. by B. Friedman and F. Hahn. Amsterdam: North-Holland, 299-356.

[13] Green, E.J. and R. Zhou (1998) “A Rudimentary Matching Model with Divisible Money and Prices,” *Journal of Economic Theory* 81, 252-271.

[14] Green, E. J. and R. Zhou (2002). “Dynamic Monetary Equilibrium in a Random-Matching Economy,” *Econometrica* 70, 929–969.

[15] Greenwood, Jeremy, Richard Rogerson and Randall Wright (1995), “Household Production in Real Business Cycle Theory,” in *Frontiers of Business Cycle Research*. Princeton: Princeton University Press.

[16] Hansen, G.D. (1985) “Indivisible Labor and the Business Cycle,” *Journal of Monetary Economics* 16, 309-327.

[17] He, P. L. Huang and R. Wright (2005) “Money and Banking in Search Equilibrium,” *International Economic Review* 46, 637-70.

[18] Hosios, A. (1990) “On the Efficiency of Matching and Related Models of Search and Unemployment,” *Review of Economic Studies* 57, 279-298.

[19] Howitt, P. (2003) “Comment on "Search, Money and Capital: A Neoclassical Dichotomy,” *Journal of Money, Credit and Banking* 35, 1107-1110.

[20] Judd, K.L. (1992) “Projection Methods for Solving Aggregate Growth Models,” *Journal of Economic Theory* 58, 410-452.

[21] Juster, F.T. and F.P. Stafford (1991) “The Allocation of Time: Empirical Findings, Behavioral Models, and Problems of Measurement,” *Journal of Economic Lieterature* 29, 471-522.

[22] Kiyotaki, N. and R. Wright (1989) “On Money as a Medium of Exchange,” *Journal of Political Economy* 97, 927-954.

[23] Kiyotaki, N. and R. Wright (1993) “A Search-Theoretic Approach to Monetary Economics,” *American Economic Review* 83, 63-77.

[24] Kocherlakota, N. (1998) “Money is Memory,” *Journal of Economic Theory* 81, 232–251.

[25] Lagos, R. (2005) “Asset Prices and Liquidity in an Exchange Economy,” *mimeo*, Federal Reserve Bank of Minneapolis.

[26] Lagos, R. and G. Rocheteau (2005) "On the Coexistence of Fiat Money and Other Assets," *mimeo*, Federal Reserve Bank of Cleveland.

[27] Lagos, R. and R. Wright (2005) "A Unified Framework for Monetary Theory and Policy Evaluation," *Journal of Political Economy* 113, 463-484.

[28] Lucas, R. (2000) "In‡ation and Welfare," *Econometrica* 68, 247-274.

[29] McGrattan, E.R. (1994) "The Macroeconomic Effects of Distortionary Taxation," *Journal of Monetary Economics* 33, 573-601.

[30] McGrattan, E.R., R. Rogerson and R.Wright (1997) "An Equilibrium Model of the Business Cycle with Household Production and Fiscal Policy," *International Economic Review* 38, 267-90.

[31] Molico, M. (1997) "The Distribution of Money and Prices in Search Equilibrium," *Ph.D. Dissertation*, University of Pennsylvania.

[32] Rocheteau, G., P. Rupert, K. Shell and R. Wright (2005) "General Equilibrium with Nonconvexities, Sunspots and Money," *mimeo*, University of Pennsylvania.

[33] Rocheteau, G. and C. Waller (2005) "Bargaining and the Value of Money," *mimeo*, University of Notre Dame.

[34] Rocheteau, G. and R. Wright (2005) "Money in Search Equilibrium, in Competitive Equilibrium and in Competitive Search Equilibrium," *Econometrica* 73, 75-202.

[35] Rogerson, R. (1988) "Indivisible Labor, Lotteries and Equilibrium," *Journal of Monetary Economics* 66, 3-16.

[36] Rogerson, R., R. Shimer and R. Wright (2005) "Search-Theoretic Models of the Labor Market: A Survey," *Journal of Economic Literature*, forthcoming.

[37] Shi S. (1995) "Money and Prices: A Model of Search and Bargaining," *Journal of Economic Theory* 67, 467-496.

[38] Shi S. (1999) "Search, Inflation and Capital Accumulation," *Journal of Monetary Economics* 44, 81-104.

[39] Shi S. (2005) "Nominal Bonds and Interest Rates," *International Economic Review* 46, 579-612.

[40] Stockman, A. (1981) "Anticipated Inflation and the Capital Stock in a Cash-in-advance Economy," *Journal of Monetary Economics* 8, 387-393.

[41] Trejos A. (1997) "Search, Bargaining, Money and Prices under Private Information," *International Economic Review* 40, 679-695.

[42] Trejos A. and R. Wright (1995) "Search, Bargaining, Money and Prices," *Journal of Political Economy* 103, 118-141.

[43] Wallace, N. (2001) "Whither Monetary Economics?" *International Economic Review* 42, 847-869.

[44] Waller, C. (2003) "Comment on "Search, Money and Capital: A Neoclassical Dichotomy," *Journal of Money, Credit and Banking* 35, 1111-1117.

[45] Williamson, S. and R. Wright (1994) "Barter and Monetary Exchange under Private Information," *American Economic Review* 84, 104-123.

[46] Zhou, R. (1999) "Individual and Aggregate Real Balances in a Random Matching Model," *International Economic Review* 40, 1009-1038.

[47] Zhu, T. (2003) "Existence of a Monetary Steady State in a Matching Model: Indivisible Money," *Journal of Economic Theory* 112, 307-324.

[48] Zhu, T. (2005). "Existence of a Monetary Steady State in a Matching Model: Divisible Money," *Journal of Economic Theory* 123, 135-160.

Table 2 - Calibration Results

	Data	(1) AW-B	(2) AW-B	(3) AWW-B	(4) AWW-B	(5) AWW-C
Calibrated Parameters						
σ		0.25	0.27	0.25	0.26	0.27
B		2.76	2.74	1.63	1.46	2.63
ψ		**1.00**	**1.00**	1.70	1.87	2.82
A		7.01	6.99	4.14	3.73	7.09
G		0.11	0.11	0.11	0.11	0.13
θ		**1.00**	0.77	**1.00**	0.73	–
Calibration Targets from the Benchmark Economy						
Markup	10.00	0.00	10.00	−1.73	10.00	0.00
K/Y	2.32	1.87	1.87	1.87	1.87	2.32
G/Y	0.25	0.25	0.25	0.25	0.25	0.25
H	0.33	0.33	0.33	0.33	0.33	0.33
Velocity	5.76	5.80	5.80	5.80	5.80	5.76
Int Elast of M/P	−0.23	−0.23	−0.23	−0.23	−0.23	−0.23
Miscellaneous						
Share of DM		4.22	4.63	4.22	4.49	4.73
Markup in DM		0.00	216.08	−41.02	222.71	0.00
Inf Elast of I	−0.023	0.000	0.000	0.000	−0.001	−0.060
Inf Elast of Y	−0.004	−0.010	−0.011	−0.010	−0.011	−0.029

Notes: AW refers to the model in Aruoba and Wright (2003), which corresponds to the case where $\psi = 1$. B refers to the bargaining version and C refers to the price-taking (competitive pricing) version of the model. The first column also corresponds to the price taking case in AW. Bold parameters show restricted parameters. Numbers in parantheses are the corresponding values computed from the data.

Table 3 - Allocations and Welfare with 10% Inflation

	(1) AW-B	(2) AW-B	(3) AWW-B	(4) AWW-B	(5) AWW-C
Compared to the Friedman Rule					
q/q^0	0.64	0.64	0.77	0.79	0.80
y/y^0	1.00	1.00	1.00	*1.00*	0.96
Y/Y^0	0.98	0.98	0.98	0.98	0.94
K/K^0	1.00	1.00	1.00	*1.00*	0.89
H/H^0	1.00	1.00	1.00	*1.00*	0.99
X/X^0	1.00	1.00	1.00	*1.00*	0.97
Compared to the Solution to Planner's Problem					
q/q^*	0.64	0.20	0.52	0.19	0.56
y/y^*	0.65	0.66	0.61	0.60	0.67
Y/Y^*	0.65	0.65	0.58	0.56	0.67
K/K^*	0.47	0.47	0.38	0.36	0.51
H/H^*	0.75	0.75	0.74	0.73	0.75
X/X^*	0.62	0.62	0.59	0.58	0.63
Welfare Gains					
10% to Friedman	0.71	2.97	0.67	3.17	1.80
during transition	0.00	0.00	0.00	−0.02	−1.72
10% to 0%	0.64	2.12	0.61	2.23	1.29
during transition	0.00	0.00	0.00	−0.02	−1.14
Steady State Welfare Loss vs. First Best					
10% to First Best	21.27	29.10	29.28	46.11	21.26
Friedman to First Best	20.41	25.37	28.42	41.58	17.13

Notes: The entries in italics are rounded up and are not identically equal to unity.

Table 4 - Policy Experiments - From 10% to 0% Inflation

	(1) AW-B	(2) AW-B	(3) AWW-B	(4) AWW-B	(5) AWW-C
Change in Lump-Sum Tax					
q_2/q_1	1.37	1.36	1.20	1.18	1.17
k_2/k_1	1.00	1.00	1.00	*1.00*	1.09
h_2/h_1	1.00	1.00	1.00	*1.00*	*1.00*
x_2/x_1	1.00	1.00	1.00	*1.00*	1.02
Y_2/Y_1	1.01	1.01	1.01	1.01	1.04
T_1/Y_1 (%)	-4.78	-4.65	-4.78	-4.67	-2.56
$T_2/Y_2 - T_1/Y_1$ (%)	1.48	1.48	1.48	1.48	1.32
Welfare Gain (%)	0.64	2.12	0.61	2.23	1.29
during the transition (%)	0.00	0.00	0.00	-0.02	-1.14
Change in Labor Income Tax					
q_2/q_1	1.37	1.36	1.19	1.16	1.15
k_2/k_1	0.97	0.97	0.97	0.97	1.05
h_2/h_1	0.97	0.97	0.97	0.97	0.98
x_2/x_1	0.95	0.95	0.95	0.95	0.98
Y_2/Y_1	0.98	0.98	0.98	0.98	1.01
New t_h	0.28	0.28	0.28	0.28	0.27
Welfare Gain (%)	-0.98	0.48	-1.08	0.52	0.02
during the transition (%)	0.39	0.40	0.41	0.40	-0.71
Change in Consumption Tax					
q_2/q_1	1.37	1.36	1.19	1.17	1.16
k_2/k_1	0.98	0.98	0.98	0.98	1.07
h_2/h_1	0.98	0.98	0.98	0.98	0.98
x_2/x_1	0.97	0.96	0.97	0.97	0.99
Y_2/Y_1	0.99	0.99	0.99	0.99	1.02
New t_x	0.11	0.11	0.11	0.11	0.10
Welfare Gain (%)	-0.49	0.97	-0.57	1.04	0.43
Loss in Transition (%)	0.28	0.28	0.29	0.28	-0.97

Notes: Subscript 1 and 2 denote before and after the change, respectively.

The entries in italics are rounded up and are not identically equal to unity.

Figure 1 - Money Demand Estimation

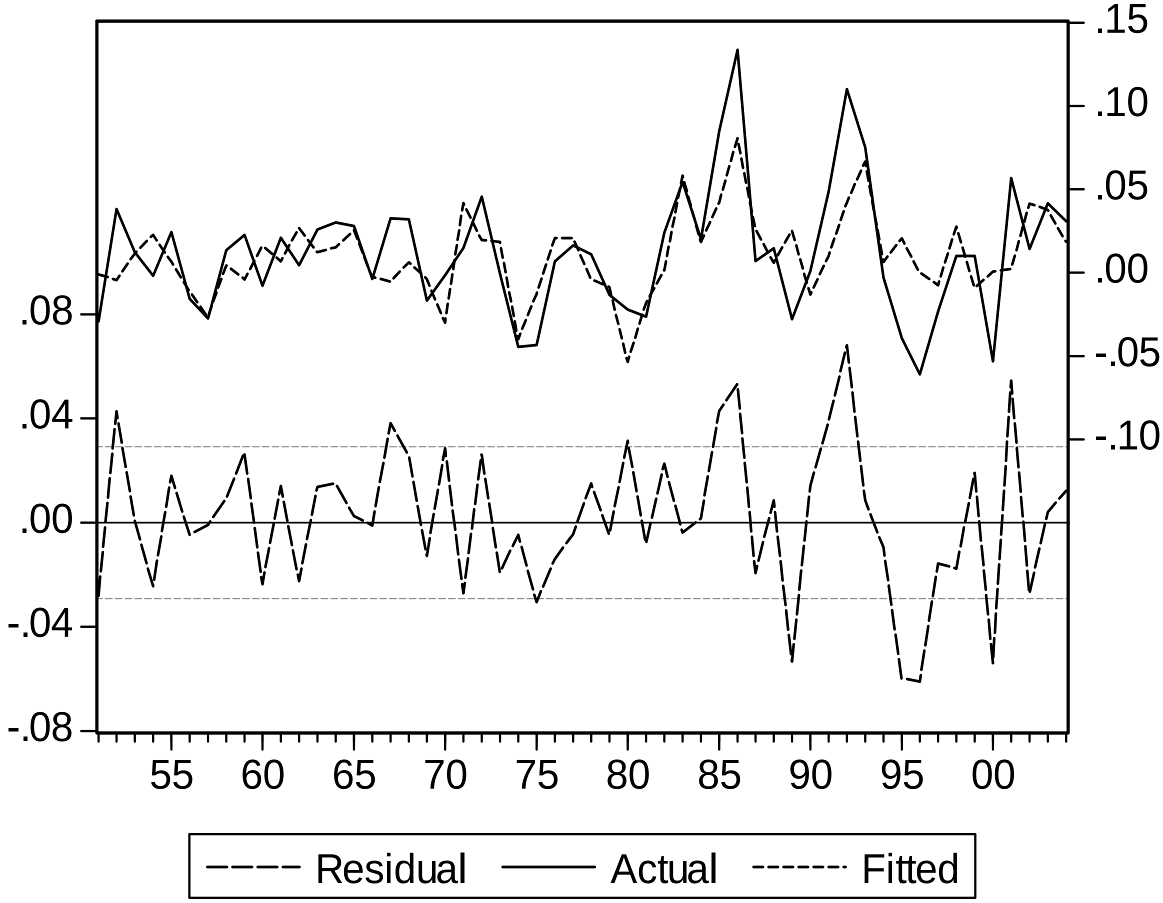
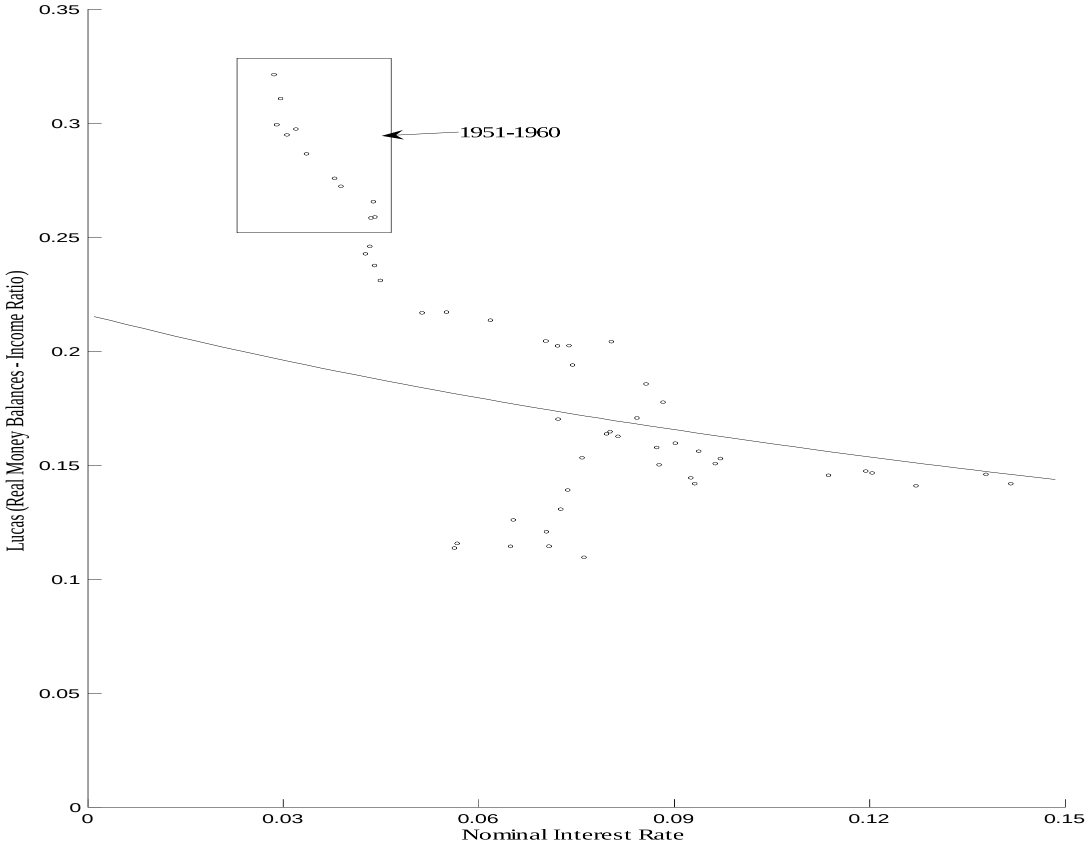
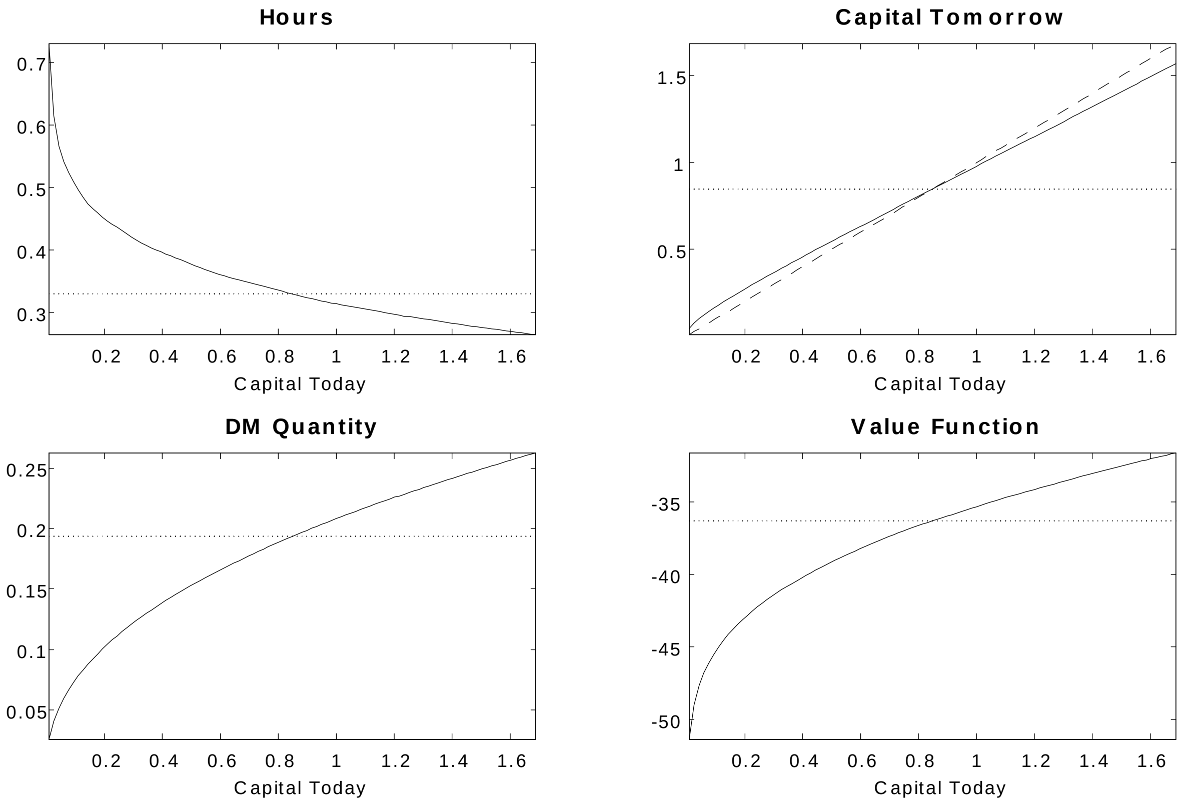


Figure 2 - Fit of Real Money Balances - Income Ratio



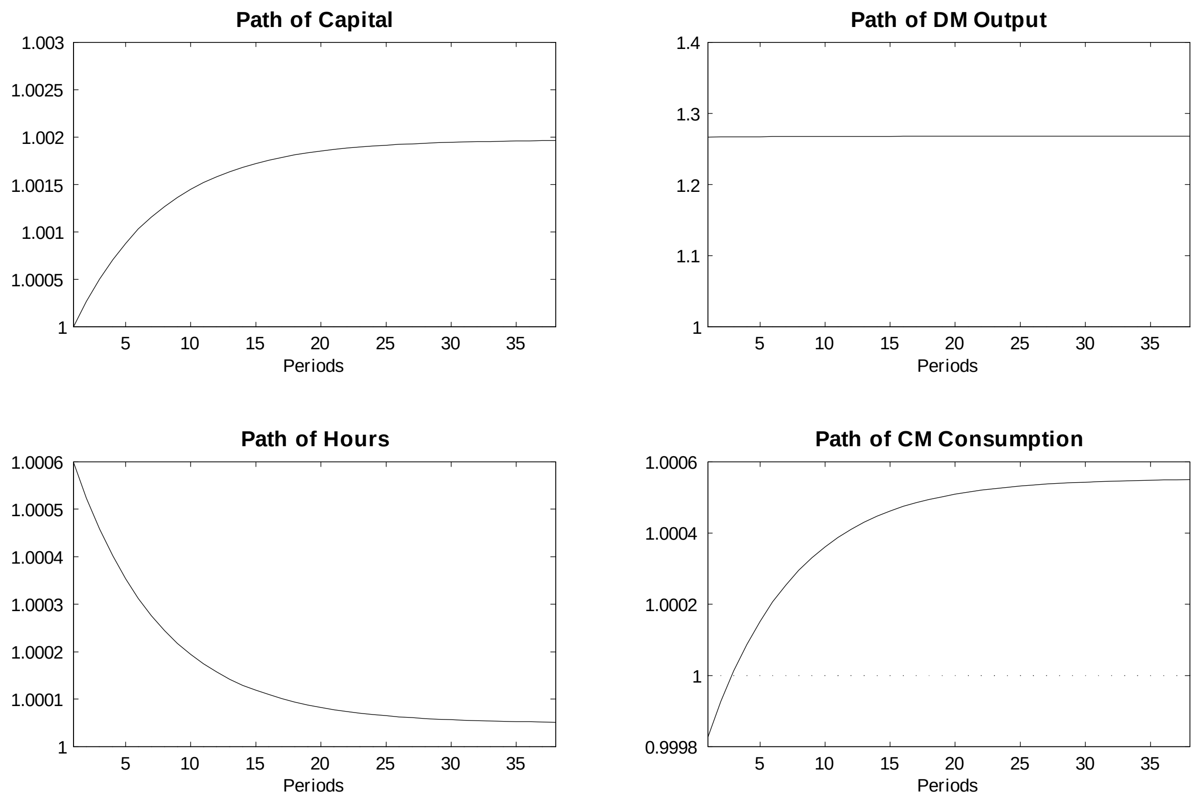
Notes: The dots are US data 1951-2004 and the solid line is the values implied by the steady state of the model, when the interest rate is changed. The rectangle shows the data for 1951-1960.

Figure 3 - Decision Rules and Value Function: Bargaining Version



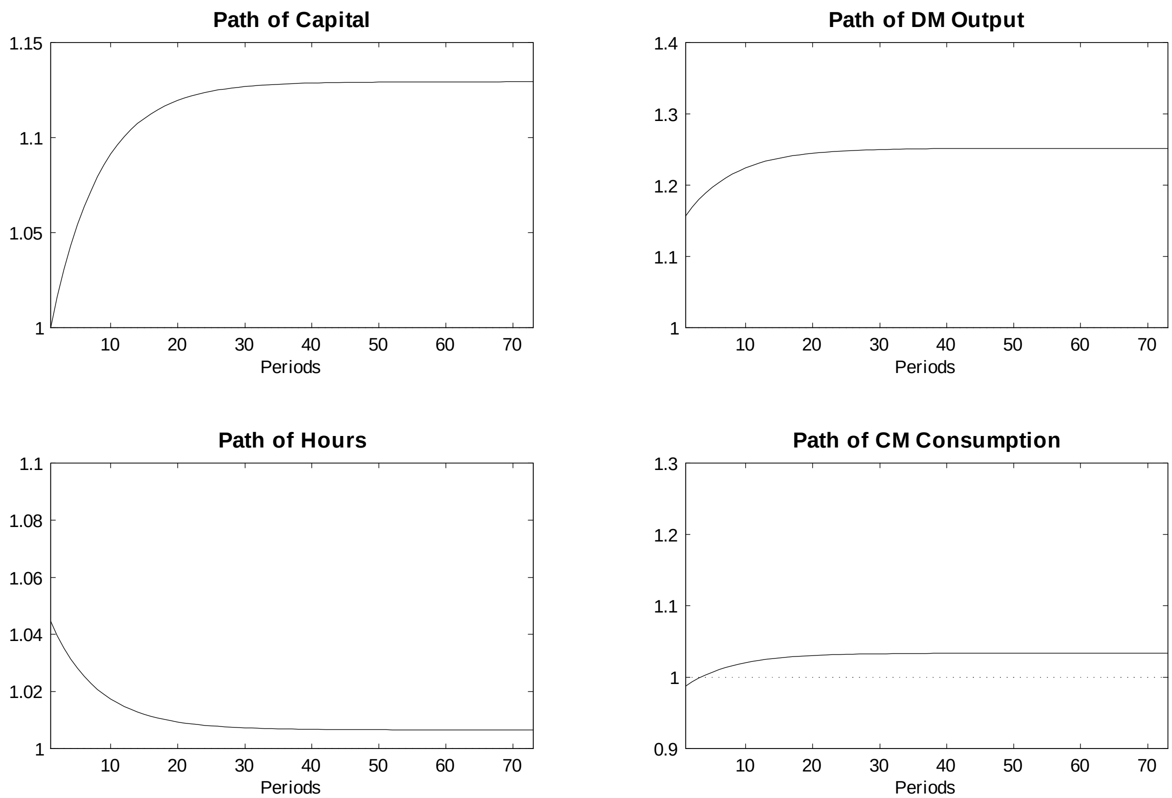
Notes: The x-axis cover $\pm 99\%$ of the steady state of capital. The thin horizontal lines show the steady state values of each variable. The dashed line in the plot of $k'(k)$ is the 45 degree line.

Figure 4 - Transition Path after Reduction in Inflation from 10% to Friedman Rule - Bargaining Version



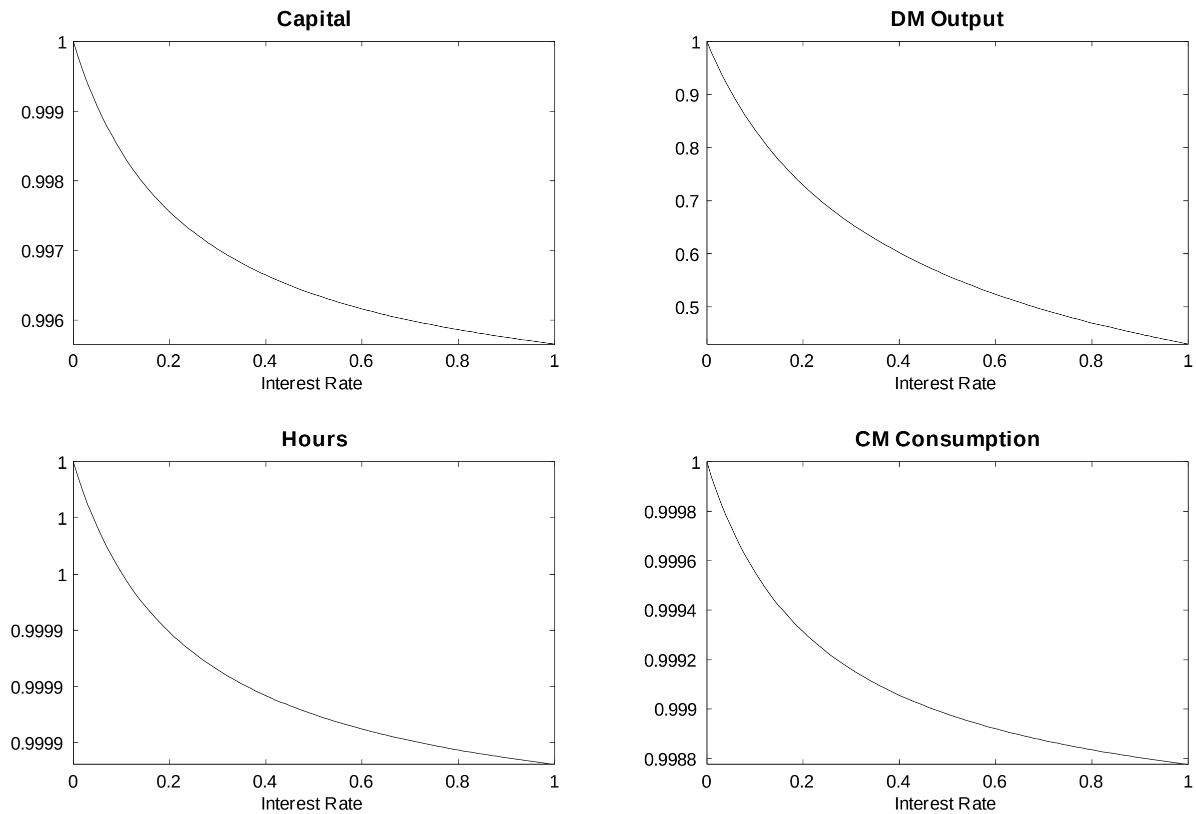
Notes: Unity on the y-axis refers to the initial steady state.

Figure 5 - Transition Path after Reduction in Inflation from 10% to Friedman Rule - Competitive Pricing Version



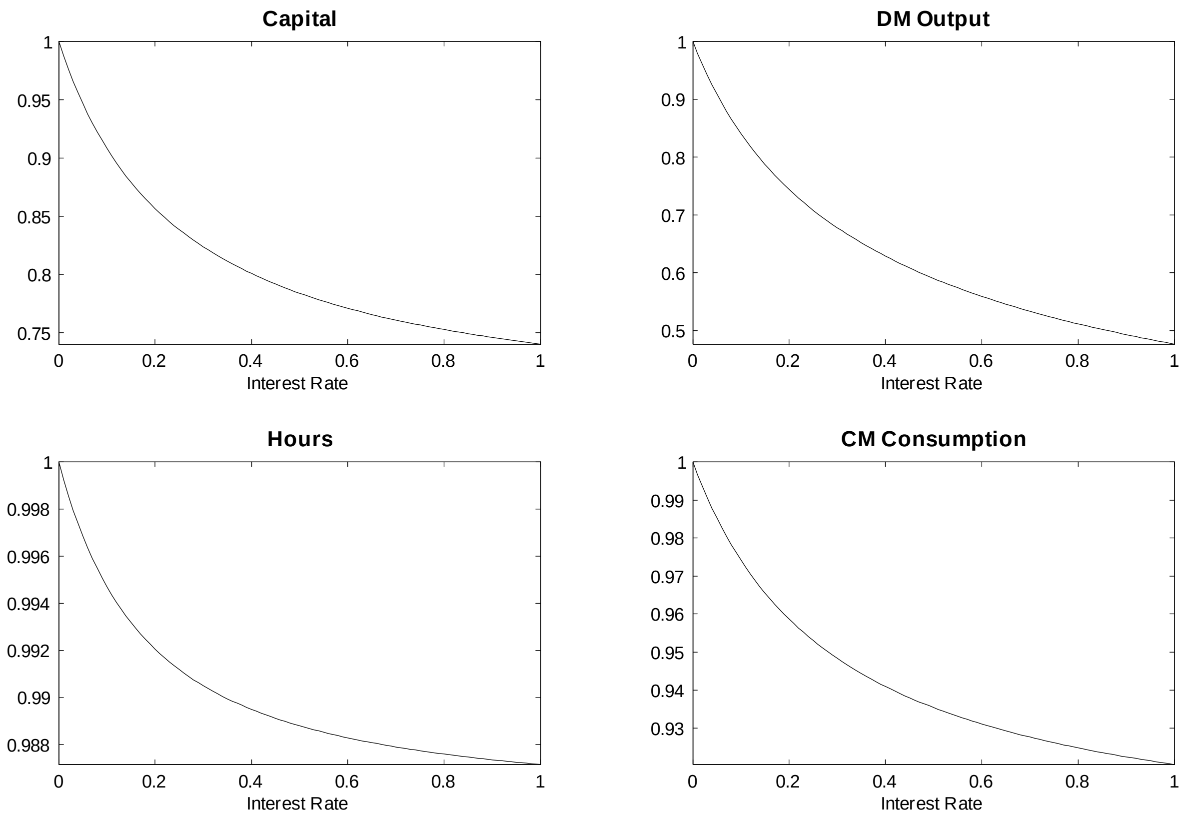
Note: Unity on the y-axis refers to the initial steady state.

Figure 6 - Allocations on a Grid of Interest Rate - Bargaining Version



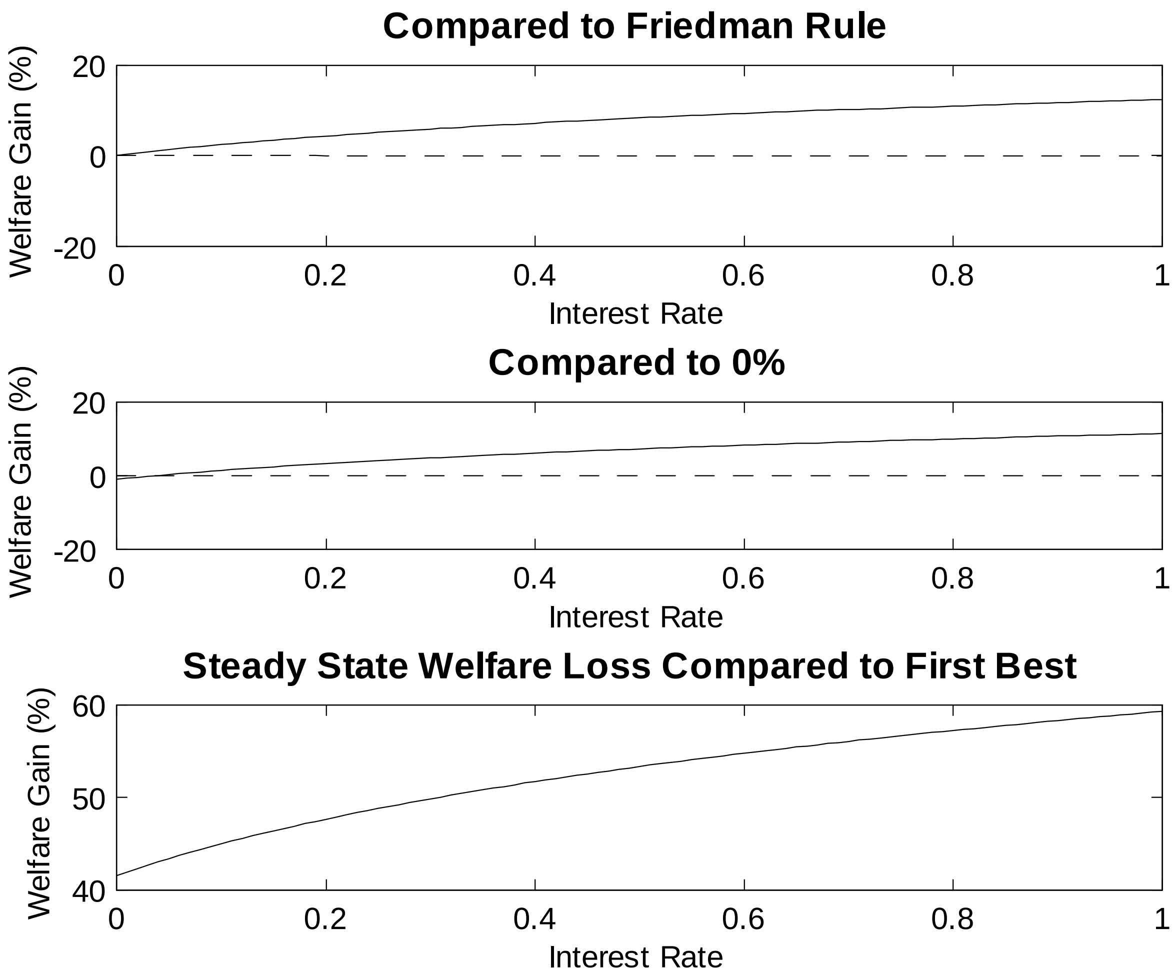
Note: Unity on the y-axis refers to the Friedman Rule.

Figure 7 - Allocations on a Grid of Interest Rate - Price-Taking Version



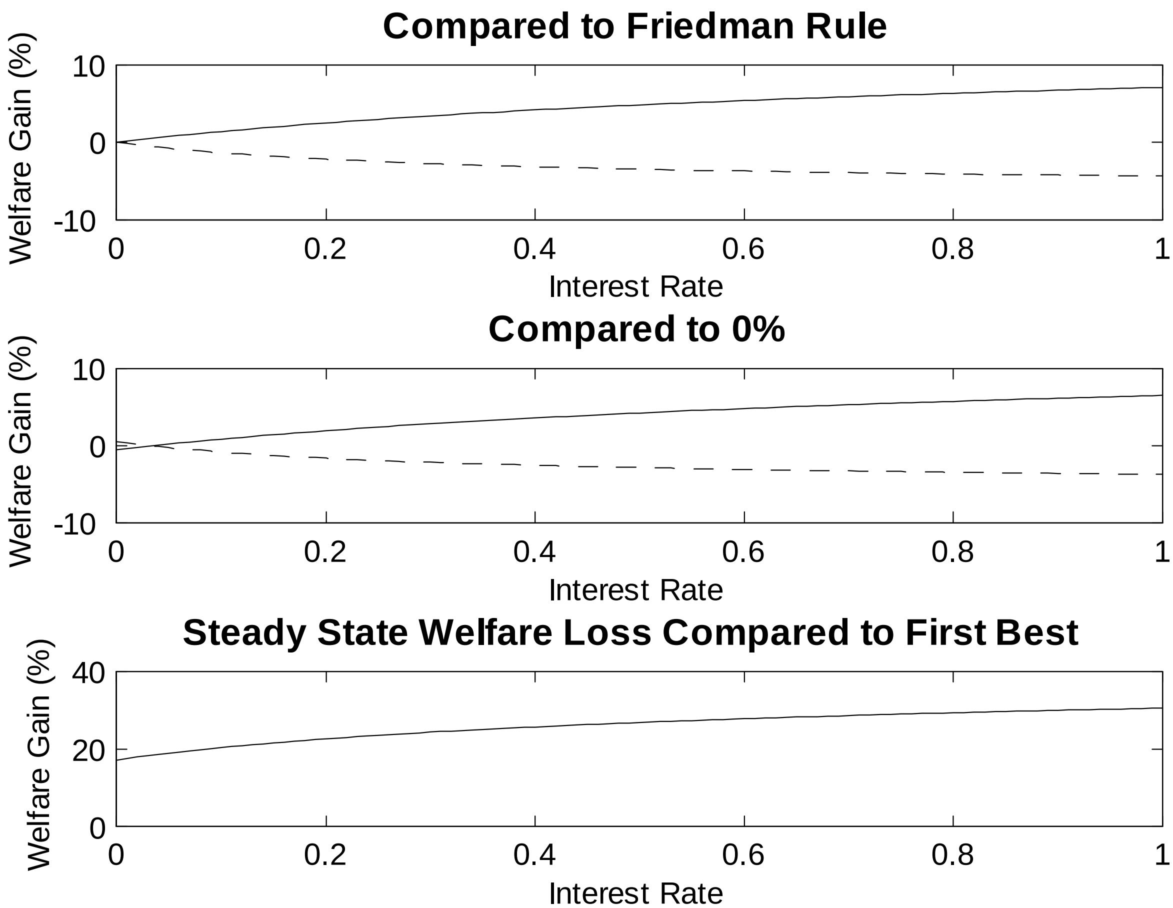
Note: Unity on the y-axis refers to the Friedman Rule.

Figure 8 - Welfare on a Grid of Interest Rate - Bargaining Version



Note: The solid line is the total welfare gain and the dashed line is the gain during the transition.

Figure 9 - Welfare on a Grid of Interest Rate - Price-Taking Version

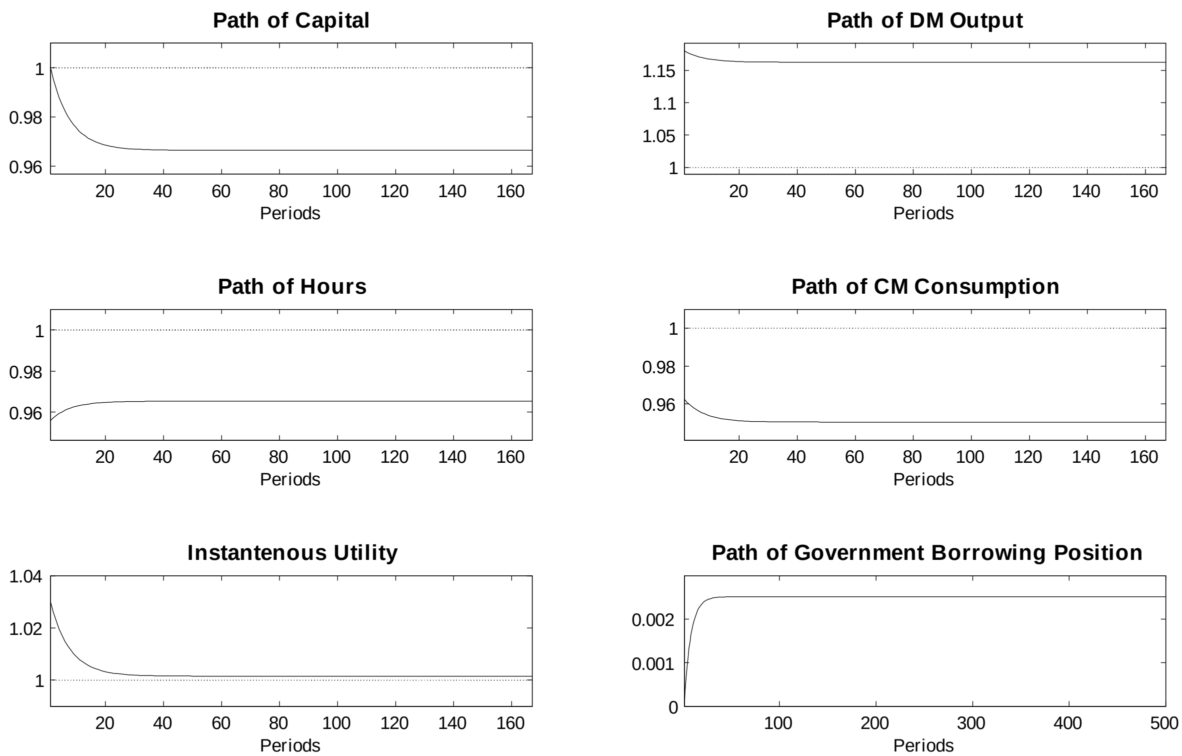


Note: The solid line is the total welfare gain and the dashed line is the gain during the transition.

Figure 9 - Transition in the Policy Experiment

Reduce Inflation From 10% to 0% and Increase Labor Income Tax

Bargaining Version

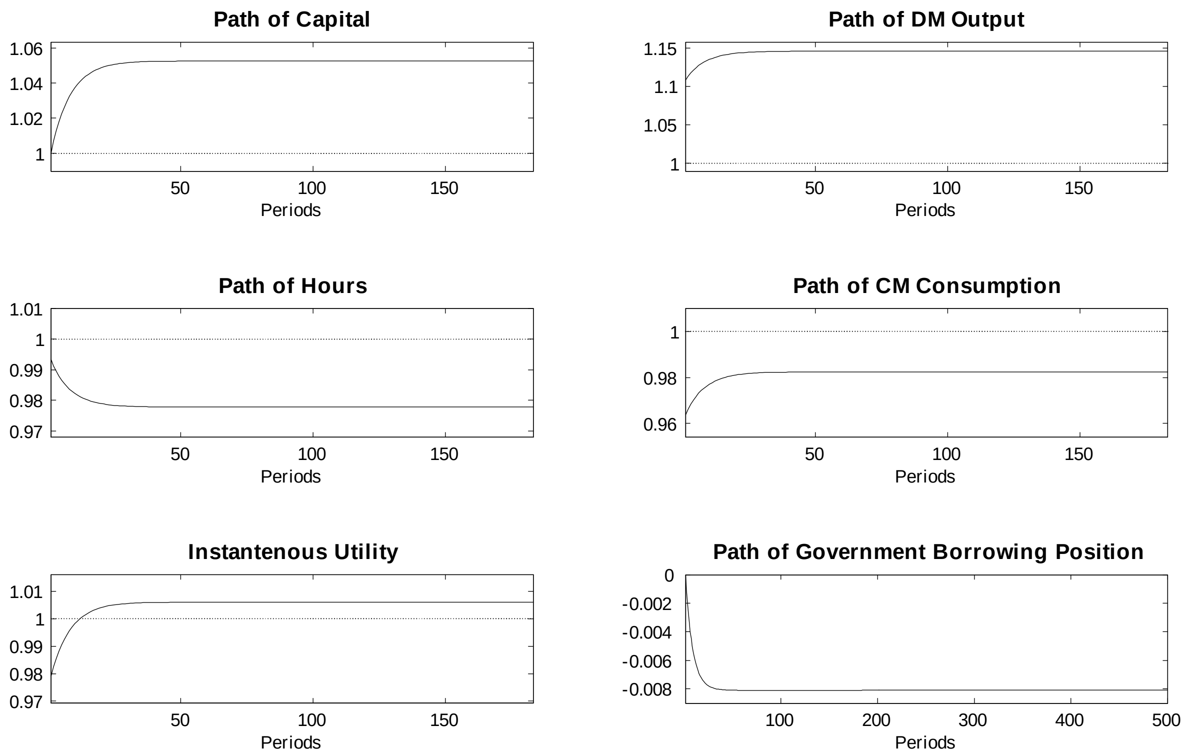


Note: Unity on the y-axis refers to the initial steady state, except for the bond position.

Figure 10 - Transition in the Policy Experiment

Reduce Inflation From 10% to 0% and Increase Labor Income Tax

Price-Taking Version



Note: Unity on the y-axis refers to the initial steady state, except for the bond position.