Efficient Interventions in Markets with Adverse Selection

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November 17, 2009

Abstract

We study cost-efficient government interventions in markets that collapse because of adverse selection. We analyze recapitalizations, debt guarantees, asset buybacks, as well as general mechanisms. We find that programs that attract all banks dominate those attracting only troubled banks, and that debt guarantees are the cheapest way to maximize new lending. If debt guarantees induce excessive risk-taking, the optimal intervention can be achieved by a menu of equity injections. Asset buybacks, on the other hand, are never optimal. We establish these results by solving a new class of mechanism design problem and our techniques may be useful for other economic applications.

PRELIMINARY AND INCOMPLETE

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An important insight of economic theory from the last forty years is that asymmetric information can lead to market collapse. George Akerlof demonstrated this phenomenon in a classic paper (Akerlof (1970)). A severe collapse may justify a government intervention to restore efficient lending and investment. Perhaps surprisingly, however, there does not exist a complete formal analysis of the optimal way to design such an intervention. Our main contributions are to provide a characterization of the costs to tax payers for a general class of optimal interventions, as well as a detailed analysis of the three programs most often used by governments: equity injections, debt guarantees, and asset buybacks. Another contribution of our paper is to develop a set of tools and strategies to solve a new class of mechanism design problems where the planner must deal with the presence of a competitive fringe and with the information conveyed by the strategic decision to participate in the program.[1]

Several features of the financial market collapse in the Fall of 2008 suggest a role for asymmetric information. Not only did spreads widen (as they would in any case given the increase in counter-party risk), but transaction stopped in many markets. In the interbank market, only overnight loans remained. Banks refrained from lending to each other because they were afraid of not being repaid, as the assets that the borrowing bank would put as collateral could be in fact worth nothing (toxic). In the OTC market, the range of acceptable forms of collateral was dramatically reduced "leaving over 80% of collateral in the form of cash during 2008", while the "repo financing of many forms of collateralized debt obligations and speculative-rate bonds became essentially impossible." (Duffie (2009)). Investors and banks were unable to agree on the price for legacy assets or for bank equity.

Governments stepped-in to try to alleviate the problem. In the US, the initial TARP program called for 700 billion to purchase illiquid assets from the banks. Subsequently other proposals were introduced and implemented with varying degrees of success. The main others were capital injection and debt guarantees. As of August 2009, there was 307 billion of outstanding debt issued by financial companies and guaranteed by the FDIC.[2] The original TARP called for 700 billion to purchase illiquid assets from the banks. It was transformed into a Capital Purchase Program (CPP) to invest $250 billion in U.S. banks. Treasury also insured 306 billions of Citibank's assets, and 118 billion of Bank of America's.

[1] To the best of our knowledge, this class of problems has not been analyzed before.

[2] http://www.fdic.gov/regulations/resources/tlgp/index.html. Citigroup sold another 5 billion of guaranteed debt in September 2009. The program is set to expire at the end of October 2009.

We compare these programs, and we derive the optimal mechanism to prevent interbank lending from freezing, in a simple tractable model where the main friction is the presence of asymmetric information about the quality of legacy assets. This analysis is important for at least three reasons. The first reason is the scale of government interventions, as explained above. The second reason is that there is no consensus about which program is better. For instance, Soros (2009) and Stiglitz (2008) argue for equity injections, Bernanke (2009) is in favor of assets buybacks and debt guarantee, Diamond, Kaplan, Kashyap, Rajan, and Thaler (2008) view assets buybacks and equity injection as best alternatives, whereas, Ausubel and Cramton (2009) argue for a careful way to 'price the assets, either implicitly or explicitly.' The third reason is methodological: as we explain below, the problem we solve is a non-standard mechanism design problem, and the techniques we had to use to solve it might be useful in other applications.

Our analysis first highlights the key ingredients that need to be present in order for asymmetric information to cause a problem: risky investment opportunities and asymmetric information about the downside risk of legacy assets. When both are present, the decentralized equilibrium can be inefficient.[3] We then seek to find the government intervention that restores efficient financing at the minimum expense of taxpayers' money. We do so from two perspectives: with symmetric information at the participation stage (when firms must opt in or out of the government program before they learn the value of their legacy assets), or with asymmetric information at the participation stage. Our two main results are to derive cost-minimizing programs for each one of these two perspectives.

These mechanism design problems are non-standard because depending on the type of intervention, the payoffs are not quasi-linear in money, since different mechanisms work through very different channels: For example for the case of equity injection, the government injects cash in return for equity, which helps because it lowers the amount that a bank need to borrow. On the other hand, in the case of debt guarantees, banks pay a fee giving them access to a specific amount that they can borrow at a low interest rate. One first key insight is analyze this problem in terms of payoffs rather than mechanisms. This gives us an unambiguous bound for the minimum cost (or maximum profit - in the case cost is

[3] We find that in normal times – the "non-crisis" regime – the quality of legacy assets is good enough to prevent the market from breaking down. In the Fall of 2008, however, it became likely that some large financial institutions may have completely worthless legacy assets.

negative) that the government can guarantee.

With symmetric information at the participation stage, the cost-minimizing government program is actually profitable. The profits equal the net welfare gain due to the program. We also show that the optimal versions of capital injection, asset-buyback and debt guarantee, all generate the maximal profits. An alternative – and perhaps more relevant – interpretation of the symmetric information outcome is that the government can exert pressure and force participation in its program. Most observers have suggested that banks were required to participate in the initial equity injection program of October 2008.[4] This type of intervention is equivalent to our model with symmetric information at the participation stage.

With asymmetric information at the participation stage, the mechanism design problem has an additional complication: The bank's mere participation decision may be enough to signal valuable information to the government. For example, if the government could design a program that attracted only banks that can always repay their loans, then all banks would face a fair interest rate, and the government could be able to make money by providing a program that works as a successful signaling devise. Our analysis shows that this is impossible. We show that all the government can do is to either design a program that attracts only the bad banks or all banks. Both these programs are costly. Again we derive the lower bound on costs using our payoffs approach. However, this insight alone is not enough to make things tractable here, as we have to address the following subtle issue: Suppose that the government is contemplating a program that attracts all banks, that is in equilibrium all banks participate. What would the market then infer if a bank opts-out? This inference is crucial because it affects bank's outside options, which ultimately affect how costly the government's program is.[5] We proceed by deriving the cost bounds for some abstract market-response rate and we show that minimum bound is often achieved by using debt-guarantees. We also show that debt-guarantee is always less costly than

[4] "The Bush administration will announce a plan to rescue frozen credit markets that includes spending about half of a total of $250 billion for preferred shares of nine major banks [...] **None of banks getting government money was given a choice** about it, said one of the people familiar with the plans." (Bloomberg, Oct. 13, 2008).

[5] The paper of Cramton and Palfrey (1995) formulates a refinement to impose restrictions on out-off-equilibrium beliefs. We chose not to select one particular belief and provide a characterization valid for all conceivable out-off-equilibrium reactions.

capital injection and asset-buybacks or even any hybrid mixed program. In the extensions, we consider the design of government programs that include menus: that is programs that consists of different options for different types of banks. Interestingly, it turns out that all incentive compatible menus for the case of debt guarantees and asset-buybacks boil down to the case of a pooling contract. The government can sustain incentive compatible menus, only in the case of equity and then the cost-minimizing menu achieves the lower bound.

We present our model in Section 1. We start by deriving some simple necessary conditions for the appearance of inefficiencies due to asymmetric information: investment opportunities must be risky, even conditional on private types, and there must be asymmetric information regarding the downside risk of legacy assets. Based on these simple initial results, we introduce our benchmark in , and we characterize its decentralized equilibria in Section 2. In Section 3 we characterize lower bounds on the costs of government interventions. Those bounds can actually be achieved by simple common interventions. This is shown in Section 4. Extensions and robustness of our findings are discussed in Section 5. We close the paper with some final remarks in Section 6.

1 The Model

In this section we present some simple general results on the role of asymmetric information, which we then use to construct our benchmark model.

1.1 Timing and technology

The model has a continuum of financial institutions indexed from 0 to 1. Financial institutions are financial companies such as commercial banks, investment banks, insurance companies, or finance companies. For simplicity, we refer to all of them as banks.

There are three dates $t = 0, 1, 2$. Banks start time 0 with some exogenously given assets, which we refer to as legacy assets. At time 1 banks learn the value of the legacy assets on their balance sheets, and they receive the opportunity to make new loans. In order to exploit these opportunities they may need borrow from each other and from outside investors. To avoid confusion, we use the word "investments" to refer to the new loans that banks make at time 1, and we use "borrowing and lending" to refer to banks borrowing from each other and from outside investors. All returns are realized at time 2, and profits are paid out to investors. We assume that investors are risk-neutral and we normalize the risk-free rate to 0.

The government announces its interventions at time 0, but the implementation can happen either at date 0 or at date 1. The difference matters because banks learn about the value of their existing assets and about their new investment opportunities at date 1. Interventions at date 1 are therefore subject to asymmetric information, while interventions at date 0 are not. We analyze both cases.

Initial assets and cash balance

Banks own two types of assets: cash and legacy assets. Cash is liquid and can be used for investments or for lending at date 1. Let c_t be cash holdings at the beginning of time t. All banks start at time 0 with c_0 in cash. Cash holdings cannot be negative:

$$c_t \geq 0 \text{ for all } t.$$

Long-term legacy assets deliver a random payoff $a \in [A_{\min}, A]$ at time 2. The upper bound A represents the book value of the assets, but some of these assets may be impaired, and their true value can be less than A. We ignore for now the issue of outstanding long term debt and deposits, so it is best to think of a as the payoff net of transfers to senior creditors. We refer the reader to Philippon and Schnabl (2009) for a model where debt overhang is the main friction.

Information and new investments at time 1

At time 1 banks learn their type θ and they receive investment opportunities. Investments cost the fixed amount x at time 1 and deliver a random payoff $v \in [0, V]$ at time 2. At time 2 total bank income y depends on the realization of the two random variables, a and v:

$$y = a + c_2(i) + v \cdot i, \tag{1}$$

where $i \in \{0, 1\}$ is a dummy for the decision to invest at time 1. The conditional distribution of (a, v) depends on the type θ and is denoted by $F(a, v|\theta)$. The type θ is revealed to the bank at time 1. Asymmetric information occurs when the market does not observe θ.

Banks can borrow at time 1 in a perfectly competitive market.[6] After learning its type

[6] This borrowing and lending could take place between banks with investment projects and banks without investment projects (interbank lending), or between banks and outside investors.

θ, a bank offers a contract $\left(l, y^l\right)$ to the competitive investors, where l is the amount raised from investors at time 1, and y^l is the schedule of repayments to investors at time 2. The schedule y^l can be contingent on the income y realized at time 2. Formally, our model involves contracts designed by informed parties and offered to competitive investors.[7] We specify later the exact nature of the contracts offered. The bank's cash at time 2 as a function of its investment decision at time 1 is

$$c_2\left(i\right) = c_1 + l - x \cdot i. \tag{2}$$

Because the credit market is competitive and investors are risk neutral, in any candidate equilibrium, the participation constraint of investors implies:

$$E\left[y^l | i = 1\right] \geq l. \tag{3}$$

1.2 Symmetric information

We first consider the case where the market observes θ. Banks raise money at time 1 to finance their investments. Banks maximize total value as of time 1:

$$E\left[y^e|\theta\right] = E\left[a|\theta\right] + c_2\left(i\right) + E\left[v|\theta\right] \cdot i - E\left[y^l|\theta\right] \cdot i,$$

subject to the cash constraint (2). The bank will go ahead with the investment if

$$E\left[a|\theta\right] + c_2(1) + E\left[v|\theta\right] - E\left[y^l|\theta\right] \geq E\left[a|\theta\right] + c_2(0), \tag{4}$$

It is easy to see that if the bank goes forward with the investment, it weakly prefers to spend all of its own cash (the bank is indifferent under symmetric information, and it strictly prefers under asymmetric information). Hence, for banks that invest we have $c_2(1) = 0$, whereas for those who do not we have that $c_2(0) = c_1$. Equation (4) simply becomes $E\left[v|\theta\right] \geq c_1 + E\left[y^l|\theta\right]$. Using (2) and (3) we have that $E\left[y^l|\theta\right] = l = x - c_1$, which we can use to write the investment condition as $E\left[v|\theta\right] \geq x$. As expected, the requirement is simply that the net present value be positive. Note that $E\left[a|\theta\right]$ is irrelevant. Investment decisions are independent of the quality of legacy assets on the banks' balance sheet.

[7] See Appendixes to Section 6 in Tirole (2006).

1.3 When does asymmetric information matter?

Now assume that the market does not observe θ. In this section, we want to shed light on the role of asymmetric information. Before doing so, we need to specify the nature of private contracts used at time 1. We assume that the random payoff $y = a + v$ satisfies the monotone likelihood ratio property with respect to the type θ. Following Innes (1990), we also assume that repayments y^l must be weakly increasing in y.[8] Under these assumptions, it is optimal for the borrower to offer a debt contract to its creditors.[9] Let r be the interest rate on the loans. The payoffs to investors are:

$$y^l = \min(y, rl).$$

We now turn to the role of asymmetric information. The following proposition presents conditions under which asymmetric information does not matter.

Proposition 1 *The symmetric information allocation is an equilibrium under asymmetric information when the information set of the bank includes the future payoff of the new project, or when the bank can issue risk free debt.*

Proof. Let us first write the investment condition, conditional on r and l:

$$i(\theta) = 1 \Longleftrightarrow E\left[\max\left\{a + v - rl, 0\right\} |\theta\right] > E\left[a|\theta\right] + c_1. \tag{5}$$

Suppose first that the type is $\theta = (\theta^a, v)$ where θ^a indexes the conditional distribution of a, $F(a|\theta^a)$. We have

$$E\left[\max(a + v - rl, 0)|\theta\right] = \int_{rl-v}^{A} (a + v - rl)\, dF(a|\theta^a).$$

If $rl > v$, then the investment condition (5) is clearly violated. So banks with $v < rl$ would never invest. But if only banks with $v > rl$ invest, then debt is risk free and $r = 1$, and, since $l = x - c_1$, the investment condition is simply

$$E\left[a|\theta^a\right] + v - l > E\left[a|\theta^a\right] + c_1 \Longleftrightarrow v > x.$$

[8] The justification is that if repayments were to decrease with y, the borrower could secretely add cash to the bank's balance sheet by borrowing from a third party, obtain the lower repayment, repay immediately the third party, and obtain strictly higher returns. See Sections 3.6 in Tirole (2006).

[9] Debt contracts dominate equity contracts for the reasons emphasized in Myers and Majluf (1984). If we allow for any contingent repayment scheme $y^l(y)$ without the monotonicity constraint, the optimal contract is $y^l = y$ up to a threshold beyond which $y^l = 0$. The monotonicity constraint introduced by Innes (1990) irons out this discontinuity and leads to a standard debt contract. See Section 6.6 in Tirole (2006).

This is the first best rule.

Suppose now that the bank can issue risk free debt, i.e., that $A_{\min} \geq x - c_0$. Recall that $y^l = \min(y, rl)$. Now, we also have that $y = a + v \geq A_{\min} > x - c_0 = l$. So $r = 1$ satisfies the participation constraint of lenders. With $r = 1$, $\max(a + v - rl, 0) = a + v - rl$ and the investment condition becomes

$$E[v|\theta] > l + c_0 = x$$

which is the first best investment rule. ■

This proposition shows that two conditions must be satisfied for asymmetric information to matter. First, there must be uncertainty in v conditional on θ. The intuition here is one of risk shifting. The low quality borrower is tempted to finance a risky project on favorable terms by pretending to be a safe borrower. If there is no risk in the project conditional on θ, then this temptation disappears, and asymmetric information is inconsequential.

Second, there must be asymmetric information with respect to the downside risk of legacy assets. As long as the balance sheet can be pledged to new lenders even under pessimistic expectations, new projects can always be financed at a low rate, and asymmetric information is irrelevant. We can think of the case $A_{\min} \geq x - c_0$ as corresponding to the normal state of interbank flows. The scale of the new investment is small relative to the size of the balance sheet, all new projects can easily be financed.

1.4 Benchmark model

We now present our benchmark model, which is a special case of the general model presented above. Proposition 1 above has established two properties that are critical for adverse selection to occur in the credit market. First, there must be risk in the new project v conditional on θ. Second, there must be private information with respect to the legacy assets' ability to cover losses from new investments. These two insights allow us to construct the simplest model where borrowing and lending is sensitive to information. We therefore assume that:

- Private types learnt at time 1 are binary and fully determine the eventual payoffs of legacy assets at time 2: $a = 0$ when $\theta = B$ (bad type), and $a = A$ when $\theta = G$ (good type). We define the ex-ante (time 0) probability of a good type as:

$$\pi \equiv \Pr(\theta = G).$$

- All new projects are identical and deliver random payoffs. The payoffs are binary $v \in \{0, V\}$ with probability of success

$$q \equiv \Pr(v = V).$$

In addition, we assume that new projects have positive NPV and that banks need to borrow in order to invest.

Assumption A1: $qV > x > c_0$

Let us briefly discuss the special features of our model. The main simplifying assumption is that the new projects are identical. This means that banks with bad assets have potentially the same lending opportunities than banks with good legacy assets. It also means that there is no asymmetric information with respect to the new opportunities. We make this assumption for two reasons. The first reason is that we need to keep the benchmark model as simple as possible, because, as will become clear, analyzing government interventions in our economy is quite complicated. Not only do we have the usual complexity of asymmetric information, but in addition our mechanism design problem is not standard because participation in a government program can by itself reveal information, and because we have to take into account a competitive fringe where banks borrow on the private credit market.

The second reason is that, based on our reading of the 2008-2009 crisis, as well as various interactions with banks and investors, it seems that there is more asymmetric information with respect to legacy assets than with respect to new opportunities. It appears possible for banks to provide good documentation on particular new loans they could make and securitize, but the sheer size and complexity of their balance sheets, as well as the ambiguity of their off-balance sheet exposures, means banks know more than outside investors about their legacy assets and liabilities.

Two other assumptions are less important. The assumption of binary types is used only to simplify the analysis, and the assumption of binary outcome for v can easily be generalized to a continuous distribution.

We use this benchmark model to establish our main results. We present extensions in Section 5.

2 Decentralized Equilibria

2.1 Private contracts

As explained earlier, we assume that the random payoff y satisfies the monotone likelihood ratio property with respect to the bank type. In our benchmark model, the income of the bank at time 2 can take on 4 values: 0, V, A, $A+V$. The likelihood ratio increases with the type if and only if $A > V$.[10] Following Innes (1990), we also impose that repayments y^l be weakly increasing in y.

Assumption A2: $A > V$ and y^l is increasing in y.

Lemma 1 *Under Assumption A2, it is optimal for banks to offer debt contracts to investors at time 1.*

Proof. See Appendix. ■

2.2 Equilibria

We have shown that debt contracts are optimal. We are now going to characterize equilibria of the benchmark model. We call an equilibrium *pooling* when all banks invest and face the same interest rate, and we denote it by $\mathcal{P}$. In such an equilibrium, the interest rate of bank debt must be:

$$r_{\mathcal{P}} \equiv \frac{1}{\pi + (1-\pi)\, q}. \tag{6}$$

Let us also define a first threshold for the cash level:

$$c_{\mathcal{P}} \equiv x - \frac{qV - x}{r_{\mathcal{P}} - 1}. \tag{7}$$

We call an equilibrium *separating* if the banks face different interest rates or chose different investment strategies. We denote by $\mathcal{S}$ a separating equilibrium where only the bad banks

[10] We can of course generalize this model. For instance, we can assume that v is uniform over $[0, V]$. In this case, the likelihood ratio is always increasing in a, even if $A < V$.

invest. We define a second threshold for the cash level:

$$c_{\mathcal{S}} \equiv x - \frac{q}{1-q} \left(qV - x \right). \tag{8}$$

The following proposition characterizes the decentralized equilibria.

Proposition 2 *There is no separating equilibrium where the good types invest alone. If $c_0 \in [0, c_{\mathcal{P}}]$, the unique equilibrium is $\mathcal{S}$. If $c_0 \in [c_{\mathcal{P}}, c_{\mathcal{S}}]$, there are multiple equilibria, either $\mathcal{S}$ or $\mathcal{P}$. If $c_0 \in [c_{\mathcal{S}}, x]$, the unique equilibrium is $\mathcal{P}$.*

Proof. The first observation is to note that there is no separating equilibrium where the good types invest alone. In such an equilibrium, the interest rate would be $r = 1$, and the bad types would always want to invest. In a pooling equilibrium $\mathcal{P}$, the interest rate must be $r_{\mathcal{P}}$. It is clearly optimal for the bad types to invest when $r = r_{\mathcal{P}}$. On the other hand, the good types chose to invest if and only if $qV - x - (r_{\mathcal{P}} - 1)(x - c_0) > 0$. Therefore there exists an equilibrium where all types of banks invest if and only if $c_0 \geq c_{\mathcal{P}}$. In a separating equilibrium $\mathcal{S}$ where only the bad banks invest, the interest rate must be $r = 1/q$. It is clearly optimal for the bad types to invest since $qV > x$. On the other hand, the good types chose not to invest if and only if $qV - x - (1/q - 1)(x - c) < 0$. Hence, there exists a separating equilibrium $\mathcal{S}$ with only bad types investing if and only if $c_0 \geq c_{\mathcal{S}}$. Finally, since $1/q > r_{\mathcal{P}}$, we have $c_{\mathcal{S}} > c_{\mathcal{P}}$. ■

The intuition for Proposition 2 is simple. A bank of high quality, $a > rl$ knows it will always repay its lenders, so it will invest if and only if $qV - rl > c_0$. A bank of low quality $a < rl$ knows that it will not repay in the low state, so it will invest if and only if $q(V + a - rl) > a + c_0$. The potential for adverse selection with respect to a exists because the investment equation is more likely to hold for lower values of a. The net value of investing for a good type facing an interest rate r is:

$$qV - x - (r-1)(x-c).$$

The term $(r-1)(x-c)$ is the informational rent paid by the good type. Conversely, a bad type earns rents because it only pays back its creditors with probability q:

$$qV - x + (1 - qr)(x - c).$$

Clearly, the rents are zero if the interest rate correctly reflect the risks of the borrower, $r = 1$ for a good type, and $r = 1/q$ for a bad type. When informational rents are too high, the good types choose not to invest.

Note that the pooling equilibrium is efficient, and the separating equilibrium is inefficient. This observation together with the characterization in Proposition 2 implies that higher cash levels improve economic efficiency. Governments might therefore seek to establish the pooling equilibrium if and when it fails to happen. In the remaining of the paper, we assume that the decentralized equilibrium is inefficient:

Assumption A3: $c_0 < c_{\mathcal{P}}$

We are now going to study interventions under symmetric information (participation decision at time 0) and under asymmetric information (participation decision at time 1).

3 Optimal interventions

In this section we analyze the general properties of optimal interventions. Our setup is different from the usual mechanism design framework in several dimensions. The informational issues are complex because we have to take into account not only the usual adverse selection problem, but also the fact that the decision to participate in the government program may itself signal private information. In addition, we do not want to assume that the government takes over the private lending market. We therefore always allow our banks to borrow in the competitive market. This means that we have to solve a mechanism design problem with a competitive fringe. In fact, we will see in Section 4 that optimal programs only provide partial funding to the banks. Efficient interventions unfreeze private markets, they do not replace them.

3.1 Objective of the government

Under Assumption A3, the decentralized equilibrium is inefficient. The government may chose to intervene to restore efficiency. We assume that the government makes take-it-or-leave-it offers. We can describe all programs in terms of the cash m injected at time 1, and the payments y^g received by the government at time 2. If a bank participates in the program, its cash at time 1 becomes $c_1 = c_0 + m$, and the payments to the shareholders at time 2 become $y_2 - y^l - y^g$. We allow the payments y^g to depend on the payoffs from legacy assets a, the payoffs from the new project v, and on the repayments to creditors y^l. This specification covers all the relevant interventions (equity injections, asset buybacks and debt guarantees) that we describe and implement in Section 4. The banks are all ex-ante identical, so the government makes the same offers to all. All our results immediately apply when there is a heterogenous population of banks. Then, governments programs are conditioned on observable characteristics, such as size, or initial cash holdings, for instance. Let Ψ be the expected cost of the government program. It is given by:

$$\Psi = E\left[m - y^g\right].$$

We assume that there is a deadweight loss χ from raising taxes. The efficiency cost of an intervention is $\chi\Psi$. The cost if 0 if the government decides not to intervene. Since there are only two types of banks, the only alternative is to implement the efficient outcome where all banks invest. In this case, the design problem is to attain the efficient outcome at the smallest cost $\chi\Psi$. Conditional on intervening, the program of the government is therefore simply:

$$\min_{\{m,y^g\}} \Psi,$$

subject to:

$$i\left(\theta\right) = 1 \text{ for } \theta = G, B. \tag{9}$$

We want to characterize the minimum cost, and to find ways to implement the minimum cost program. Let us first define V_{in}^{θ} to be the value for type θ of participating in the government program:

$$V_{in}^{\theta} = E\left[y_2 - y^l - y^g|\theta\right] \tag{10}$$

The following proposition characterizes the cost of any intervention as a function of the inside values:

Proposition 3 *In any program where all banks participate, the cost is*

$$\Psi = E\left[V_{in}^{\theta}\right] - W,$$

where $W \equiv \pi A + c_0 + qV - x$.

Proof. In any program where $i(\theta) = 1$ for $\theta = G, B$, we must have $E[y_2|\theta] = E[a|\theta] + qV + c_2(1)$. From (2), we get $c_2(1) = c_0 + l - x + m$. Taking unconditional expectations of (10), we get

$$E\left[V_{in}^{\theta}\right] = \pi A + qV + E\left[c_0 + l - x + m - y^l - y^g\right].$$

The break even constraint of investors is $E\left[l - y^l\right] = 0$ and the expected cost of the government is by definition $\Psi = E[m - y^g]$. Therefore

$$E\left[V_{in}^{\theta}\right] = \pi A + c_0 + qV - x + \Psi.$$

■

We now proceed to study interventions at time 0 and at time 1.

3.2 Interventions under symmetric information

Let us now study interventions at time 0, i.e. before banks learn their types. At this point there is no asymmetric information between the government and the banks, so the government program must be designed in such a way, so as to attract banks voluntarily and to ensure that banks want to invest given the government intervention. Given that banks do not know their types, the participation constraint is simply

$$E\left[V_{in}^{\theta}\right] \geq E\left[V_{out}^{\theta}\right] \tag{11}$$

where V_{out}^{θ} is the value of staying outside the government program.

Proposition 4 *If banks opt in the government program before they learn the quality of their legacy assets, then the program delivers a profit of $\pi(qV - x)$ to the government.*

Proof. Because banks decide to participate before they learn their type, their decision to opt in or out does not convey any information. Under assumption A3, a bank that opts out would end up in the separating equilibrium $\mathcal{S}$, where only the bad types invest. The outside value is therefore:

$$E\left[V_{out}^{\theta}\right] = \pi(A + c_0) + (1 - \pi)(c_0 + qV - x) = W - \pi(qV - x).$$

From the participation constraint and Proposition 3, we get $\Psi \geq E\left[V_{out}^{\theta}\right] - W$. The government can always reduce its costs by uniformly increasing y^g so the participation constraint binds, and we get: $\Psi = -\pi\left(qV - x\right)$. ■

The intuition behind Proposition 4 is simple. In the inefficient separating equilibrium $\mathcal{S}$, the good types do not invest. The government intervention enables all banks to invest. The net welfare gain is equal to $\pi\left(qV - x\right)$. Since the new lenders who come in at time 1 must break even on average, the welfare gains must accrue to the government and the initial shareholders. However, because the government makes a take-it-or-leave-it offer at time 0, it can extract all the surplus. We will show in Section 4 that equity injections, asset buybacks and debt guarantees can all be designed to achieve this maximum profit.

3.3 Interventions under asymmetric information

Let us now consider interventions at time 1, when banks have private information regarding their legacy assets. These interventions are more difficult to analyze because banks know how much their assets are worth but the government does not. Not only does this create adverse selection issues for the government, but it also implies that the decision to participate in the government program may signal some information about the value of their assets, and therefore influence the market rates offered to participating and non participating banks.

We assume that government can design a program where the repayments depend on a, v and y^l. We allow for explicit dependence on a because we want to be able to discuss asset buyback programs. To be consistent with our assumption on private contracts, we restrict $y^g\left(a, v, rl\right)$ to be increasing in $a + v$.

Assumption A4: $y^g\left(a, v, y^l\right)$ is increasing in $a + v$.

3.3.1 Programs that attract only one type of banks

The decision to participate in the government program can signal the type of the bank. Hence the mere ability of the government to design a program that attracts only a subset of types of banks, alleviates the asymmetric information problem for non-participating banks as well. In fact, in the two-type model we are considering it solves it completely. Particularly appealing seem to be interventions that attract good banks: This is because good banks would be willing to pay to participate in the government program, because by doing do they can borrow at a low interest rate, since they are separated from the bad banks. Our first result demonstrates that such government interventions do not exist: if a program is designed to attract good banks it will necessarily attract bad banks as well.

Proposition 5 *There cannot exist a government program that attracts only good banks.*

Proof. Given a program designed to attract only good banks, participation would reveal good type, non participation reveals bad type. The interest rate would be 1 for participating banks, and $1/q$ for non-participating banks. Since $c_0 < c_{\mathcal{S}}$, the outside option of a good bank is to not invest. The fact that good banks participate implies that $A + qV - (x - c_0 - m) - E\left[y^g\left(A, v, l\right)\right] > A + c_0$ which we can write as

$$qV - x + m > E\left[y^g\left(A, v, l\right)\right].$$

Since $c_0 \leq c_{\mathcal{S}}$, we know that $qV - x < (1-q)\left(x - c_0\right)/q$, which together with the previous inequality implies that

$$\frac{(1-q)}{q}\left(x - c_0\right) + m > E\left[y^g\left(A, v, l\right)\right].$$

Multiplying by q and adding $qV - x + c_0$ on both sides, we get

$$qV - q\left(x - c_0 - m\right) - qE\left[y^g\left(A, v, l\right)\right] > qV - x + c_0. \tag{12}$$

Consider now the decision of a bad bank. Limited liability implies $y^g\left(0, 0, rl\right) = 0$. The participation constraint for bad banks is therefore:

$$qV - q\left(x - c_0 - m\right) - qy^g\left(0, V, l\right) > qV - x + c_0.$$

Since under Assumption A4 $y^g\left(a, v, rl\right)$ is increasing in $a + v$, we see that $E\left[y^g\left(A, v, l\right)\right] \geq y^g\left(0, V, l\right)$, and therefore bad banks want to participate. We conclude that whenever a program is designed to attract only good banks it will necessarily attract bad banks as well.

■

Consider now a program designed to attract only bad banks. Given such a program, which we index by $\mathcal{B}$, participation reveals bad type, whereas, non participation reveals good type. Then, the non participation value for a good bank is $V_{out}^{G} = A + c_0 + qV - x$ and the non participation value for a bad bank is

$$V_{out}^{B} = c_0 + qV - x + (1-q)(x - c_0).$$

Given these values, we can derive a lower bound for the cost of designing such a program:

Proposition 6 *The minimum cost of a program that attracts only bad banks is equal to the informational rents of the bad banks:*

$$\Psi_{\min}^{\mathcal{B}} = (1-\pi)(1-q)(x - c_0).$$

Proof. Calculations similar to the ones done in the proof of Proposition 3 show that

$$\Psi(\mathcal{B}) = (1-\pi)\left(V_{in}^{B} - (c_0 + qV - x)\right).$$

The participation constraint of the bad type is $V_{in}^{B} \geq V_{out}^{B}$. Therefore

$$\Psi(\mathcal{B}) \geq (1-\pi)(1-q)(x - c_0).$$

■

The intuition is straightforward. Separating types requires paying informational rents to the bad types, so the cost of the government program is at least as big as these informational rents.

3.3.2 Programs that attract all banks

In a program designed to attract all banks the private interest rate conditional on participation must be $r_{\mathcal{P}}$. We index these programs by $\mathcal{A}$. Given such a program, deriving the banks' nonparticipating payoffs is delicate, because they depend on the out-of-equilirium belief of investors regarding a bank that would unexpectedly opt out of the program. Let $\tilde{r}$ be the interest rate a bank would face if it decided to opt out of the government program. In general, this rate $\tilde{r}$ could be anywhere between 1 and $1/q$. The outside option of a good bank is

$$V_{out}^{G}(\tilde{r}) = A + \max\{qV - \tilde{r}(x - c_0), c_0\} \quad (13)$$

and the outside option of a bad bank would be

$$V_{out}^{B}(\tilde{r}) = q\left(V - \tilde{r}\left(x - c_0\right)\right). \tag{14}$$

In what follows we assume that the market perception about a bank dropping out from the government program is favorable enough to induce an interest rate $\tilde{r}$ that is low enough for good banks to invest:

Assumption A5: $\tilde{r}$ is such that $qV - \tilde{r}\left(x - c_0\right) > c_0$

Note that A5 is a conservative assumption. It makes it harder for the government to attract good types. By making this assumption, we increase the cost of the programs that attract all banks. We are going to show that programs designed to attract all banks are cheaper than programs that attract just troubled banks, *even* under this conservative assumption.

A program designed to attract all banks is feasible if all banks find it in their interest to participate, the program is incentive compatible, and it induces all banks to invest. Formally:

Definition 1 *A program $\mathcal{A}$ is **feasible** if it satisfies voluntary participation:*

$$V_{in}^{\theta}\left(r_{\mathcal{P}}, \mathcal{A}\left(\theta\right)\right) \geq V_{out}^{\theta}\left(\tilde{r}\right) \text{ for } \theta = B, G, \tag{15}$$

the incentive constraints

$$V_{in}^{\theta}\left(r_{\mathcal{P}}, \mathcal{A}\left(\theta\right)\right) \geq V_{in}^{\theta}\left(r_{\mathcal{P}}, \mathcal{A}\left(\theta'\right)\right) \text{ for } \theta, \theta' = B, G, \tag{16}$$

and the investment constraints:

$$V_{in}^{\theta}\left(r_{\mathcal{P}}, \mathcal{A}\left(\theta\right), i = 1\right) \geq V_{in}^{\theta}\left(r_{\mathcal{P}}, \mathcal{A}\left(\theta\right), i = 0\right) \text{ for } \theta = B, G. \tag{17}$$

The optimal pooling program $\mathcal{A}$ minimizes the cost Ψ among feasible contracts. As we did before, we can obtain a lower bound on the cost for a program designed to attract both kinds of banks:

Proposition 7 . *The lowest possible cost for a program that attracts all banks is*

$$\Psi_{\min}^{\mathcal{A}} = \left(x - c_0\right)\left(1 - \frac{\tilde{r}}{r_{\mathcal{P}}}\right).$$

This minimum cost is lower than the minimum cost for a program that attracts only bad banks.

Proof. Since all banks participate in a pooling equilibrium, we know from Proposition 3 that

$$\Psi = E\left[V_{in}^{\theta}\left(\Gamma\right)\right] - W$$

Using the participation constraints and the outside options (13) and (14).

$$\Psi \geq E\left[V_{out}^{\theta}\left(\tilde{r}\right)\right] - W$$

The lowest bound for the cost is given by

$$\begin{aligned}
E\left[V_{out}^{\theta}\left(\tilde{r}\right)\right] - W &= \pi A + \pi\left(qV - \tilde{r}\left(x - c_0\right)\right) + \left(1-\pi\right)q\left(V - \tilde{r}\left(x - c_0\right)\right) - \left[\pi A + c_0 + qV - x\right] \\
&= \pi A + qV - \tilde{r}\left(x - c_0\right)\left(\pi + \left(1-\pi\right)q - \frac{1}{\tilde{r}}\right) - \pi A + qV \\
&= \left(x - c_0\right)\left(1 - \frac{\tilde{r}}{r_{\mathcal{P}}}\right)
\end{aligned}$$

Since $\tilde{r} \geq 1$, we have

$$\Psi_{\min}^{\mathcal{A}} = \left(x - c_0\right)\left(1 - \frac{\tilde{r}}{r_{\mathcal{P}}}\right) \leq \left(x - c_0\right)\left(1 - \frac{1}{r_{\mathcal{P}}}\right) = \left(x - c_0\right)\left(1-\pi\right)\left(1-q\right) = \Psi_{\min}^{\mathcal{B}}$$

■

This proposition suggests that programs that attract all banks have the potential to dominate programs that attract only bad banks. The reason is that programs that attract only bad banks have the perverse effect of creating the most attractive outside option for banks that consider opting out of the program. This makes it costly for the government to attract the bad banks.

Notice, however, the lower bound for programs that attract all banks might be harder to achieve than that of programs that attract only bad banks. This is because reaching the lower bound requires that the government program be designed to make the participation constraints of *both* types of banks binding. It also presupposes that the constraints that matter for the design are the participation ones and not the incentive, nor the investment constraints.

So far we have derived bounds for what the government can expect to achieve at the best possible program that is designed to attract troubled banks and at the best one designed to attract all banks. We now proceed to ask whether, and under which circumstances, capital injections, asset buybacks and debt guarantees can reach these optimal bounds.

4 Implementation

In this section we study three government interventions that are often used during financial crisis: recapitalization (equity injections), asset buybacks, and debt guarantees. Under symmetric information, we find that each program can be designed to achieve the lowest cost implementation derived in Proposition 4. We then show that this equivalence breaks down under asymmetric information, and that debt guarantees are then optimal.

4.1 Descriptions of programs

We start by describing each intervention. Note that, at this point, we only consider simple programs. In Section 5 we allow the government to offer menus of contracts.

- **Equity injection:** the government offers cash m_α against a share α of equity returns, $y^g = \alpha \left(y_2 - y^l\right)$

- **Asset buyback:** the government offers to buy an amount Z of legacy assets for cash m_z. If a bank opts in the program, the face value of its legacy assets decreases by Z. The payoffs to the government are $y^g = a\frac{Z}{A}$.

- **Debt guarantee:** the government offers to guarantee debt issuance up to an amount S for a fee ϕ paid up-front: $m = -\phi S$. Private lenders accept an interest rate of 1 on the guaranteed debt, so the date 1 budget constraint becomes

  $$x = c_0 + (1 - \phi) S + l^u,$$

  where l^u is the unsecured loan. The government will have to make payments in case of default: $y^g = -S$ if $a + v = 0$.[11]

Note that these specific programs all belong to the general class of mechanisms we wrote down earlier. In particular, all these interventions satisfy the monotonicity condition A4 that y^g be increasing in $a + v$.

[11] This is also equivalent to the goverment providing a junior loan with face value S and an interest rate of $1/(1 - \phi)$.

4.2 Symmetric information at participation stage

We start by considering the performance of optimal versions of these programs when banks make their participation decisions $t = 0$: At that point banks do not have any private information when they decide whether to participate in the program or not. Our first main result is:

Theorem 1 *When banks make their participation decisions* $t = 0$ *equity injection, asset buybacks and debt guarantee are all optimal and achieve the maximum profits for the government.*

Proof. See Appendix ■

The critical point of the Theorem is that the interventions can actually make sure that (9) is satisfied and at the same time the participation constraint (11) holds with equality. For each program, we first make sure the banks have enough liquid assets (cash) to invest at time 1 even under asymmetric information. Once this is achieved the government simply sets the other part of the program to make the participation constraint binding, by asking for the right amount of equity or assets. Of course, if interventions under symmetric information were feasible, the banks could also raise private money from investors by issuing debt or equity before learning their types.

An alternative – and perhaps more relevant – interpretation of the symmetric information outcome is that the government can exert pressure and force participation in its program. Most observers have suggested that banks were required to participate in the initial equity injection program of October 2008.[12] Since the government has no reason to impose a loss on the industry as a whole, we can assume forced participation subject to the average bank breaking even (or subject to a diversified household owning shares and bonds of the banks breaking even). This is then formally equivalent to our model with symmetric information at the participation stage. The type of arm-twisting intervention is obviously not feasible in the private sector, and this might explain why banks did not raise money by themselves on private markets.

[12] "The Bush administration will announce a plan to rescue frozen credit markets that includes spending about half of a total of $250 billion for preferred shares of nine major banks [...] **None of banks getting government money was given a choice** about it, said one of the people familiar with the plans." (Bloomberg, Oct. 13, 2008).

4.3 Asymmetric information

We now examine the implementation of optimal programs when the banks make their participation decisions $t = 1$, once they know their value of their assets. We have shown in Proposition 5 that there does not exist a program that can attract only good banks. We therefore only need to consider programs that attract only bad banks, and programs that attract all banks.

Proposition 8 *All three interventions are optimal among the programs that only attract bad banks*

Proof. See Appendix ■

The intuition for this result is based on the fact that the program reveals the type of the bank: The good banks do not participate and raise money on private markets at a low interest rate. The bad banks must be convinced to participate. Hence, the program must be generous enough. But once participation is ensured, investment follows automatically because all banks face fair interest rates. The three programs can be designed to give the bad banks their expected informational rents.

Let us now consider programs that attract all banks. For such programs, our main result is that debt guarantees are optimal: debt guarantees achieve efficiency at the minimum cost for the government.

Proposition 9 *Guaranteeing new debt is the optimal intervention among the programs that attracts all banks.*

Proof. Notice first that no bank wants to issue guaranteed debt without investing: if the bank does not invest, shareholders receive $a + c_0 - \phi S$ which is decreasing in S irrespective of a as long as $\phi \geq 0$. After joining the program, the choice is therefore to invest with guarantee, or not to invest at all. The net value of debt guarantee for the good type is:

$$\Sigma(\phi, S) \equiv ((1 - \phi) r_{\mathcal{P}} - 1) S. \tag{18}$$

This net value corresponds to the interest payments saved thanks to the guarantee, net of the fees paid to the government. It is optimal for the good type to use the guarantee and invest if and only if:

$$qV + \Sigma - r_{\mathcal{P}} (x - c_0) > c_0. \tag{19}$$

The participation constraint of the good type is

$$qV + \Sigma - r_{\mathcal{P}} (x - c_0) \geq qV - \tilde{r} (x - c_0) \tag{20}$$

We can see immediately from Assumption A5 that (20) implies (19).[13] For the bad type, the participation constraint is

$$q (V + \Sigma + r_{\mathcal{P}} (x - c_0)) \geq q (V - \tilde{r} (x - c_0)). \tag{21}$$

Both participation constraints (20) and (21) are equivalent to $\Sigma \geq (r_{\mathcal{P}} - \tilde{r}) (x - c_0)$. Moreover, the government can ensure that these constraints bind by increasing the fee ϕ. This implies that

$$\Psi (\mathcal{D}) = \Psi^{\mathcal{A}}_{\min}.$$

which says that the debt guarantee program achieves the minimum cost among all the programs $\mathcal{A}$ that attract all types. ■

Notice that debt guarantees achieve the lower bound for the cost function. They are thus optimal among all conceivable programs that attract all the banks, not just among the three programs for which we provide detailed analysis. The intuition behind Proposition 9 is contained in the proof. The first idea is that the investment constraints are implied by the participation constraints. The second idea is that the participation constraints of both types are equivalent. Hence a debt guarantee program achieves the bound derived in Proposition 7.

So far we have shown that debt guarantees achieve the lowest cost intervention. We now turn to equity injections and asset buybacks. To be fully general we allow the government to construct a hybrid program that includes some equity injection, some asset buybacks and some debt guarantee.

[13] This property holds even when Assumption A5 fails. In that case, the left-hand-side of the participation constraint (after we cancel-out A) is c_0, implying that the participation and investment constraints remain equivalent.

Theorem 2 *As long as debt guarantees are available, an optimal intervention never includes any asset buyback or any equity injection. If debt guarantees are exogenously ruled out, a pure equity injection program dominates a pure asset buyback program.*

Proof. See Appendix. ■

Finally, we can compare the cost of designing government interventions that attract only bad banks (separating), with the ones that are designed to attract all banks (pooling).

Theorem 3 *The optimal government intervention to achieve the efficient level of investment is to set up a pure debt guarantee program in which all banks participate.*

Proof. Proposition 7 shows that the minimum cost is lower for programs that attract all banks that for programs that only attract bad banks. Proposition 9 shows that debt guarantees achieve this minimum cost. Theorem 2 shows that assets buybacks and equity injections are never optimal. ■

Our Theorem establishes that the optimal way to restore efficient lending are debt-guarantees. This is the best not only among equity injection or assets buybacks, but among all conceivable mechanisms.

5 Extensions

In this section we provide two important extensions to our main results. The first extension is to consider menus of contracts. The second extension is to consider the consequences of moral hazard in addition to adverse selection.

5.1 Menus

We have assumed in Section 4 that the government offers a unique contract. Given this assumption we have shown that debt guarantees are optimal, while equity injections and asset buybacks are not. We now allow the government to offer menus of contracts where banks can select various levels for the parameters of the program. For instance, a larger injection of cash against a larger share of equity. It is important to emphasize that the results of Section 3 are valid with or without menus, since they only rely on the participation constraints and the competitive fringe. Hence we know we will not improve upon the simple debt guarantee program, but we can hope to obtain the same lower bound for the cost.

In what follows we appeal to the revelation principle and look menus with two options: one for good banks, another for bad banks. We first establish in the case of debt guarantee and asset-buybacks that there does not exist incentive compatible type-dependent menus: The only incentive compatible contract is a pooling one.

Proposition 10 *For asset-buybacks and debt guarantees the only incentive compatible menus are menus offering only one option.*

Proof. See Appendix ■

Now we move on to examine the optimal menu for the case of equity. The revelation principle tells us that the program can be taken to consist of an option for each type:

$$\alpha_G, m_G \text{ and } \alpha_B, m_{B.}$$

In order to be feasible each of these options must satisfy the participation constraints as before, and additionally incentive-compatibility constraints. However, observe that now, even though the government is designing a program to attract all banks, the interest rate that banks face is not $r_{\mathcal{P}}$. Good banks choose the option α_G, m_G and face an interest rate of 1, whereas banks choose option α_B, m_B and face an interest rate of $1/q$. Given that banks face a fair interest rate, when they choose to participate, they will always invest, hence there is no need to consider investment constraints. The cost-minimizing program for the government solves:

$$\min_{\alpha_G, m_G, \alpha_B, m_B} \pi \left\{ (1-\alpha_G) \, m_G - \alpha_G \, (\pi A + c_0 + N) \right\} + (1-\pi) \left\{ (1-\alpha_B) \, m_B - \alpha_B \, (c_0 + N) \right\}$$

subject to incentive constraints (IC_B) and (IC_G), participation constraints (PC_G) and (PC_B), and constraints on equity shares α_G and α_G being positive. The constraints can be found in the Appendix. Note that the constraints that the $\alpha's$ must be less than 1 are ignored because they are implied by the participation constraints.

Proposition 11 *The optimal menu is given by* $\alpha_G^* = 0$ *and* $m_G^* = -(\tilde{r}-1)(x-c_0)$*, and* α_B^* *and* m_B^* *are such that*

$$(1-\alpha_B) m_B^* - \alpha_B^* \, (N + c_0) = (1 - q\tilde{r}) \, (x - c_0) \, ,$$

which ensures that both IC_B and PC_B hold with equality. This menu achieves the minimal cost, exactly as the simple debt-guarantee program, namely

$$\Psi^* = \frac{r_{\mathcal{P}} - \tilde{r}}{r_{\mathcal{P}}} (x - c_0) = \Psi_{\min}(\mathcal{A}).$$

Proof. See Appendix. ■

We conclude that equity injections can be designed to be as efficient as debt guarantee programs, but they involve complex menus of programs, each one designed for a particular type of bank.

5.2 Moral hazard

So far we have assumed that all new projects are identical and all have positive NPV. Under these assumptions, the government objective is to obtain the maximum amount of lending at the minimum cost, and we have shown that a debt guarantee program designed to attract all banks is the most cost-effective intervention. In reality, however, banks can sometimes control the characteristics of their new lending opportunities. In particular, they can certainly choose various degrees of riskiness in the new loans that they extend. It is therefore important to understand how endogenous risk-taking could affect our results on the optimality of debt guarantees.

We introduce a new project with random binary payoff $v' = 0, V'$. The investment cost is the same x as in the project v. The probability of success is $q\prime \equiv \Pr(v' = V')$, and we assume that $V' > V$ while $qV' < qV$. Project $v\prime$ is therefore riskier and less efficient than project v. We assume that the choice of the project is observable by the private agents, but cannot be enforced by the government.

Assumption A6: **Moral hazard**. The choice of project $v\prime$ is observable by private lenders but cannot be controlled by the government.

Assumption 6 implies that the government's debt guarantee program can increase risk taking as banks and private investors take advantage of the implicit subsidy. Note that A6 is the worst case for the government. The alternative assumption of hidden risk choice by the bank would imply risk shifting with or without the government's program. While risk shifting would be a greater issue in that case, it would be unrelated to the government's intervention, and would therefore not lower the efficiency of debt guarantee programs.

Proposition 12 *The equilibria with equity injection or asset buyback are unaffected by the availability of project $v\prime$.*

Proof. Consider first the equilibria without interventions. Because $v\prime$ is observable, the lending rates from the private sector depend on whether v or $v\prime$ is chosen. In a separating equilibrium, the banks obtain the fair interest rates. Therefore they always chose the projects with the higher NPV. In the pooling case, it is easy to see that good types dislike project $v\prime$ because it generates a higher interest rate and because good types always repay their debts. Bad types cannot chose $v\prime$ without revealing their types. Hence the project $v\prime$ is never chosen in a decentralized equilibrium. Asset buybacks do not change this result because the net payments from the government do not depend on the project choice. In the case of equity they do, but the banks still seek to maximize shareholder value, and they make the same project choices as in the decentralized equilibrium. ■

Under equity injection or asset buyback, risky projects do not interfere with the government program. In fact, it is easy to see that good types would be willing to chose safer project even if they had lower NPV. This anti risk shifting could be interpreted as a costly signalling device to reveal their types.

We now turn to debt guarantees. The issue with debt guarantees is that the subsidies are higher when the projects are riskier. Consider first the case of programs that attract only bad banks.

Lemma 2 *Programs designed to attract bad banks induce risk shifting when $(q - q')(x - c_0) > qV - q'V'$.*

Proof. In the separating case the government must offer $\phi = 0$ and $S = x - c_0$ (see proof of Proposition 8 in the Appendix). The participating bad banks borrow the full amount with guarantee at an interest rate of one. They would chose $v\prime$ over v if and only if:

$$q'\left(V' - (x - c_0)\right) > q\left(V - (x - c_0)\right).$$

■

Since we have shown that interventions that attract all banks dominate the ones that only attract bad banks, we are more interested in consequences of moral hazard in this case. Consider first the benchmark model where the good type knows for sure that $a = A$. The key point here is that they have no incentives to risk shift at the cost of the government because they always repay their debts. The same is not true for the bad types. Choosing the risky project is tempting for the bad types because of the implicit subsidy, but it is costly because it reveals their type. If they choose v, they pool and obtain the value function $V_{in}^{B} = qV + q\left((1-\phi) r_{\mathcal{P}} - 1\right) S - q r_{\mathcal{P}} (x - c_0)$. If they choose $v\prime$ they obtain

$$V_{in}^{B\prime} = q'V' + \left((1-\phi) - q'\right) S - (x - c_0) .$$

So they chose $v\prime$ if and only if $V_{in}^{B\prime} > V_{in}^{B}$ which yields

$$\left(q - q'\right) S > qV - q'V' + (x - c_0 - (1-\phi) S)(1 - q r_{\mathcal{P}}) \tag{22}$$

The LHS is the net benefit from risk shifting. The RHS is the cost, which has two parts. The term $qV - q'V'$ is the NPV loss. The last term is the revelation cost of facing a high interest rate and loosing the information subsidy $1 - q r_{\mathcal{P}}$ from the pooling equilibrium. This reputation cost applies to the part of borrowing that is not insured, $x - c_0 - (1-\phi) S$. The formula shows that risk shifting is more likely when $q\prime$ is small, $V\prime$ is large, and S is large. We can summarize our discussion by the following proposition.

Proposition 13 *In the benchmark model, risk shifting occurs when the debt guarantee is large enough to induce the bad types to select the risky project even though this choice reveals their type. This happens when condition (22) is satisfied. Risk shifting is less likely to occur in the efficient program than in a program attracting only the bad banks.*

The good news here is that the optimal debt guarantee program (attracting all banks with as little guarantee as possible) is the least likely to induce risk shifting. Risk shifting only occurs when the subsidy is high enough to dominate both the NPV loss and the negative signalling. Note that in the benchmark model, good types still do not engage in risk shifting because they know the quality of their legacy assets.

If we move away from the benchmark model and assume that even good types have some credit risk, then risk shifting could potentially happen for both types. However, even in this case, the benefits from risk shifting at the expense of tax payers are limited by the negative signalling impact of risk shifting on the private lending rate. Risk shifting is always more tempting for bad banks, and choosing the safer project signals a good type. Therefore when we check the existence of a pooling risk-shifting equilibrium, we must allow the good type to deviate, choose the safer project, and face the low market rate of $\tilde{r}$. This endogenous response makes risk shifting less appealing. (See the Appendix for a brief analysis of this case).

We conclude that the risk shifting problem might not be as damaging for government interventions as one would have predicted before our analysis. Either risk shifting is observed by market participants and the endogenous response of private lending rates puts discipline on the banks, or risk shifting is private information to the bank, but then the moral hazard problems occurs with or without the government. If the risk shifting problem is so severe that even the NPV loss and the negative signalling cannot prevent it, then the government should implement the optimal outcome using a menu of equity injections. We have see that these menus are as efficient as the simple debt guarantee program (Proposition 11), and that equity injections do not create incentives for risk shifting (Proposition 12).

6 Conclusion

We provide a complete characterization of the most cost-effective interventions to restore efficient lending and investment in markets that collapse due to asymmetric information about the credit-worthiness of borrowers. In doing so we make two contributions. On the technical side, we solve a non standard mechanism design problem with two information sensitive decisions (investing or not, participating or not) and in the presence of a competitive fringe (the government does not shut down the private markets).

On the normative side, our main results are as follows. If participation decisions occur under symmetric information (or equivalently under forced participation), then all common interventions (capital injections, asset buyback and debt guarantees) are equivalent and optimal. In the more interesting case where government programs are subject to adverse selection, we find that it is less costly to design programs that attract all banks. The optimal way of doing so is by offering a simple program of debt guarantees. This is the most cost-effective intervention among all possible government interventions aimed at increasing bank lending. Moral hazard and endogenous risk taking may overturn the optimality of debt guarantees. If risk shifting is a material concern, we show that an optimal intervention can be implemented by offering a menu of recapitalization contracts.

A Proof of Lemma 1: Private Contracts

Let E^θ denote expectations under distribution for type $\theta = B, G$. Ignoring the monotonicity constraint, the program of a good bank trying to separate from a bad bank is:

$$\max_{y^l \in [0,y]} E^G \left[y - y^l \right]$$

$$\begin{aligned} E^G \left[y^l \right] &\geq x - c_0 \\ E^B \left[y - y^l \right] &\leq \tilde{B} \end{aligned}$$

where $\tilde{B}$ is the outside option of the bad type. Using the density functions f^θ, we can write the Lagrangian as

$$\begin{aligned} \mathcal{L} &= \int \left(y - y^l \right) f^G(y)\, dy + \lambda \left(\int y^l f^G(y)\, dy - (x - c_0) \right) - \mu \left(\int \left(y - y^l \right) f^B(y)\, dy - \tilde{B} \right) \\ &= \int \left(1 - \lambda - \mu \frac{f^B(y)}{f^G(y)} \right) \left(y - y^l \right) f^G(y)\, dy + \lambda \left(E[y] + x - c_0 \right) + \mu \tilde{B} \end{aligned}$$

Under A2, f^G / f^B is increasing in y, so f^B / f^G is decreasing, and $1 - \lambda - \mu f^B(y) / f^G(y)$ is increasing. When it is negative, it is optimal to set $y - y^l = 0$. When it turns positive, it is optimal to set $y^l = 0$. This is the well known result of a "live or die" contract. If we know introduce the monotonicity constraint of Innes (1990), it is easy to see that that as long as the contract is strictly increasing, the monotonicity does not bind, and when the contract tends to decrease, the monotonicity constraint forces it to be constant. We therefore obtain a debt contract.

If the good bank cannot separate and is forced to pool, then the program becomes

$$\max_{y^l \in [0,y]} E^G \left[y - y^l \right]$$

$$E^{\mathcal{P}} \left[y^l \right] \geq x - c_0$$

where $\mathcal{P}$ denotes that pooling distribution. We can then write the Lagrangian as

$$\mathcal{L} = \int \left(1 - \lambda \frac{f^{\mathcal{P}}(y)}{f^G(y)} \right) \left(y - y^l \right) f^G(y)\, dy + \lambda \left(E^{\mathcal{P}}[y] + x - c_0 \right)$$

Again, since $f^{\mathcal{P}} = \pi f^G + (1 - \pi) f^B$, we get the "live or die" contract if we do not impose monotonicity, and the debt contract when we impose that y^l be increasing in y.

B Proof of Theorem 1: Equivalence of Interventions under Symmetric Information

B.1 Equity injection

The government cost function is

$$\Psi_0^E = m_\alpha - \alpha E \left[a + c_2 + v \cdot i - y^l \right]$$

To make sure that all banks invest, the government must inject $m_\alpha = c_{\mathcal{P}} - c_0$. Given such a government program, all firms invest, so $i = 1$ and $c_2 = 0$ for all types. Then, the cost function becomes

$$\Psi_0^E = (1-\alpha)\, m_\alpha - \alpha\left(\pi A + c_0 + qV - x\right).$$

By participating, a bank knows that it will be able to invest irrespective of its type, and that it will receive a cash m_α. In return, it will give up a fraction α of its equity. The participation constraint at time 0 then implies:

$$\alpha\left(\pi A + c_0 + qV - x\right) = (1-\alpha)\, m_\alpha + \pi\left(qV - x\right).$$

Therefore we see that the cost is negative:

$$\Psi_0^E = -\pi\left(qV - x\right).$$

B.2 Asset buyback program

Let $\tilde{r}_{\mathcal{P}}$ be the pooling rate in the modified equilibrium with assets reduced to $A - Z$. We consider two cases, depending on the size of the asset purchase.
Case 1: $A - Z > r_{\mathcal{P}} l$. Then the good type is not risky and the equilibrium conditions are the same as in the equilibrium without intervention. This means that $\tilde{r}_{\mathcal{P}} = r_{\mathcal{P}}$. The cash injection needed is $m_z = c_{\mathcal{P}} - c_0$. The government cost is

$$\Psi_0^A = m_z - \pi Z$$

The participation constraint at time 0 is simply $\pi\left(qV - x\right) \geq \pi Z - m_z$, therefore

$$\pi Z = \pi\left(qV - x\right) + m_z$$

The cost function is the same as with capital injections:

$$\Psi_0^A = -\pi\left(qV - x\right).$$

Case 2: $A - Z < r_{\mathcal{P}} l$. Then the good type is risky and $\tilde{r}_{\mathcal{P}} > r_{\mathcal{P}}$. We can find the new rate using $E\left[y^l\right] = l$:

$$\tilde{r}_{\mathcal{P}} = \frac{l - (1-q)\,\pi\left(A - Z\right)}{ql}.$$

Assuming good types invest, the participation constraint at time 0 is

$$\pi\left(A - Z\right) + qV - \left(x - c_0 - m_z\right) \geq \pi\left(A + c_0\right) + (1-\pi)\left(qV - \left(x - c_0\right)\right)$$

Binding participation constraint means that $\pi Z = \Delta^{NPV} + m_z$, and once again we find $\Psi^a = -\Delta^{NPV}$. Given the rate, the good type wants to invest iff $q\left(V - r_{\mathcal{P}} l + A - Z\right) > A - Z + c_0 + m_z$. Using $l = x - c_0 - m_z$, this is equivalent to

$$qV > x + (1-q)(1-\pi)\left(A - Z\right)$$

We can always choose Z to satisfy the investment constraint, and then m_z to satisfy the participation constraint.

B.3 Debt guarantee program

With this program, the firm has a different capital structure at time 1. It has debt on its balance sheet. If this debt is super senior, it can create debt overhang, which would be inefficient. So the government should make sure that the firm can issue new debt l^u which is senior to S. If l^u is senior, then the repayments to new lenders do not depend on S, so the pooling rate is the same as without the program, namely $r_{\mathcal{P}}$. The good type chooses to invest if and only if:

$$q\left(A+V-S-r_{\mathcal{P}}l^u\right)+(1-q)\max\left(A-S-r_{\mathcal{P}}l^u,0\right)>A-S+c_0+(1-\phi)\,S.$$

Once again, there are two cases, depending on whether the high type remains credit worthy with probability one.

Case 1:$A>S+r_{\mathcal{P}}l^u$. The investment condition becomes

$$qV>r_{\mathcal{P}}l^u+c_0+(1-\phi)\,S. \tag{23}$$

So if $m_\alpha=(1-\phi)\,S$, we get exactly same as in equity injection. The expected cost for the government is

$$\Psi_0^S=(1-\phi)\,S-\pi S-(1-\pi)\left(q\min\left(V-r_{\mathcal{P}}l^u,S\right)+(1-q)\,0\right)$$

Assume for now that $V>S+r_{\mathcal{P}}l^u$, we get

$$\Psi_0^S=(1-\phi)\,S-\pi S-(1-\pi)\,qS$$

The participation constraint at time 0 is

$$\pi A+qV-\left(x-c_0-(1-\phi)\,S\right)-\pi S-(1-\pi)\,qS\geq\pi\left(A+c_0\right)+(1-\pi)\left(qV-\left(x-c_0\right)\right)$$

So

$$\pi\left(qV-x\right)+(1-\phi)\,S=\pi S+(1-\pi)\,qS$$

And we get the same as above, $\Psi^s=-\pi\left(qV-x\right)$. We must now check that we can implement the program with $V>S+r_{\mathcal{P}}l^u$. To sustain investment by all kinds of banks, the cash injection must be such that $c_0+(1-\phi)\,S=c_{\mathcal{P}}=x-\frac{qV-x}{r_{\mathcal{P}}-1}$. So $l=\frac{qV-x}{r_{\mathcal{P}}-1}$ and $r_{\mathcal{P}}l^u=qV-x+l^u$. So we want $V>S+N+l^u$, or $(1-q)\,V>\phi S-c_0$. This is clearly satisfied as long as $\phi\leq1-q$. This simply means that the credit premium cannot be tougher than the premium the market would charge to a low type. It can be equal, however. This means that the government can always implement with at least a fair premium, and that the constraint $V>S+r_{\mathcal{P}}l^u$ is not binding.

Case 2: $A<S+r_{\mathcal{P}}l^u$. The investment condition becomes

$$qV>qr_{\mathcal{P}}l^u+(1-q)\left(A-S\right)+c_0+(1-\phi)\,S \tag{24}$$

but since $A<S+r_{\mathcal{P}}l^u$ we know that

$$r_{\mathcal{P}}l^u>qr_{\mathcal{P}}l^u+(1-q)\left(A-S\right)$$

therefore if (23) is satisfied, then (24) is satisfied. In other words, the investment condition is easier to satisfy. In expected value, however, we still get same cost, because the participation constraint at time 0 is

$$\begin{aligned}&\pi A+qV-\left(x-c_0-(1-\phi)\,S\right)-\pi\left(qS+(1-q)\left(A-r_{\mathcal{P}}l^u\right)\right)-(1-\pi)\,qS\\\geq\;&\pi\left(A+c_0\right)+(1-\pi)\left(qV-\left(x-c_0\right)\right)\end{aligned}$$

So

$$\pi(qV - x) + (1-\phi)S = \pi(qS + (1-q)(A - r_{\mathcal{P}} l^u)) + (1-\pi) qS$$

But from $\Psi^s = (1-\phi)S - \pi(qS + (1-q)(A - r_{\mathcal{P}} l^u)) - (1-\pi) qS$ we obtain once again

$$\Psi_0^S = -\pi(qV - x).$$

C Proof of Proposition 8: Implementation of Program for Bad Banks

C.1 Equity injection

In what follows, we use

$$N \equiv qV - x$$

to denote the NPV of the project. The participation constraint for bad banks is $(1-\alpha)(c_0 + m_\alpha + N) > c_0 + N + (1-q)(x - c_0)$, which implies that

$$\alpha < \alpha^B(B) \equiv \frac{m_\alpha - (1-q)(x-c_0)}{N + c_0 + m_\alpha}.$$

If good banks participate in this equilibrium they are perceived as bad, and face the high interest rate. Then good banks would not participate as long as $(1-\alpha)(A + c_0 + m_\alpha) < A + c_0 + N$, which is equivalent to

$$\alpha > \alpha^G(B) \equiv \frac{m_\alpha - N}{A + c_0 + m_\alpha}.$$

The government can design an equity injection program that only attracts bad banks, if one can find α such that bad banks opt in and good ones drop out:

$$\begin{aligned} \alpha^G(B) \quad &< \quad \alpha^B(B) \Longleftrightarrow \frac{m_\alpha - N}{A + c_0 + m_\alpha} \leq \frac{m_\alpha - (1-q)(x-c_0)}{N + c_0 + m_\alpha} \\ &\Longleftrightarrow \quad (c_0 + m_\alpha)(1-q)(x-c_0) \leq A(m_\alpha - (1-q)(x-c_0)) + N(N + c_0) \end{aligned}$$

Suppose we set $m = x - c_0$, then we get

$$\alpha^G(B) < \alpha^B(B) \Longleftrightarrow x(1-q)(x-c_0) \leq Aq(x-c_0) + N(N+c_0)$$

But since $A + c_0 > x/q$ and $c_0 < x$, then $A > x/q - x$, and we always get $\alpha^G(B) < \alpha^B(B)$. Finally, we need to check that $\alpha < 1$. Indeed, when $m = x - c_0$

$$\alpha^B(B) = \frac{x - c_0}{V} < 1.$$

Hence, for the range of parameters specified in Assumptions A1, A2 and A3, the government can design an equity program that attracts bad banks.

We can compute the expected cost of the government program:

$$\begin{aligned} \Psi^E(\mathcal{B}) &= (1-\pi)\left(m_\alpha - \alpha_b^b(c_0 + m_\alpha + N)\right) \\ &= (1-\pi)\left(m_\alpha - \frac{m_\alpha - (1-q)(x-c_0)}{N + c_0 + m_\alpha}(c_0 + m_\alpha + N)\right) \\ &= (1-\pi)(1-q)(x-c_0). \end{aligned}$$

C.2 Assets buyback

In asset buyback program designed to attract only bad banks, participation reveals bad type, non participation reveals good type. A good type who participates would not invest. Therefore, in this case a bank with good assets would not participate in the government program if:

$$\begin{aligned} A - Z + c_0 + m_z &< A + c_0 + N \\ m_z - Z &< N. \end{aligned}$$

Banks with bad assets participate in the government program if:

$$\begin{aligned} c_0 + m_z + N &> c_0 + N + (1-q)(x - c_0) \\ m_z &\geq (1-q)(x - c_0). \end{aligned}$$

Then, the government can design an asset buyback program that only attracts bad banks by setting

$$Z = (1-q)(x - c_0) - N + \varepsilon, \text{ for some } \varepsilon > 0.$$

Then, in regards to choosing m_z the best that government can do is to set just high enough for bad banks to participate, that is:

$$m_z = (1-q)(x - c_0),$$

then it is immediate to see that the expected cost to the government is

$$\Psi^{ABB}(\mathcal{B}) = (1-\pi)(1-q)(x - c_0).$$

C.3 Debt guarantee

Suppose that the government wants to design a debt guarantee program that only attracts bad banks. Then the participation constraint for bad banks is given by:

$$\begin{aligned} q(V - S) - l^u &= c_0 + N + (1 - q - \phi) S \\ &> c_0 + N + (1-q)(x - c_0) - \phi S \\ &> (1-q)(x - c_0 - S) \end{aligned}$$

On the other hand, the participation constraint for good banks is given by:

$$A + qV - S - \frac{(x - (c_0 - \phi S) - S)}{q} > A + qV - x$$

which requires that

$$-\phi S + (1-q) S > (1-q) x - c_0$$

Now, the government can design a program that attracts only bad banks by setting

$$-\phi S + (1-q) S = (1-q)(x - c_0).$$

This makes the participation constraint of bad banks bind and the total cost is

$$\Psi^{DG}(\mathcal{B}) = (1-\pi)(1-q)(x - c_0),$$

as before.

D Proof of Theorem 2: Comparisons

D.1 Pure debt guarantee dominates any hybrid program

We allow the government to offer a hybrid program that consists of a mixture of capital injections, debt guarantees and assets buybacks: $\Gamma = \{\alpha, Z, S, m_\alpha, m_z, \phi\}$. This formulation allows us to establish a number of results that hold for all programs. These results entail to which of the constraint bind.

The government seeks to minimize its cost subject to a set of constraints. We first derive an expression of the costs given a hybrid program Γ :

Lemma 3 *The expected cost of the government program is*

$$\Psi = x - c_0 + (1-\alpha)\left(\frac{\Sigma}{r_{\mathcal{P}}} - \pi Z + c_1 - x\right) - \alpha\left(\pi A + qV\right), \tag{25}$$

where $c_1 = c_0 + m_\alpha + m_z$, and the net value of debt guarantee is

$$\Sigma\left(\phi, S\right) = \left(\left(1-\phi\right) r_{\mathcal{P}} - 1\right) S.$$

Proof. The government finances $c_1 - c_0$ up-front. The expected default loss on the credit insurance is $(1-\pi)(1-q)$ since bad type defaults when their new projects fail. The net cost of the insurance liability is therefore:

$$\left(\left(1-\pi\right)\left(1-q\right) - \phi\right) S = \left(1 - \frac{1}{r_{\mathcal{P}}} - \phi\right) S = \frac{\Sigma}{r_{\mathcal{P}}}$$

From the good type, the government receives $Z + \alpha\left(A - Z + \Sigma + qV + r_{\mathcal{P}}\left(c_1 - x\right)\right)$. From the bad type the government receives $\alpha q\left(V + \Sigma + r_{\mathcal{P}}\left(c_1 - x\right)\right)$. The net cost to the government is therefore

$$\begin{aligned}\Psi\left(\Omega\right) &= c_1 - c_0 + \frac{\Sigma}{r_{\mathcal{P}}} - (1-\alpha)\pi Z - \alpha\left(\pi A + qV + \left(\pi + (1-\pi) q\right)\left(\Sigma + r_{\mathcal{P}}\left(c_1 - x\right)\right)\right) \\ &= c_1 - c_0 + \frac{\Sigma}{r_{\mathcal{P}}} - (1-\alpha)\pi Z - \alpha\left(\pi A + qV + \frac{\Sigma}{r_{\mathcal{P}}} + (c_1 - x)\right).\end{aligned}$$

The formula then follows from the definition of $r_{\mathcal{P}}$ in equation (6) and Σ in (18). ■

Using the fact that $\pi r_{\mathcal{P}} + (1-\pi) q r_{\mathcal{P}} = 1$, we can also rewrite the cost function as

$$\Psi = (1-\alpha)\left(c_1 - c_0 + \left((1-\pi)(1-q) - \phi\right) S - \pi Z\right) - \alpha\left(\pi A + c_0 + N\right).$$

The intuition is clear. The government gets a share α of the equity, whose net value (averaged across types) is $\pi A + c_0 + N$. On the other hand, the government injects cash $c_1 - c_0$, receives assets worth πZ and provides credit guarantee at price ϕ against expected loss $(1-\pi)(1-q)$.

In order to simplify the expression of the Lagrangian we move on to establish a few results regarding which constraints are binding. In order to do so, we first need to characterize the inside values V_{in}^{θ} for the general program.

Lemma 4 ***Inside values***. *The inside value of a good bank in a pooling equilibrium*

$$V_{in}^{G}\left(r_{\mathcal{P}},\Gamma\right)=\left(1-\alpha\right)\left(A-Z+\Sigma+qV+r_{\mathcal{P}}\left(c_{1}-x\right)\right) \tag{26}$$

and the inside value of a bad bank is

$$V_{in}^{B}\left(r_{\mathcal{P}},\Gamma\right)=(1-\alpha)q\left(V+\Sigma+r_{\mathcal{P}}\left(c_{1}-x\right)\right). \tag{27}$$

After joining the program a bad bank always wants to invest, and a good bank wants to invest if and only if

$$\left(r_{\mathcal{P}}-1\right)c_{1}+\Sigma\geq r_{\mathcal{P}}x-qV \tag{28}$$

Proof. First note that as long as $\phi>0$, it is never optimal to use the debt guarantee without investing. If the bank does not invest, shareholders receive

$$a+c_{0}-\phi S.$$

This is decreasing in S irrespective of a. By entering the program, the bank receives $m_{\alpha}+m_{z}$ in cash. It issues guaranteed debt S at an interest rate of 1 and pays ϕS to the government. Its new cash balance is then $c_{1}+\left(1-\phi\right)S$. Its unsecured borrowing, at rate $r_{\mathcal{P}}$, is therefore:

$$l^{u}=x-c_{1}+\left(1-\phi\right)S.$$

Now consider the total shareholder value at time 2. A good bank always repays all its loans, therefore total shareholder value is

$$A-Z+qV-S-r_{\mathcal{P}}l^{u}=A-Z+qV-r_{\mathcal{P}}\left(x-c_{1}\right)+\left(r_{\mathcal{P}}\left(1-\phi\right)-1\right)S$$

If it does not invest, its shareholder value is $A-Z+c_{1}$. Comparing with the previous equation leads to condition (28). Since initial shareholders only keep a fraction $1-\alpha$ of the total at time 2, we obtain (26). A bad bank, by contrast, knows it will default with probability q. Total shareholder value at time 2 is then

$$q\left(V-S-r_{\mathcal{P}}l^{u}\right)=qV-qr_{\mathcal{P}}\left(x-c_{1}\right)+q\left(r_{\mathcal{P}}\left(1-\phi\right)-1\right)S$$

which leads to (27). If it does not invest, its shareholder value is c_{1}. ■

Let us now turn to the set of constraints. We must now try to simplify the program by figuring out which constraints are binding, and which ones are not. We already know that the investment constraint is slack for bad types. Let us now compare the participation constraints across types:

Lemma 5 *For any outside market rate $\tilde{r}$, if the participation constraint (15) holds for the good type, then it holds for the bad type.*

Proof. Suppose the participation constraint holds for $\theta=G$. Then, $V_{in}^{G}\left(\mathcal{P},\Omega\right)\geq V_{out}^{G}\left(\tilde{r}\right)\geq A+qV-\tilde{r}\left(x-c_{0}\right)$ which we can write as

$$\tilde{r}\left(x-c_{0}\right)-r_{\mathcal{P}}\left(x-c_{1}\right)+\Sigma\geq\alpha\left(A+\Sigma+qV+r_{\mathcal{P}}\left(c_{1}-x\right)\right)+\left(1-\alpha\right)Z.$$

Now notice that $A+c_{0}>V$ and $c_{0}<x<qV$ implies $A>\left(1-q\right)V$. Therefore, since $0\leq\alpha\leq1$ and $Z\geq0$, we see that the previous equation implies

$$\tilde{r}\left(x-c_{0}\right)-r_{\mathcal{P}}\left(x-c_{1}\right)+\Sigma>\alpha\left(V+r_{\mathcal{P}}\left(c_{1}-x\right)+\Sigma\right)$$

which implies that the participation constraint holds for $\theta = B$. ■

Intuition: The fact that the participation constraint of the good type binds is intuitive. The investment constraint is simply $(r_{\mathcal{P}} - 1) c_1 + \Sigma \geq r_{\mathcal{P}} x - qV$ and we can rewrite the participation constraint as

$$(r_{\mathcal{P}} - 1) c_1 + \Sigma \geq r_{\mathcal{P}} x - qV + Z + \frac{\alpha A + qV - \tilde{r}(x - c_0)}{1 - \alpha} - c_1$$

As we have already discussed in the proof of Theorem **??**, in the pure debt guarantee case the two are equivalent. For equity injection and asset buybacks, the government can always increase α or Z to make the participation constraint binding, without changing the investment constraint, but also lowering its cost Ψ. So clearly the participation constraint must always bind.

We have implicitly assumed that the solvency constraint is satisfied for the good type. The solvency constraint says that legacy assets are enough to repay the creditors:

$$A - Z > S + r_{\mathcal{P}} l^u. \tag{29}$$

The following Lemma explains why we were justified in ignoring this constraint.

Lemma 6 *The participation constraint is always tighter than the solvency constraint.*

Proof. First, we can write the solvency constraint (29) as:

$$A - Z + \Sigma - r_{\mathcal{P}} (x - c_1) \geq 0$$

Suppose it is violated. Then it must mean that good types get nothing if $v = 0$. The participation constraint of good types would then be $(1 - \alpha) q (A - Z + V + r_{\mathcal{P}} (c_1 - x) + \Sigma) \geq V_{out}^G (\tilde{r})$. In particular, it implies:

$$\begin{aligned}(1 - \alpha) q (A - Z + V + r_{\mathcal{P}} (c_1 - x) + \Sigma) &\geq A + qV - \tilde{r}(x - c_0) \\ q (A - Z + r_{\mathcal{P}} (c_1 - x) + \Sigma) &\geq A - \tilde{r}(x - c_0) + \alpha q (A - Z + V + r_{\mathcal{P}} (c_1 - x) + \Sigma).\end{aligned}$$

Assumptions A1 and A2 ensures that $A > (x - c_0)/q$ and since $\tilde{r} \leq 1/q$, this means that the RHS is strictly positive, which contradicts the assumption that $A - Z + \Sigma - r_{\mathcal{P}} (x - c_1) < 0$.
■

Having simplified all the constraints, we can write the Lagrangian of the government program as:

$$\begin{aligned} L = {} & x - c_0 + (1 - \alpha) \left(\frac{\Sigma}{r_{\mathcal{P}}} - \pi Z + c_1 - x \right) - \alpha (\pi A + qV) \\ & - \lambda_{part} \left((1 - \alpha)(A - Z + \Sigma + qV + r_{\mathcal{P}} (c_1 - x)) - V_{out}^G (\tilde{r}) \right) \\ & - \lambda_{inv} \left((r_{\mathcal{P}} - 1) c_1 + \Sigma - (r_{\mathcal{P}} x - qV) \right) \\ & - \lambda_\alpha \alpha - \lambda_z Z - \lambda_\Sigma \Sigma - \lambda_{c_1} c_1 \end{aligned}$$

The first order conditions are

$$\begin{aligned} \frac{\partial L}{\partial \alpha} &= 0 : \lambda_{part} = \frac{\frac{\Sigma}{r_{\mathcal{P}}} - \pi Z + c_1 - x + \pi A + qV + \lambda_\alpha}{A - Z + \Sigma + qV + r_{\mathcal{P}} (c_1 - x)} \\ \frac{\partial L}{\partial \Sigma} &= 0 : \lambda_\Sigma = (1 - \alpha) \left(\frac{1}{r_{\mathcal{P}}} - \lambda_{part} \right) - \lambda_{inv} \\ \frac{\partial L}{\partial c_1} &= 0 : \lambda_{c_1} = (1 - \alpha)(1 - \lambda_{part} r_{\mathcal{P}}) - \lambda_{inv} (r_{\mathcal{P}} - 1) \\ \frac{\partial L}{\partial Z} &= 0 : \lambda_z = (1 - \alpha)(r_{\mathcal{P}} \lambda_{part} - \pi) \end{aligned}$$

Plugging $\frac{\partial L}{\partial c_1}$ into $\frac{\partial L}{\partial \Sigma}$ leads to

$$\lambda_{c_1} = r_{\mathcal{P}} \lambda_{\Sigma} + \lambda_{inv} \tag{30}$$

The following Lemma shows that debt guarantee cannot be dominated and that the investment constraint is slack.

Lemma 7 *The constraint $\Sigma \geq 0$ and investment constraint cannot be binding. The asset buyback and the equity injection are never used: $\alpha = 0$ and $Z = 0$.*

Proof. Suppose the constraint $\Sigma \geq 0$ binds. Then $\lambda_{\Sigma} > 0$. But (30) then implies that $\lambda_{c_1} > 0$ and $c_1 = 0$. But in this case the investment constraint is always violated. Suppose investment constraint binds. Then $\lambda_{inv} > 0$ and from (30) we have $c_1 = 0$. So the binding investment constraint means $\Sigma = r_{\mathcal{P}} x - qV$. But with $c_1 = 0$ and $\Sigma = r_{\mathcal{P}} x - qV$, the participation constraint is always violated. Since $\lambda_{\Sigma} = 0$ and $\lambda_{inv} = 0$, we know that $\lambda_{c_1} = 0$. This implies that $\lambda_{part} r_{\mathcal{P}} = 1$, and therefore $\lambda_z = (1-\alpha)(1-\pi) > 0$. Therefore $Z = 0$. Using $\frac{\partial L}{\partial \alpha} = 0$ and $\lambda_{part} r_{\mathcal{P}} = 1$ we get

$$\frac{1}{r_{\mathcal{P}}} = \frac{\frac{\Sigma}{r_{\mathcal{P}}} + c_1 - x + \pi A + qV + \lambda_{\alpha}}{A + \Sigma + qV + r_{\mathcal{P}}(c_1 - x)}$$

which leads to

$$A\left(\frac{1}{r_{\mathcal{P}}} - \pi\right) = qV\left(1 - \frac{1}{r_{\mathcal{P}}}\right) + \lambda_{\alpha}$$

and finally to

$$\lambda_{\alpha} = (1-\pi)\, q\,(A - (1-q)\, V)$$

Therefore $\lambda_{\alpha} > 0$ and $\alpha = 0$. ■

With a pure debt program the cost is

$$\Psi = x - c_0 + \left(\frac{\Sigma}{r_{\mathcal{P}}} + c_0 - x\right) = \frac{r_{\mathcal{P}} - \tilde{r}}{r_{\mathcal{P}}}(x - c_0),$$

where the second equality follows from the fact that the participation constraint for good banks binds. This last step is already in the proof of Theorem **??**.

Now we examine the minimal cost of capital injections and asset buybacks.

D.2 Equity Injection ($\mathcal{CI}$)

Lemma 8 *Pure equity injections: If $\tilde{r}(x - c_0) > \frac{r_{\mathcal{P}}}{r_{\mathcal{P}}-1} N$, then*

$$\alpha = \frac{\tilde{r}(x-c_0) - \frac{r_{\mathcal{P}}}{r_{\mathcal{P}}-1} N}{A + qV - \frac{r_{\mathcal{P}}}{r_{\mathcal{P}}-1} N}$$

is strictly positive, $m = x - c_0 - \frac{N}{r_{\mathcal{P}}-1}$, and

$$\Psi(\mathcal{CI}) = (1-\alpha)\, m - \alpha(\pi A + c_0 + N).$$

Otherwise, $\alpha = 0$ and $\Psi(\mathcal{C}) = m = \frac{(r_{\mathcal{P}} - \tilde{r})(x - c_0)}{r_{\mathcal{P}}}$.

Proof. Assume first that the investment constraint binds. Then

$$\hat{c}_1 = \frac{r_{\mathcal{P}} x - qV}{r_{\mathcal{P}} - 1} = x - \frac{N}{r_{\mathcal{P}} - 1}$$

As long as $r_{\mathcal{P}}\hat{c}_1 - \tilde{r}c_0 - (r_{\mathcal{P}} - \tilde{r})\, x > 0$, we get α from the participation constraint of good banks. Using

$$r_{\mathcal{P}}\hat{c}_1 - \tilde{r}c_0 - (r_{\mathcal{P}} - \tilde{r})\, x = \tilde{r}\,(x - c_0) - \frac{r_{\mathcal{P}}}{r_{\mathcal{P}} - 1}(qV - x)$$

we get

$$\alpha = \frac{\tilde{r}\,(x - c_0) - \frac{r_{\mathcal{P}}}{r_{\mathcal{P}}-1}N}{A + qV - \frac{r_{\mathcal{P}}}{r_{\mathcal{P}}-1}N}$$

Otherwise, the constraint $\alpha = 0$ binds and we get c_1 from the investment constraint

$$c_1 = \frac{\tilde{r}c_0 + (r_{\mathcal{P}} - \tilde{r})\, x}{r_{\mathcal{P}}}.$$

■

D.3 Asset buybacks ($\mathcal{ABB}$)

First observe that Lemma 6 allows us to just focus on the investment and the participation constraints.

Lemma 9 *Pure asset buyback program: If* $\tilde{r}\,(x - c_0) > \frac{r_{\mathcal{P}}}{r_{\mathcal{P}}-1}N$, *then*

$$Z = \tilde{r}\,(x - c_0) - \frac{r_{\mathcal{P}}}{r_{\mathcal{P}} - 1}N$$

is strictly positive, $m = x - c_0 - \frac{N}{r_{\mathcal{P}}-1}$ *and*

$$\Psi\,(\mathcal{ABB}) = m - \pi Z.$$

Otherwise $Z = 0$ *and* $\Psi\,(\mathcal{ABB}) = m = \frac{(r_{\mathcal{P}}-\tilde{r})(x-c_0)}{r_{\mathcal{P}}}$.

Proof. Assume first that the investment constraint binds. Then

$$\hat{c}_1 = \frac{r_{\mathcal{P}} x - qV}{r_{\mathcal{P}} - 1}$$

As long as $r_{\mathcal{P}}\hat{c}_1 - \tilde{r}c_0 - (r_{\mathcal{P}} - \tilde{r})\, x > 0$, we get Z from the participation constraint. Using

$$r_{\mathcal{P}}\hat{c}_1 - \tilde{r}c_0 - (r_{\mathcal{P}} - \tilde{r})\, x = \tilde{r}\,(x - c_0) - \frac{r_{\mathcal{P}}}{r_{\mathcal{P}} - 1}(qV - x)$$

we get

$$Z = \tilde{r}\,(x - c_0) - \frac{r_{\mathcal{P}}}{r_{\mathcal{P}} - 1}N$$

Otherwise, the constraint $Z = 0$ binds and we get c_1 from the investment constraint

$$c_1 = \frac{\tilde{r}c_0 + (r_{\mathcal{P}} - \tilde{r})\, x}{r_{\mathcal{P}}}.$$

■

D.4 Comparisons

We can now show that pure equity injection is cheaper than pure asset buyback. In the case where $\tilde{r}(x-c_0) < \frac{r_{\mathcal{P}}}{r_{\mathcal{P}}-1}N$, all the programs are equivalent. If $\tilde{r}(x-c_0) > \frac{r_{\mathcal{P}}}{r_{\mathcal{P}}-1}N$, we have $\Psi(\mathcal{CI}) < \Psi(\mathcal{ABB})$ if and only if

$$\begin{aligned}
\alpha(\pi A + c_0 + N + m) &> \pi Z \\
&\Leftrightarrow (1-\pi)\, qV > \frac{1-\pi r_{\mathcal{P}}}{r_{\mathcal{P}}-1}N \\
&\Leftrightarrow \frac{N}{V} < \frac{(1-\pi)\, q\, (1-1/r_{\mathcal{P}})}{1/r_{\mathcal{P}}-\pi} \\
&\Leftrightarrow \frac{N}{V} < (1-q)(1-\pi)
\end{aligned}$$

But the condition $\tilde{r}(x-c_0) > \frac{r_{\mathcal{P}}}{r_{\mathcal{P}}-1}N$ is equivalent to

$$(1-\pi)(1-q) > \frac{N}{\tilde{r}(x-c_0)}$$

Since $V > \tilde{r}(x-c_0)$, $\frac{N}{V} < \frac{N}{\tilde{r}(x-c_0)} < (1-\pi)(1-q)$ so the last condition is always satisfied, and equity injection is always cheaper.

Finally, we can verify that the pure equity injection program is more expensive than the pure debt program. The pure debt program costs $\Psi(\mathcal{DG}) = \frac{r_{\mathcal{P}}-\tilde{r}}{r_{\mathcal{P}}}(x-c_0)$. Therefore

$$\Psi(\mathcal{CI}) = m - \alpha(\pi A + c_0 + N + m) > \Psi(\mathcal{DG}) = \frac{(r_{\mathcal{P}}-\tilde{r})(x-c_0)}{r_{\mathcal{P}}}$$

if and only if

$$\begin{aligned}
x - c_0 - \frac{N}{r_{\mathcal{P}}-1} - \frac{\tilde{r}(x-c_0) - \frac{r_{\mathcal{P}}}{r_{\mathcal{P}}-1}N}{A + qV - \frac{r_{\mathcal{P}}}{r_{\mathcal{P}}-1}N}\left(\pi A + qV - \frac{N}{r_{\mathcal{P}}-1}\right) &> \frac{(r_{\mathcal{P}}-\tilde{r})(x-c_0)}{r_{\mathcal{P}}} \\
&\Leftrightarrow A + qV > r_{\mathcal{P}}\pi A + r_{\mathcal{P}} qV \\
&\Leftrightarrow A\left(\frac{1}{r_{\mathcal{P}}} - \pi\right) > \left(1 - \frac{1}{r_{\mathcal{P}}}\right) qV \\
&\Leftrightarrow A > (1-q)\, V
\end{aligned}$$

which is always true.

E Menus

E.1 Proof of Proposition 10

We first look at asset-buyback programs. The revelation principle implies that without loss we can assume that each program consists of a option for good banks and an option for bad banks:

$$m_G, Z_G \text{ and } m_B, Z_B.$$

Then, since bad banks have no assets we immediately get that $Z_B = 0$. Then, the incentive constraints for the two types of banks are as follows: The incentive compatibility constraint for good banks is:

$$A - Z_G + m_G + c_0 + N > A + m_B + c_0 + \max\left\{0, N - \frac{(1-q)}{q}(x - c_0 - m_B)\right\},$$

which reduces to:

$$m_G + N - Z_G > m_B \text{ if } 0 = \max\left\{0, N - \frac{(1-q)}{q}(x - c_0 - m_B)\right\} \tag{31}$$

$$q m_G + (1-q)(x - c_0) - q Z_G > m_B \text{ otherwise.} \tag{32}$$

The incentive compatibility constraint for bad banks is:

$$\begin{aligned} c_0 + m_B + N &> c_0 + m_G + N + (1-q)(x - m_G - c_0) \\ m_B &> m_G + N + (1-q)(x - m_G - c_0) \\ m_B &> q m_G + N + (1-q)(x - c_0). \end{aligned} \tag{33}$$

But, then it is immediate that (31) (or (32)) and (33) cannot be satisfied simultaneously, so there does not exist an incentive compatible two-option menu asset-buyback program.

Now to turn to examine debt guarantee programs. Here the two menus are

$$\phi_G, S_G \text{ and } \phi_B, S_B.$$

IC for bad banks:

$$\begin{aligned} qV - qS_B - [x - (c_0 - \phi_B S_B) - S_B] &\geq qV - qS_G - q[x - (c_0 - \phi_G S_G) - S_G] \\ -qS_B - [x - (c_0 - \phi_B S_B) - S_B] &\geq -qS_G - q[x - (c_0 - \phi_G S_G) - S_G] \\ -S_B - \frac{[x - (c_0 - \phi_B S_B) - S_B]}{q} &\geq -S_G - [x - (c_0 - \phi_G S_G) - S_G] \end{aligned} \tag{34}$$

General IC for good banks to take care of investment decision in the event of deviation:

$$A + qV - S_G - [x - (c_0 - \phi_G S_G) - S_G] \geq A + \max\{c_0, qV - S_B - r_B[x - (c_0 - \phi_B S_B) - S_B]\}$$

If $c_0 \leq qV - S_B - r_B[x - (c_0 - \phi_B S_B) - S_B]$, then the IC for the good banks implies:

$$\begin{aligned} A + qV - S_G - [x - (c_0 - \phi_G S_G) - S_G] &\geq A + qV - S_B - r_B[x - (c_0 - \phi_B S_B) - S_B] \\ -S_G - [x - (c_0 - \phi_G S_G) - S_G] &\geq -S_B - r_B[x - (c_0 - \phi_B S_B) - S_B] \end{aligned} \tag{35}$$

But because $r_B = \frac{1}{q}$ they can hold simultaneously only if they hold with equality. Hence two-option menus boil down to one-option menus. Now, if $c_0 > qV - S_B - r_B[x - (c_0 - \phi_B S_B) - S_B]$, the inequalities is (35) are strict, which together with (34) imply that two-option menus are not feasible.

E.2 Proof of Proposition 11

The constraints are

$$
\begin{aligned}
IC_B &: (1-\alpha_B)(qV+c_0+m_B-x) \geq (1-\alpha_G)q(V+c_0+m_G-x) \\
IC_G &: (1-\alpha_G)(A+qV+c_0+m_G-x) \geq (1-\alpha_B)\left(A+\max\left\{c_0+m_B, qV-\frac{(x-c_0-m_B)}{q}\right\}\right) \\
PC_G &: (1-\alpha_G)(A+qV+c_0+m_G-x) \geq A+qV-\tilde{r}(x-c_0) \\
PC_B &: (1-\alpha_B)(qV+c_0+m_B-x) \geq q(V-\tilde{r}(x-c_0)) \\
A_G &: \alpha_G \geq 0 \\
A_B &: \alpha_B \geq 0
\end{aligned}
$$

Observe that the incentive constraint for good banks depends on whether m_B is high enough to make them willing to invest even though, when they deviate they are perceived as bad banks and face an interest rate of $\frac{1}{q}$. Hence, depending on the $\max\left\{c_0+m_B, qV-\frac{(x-c_0-m_B)}{q}\right\}$ there are two cases to consider. Notice that m_B is one of the unknowns, so a good approach seems to be to solve the problem in each case and check which one is internally consistent In what follows we show that the solution is the same in both cases.

Case 1: $c_0+m_B = \max\left\{c_0+m_B, qV-\frac{(x-c_0-m_B)}{q}\right\}$

In this case the Lagrangian is

$$
\begin{aligned}
L = \; & \pi\{(1-\alpha_G)m_G-\alpha_G(\pi A+c_0+N)\}+(1-\pi)\{(1-\alpha_B)m_B-\alpha_B(c_0+N)\} \\
& -\lambda_{IC_G}\left[(1-\alpha_G)(A+qV+c_0+m_G-x)-(1-\alpha_B)(A+c_0+m_B)\right] \\
& -\lambda_{IC_B}\left[(1-\alpha_B)(qV+c_0+m_B-x)-(1-\alpha_G)q(V+c_0+m_G-x)\right] \\
& -\lambda_{PC_B}\left[(1-\alpha_B)(c_0+m_B+qV-x)-q(V-\tilde{r}(x-c_0))\right] \\
& -\lambda_{PC_G}\left[(1-\alpha_G)(A+qV+c_0+m_G-x)-A-qV+\tilde{r}(x-c_0)\right] \\
& -\lambda_{A_G}\alpha_G \\
& -\lambda_{A_B}\alpha_B
\end{aligned}
$$

Combining $\frac{\partial L}{\partial m_G}=0$ and $\frac{\partial L}{\partial m_B}=0$ we get that $\pi+\lambda_{IC_B}q-\lambda_{PC_G}=\lambda_{IC_G}$ and $\lambda_{IC_G}=\lambda_{IC_B}+\lambda_{PC_B}-1+\pi$ or

$$1-\lambda_{PC_G}-\lambda_{IC_B}(1-q)=\lambda_{PC_B} \text{ and } \lambda_{IC_G}=q\lambda_{IC_B}-\lambda_{PC_G}+\pi.$$

From these constraints it follows that either $\lambda_{IC_B}>0$ or $\lambda_{PC_B}>0$ and that either $\lambda_{IC_G}>0$ or $\lambda_{PC_G}>0$.

Conjecture: $\lambda_{IC_B}>0$ or $\lambda_{PC_B}=0$ and $\lambda_{IC_G}=0$ or $\lambda_{PC_G}>0$. Then, from the first-order conditions we get that:

$$\lambda_{PC_G}(A+qV+c_0+m_G-x)=\lambda_{IC_B}q(V+c_0+m_G-x)+\lambda_{A_G}+\pi(m_G+\pi A+c_0+N) \tag{36}$$

$$\frac{(1-\pi)(m_B+c_0+N)+\lambda_{A_B}}{(qV+c_0+m_B-x)}=\lambda_{IC_B} \tag{37}$$

$$\pi+\lambda_{IC_B}q-\lambda_{PC_G}=0 \tag{38}$$

$$\lambda^*_{IC_B}=(1-\pi) \tag{39}$$

From (38) and (37) we get that $(1-\pi) = \lambda_{IC_B} > 0$ and that $\lambda^*_{A_B} = 0$. Then combining these results with (38) we get that $\lambda^*_{PC_G} = \pi + (1-\pi)\,q$, which with the help of (36) results to:

$$\left(\pi + (1-\pi)\,q - \pi^2\right) A - (1-\pi)\,q\,(1-q)\,V = \lambda_{A_G}$$

From here we get that $\lambda_{A_G} = \begin{cases} 0 \text{ if } \pi = 1 \\ > 0 \text{ if } \pi < 1 \end{cases}$, which implies that $\alpha^*_G = 0$. Then we can get m_G from the participation constraint of good banks: $A + qV + c_0 + m_G - x = A + qV - \tilde{r}\,(x - c_0)$, or

$$m^*_G = -\left(\tilde{r} - 1\right)(x - c_0), \tag{40}$$

that is good banks pay to participate.

Substituting these optimal values in IC_B we get that

$$(1-\alpha_B)m_B - \alpha_B\,(N + c_0) = (1 - q\tilde{r})\,(x - c_0)\,. \tag{41}$$

From (41) and (40) we see that the cost of this menu to the government is given by

$$\begin{aligned}\Psi^* &= -\pi\,(\tilde{r}-1)\,(x-c_0) + (1-\pi)\,(1-q\tilde{r})\,(x-c_0) \\ &= \frac{r_{\mathcal{P}} - \tilde{r}}{r_{\mathcal{P}}}\,(x-c_0)\end{aligned}$$

It is straightforward to check that there are no other solutions.

Case 2: $qV - \frac{(x-c_0-m_B)}{q} = \max\left\{c_0 + m_B, qV - \frac{(x-c_0-m_B)}{q}\right\}$

In this case the Lagrangian is exactly the same as before but with the second line replaced by $-\lambda_{IC_G}\left[(1-\alpha_G)\,(A + qV + c_0 + m_G - x) - (1-\alpha_B)\left(A + qV - \frac{(x-c_0-m_B)}{q}\right)\right]$.

By rearranging and combining $\frac{\partial L}{\partial m_G} = 0$ and $\frac{\partial L}{\partial m_B} = 0$ we get that

$$\pi\,(1-q) + q = q\lambda_{PC_B} + \lambda_{PC_G}.$$

Conjecture: $\lambda_{IC_B} > 0$ or $\lambda_{PC_B} = 0$ and $\lambda_{IC_G} = 0$ or $\lambda_{PC_G} > 0$. Then we get $\pi\,(1-q) + q = \lambda_{PC_G}$ and $\lambda_{IC_B} = (1-\pi)$. Then substituting all our conjectures to $\frac{\partial L}{\partial \alpha_B} = 0$ we we get that $\lambda_{A_B} = 0$ and everything is exactly as we have derived before.

It is straightforward to check that there are no other solutions, as in the previous case.[14]

F Moral Hazard

Assume that good types are a probability p of $a = A$, and $1-p$ of $a = 0$. The pooling rate without risk shifting becomes

$$r_{\mathcal{P}} \equiv \frac{1}{\pi\,(p + q - pq) + (1-\pi)\,q}$$

We are going to check the existence of a pooling risk-shifting equilibrium. Conditional on risk shifting by all banks, the pooling rate is

$$r'_{\mathcal{P}} \equiv \frac{1}{\pi\,(p + q' - pq') + (1-\pi)\,q'}$$

[14] All missing details available upon request.

If good types opt in the government program and risk shift, their get

$$\begin{aligned} V_{in}^{G\prime} &= p\left(A + q'V' + \left((1-\phi)\, r'_{\mathcal{P}} - 1\right) S - r'_{\mathcal{P}}\,(x - c_0)\right) + (1-p)\, q' \left(V' + \left((1-\phi)\, r'_{\mathcal{P}} - 1\right) S - r'_{\mathcal{P}}\,(x - c_0)\right) \\ &= pA + q'V' + \left(p + (1-p)\, q'\right)\left(S\left((1-\phi)\, r'_{\mathcal{P}} - 1\right) - r'_{\mathcal{P}}\,(x - c_0)\right) \end{aligned}$$

The outside option of the good type depends on the perception of the market after an out of equilibrium move where they choose the safer project. As before, the market's beliefs are captured by the rate $\tilde{r}$. As long as $(1-\phi)\,\tilde{r} > 1$ it is still optimal to use the government debt guarantee, and the value of deviating for the good type is:

$$\begin{aligned} V_{in}^{G} &= p\,(A + qV + ((1-\phi)\,\tilde{r} - 1)\, S - \tilde{r}\,(x - c_0)) + (1-p)\, q\,(V + ((1-\phi)\,\tilde{r} - 1)\, S - \tilde{r}\,(x - c_0)) \\ &= pA + qV + (p + (1-p)\, q)\,(S\,((1-\phi)\,\tilde{r} - 1) - \tilde{r}\,(x - c_0)) \end{aligned}$$

So good types choose to risk shift iff

$$(1-p)\left(q - q'\right) S > qV - q'V' + \left(\left(p + (1-p)\, q'\right) r'_{\mathcal{P}} - (p + (1-p)\, q)\,\tilde{r}\right)(x - c_0 - (1-\phi)\, S)$$

The LHS is the net benefit from risk shifting. The RHS is the cost, which has two parts. The term $qV - q'V'$ is the NPV loss. The last term is the opportunity cost of not facing a low interest rate $\tilde{r}$. This reputation cost applies to the part of borrowing that is not insured, $x - c_0 - (1-\phi)\, S$.

References

Akerlof, G. A. (1970): “The Market for ’Lemons’: Quality Uncertainty and the Market Mechanism,” *Quarterly Journal of Economics*, 84, 488Ű500.

Ausubel, L. M., and P. Cramton (2009): “No Substitute for the "P" Word in Financial Rescue,” *Economist’s Voice*, 6(2).

Bernanke, B. (2009): “The Crisis and the Policy Response,” Stamp Lecture, London School of Economics.

Cramton, P., and T. R. Palfrey (1995): “Ratifiable Mechanisms: Learning from Disagreement,” *Games and Economic Behavior*, 10, 255–283.

Diamond, D., S. Kaplan, A. Kashyap, R. Rajan, and R. Thaler (2008): “Fixing the Paulson Plan,” *Wall Street Journal*.

Duffie, D. (2009): “The Failure Mechanics of Dealer Banks,” Working Paper, Stanford GSB.

Innes, R. (1990): “Limited Liability and incentive contracting with ex ante action choices,” *Journal of Economic Theory*, 52, 45–67.

Myers, S., and N. Majluf (1984): “Corporate Financing and Investment Decisions when Firms Have Information that Investors do not have,” *Journal of Financial Economics*, 13, 187–221.

Philippon, T., and P. Schnabl (2009): “Efficient Recapitalization,” NBER WP 14929.

Soros, G. (2009): “The right and wrong way to bail out the banks,” *Financial Times*, January 22.

Stiglitz, J. (2008): “We Aren’t Done Yet: Comments on the Financial Crisis and Bailout,” *The Economists’ Voice*, 5 (5).

Tirole, J. (2006): *The Theory of Corporate Finance*. Princeton University Press.