A Model of Central Bank Liquidity Provision

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Policy Questions

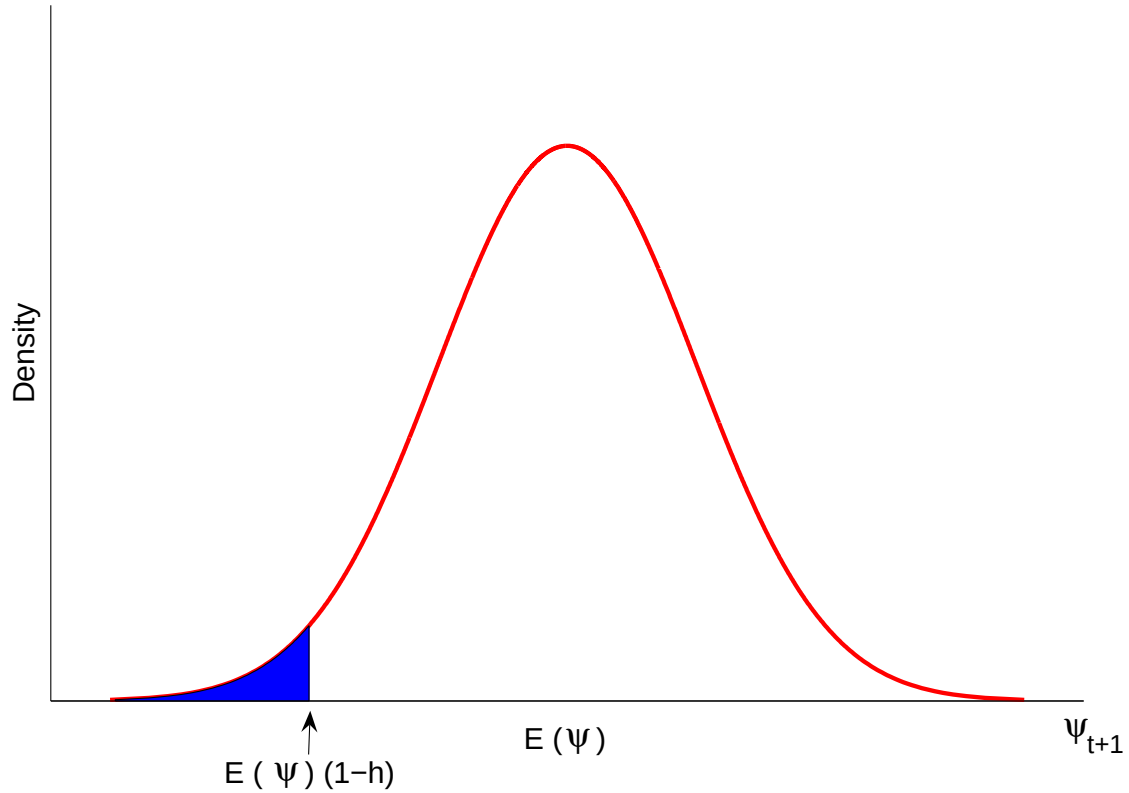
When a central bank provides liquidity through collateralized loans (e.g. intraday central bank liquidity, overnight liquidity facility) such as in the Canadian case through SLF or SPRA/SRA:

- How should it design its collateral policy?
- In particular, how should it determine its *haircut* policy?

What are "haircuts"?

Borrowing Constraint: $L_t \leq A_t \psi_t (1 - h)$

where L : loans, A : asset, ψ : asset price, h : haircut



Motivation

Research into haircut policy is motivated by the following questions

- What is the essential trade-off involved in setting haircuts?
- What are the equilibrium effects of changing haircuts?
- What are the welfare implications of collateralized lending policy?
- What are the key factors that determine an optimal haircut? (e.g. collateral types, borrowers, lending mechanism)

Motivation

payment systems all transactions in most settlement systems are subject to "collateral-in-advance" constraints

liquidity provision Central banks need guidance for their collateral policy

Develop A Tractable Model of Liquidity Provision in a Settlement System

Four building blocks:

1. Portfolio choice: liquid vs illiquid assets
2. Uncertain liquidity needs $\Rightarrow$ CB liquidity provision
3. Potential for default $\Rightarrow$ Collateral requirement
4. Asset price uncertainty $\Rightarrow$ Haircuts

Findings

- A central bank liquidity facility is a portfolio of two types of insurance:
  1. Insurance against liquidity risk
  2. Insurance against downside risk of asset
- Setting a haircut involves a trade-off between:
  - Relax liquidity constraint of illiquid agents
  - Tighten liquidity constraint of liquid agents through:
    1. Lower value of liquid asset
    2. Increased opportunity cost of holding liquid asset
    3. Distortion of the portfolio choice
  - The optimal haircut is higher when:
    - Default incentives and portfolio choices respond strongly to haircut change
    - Volatility of asset prices is higher
    - Unable to target lending to agents who really need liquidity

Model

- Time is discrete: $t = 0, 1, 2, ...$
- Continuum of infinitely lived agents
- Three consecutive sub-periods (denoted by s):

  AM centralized asset market (portfolio choice) ($s = 1$)
  DM decentralized goods market (liquidity need) ($s = 2$)
  CM centralized market (settlement) ($s = 3$)

Preference

Period utility of an agent

$$u(q_2^b) - q_2^s - h_3,$$

where

- q_2^b: consumption of the DM goods when the agent is a buyer
- q_2^s: production of the DM goods when the agent is a seller
- h_3: production (net of consumption) of the CM goods
- β: discount factor

Portfolio: Money and Asset

- M_t: liquid asset (e.g. fiat money/ bank reserves)
  - exogenous growth rate γ
- A_t: illiquid asset (e.g. claims to investment projects)
  - endowed with A projects at the beginning of a period
  - each unit yields δ_t units of CM goods at the end of a period
  - δ is a random i.i.d. (over time and across owners) variable: $\delta_t \sim U(\bar{\delta}(1-\varepsilon), \bar{\delta}(1+\varepsilon))$, and with $\bar{\delta} < 1$
  - Price of asset: $\psi_s, s = 1, 2, 3$

Sub-Period 1: Asset Market AM

- An agent starts with (m_1, A) and receives signal $S \in \{H, L\}$:
  - H: likely to become a buyer in the DM (high liquidity need)
  - L: likely to become a seller in the DM (low liquidity need)
- Given the signal, agents trade in AM and make portfolio choice (m_2, a_2)
- The signal turns out to be incorrect with a probability $\theta < \frac{1}{2}$
  - an agent with H signal will be a buyer with prob. $\sigma^H = 1 - \theta$
  - an agent with L signal will be a buyer with prob. $\sigma^L = \theta$

Sub-Period 2: Decentralized Trading DM

An agent starts with (m_2, a_2)

- The trading status realizes: buyer or seller
- Trading subject to liquidity constraint (only m is accepted)
- Before trade, agents have access to central bank lending facilities:
  - Borrow a nominal loan l_2 by posting asset as collateral
  - The loan has to be settled in the next CM

Sub-Period 3: Settlement CM

- An agent starts with (m_3, a_3, l_2), and δ_t is realized
- Agents decide whether to settle the loan l_2 or to default
- Agents trade h_3, and choose m_{+1} for next period

Reducing The Value of Holding Liquid Asset

MC of liquidity = MB of liquidity

$$\phi_3(1+i) = \frac{1}{2}(\lambda^H + \lambda^L)$$

where

$$\lambda^H = \phi_3(1+\sigma^H\Delta^H)$$

$$\lambda^L = \frac{\bar{\delta}}{\psi_1}\{1+\sigma^L[\Delta^L(h)+S(h)](1-h)\}$$

$$\Delta^j = u'(q^j) - 1$$

$$q^H = 2M\phi_3$$

$$q^L = 2A\bar{\delta}(1-h)$$

$h \downarrow \Rightarrow q^L \uparrow \Rightarrow \Delta^L \downarrow$ (relax L-type liquidity constraint)

$\Rightarrow \lambda^L \downarrow \Rightarrow \phi_3 \downarrow \Rightarrow q_H \downarrow$ (tighten H-type liquidity constraint)

Increasing The Opportunity Cost of Holding Liquid Asset

$$\text{Fisher's equation} \qquad 1 + i = \frac{\gamma}{\beta} \geq \frac{\sigma^L A}{\beta M} \mathrm{E}[S(h)] + \frac{1}{\beta},$$

$$\text{where} \qquad \mathrm{E}[S(h)] = \frac{\bar{\delta}}{4\phi_3 \varepsilon}(\varepsilon - h)^2$$

$h \downarrow \quad \Rightarrow \quad \mathrm{E}[S(h)] \uparrow$ (insure against downside risk)

$\Rightarrow \quad \gamma \uparrow \Rightarrow i \uparrow \Rightarrow \phi_3 \downarrow \Rightarrow q_H \downarrow$ (tighten H-type liquidity constraint)

Distorting Portfolio Choice

$$\phi_3(1+i) = \frac{1}{2}(\lambda^H + \lambda^L)$$

where $$\lambda^L = \frac{\bar{\delta}}{\psi_1}\{1 + \sigma^L(\Delta^L(h) + S(h))(1-h)\}$$

$h \downarrow \Rightarrow$ induce H-type to hold more illiquid asset $\Rightarrow q_H \downarrow$

$\Rightarrow \psi_1 \uparrow \Rightarrow \phi_3 \downarrow \Rightarrow q_H \downarrow$ (tighten H-type liquidity constraint)

Conclusion

We have developed a model of collateralized central bank lending and shown the:

1. Equilibrium effects of reducing haircuts

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\begin{aligned}
h \downarrow \quad &\Rightarrow \quad \text{provide liquidity insurance} \\
h \downarrow \quad &\Rightarrow \quad \text{(i) lower value of liquid asset} \\
&\Rightarrow \quad \text{(ii) increase opp. cost of holding liquid asset} \\
&\Rightarrow \quad \text{(iii) distort portfolio choice}
\end{aligned}
$$

2. Optimal haircut is lower if
   - Downside risk of collateral is low [small (ii)]
   - Perfect enforcement [no (ii)] or exogenous default [small (ii)]
   - CB can target lending to agents really in need of liquidity [small (ii)]
   - Portfolio choice insensitive to haircut change [small (iii)]
   - It is an unanticipated, temporary cut in h [no (i), (iii), small (ii)]