Interbank Lending, Credit Risk Premia and Collateral*

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Abstract

We study the functioning of secured and unsecured interbank markets in the presence of credit risk. We allow for safe collateral, e.g. government bonds, as well as risky collateral, e.g. mortgage-backed securities, in secured interbank transactions. Acquiring liquidity in the unsecured market is costly due to credit risk premia, while the secured segment is subject to market risk. The model illustrates how tensions in the unsecured market and the market secured by risky collateral affect repo rates in the market secured by safe collateral. The volatility of repo rates is exacerbated by the scarcity of high-quality collateral. The model generates empirical predictions that are in line with developments during the 2007-2009 financial crisis. Interest rates decouple across secured and unsecured markets as well as across different collateral classes. We use the model to discuss various policy responses.

JEL classification: G01, G21, D82

Keywords: Financial crisis; Interbank market; Liquidity; Credit risk

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1 Introduction

Interbank markets play a key role in the financial system. They are vital for banks' liquidity management and the transmission of monetary policy. The interest rate in the unsecured three-month interbank market acts as a benchmark for pricing fixed-income securities throughout the economy. Secured, or repo, markets have been a fast-growing segment of money markets: They have doubled in size since 2002 with gross amounts outstanding of about $10 trillion in the United States and comparable amounts in the euro area just prior to the start of the crisis in August 2007. Since repo transactions are backed by collateral securities similar to those used in the central bank's refinancing operations, repo markets are a key tool for the implementation of monetary policy.

In normal times, interbank markets are among the most liquid in the financial sector. Rates are usually stable across secured and unsecured segments, as well as across different collateral classes. Since August 2007, however, the functioning of interbank markets has become severely impaired around the world. The frictions in the interbank market have become a key feature of the 2007-09 crisis (see, for example, Allen and Carletti, 2008, and Brunnermeier, 2009).

One striking manifestation of the frictions in the interbank market has been the decoupling of interest rates between secured and unsecured markets. Figure 1 shows the unsecured and secured interbank market rates for the euro area since January 2007. Prior to the outbreak of the crisis in August 2007, the rates were closely tied together. Since August 2007, they have moved in opposite directions with the unsecured rate increasing and the secured rate decreasing. The decoupling further deepened after the Lehman bankruptcy, and to a lesser extent, just prior to the sale of Bear Stearns.

A second, related important feature of the tensions in the interbank market has been the difference in the severity of the disruptions in the United States and in Europe. Figure 2 shows rates in secured and unsecured interbank markets in the United States. As in Europe, there is a decoupling of the rates at the start of the financial crisis and a further deepening

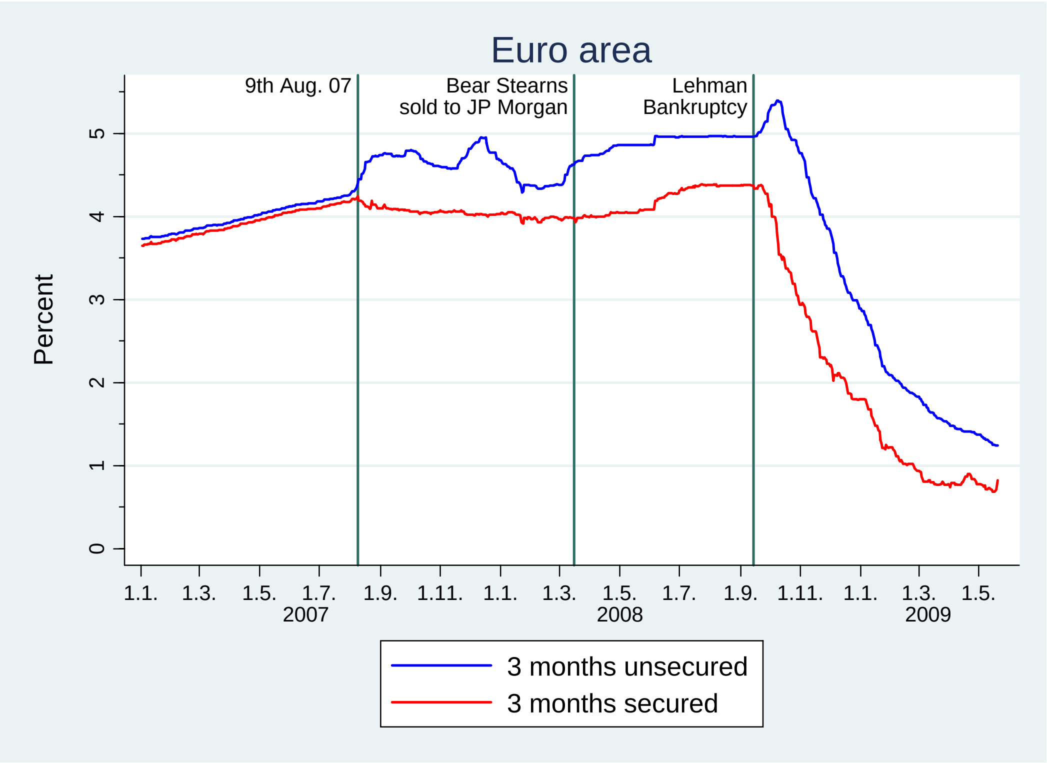

Figure 1: Decoupling of secured and unsecured interbank rates in the EA

after the sale of Bear Stearns and the bankruptcy of Lehman. However, the decoupling and the volatility of the rates is much stronger than in Europe.

Why then have secured and unsecured interbank interest rates decoupled? Why has the US repo market experienced significantly more disruptions than the euro area market? What underlying friction can explain these developments? And what policy responses are possible to tackle the tensions in interbank markets?

To examine these questions, this paper provides a model of interbank markets with both secured and unsecured lending in the presence of credit risk. We model the interbank market in the spirit of Diamond and Dybvig (1983). Banks may need to realize cash quickly due to demands of customers who draw on committed lines of credit or on their demandable

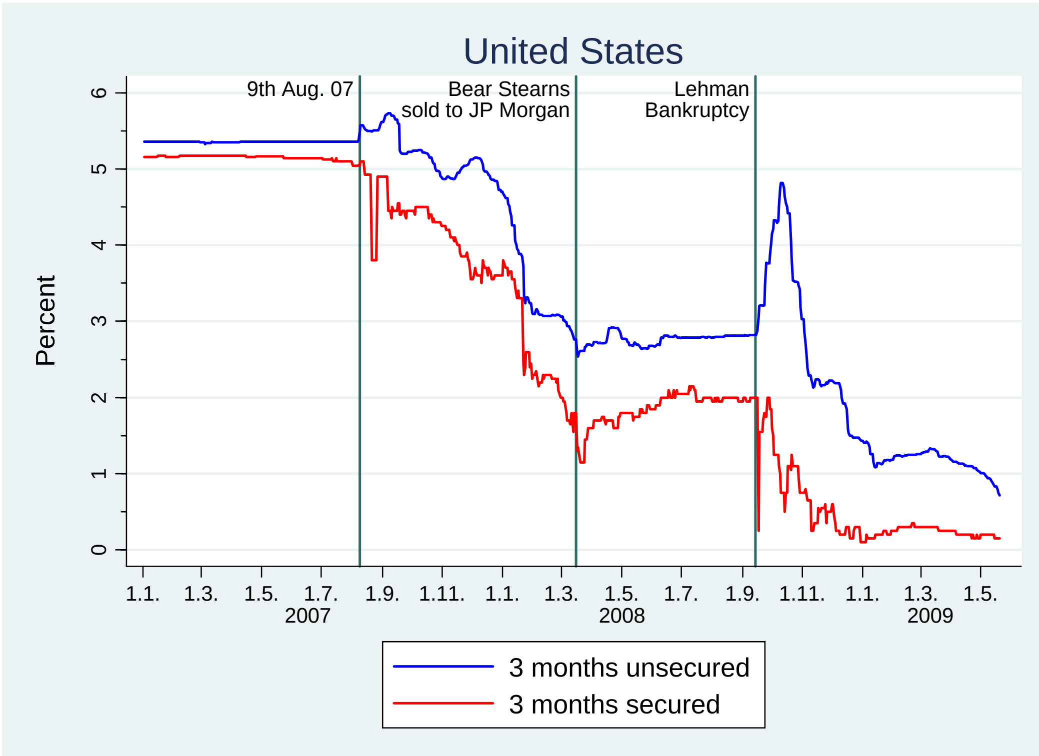

Figure 2: Decoupling of secured and unsecured interbank rates in the US

deposits. Banks in need of liquidity can borrow from banks with a surplus of liquidity as in Bhattacharya and Gale (1987) and Bhattacharya and Fulghieri (1994). Banks' profitable but illiquid assets are risky. Hence, banks may not be able to repay their interbank loan giving rise to credit risk. To compensate lenders, borrowers have to pay a premium for funds obtained in the unsecured interbank market.

In addition to the choice between the liquid (cash) and the illiquid asset (loans), banks can invest in bonds. Bonds provide a long-run return but unlike the illiquid asset, they can also be traded for liquidity in the short-term. We first consider the case of safe bonds, e.g. government bonds. Since unsecured borrowing is costly due to credit risk, banks in need of liquidity will sell bonds to reduce their borrowing needs. We assume that government bonds are in fixed supply and that they are scarce enough not to crowd out the unsecured market. Credit risk will affect the price of safe government bonds since banks with a liquidity surplus must be willing to both buy the bonds offered and lend in the unsecured interbank market. In equilibrium there must not be an arbitrage opportunity between secured and unsecured lending.

We then introduce risky bonds, e.g. mortgage-backed securities. The realization of the risky bond return becomes known when banks trade liquidity and safe bonds. Risky bond returns lead to aggregate risk that spills over to the market for safe bonds. In consequence, even the price of safe bonds will be volatile.

Our modeling assumptions are designed to reflect the insights from broad analyses of the 2007-09 financial crisis. First, risk, and the accompanying fear of credit default, which was created by the complexity of securitization, is at the heart of the financial crisis (see Gorton, 2008, 2009). Second, illiquidity is a key factor contributing to the fragility of modern financial systems (see, for example, Diamond and Rajan, 2008a, and Brunnermeier, 2009). Hence, we employ the model of banking introduced by Diamond and Dybvig (1983) that allows us to consider the tradeoff between liquidity and return in bank's portfolio decisions. A further advantage of this model is that it naturally creates a scope for interbank markets (see Bhattacharya and Gale, 1987, and Bhattacharya and Fulghieri, 1994).[1]

This paper is part of the growing literature analyzing the ability of interbank market to smooth out liquidity shocks. Our model builds on Freixas and Holthausen (2004) who examine the scope for the integration of unsecured interbank markets when cross-country information in noisy. They show that introducing secured interbank markets reduces interest rates and improves conditions when unsecured markets are not integrated, however their introduction may hinder the integration process.

The role of asymmetric information about credit risk as a factor behind tensions in the unsecured interbank markets is emphasized in Heider, Hoerova and Holthausen (2009).

[1] An important complement to liquidity within the financial sector is the demand and supply of liquidity within the real sector (see Holmström and Tirole, 1998).

They derive various regimes in the interbank markets akin to the developments prior to and during the 2007-2009 financial crisis. Bolton, Santos, and Scheinkman (2009) also examine the role of asymmetric information and distinguish between outside and inside liquidity (asset sales versus cash), which connects to our analysis where banks hold liquid and illiquid securities, and where safe and risky claims on illiquid assets can be traded in exchange for liquidity. Brunnermeier and Pedersen (2009) similarly distinguish between market liquidity and funding liquidity. In our model, banks can obtain funding liquidity in the secured and unsecured interbank markets by issuing claims on illiquid assets, i.e. assets with limited market liquidity.

A recent paper by Allen, Carletti, and Gale (2009) presents a model of a market freeze without credit risk or unsecured interbank markets. Banks can stop trading due to aggregate liquidity risk, i.e. banks hold similar rather than offsetting positions. Aggregate shortages are also examined in Diamond and Rajan (2005) where bank failures can be contagious due to a shrinking of the pool of available liquidity. Freixas, Parigi, and Rochet (2000) analyze systemic risk and contagion in a financial network and its ability to withstand the insolvency of one bank. In Allen and Gale (2000), the financial connections leading to contagion arise endogenously as a means of insurance against liquidity shocks. Illiquidity can depress lending and low prices for illiquid assets go hand in hand with high returns on holding liquidity in Diamond and Rajan (2009). Potential buyers may want to wait for asset prices to decline further. At the same time, the managers of selling banks may want to gamble for resurrection. These two effects feed on each other and may lead to a market freeze.

Rationales for central bank intervention in the interbank market are examined in Acharya, Gromb, and Yorulmazer (2008) and Freixas, Martin, and Skeie (2008). In Acharya et al., market power makes it possible for liquidity-rich banks to extract surplus from banks that need liquidity. A central bank provides an outside option for the banks suffering from such liquidity squeezes. In Freixas et al., multiple equilibria exist in interbank markets, some of which are more efficient than others. By steering interest rates, a central bank can act as a coordination device for market participants and ensure that a more efficient equilibrium is reached. Freixas and Jorge (2008) examine how financial imperfections in the interbank market affect the monetary policy transmission mechanism beyond the classical money channel.

The remainder of the paper is organized as follows. In Section 2, we describe the set-up of the model. In section 3, we solve the benchmark case when banks can only trade in the unsecured market. In Section 4, we allow banks to invest in safe bonds. In Section 5, we introduce risky bonds and market risk. In Section 6, we present empirical implications and relate them to the developments during the 2007-09 financial crisis. In Section 7, we discuss policy responses to mitigate the tensions in interbank markets and in Section 8 we offer concluding remarks. All proofs are in the Appendix.

2 The model

There are three dates, $t = 0, 1$, and 2, and a single homogeneous good that can be used for consumption and investment. There is no discounting between dates.

Consumers and banks. There is a $[0, 1]$ continuum of consumers. Every consumer has an endowment of 1 unit of the good at $t = 0$. Consumers deposit their endowment with a bank at $t = 0$ in exchange for a demand deposit contract which promises them consumption c_1 if they withdraw at $t = 1$ ("impatient" consumers) or c_2 if they withdraw at $t = 2$ ("patient" consumers), as in Diamond and Dybvig (1983).

There is a $[0, 1]$ continuum of risk neutral, profit maximizing banks. We assume that the banking industry is perfectly competitive. Thus, banks make zero profits in equilibrium and maximize pay-out to depositors. Deposits are fully insured by deposit insurance and no bank runs occur.[2]

Liquidity shocks. Banks are uncertain about the liquidity demand they will face at

[2]We abstract from any risk sharing issues and take the institutions of banking (and interbank markets) as given.

$t = 1$. For a fraction π_h of banks, a high fraction of consumers, denoted by λ_h, wishes to withdraw at $t = 1$. The remaining fraction $\pi_l = 1 - \pi_h$ of banks faces a low liquidity demand λ_l, with $\lambda_l < \lambda_h$. The aggregate demand for liquidity at $t = 1$, denoted by $\lambda = \pi_h \lambda_h + \pi_l \lambda_l$, is known. Let the subscript $k = l, h$ denote whether a bank faces a low or high need for liquidity.

Assets and banks' portfolio decision. At $t = 0$, banks can use the consumers' endowment to invest in a long-term illiquid asset *(loans)*, a short-term liquid asset *(cash)*, or to buy government bonds at price P_0. Let B denote the supply of government bonds to the banking sector at $t = 0$.

Each unit invested in the liquid asset offers a return equal to 1 unit of the good after one period (costless storage). Each unit invested in the illiquid asset yields an uncertain payoff at $t = 2$. The illiquid asset can either succeed and return R or fail and return zero. In the latter case, a bank is insolvent and it is taken over by the deposit insurance fund. The government bonds yield a return equal to Y at $t = 2$ with certainty. They can be sold at $t = 1$ at the prevailing market price P_1 (repo market secured by government bonds). We assume that $pR > Y > 1$ holds (loans are more productive than bonds or cash over the long-run). Moreover, loans are fully illiquid, i.e. in case of early liquidation, the illiquid asset returns zero. Hence, banks face a trade-off between liquidity and return when making their portfolio decisions. Let α denote the fraction invested in the illiquid asset and β fraction invested in government bonds at $t = 0$. The remaining fraction $1 - \alpha - \beta$ is invested in the liquid asset.

Interbank market and liquidity management. Given that banks face differing liquidity demands at $t = 1$, an interbank market can develop. Banks with low level of withdrawals can lend any excess liquidity to banks with high level of withdrawals. Let L_l and L_h denote the amount lent and borrowed, respectively, and let r denote the interest rate on interbank loans. We assume that the interbank market is competitive, i.e. banks act as price takers.

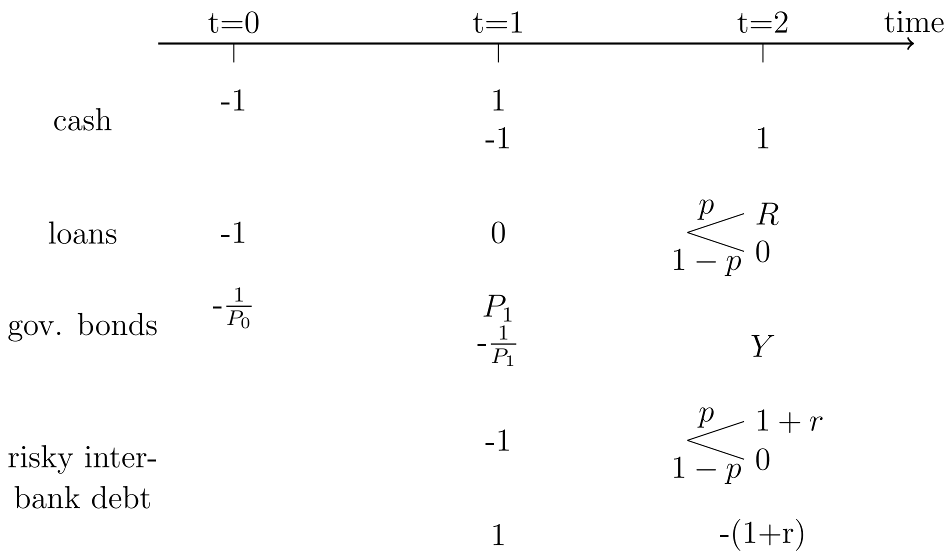

Figure 3: Assets and financial claims

Due to the risk of the illiquid asset, a borrower as well as a lender in the interbank market may be insolvent at $t = 2$ when the loan is repaid. A solvent borrower must always repay his interbank loan. If his lender is insolvent, the repayment goes to the deposit insurance fund. In contrast, a solvent lender is only repaid if his borrower is solvent, too. Figure 3 summarizes the (expected) payoffs of assets and financial claims.

In sum, a bank can manage its liquidity at $t = 1$ in three ways: 1) by borrowing/lending in the interbank market, 2) by buying and selling government bonds on the repo market, and 3) by investing in the liquid asset for another period.

The sequence of events is summarized in Figure 4 below.

3 Benchmark: no government bonds

In this section we solve the model without government bonds (i.e. $\beta = 0$). The analysis clarifies how the model works and provides a benchmark. We proceed backwards by first considering banks' liquidity management at $t = 1$ and then examining their portfolio allocation at $t = 0$.

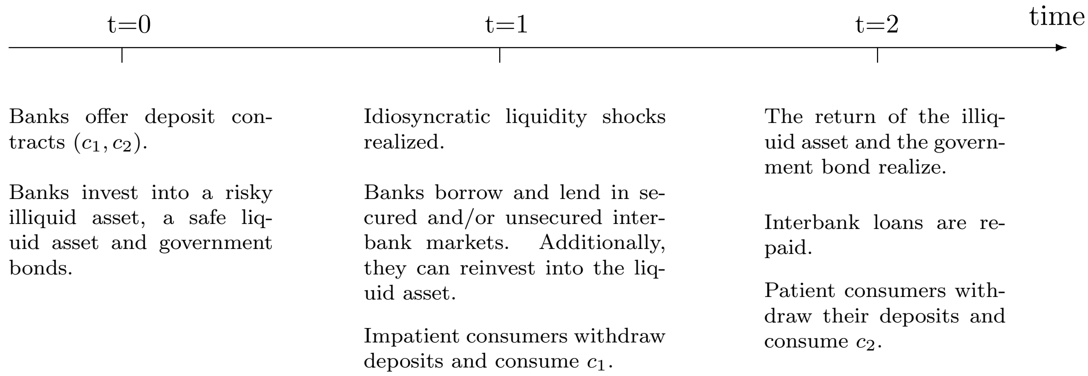

Figure 4: The timing of events

Liquidity management. Having received liquidity shocks, $k = l, h$, banks manage their liquidity at $t = 1$ while taking their portfolio allocation $(\alpha, 1-\alpha)$ and their liabilities (c_1, c_2) as given.

A bank that faces a low level of withdrawals by impatient consumers, $k = l$, maximizes $t = 2$ profits:

$$\max_{\gamma_l^1, L_l} p[R\alpha + \gamma_l^1(1-\alpha) + p(1+r)L_l - (1-\lambda_l)c_2] \tag{1}$$

subject to

$$\lambda_l c_1 + L_l + \gamma_l^1(1-\alpha) \leq (1-\alpha).$$

and feasibility constraints: $0 \leq \gamma_l^1 \leq 1$ and $L_l \geq 0$.

A type-l bank has spare liquidity since the level of early withdrawals is lower than expected at $t = 0$, $\lambda_l < \lambda$. The bank can thus lend L_l at a rate r in the interbank market. The bank can also reinvest a fraction γ_l^1 of cash leftover in the liquid asset.

Conditional on being solvent, the profits at $t = 2$ of a bank with a surplus of liquidity at $t = 1$ are the sum of the proceeds from the illiquid investment, from the reinvestment into the liquid asset and the repayment of the risky interbank loan minus the pay-out to patient consumers.

The budget constraint requires that the outflow of liquidity at $t = 1$ (deposit withdrawals, reinvestment into the liquid asset and interbank lending) is matched by the inflow (return on the liquid asset).

A bank that has received a high liquidity shock, $k = h$, will be a borrower in the interbank market, solving:

$$\max_{\gamma_h^1, L_h} p[R\alpha + \gamma_h^1(1-\alpha) - (1+r)L_h - (1-\lambda_h)c_2] \tag{2}$$

subject to

$$\lambda_h c_1 + \gamma_l^1(1-\alpha) \leq (1-\alpha) + L_h.$$

and feasibility constraints: $0 \leq \gamma_h^1 \leq 1$ and $L_h \geq 0$.

A type-h bank has a liquidity shortage. It can borrow an amount L_h in the interbank market. It could also reinvest into the liquid asset.

There are two key differences between the optimization problems of a lender and a borrower. The first difference is in the objective function. A borrower expects having to repay $p(1+r)L_h$ while a lenders expects a repayment $p^2(1+r)L_l$. A lender will not be repaid if the illiquid investment of his counterparty fails. The second difference is in the budget constraint. The interbank loan is an outflow for a lender and an inflow for a borrower.

Given that the investment in the illiquid investment is profitable and that there are no aggregate liquidity shocks, it will be optimal to trade liquidity in the interbank market, $L_l > 0$ and $L_h > 0$.

The marginal value of (inside) liquidity, $1-\alpha$, is given by the Lagrange multiplier, denoted by μ^k, on the budget constraints of the optimization problems (1) and (2).

Lemma 1 (Marginal value of liquidity) *The marginal value of liquidity is* $\mu^l = p^2(1+r)$ *for a lender and* $\mu^h = p(1+r)$ *for a borrower.*

A lender values liquidity at $t = 1$ since he can lend it out at an expected return of $p^2(1+r)$. A borrower values liquidity since it saves the cost of borrowing in the interbank market, $p(1+r)$. The marginal value of liquidity is lower for a lender because of credit risk.

The following result describes banks' decision to reinvest into the liquid asset.

Lemma 2 (Reinvestment into the liquid asset) *A borrower does not reinvest in the liquid asset at* $t = 1$*:* $\gamma_h^1 = 0$*. A lender does not reinvest in the liquid asset if and only if* $p(1 + r) \geq 1$*.*

It cannot be optimal for a bank with a shortage of liquidity to borrow in the interbank market at rate $1 + r$ and to reinvest the obtained liquidity in the liquid asset since it would yield a negative net return. The same is not true for a lender since his rate of return on the lending in the interbank market is only $p(1 + r)$ due to credit risk. If a lender stores his liquidity instead of lending it out, then the interbank market cannot be active. Thus, we will have to check whether $p(1 + r) \geq 1$ once we have obtained the interest rate in the interbank market.

Market clearing in the interbank market, $\pi_l L_l = \pi_h L_H$, yields:

Lemma 3 (Interbank market clearing) *The amount of cash held by banks exactly balances the aggregate pay-out at* $t = 1$*:*

$$\lambda c_1 = 1 - \alpha.$$

The interbank market fully smoothes out the idiosyncratic liquidity shocks, λ_k.

The following proposition links returns on the liquid and on the illiquid asset to the amounts invested into them. Note that the return on the liquid asset is $1 + r$ since liquidity is traded in the interbank market.

Proposition 1 *The ratio of the total returns on the illiquid and the liquid asset is equal to the ratio of patient to impatient consumers adjusted for credit risk:*

$$\frac{R}{1+r}\,\frac{\alpha}{1-\alpha} = \frac{(1-\lambda_l)\pi_l + (1-\lambda_h)\pi_h p}{\lambda_l \pi_l + \lambda_h \pi_h} \tag{3}$$

Note that higher credit risk (lower p) leads to a less illiquid portfolio ceteris paribus. Higher credit risk means less profits for lenders at $t = 2$ and consequently a lower pay-out to their patient consumers. Lenders have more patient consumers than borrowers but banks' liabilities cannot be made contingent on banks' idiosyncratic liquidity shocks. More liquidity allows banks to lend more in case of a low liquidity shock and to increase their late liabilities in case of a high liquidity shock. Both counter the effect of higher credit risk across lenders and borrowers at $t = 2$.

If a bank is more likely to have a liquidity surplus at $t = 1$ (higher π_l), and hence face relatively more late withdrawals, it allocates a higher proportion of its portfolio to the illiquid asset ceteris paribus.[3]

Pricing liquidity. The return on liquidity, $1 + r$, is determined by banks' portfolio allocation. At $t = 0$ banks decide how much to invest in the illiquid asset, fraction α, in order to maximize expected profits, not knowing whether they will end up having a surplus or a shortage of liquidity at $t = 1$:

$$\max_{\alpha} \quad \pi_l p\big[R\alpha + p(1+r)L_l - (1-\lambda_l)c_2\big] + \pi_h p\big[R\alpha - (1+r)L_h - (1-\lambda_h)c_2\big] \tag{4}$$

subject to

$$L_l = (1-\alpha^I) - \lambda_l c_1 \tag{5}$$

$$L_h = \lambda_h c_1 - (1-\alpha^I). \tag{6}$$

where we have used that $\gamma_k^1 = 0$ (Lemma 2).

The first-order condition for a bank's optimal portfolio allocation across the liquid and

[3] The derivative with respect to π_l of the right-hand side of the expression in proposition 1 is positive if and only if $\lambda_h(1-\lambda_l) > p\lambda_l(1-\lambda_h)$. This always holds since $\lambda_h > \lambda_l$.

illiquid assets requires that:[4]

$$\pi_h p(1+r) + \pi_l p^2(1+r) = \pi_h pR + \pi_l pR$$

or, equivalently,

$$(\pi_h + \pi_l p)(1+r) = R. \tag{7}$$

The interbank interest rate r, the price of liquidity traded in the interbank market, is given by a no-arbitrage condition. The right-hand side is the expected return from investing an additional unit into the illiquid asset, R. The left-hand side is the expected return from investing an additional unit into the liquid asset. With probability π_h, a bank will have a shortage of liquidity at $t = 1$ and one more unit of the liquid asset saves on borrowing in the interbank market at an expected cost of $p(1+r)$. With probability π_l, a bank will have excess liquidity and one more unit of the liquid asset can be lent out at an expected return $p^2(1+r)$. Note that banks' own probability of being solvent at $t = 2$, p, cancels out in (7) since it affects the expected return on the liquid and the illiquid investment symmetrically.

We rewrite (7) as:

$$\delta(1+r) = R \tag{8}$$

where

$$\frac{1}{\delta} \equiv \frac{1}{\pi_h + \pi_l p} > 1 \tag{9}$$

is the premium of lending in the interbank market due to credit risk. Liquidity becomes more costly when i) credit risk increases (lower p) and ii) a bank is more likely to become a lender (higher π_l) and thus is more likely to be subject to credit risk

Given the price of liquidity (8), a bank with a surplus of liquidity will always want to lend it out rather than store it. That is, the condition in Lemma 2 is always satisfied: $p\frac{R}{\delta} > 1$

[4] It is straightforward to show that a corner solution cannot be optimal. The profitability of the illiquid asset implies a strictly positive investment in it. The presence of liquidity shocks implies a non-zero investment in the liquid asset.

since $pR > 1$ and $\delta < 1$.

Optimal portfolio allocation. We can now use the information about the return on liquidity (8) in proposition (1) to obtain the following result:

Proposition 2 (Portfolio allocation) *Banks' optimal portfolio allocation across the liquid and the illiquid asset satisfies:*

$$\frac{\alpha}{1-\alpha} = \frac{1}{\delta}\frac{(1-\lambda_l)\pi_l + (1-\lambda_h)\pi_h p}{\lambda_l \pi_l + \lambda_h \pi_h}. \tag{10}$$

It is easy to see that a bank chooses to hold a more liquid portfolio if it expects a higher level of early withdrawals (λ_k increases). With respect to the probability of becoming a lender, π_l, and credit risk, p, there are two effects at play: the credit risk premium and the relative fraction of patient versus impatient consumers. With respect to the probability of becoming a lender, both effects go in the same direction: higher π_l increases the credit risk premium and the relative proportion of patient versus impatient consumers. Consequently, a higher probability of having a liquidity surplus at $t = 1$ leads to a less liquid portfolio at $t = 0$.

With respect to credit risk, the two effects work in opposite directions. More credit risk increases the credit risk premium in the unsecured market but lowers the ratio of patient versus impatient consumers (see the discussion following propostion 1). The derivative of the right-hand side of equation (10) with respect to p is negative if and if

$$(1-\lambda_h)\pi_h^2 < (1-\lambda_l)\pi_l^2. \tag{11}$$

A sufficient condition for more credit risk leading to less liquid investments is that banks are (weakly) more likely to have a liquidity surplus than a shortage, $\pi_l \geq \pi_h$ or $\pi_l \geq \frac{1}{2}$.

Note that we can write condition (10) also as

$$\frac{\alpha}{1-\alpha} = \frac{(1-\lambda_l)\hat{\pi}_l + (1-\lambda_h)\hat{\pi}_h}{\lambda_l \pi_l + \lambda_h \pi_h}$$

where credit risk p and the associated risk premium induce new state probabilities $\hat{\pi}_l = \frac{\pi_l}{p\pi_l+\pi_h}$ and $\hat{\pi}_h = \frac{p\pi_h}{p\pi_l+\pi_h}$. The probabilities add up to one, $\hat{\pi}_l + \hat{\pi}_h = 1$, i.e. there is no aggregate shock, if and only if banks are equally likely to be a lender or a borrower (equally likely to have a liquidity shortage or a surplus at $t = 1$), $\pi_l = \pi_h$.

A benchmark - no credit risk It is useful to consider the benchmark case when there is no credit risk. Substituting $p = 1$ into (10) yields the following result:

Corollary 1 (No credit risk) *Without credit risk, the interest rate in the unsecured interbank market is equal to R, and the fraction invested in the illiquid asset is equal to expected amount of late withdrawals: $\alpha^* = 1 - \lambda$.*

Without credit risk there is no friction in the economy. The amount invested in the liquid asset exactly covers the expected amount of early withdrawals since the interbank market fully smoothes out the problem of uneven demand for liquidity across banks. The fraction invested in the illiquid investment exactly covers the expected amount of late withdrawals.[5]

4 Access to government bonds

In this section we allow banks at $t = 0$ to allocate a fraction β of their portfolio into government bonds and to trade these bonds at $t = 1$. To solve the model we follow the same steps as in the previous section.

Liquidity management. At $t = 1$, when banks need to manage their liquidity needs, their investment portfolio is a triple $(\alpha, \beta, 1 - \alpha - \beta)$ (a fraction of deposits invested in the illiquid asset, a fraction used to buy bonds, and a fraction of deposits stored). The bank chooses a fraction of government bond holdings to sell, β_k^S, a fraction of liquid asset holdings to be reinvested in the liquid asset, γ_k^1, a fraction of liquid asset holdings to be used to acquire more government bonds, γ_k^2, and how much to borrow/lend in the interbank market, L_k.

[5]It is easy to see that the pay-out to impatient and patient depositors is $c_1^* = 1$, $c_2^* = R$, respectively.

A bank that faces a low level of withdrawals by impatient consumers, type-l, solves the following problem:

$$\max_{\beta_l^S,\gamma_l^1,\gamma_l^2,L_l} p\big[R\alpha+(\gamma_l^1+\gamma_l^2\frac{Y}{P_1})\big((1-\alpha-\beta)+\beta_l^S\frac{\beta}{P_0}P_1\big)+(1-\beta_l^S)\frac{\beta}{P_0}Y+p(1+r)L_l-(1-\lambda_l)c_2\big] \quad (12)$$

subject to

$$\lambda_l c_1 + L_l + (\gamma_l^1 + \gamma_l^2)\big((1-\alpha-\beta)+\beta_l^S\frac{\beta}{P_0}P_1\big) \leq (1-\alpha-\beta)+\beta_l^S\frac{\beta}{P_0}P_1 \quad (13)$$

and feasibility constraints: $0 \leq \beta_l^S \leq 1$, $0 \leq \gamma_l^1$, $0 \leq \gamma_l^2$, $\gamma_l^1+\gamma_l^2 \leq 1$ and $L_l \geq 0$.

A bank that has received a high liquidity shock, type-h, will be a borrower in the interbank market, solving:

$$\max_{\beta_h^S,\gamma_h^1,\gamma_h^2,L_h} p\big[R\alpha+(\gamma_h^1+\gamma_h^2\frac{Y}{P_1})\big((1-\alpha-\beta)+\beta_h^S\frac{\beta}{P_0}P_1\big)+(1-\beta_h^S)\frac{\beta}{P_0}Y-(1+r)L_h-(1-\lambda_h)c_2\big] \quad (14)$$

subject to

$$\lambda_h c_1 + (\gamma_h^1 + \gamma_h^2)\big((1-\alpha-\beta)+\beta_h^S\frac{\beta}{P_0}P_1\big) \leq (1-\alpha-\beta)+\beta_h^S\frac{\beta}{P_0}P_1 + L_h \quad (15)$$

and feasibility constraints: $0 \leq \beta_h^S \leq 1$, $0 \leq \gamma_h^1$, $0 \leq \gamma_h^2$, $\gamma_h^1+\gamma_h^2 \leq 1$ and $L_h \geq 0$.

Access to bonds changes the liquidity management of banks as follows. Banks holds $\frac{\beta}{P_0}$ units of bonds. They can sell a fraction β_k^S of their bond holdings at price P_1. Hence, the amount of cash available at $t=1$ is the sum of cash holdings, $1-\alpha-\beta$, and the proceeds from selling bonds, $\beta_k^S\frac{\beta}{P_0}P_1$. Banks can also acquire new bonds using γ_k^2 fraction of their cash holdings.

At $t=2$, banks have extra proceeds Y from holding bonds. The proceeds come both from bonds bought at $t=0$ that were not sold at $t=1$, $(1-\beta_k^S)\frac{\beta}{P_0}$ units, and from additional bonds bought at $t=1$, $\frac{\gamma_k^2(1-\alpha-\beta)}{P_1}$ units.

Market clearing in the bond market requires that

$$(\pi_l \beta_l^S + \pi_h \beta_h^S)\frac{\beta}{P_0}P_1 = \pi_l \gamma_l^2\big((1-\alpha-\beta) + \beta_l^S \frac{\beta}{P_0}P_1\big) + \pi_h \gamma_h^2\big((1-\alpha-\beta) + \beta_h^S \frac{\beta}{P_0}P_1\big) \quad (16)$$

The left-hand side of (16) is the amount of bonds sold by banks at $t = 1$ while the right-hand side is the amount bought. The demand for bonds at $t = 1$ will depend on how much cash banks decide to hold at $t = 0$, $1 - \alpha - \beta$.

As before, the unsecured interbank market will be active in order to trade away the idiosyncratic liquidity shock. Access to safe government bonds will however reduce the amount that banks in need of liquidity borrow. Selling government bonds is cheaper since the provider of liquidity (the buyer of the bond) does not need to be compensated for credit risk. Given that government bonds are in fixed supply, i.e. they are scarce, we focus on the more interesting case in which there are not enough bonds to fully cover banks' liquidity shortage.

The introduction of bonds does not change the marginal value of liquidity (in the unsecured interbank market). It is still given by Lemma 1.

A bank with a shortage of liquidity at $t = 1$ that has no access to bonds does not reinvest into the liquid asset (Lemma 2). When such a bank holds bonds, it will not sell these bonds to reinvest into cash. Moreover, a bank with a shortage of liquidity will use the bonds to reduce the amount it needs to borrow in the interbank market rather than hold on to them.

Lemma 4 (Liquidity management of a bank with a shortage) *A bank with a liquidity shortage will not reinvest, neither in bonds nor in cash, $\gamma_h^1 = 0, \gamma_h^2 = 0$, and it will sell all its bonds: $\beta_h^S = 1$.*

The intuition for the result is that since bonds are scarce and the unsecured market is active, banks with a surplus of liquidity must still find it attractive to lend. The return on bonds must not be larger than the return on unsecured lending. Since lenders need to be compensated for credit risk in unsecured lending, banks with a shortage of liquidity will sell all their bonds first and then borrow the remaining amount.

Given that banks with a liquidity shortage sell bonds and borrow in the unsecured market, banks with a liquidity surplus must buy bonds and lend.

Lemma 5 (Liquidity management of a bank with a surplus) *A bank with a liquidity surplus will reinvest by buying additional bonds:* $\gamma_l^1 = 0$, $\gamma_l^2 > 0$ *and* $\beta_l^S = 0$.

Both the secured and the unsecured market are therefore open. There cannot be an arbitrage opportunity between the two markets at $t = 1$ so that

$$\frac{Y}{P_1} = p(1 + r) \geq 1. \tag{17}$$

Secured and unsecured lending must offer the same rate of return. In order to establish whether the rate of return is greater than one, we will have to obtain $1 + r$ by solving banks' portfolio choice at $t = 0$ as before in the case without access to bonds. But before doing so, we establish the analogous result to Lemma 3 and Proposition 1 when banks have access to bonds.

Using the results in Lemma 4 and 5, we can simplify the market clearing condition in the bond market (16):

$$\pi_h \frac{\beta}{P_0} P_1 = \pi_l \gamma_l^2 (1 - \alpha - \beta) \tag{18}$$

Market clearing together with the price of bonds P_1 determines how many bonds banks with a liquidity surplus will buy, γ_l^2.

Market clearing in the unsecured and the secured market yields:

Lemma 6 (Interbank market clearing) *The amount of cash held by banks exactly balances the aggregate pay-out to impatient consumers:*

$$\lambda c_1 = 1 - \alpha - \beta.$$

As before, trading at $t = 1$ fully smoothes out the idiosyncratic liquidity shocks.

Proposition 3 *The ratio of the total returns on the illiquid and the liquid asset is equal to the ratio of patient to impatient consumers (adjusted for credit risk) minus the (adjusted) value of bonds at $t = 1$:*

$$\frac{R}{1+r}\frac{\alpha}{1-\alpha-\beta} = \frac{(1-\lambda_l)\pi_l + (1-\lambda_h)\pi_h p}{\lambda_l \pi_l + \lambda_h \pi_h} - \frac{BP_1}{1-\alpha-\beta}\frac{(1-\lambda_l)-(1-\lambda_h)p}{\lambda_h - \lambda_l}. \tag{19}$$

The left-hand side of (19) is as in (3), except that we have to subtract the bond holdings β to obtain the cash holdings. The first-term on the right-hand side is the same as in the case without bonds. The access to bonds means that banks hold a more liquid portfolio ceteris paribus. Bonds are not subject to credit risk and therefore provide a safe late return at $t = 2$. Moreover, bonds can be used to mitigate liquidity shortages at $t = 1$.

The effect of bonds on ex-ante liquidity holdings is stronger when it is more difficult to trade liquidity ex-post in the unsecured market, i.e if there is more credit risk (lower p), more withdrawals at banks with a liquidity surplus (higher λ_l) and less withdrawals at banks with a liquidity shortage (lower λ_h).

Pricing liquidity. The return on trading liquidity, $1 + r$, and the price of bonds, P_1, at $t = 1$ is determined by banks' portfolio allocation at $t = 0$. Banks invest into the illiquid asset and into bonds in order to:

$$\max_{\alpha,\beta} \quad \pi_l p[R\alpha + \left(\frac{\pi_h}{\pi_l} + 1\right)\frac{Y}{P_0}\beta + p(1+r)L_l - (1-\lambda_l)c_2] \tag{20}$$

$$+ \pi_h p[R\alpha - (1+r)L_h - (1-\lambda_h)c_2] \tag{21}$$

subject to

$$\begin{aligned} L_l &= (1-\alpha-\beta) - \frac{\pi_h}{\pi_l}\frac{P_1}{P_0}\beta - \lambda_l c_1 \\ L_h &= \lambda_h c_1 - (1-\alpha-\beta) - \frac{P_1}{P_0}\beta. \end{aligned}$$

where we have used the results in Lemma 4 and 5 on banks' liquidity management and market clearing in the bond market (18) in order substitute for γ_l^2.

The first-order condition for a bank's optimal portfolio allocation with respect to the allocation into the illiquid asset, α, is

$$p[R - \pi_l p(1 + r) - \pi_h(1 + r)] = 0,$$

which simplifies to

$$1 + r = \frac{R}{\delta}. \tag{22}$$

The cost of unsecured borrowing is not affected by the access to bonds. Condition (22) is identical to condition (8). It is given by a no-arbitrage condition at $t = 0$ between investing into the liquid asset, i.e. holding cash and lending it out, and investing into the illiquid asset.

Due the no-arbitrage condition between unsecured and secured interbank lending at $t = 1$ (equation (17)), condition (22) also ties down the price of bonds at $t = 1$:

$$\frac{Y}{P_1} = \frac{pR}{\delta} \tag{23}$$

The condition immediately implies that $\frac{Y}{P_1} > 1$ since $pR > 1$ and $\delta < 1$. That is, bonds trade at a discount at $t = 1$. If they did not, then holding cash to lend it out is not very attractive. Banks would invest everything into the illiquid asset. Alternatively, banks could invest in both liquid and illiquid assets ex-ante, but if bonds are priced at par, banks with a surplus of liquidity would not be willing to buy them at $t = 1$ from banks that need to sell them to cover their liquidity shortage. It is interesting to note that higher credit risk (lower p) increases the price of bonds P_1.

The optimal choice of bond holdings ties down the price of bonds at $t = 0$.

Lemma 7 *The price of bonds at $t = 0$ is equal to the price at $t = 1$:*

$$P_0 = P_1. \tag{24}$$

If the first-period yield would be less than one, then the bond would be dominated by cash.

The following proposition summarizes prices and rates when banks have access to government bonds:

Proposition 4 (Pricing) *The interest rate in the unsecured market is* $1+r = \frac{R}{\delta}$. *The yield of the bond at t=0 and at t=1 is identical and it is given by* $\frac{Y}{P_0} = \frac{Y}{P_1} = \frac{pR}{\delta}$.

We can now combine the results in Propositions 3 and 4 to derive banks' optimal portfolio allocation at $t = 0$:

Proposition 5 (Portfolio allocation) *Banks' portfolio allocation into the liquid and illiquid asset satisfies:*

$$\frac{\alpha}{1-\alpha-\beta} = \frac{1}{\delta}\frac{(1-\lambda_l)\pi_l + (1-\lambda_h)\pi_h p}{\lambda_l\pi_l + \lambda_h\pi_h} - \frac{BY}{pR}\frac{1}{1-\alpha-\beta}\frac{(1-\lambda_l)-(1-\lambda_h)p}{\lambda_h - \lambda_l} \tag{25}$$

The fraction invested in bonds is given by market clearing at $t = 0$: $\beta = BP_0 = B\delta\frac{Y}{pR}$.

The first term of the left-hand side is as in Proposition 2. The second term reflects the fact that the access to bonds allows banks to hold more liquid assets (see also the discussion after Proposition 3). Note also that the size of the banking sector relative to the amount of collateral matters. The effect of bonds is stronger when the ratio of banks' productive assets to the value of bonds, $\frac{BY}{pR}$, is larger.

Suppose that banks are equally likely to have a liquidity shortage or a liquidity surplus. Then more credit risk increases the first right-hand side term (see condition (11)). It also increases the second right-hand side term, making the overall impact of more credit risk on banks' portfolio choice ambiguous.

5 Access to both government and risky bonds

Suppose that in addition to the safe government bonds, there is a second, risky bond, e.g. a mortgage-backed security. Let this risky bond pay Y_H with probability $1-\epsilon$ and $Y_L < Y_H$ with probability ϵ at $t=2$, with the expected payoff equal to Y (the payoff on the government bond). The shock to the return of the risky bond realizes at $t=1$, at the same time as the liquidity shock. Denote the fraction banks invest into the risky bond at $t=0$ as γ. The bond is in fixed supply at $t=0$, B^r. The price of the bond at $t=0$ is denoted by P_0^r.

The shock to the bond return is observable. We therefore index all variables at $t=1$ by the state that realizes. With a slight abuse of notation, we then have $c_2(\epsilon)$, $P_1(\epsilon)$, $1+r(\epsilon)$, $\gamma_k^1(\epsilon)$, $\gamma_k^2(\epsilon)$, $\beta_k^S(\epsilon)$ and $L_k(\epsilon)$. Furthermore, let $\gamma_k^3(\epsilon)$ be the fraction of cash used to acquire more risky bonds at $t=1$ and $P_1^r(\epsilon)$ be the price of the risky bond at $t=1$. Finally, denote the pay-off of the risky bond at $t=2$ as $Y(\epsilon)$.

As before, a bank with a shortage of liquidity at $t=1$ will not reinvest its liquidity into the liquid asset, or use it to acquire more bonds, $\gamma_h^1(\epsilon)=\gamma_h^2(\epsilon)=\gamma_h^3(\epsilon)=0$. Instead, it will try to minimize unsecured lending by selling all its bonds. Its objective function at $t=1$ is therefore given by:

$$p[R\alpha-(1+r(\epsilon))L_h(\epsilon)-(1-\lambda_h)c_2(\epsilon)]$$

and its budget constraint is

$$L_h(\epsilon)=\lambda_h c_1-(1-\alpha-\beta-\gamma)-\frac{P_1(\epsilon)}{P_0}\beta-\frac{P_1^r(\epsilon)}{P_0^r}\gamma.$$

A bank with a surplus of liquidity at $t=1$ will buy additional bonds: $\gamma_l^2(\epsilon)>0$ and $\gamma_l^3(\epsilon)>0$. Its objective function is given by:

$$R\alpha+\left(\gamma_l^2(\epsilon)\frac{Y}{P_1(\epsilon)}+\gamma_l^3(\epsilon)\frac{Y(\epsilon)}{P_1^r(\epsilon)}\right)(1-\alpha-\beta-\gamma)+\frac{\beta}{P_0}Y+\frac{\gamma}{P_0^r}Y(\epsilon)+p(1+r(\epsilon))L_l(\epsilon)-(1-\lambda_l)c_2(\epsilon)$$

and its budget constraint is

$$L_l(\epsilon) = (1-\alpha-\beta-\gamma)(1-\gamma_l^2(\epsilon)-\gamma_l^3(\epsilon)) - \lambda_h c_1.$$

Moreover, the bond markets have to clear

$$\begin{aligned}
\pi_l(1-\alpha-\beta-\gamma)\gamma_l^2(\epsilon) &= \pi_h \frac{\beta}{P_0} P_1(\epsilon) \\
\pi_l(1-\alpha-\beta-\gamma)\gamma_l^3(\epsilon) &= \pi_h \frac{\gamma}{P_0^r} P_1^r(\epsilon)
\end{aligned}$$

and there must be no-arbitrage between safe and risky bonds at $t=1$:

$$\frac{Y}{P_1(\epsilon)} = \frac{Y(\epsilon)}{P_1^r(\epsilon)}. \tag{26}$$

Market clearing in the unsecured market, $\pi_l L_l = \pi_h L_h$, yields

$$\lambda c_1 = (1-\alpha-\beta-\gamma).$$

The equivalent of the condition in propositions 1 and 3 is:

$$\frac{R}{1+r(\epsilon)} \frac{\alpha}{1-\alpha-\beta} = \frac{(1-\lambda_l)\pi_l + (1-\lambda_h)\pi_h p}{\lambda_l \pi_l + \lambda_h \pi_h} - \frac{BP_1(\epsilon) + B^r P_1^r(\epsilon)}{1-\alpha-\beta} \frac{(1-\lambda_l) - (1-\lambda_h)p}{\lambda_h - \lambda_l}. \tag{27}$$

The first order conditions with respect to α, β and γ are, respectively:

$$\begin{aligned}
R &= \delta E[1+r(\epsilon)] \\
\frac{1}{P_0} &= E[\frac{1}{P_1(\epsilon)}] \\
\frac{1}{P_0^r} &= E[\frac{1}{P_1^r(\epsilon)}].
\end{aligned}$$

Next, we examine more closely how prices of safe and risky bonds are related. The no-arbitrage condition (26) together with the two bond market clearing conditions

$$P_1(\epsilon) = \frac{\pi_l \gamma_l^2(\epsilon)(1-\alpha-\beta-\gamma)}{\pi_h B} \text{ and } P_1^r(\epsilon) = \frac{\pi_l \gamma_l^3(\epsilon)(1-\alpha-\beta-\gamma)}{\pi_h B^r}$$

imply that banks with excess liquidity will spend a relatively higher proportion of their cash on buying government bonds than risky bonds in the low state:

$$\frac{\gamma_l^2(L)}{\gamma_l^3(L)} > \frac{\gamma_l^2(H)}{\gamma_l^3(H)}.$$

This is because government bonds offer a relatively higher return in the low state.

If a bank chooses to acquire liquidity at $t=1$ through a sale of a government bond, its per unit return is given by:

$$\frac{P_1(\epsilon)}{P_0} = P_1(\epsilon)\left[(1-\epsilon)\frac{1}{P_1(H)} + \epsilon\frac{1}{P_1(L)}\right] \tag{28}$$

in state ϵ. The analogue of equation (25) for the environment with both government and risky bonds is:

$$\frac{R\alpha}{1-\alpha-\beta-\gamma} = \frac{1}{p}\frac{Y}{P_1(\epsilon)}\frac{(1-\lambda_l)+(1-\lambda_h)\frac{\pi_h}{\pi_l}p}{\lambda_l+\lambda_h\frac{\pi_h}{\pi_l}} - \frac{1}{p}\frac{BY+B^rY(\epsilon)}{1-\alpha-\beta-\gamma}\frac{(1-\lambda_l)-(1-\lambda_h)p}{\lambda_h-\lambda_l}.$$

It follows that

$$\frac{Y}{P_1(L)} < \frac{Y}{P_1(H)} \tag{29}$$

and hence

$$P_1(L) > P_1(H).$$

Thus, we can re-write equation (28) to conclude that

$$\frac{P_1(L)}{P_0} = (1-\epsilon)\frac{P_1(L)}{P_1(H)} + \epsilon > 1.$$

Government bonds are a particularly valuable source of liquidity in the low state as they return more than cash.

If, on the other hand, a bank chooses to acquire liquidity at $t = 1$ through a sale of a risky bond, its per unit return is given by:

$$\frac{P_1^r(\epsilon)}{P_0^r} = P_1^r(\epsilon)\left[(1-\epsilon)\frac{1}{P_1^r(H)} + \epsilon\frac{1}{P_1^r(L)}\right]$$

in state ϵ. It is easy to see that the price of the risky bond is lower when the low return Y_L is realized as compared to when the high return is realized: $P_1(H) > P_1(L)$. It follows that the return from selling a risky bond in the low state is equal to:

$$\frac{P_1^r(L)}{P_0^r} = (1-\epsilon)\frac{P_1^r(L)}{P_1^r(H)} + \epsilon < 1.$$

Thus, risky bonds are a less valuable source of liquidity in a low state as their return is lower than the return on cash. Naturally, it is also the case that government bonds are more valuable than risky bonds in the low state: $P_1^r(L) < P_1(L)$.

6 Empirical implications

Looking at Figures 1 and 2, it seems that the repo markets secured by government bonds in the US and in the euro area followed a different dynamics between August 2007 and May 2009. Below, we discuss additional empirical predictions of the model that may help explain such distinct developments.

De-coupling of secured and unsecured rates

If the credit risk problem becomes more severe, p decreases, government bonds become relatively more valuable, i.e. $\frac{\partial E\left[\frac{1}{P_1(\epsilon)}\right]}{\partial p} > 0$. To see this, note that the following no-arbitrage conditions hold ex ante:

$$pR = p\delta E[1 + r(\epsilon)] = \delta Y E\left[\frac{1}{P_1(\epsilon)}\right].$$

Lower p puts an upward pressure on the unsecured rate by increasing the premium for unsecured borrowing, $\frac{1}{\delta}$. Hence, $E[1 + r(\epsilon)]$ increases but less than proportionally with p. At the same time, expected value of government bonds increases, implying that borrowing in the repo market secured by government bonds will be cheaper. In other words, following a shock to credit risk, unsecured rates and rates secured by government bonds move in the opposite direction.

De-coupling of rates secured by government bonds and risky bonds

The model implies that while the price of a risky bond decreases in the low state compared to the high state, the price of a government bond increases compared to the high state. Hence, if there is a shock to the return on a risky bond, repo rates secured by government bonds decrease whereas repo rates secured by risky securities increase.

When sub-prime mortgages were discovered in the portfolio of banks and bank-sponsored conduits in the summer of 2007, it led to a market-wide reassessment of risk. Interbank interest rates in the unsecured market rose. Moreover, repo rates for riskier type of collateral in the US rose as well.

Relative scarcity of collateral

How do differences in the scarcity of the underlying securities affect the dynamics of the repo rates when credit risk increases? Our model implies that the sensitivity of the price of government bonds to the unsecured market developments is lower in a country with a high supply of government bonds. In other words, we have that, ex post,

$$\frac{\partial^2 P_1}{\partial B \partial p} > 0$$

while, as is intuitive, $\frac{\partial P_1}{\partial B} < 0$.

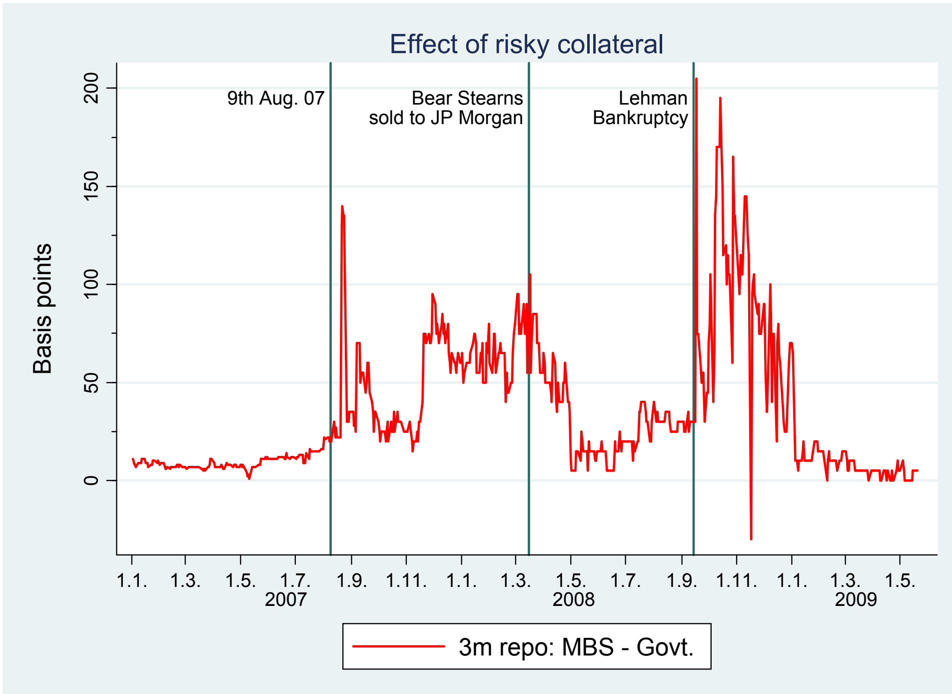

Figure 5: Decoupling of interbank rates secured by government bonds and mortgage-backed securities

With the onset of the crisis in August 2007, repo rates in the US became much more volatile than in the euro area. Hördahl and King (2008) argue that this difference can be partly explained by the increased safe haven demand for the US Treasury securities, thus making Treasuries relatively scarce.

Spillovers between the secured and unsecured markets are lowest for low levels of credit risk

The potential for spillover effects from the secured to the unsecured market increases as the level of credit risk increases. To see this, note that

$$R = \frac{Y}{P_1}\left[\pi_l + \frac{1}{p}\pi_h\right]$$

must hold ex ante implying that

$$-\frac{\frac{\partial P_1}{\partial p}}{\frac{P_1}{p}} = \frac{\pi_h}{\delta} < 1$$

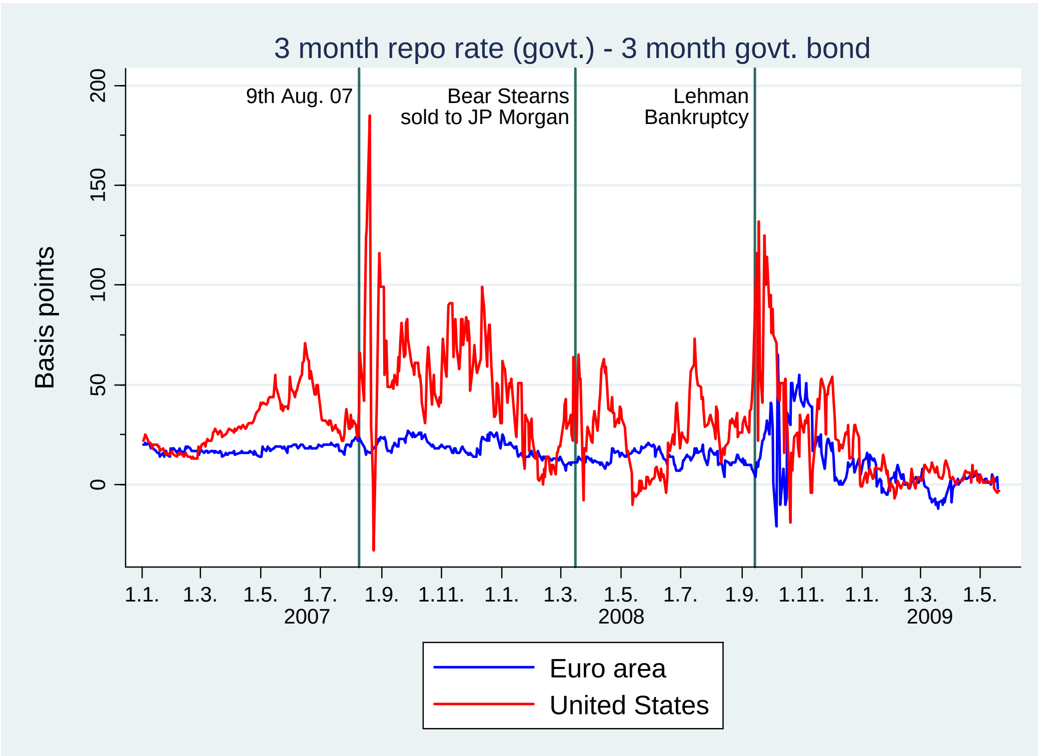

Figure 6: Tensions in Treasury-backed repo markets, EA and US

for all $p > 0$. It follows that the elasticity of the price of government bonds to credit risk is the lowest for $p = 1$ (no credit risk).

Aggregate liquidity shocks

If there is an unanticipated shock to the relative proportion of high and low liquidity demand banks, i.e. $\frac{\pi_h}{\pi_l}$, then liquidity becomes more scarce and even the price of government bonds declines. To see this, note that:

$$\frac{R\alpha}{1-\alpha-\beta-\gamma} = \frac{1}{p}\frac{Y}{P_1(\epsilon)}\frac{(1-\lambda_l)+(1-\lambda_h)\frac{\pi_h}{\pi_l}p}{\lambda_l+\lambda_h\frac{\pi_h}{\pi_l}} - \frac{1}{p}\frac{BY+B^rY(\epsilon)}{1-\alpha-\beta-\gamma}\frac{(1-\lambda_l)-(1-\lambda_h)p}{\lambda_h-\lambda_l}.$$

It is easy to show that higher $\frac{\pi_h}{\pi_l}$ always leads to lower $P_1(\epsilon)$. The only term in the equation above that depends on $\frac{\pi_h}{\pi_l}$ is the first term on the right-hand side. Its derivative with respect to $\frac{\pi_h}{\pi_l}$ is given by:

$$\frac{\partial}{\partial \frac{\pi_h}{\pi_l}} \frac{(1-\lambda_l) + (1-\lambda_h)\frac{\pi_h}{\pi_l} p}{\lambda_l + \lambda_h \frac{\pi_h}{\pi_l}} = \frac{p(1-\lambda_h)\lambda_l - \lambda_h(1-\lambda_l)}{\left(\lambda_l + \lambda_h \frac{\pi_h}{\pi_l}\right)^2}.$$

As long as

$$p < \frac{\lambda_h(1-\lambda_l)}{\lambda_l(1-\lambda_h)},$$

we have that this derivative is negative and hence $P_1(\epsilon)$ must decline. Note that for $\lambda_h > \lambda_l$, which is what we assume, $\frac{\lambda_h(1-\lambda_l)}{\lambda_l(1-\lambda_h)} > 1$ and thus the inequality above always holds.

Relative importance of the market secured by risky bonds

The model implies that in the low state, government bonds are relatively more valuable, $P_1(L) > P_1(H)$. Moreover, bonds become relatively more valuable if the risk premium in the unsecured market increases due to a shock to the level of credit risk (lower p). Hence, if a shock to p and the low return on risky bonds are realized simultaneously, then the price of a government bond will increase by more in a country in which risky bonds are used to obtain liquidity as compared to a country in which mostly unsecured market and market secured by government bonds are used to obtain liquidity.

In general, government bonds are predominantly used as collateral in private euro area repo transactions, while usage of illiquid and risky securities, such as asset-backed securities, is uncommon. In the initial phase of the 2007-09 financial crisis, the share of government bonds as collateral was around around 81 percent, compared to 83.7 percent in June 2007, according to the International Capital Markets Association (ICMA) semi-annual survey of financial institution.[6] After the dramatic market events in September 2008, the share of government collateral increased to 83.6 percent in December 2008. A significant increase in the share of German collateral was also reported, supporting a flight to quality hypothesis, given the increasing differentiation of the euro area government debt.

While the repo market in the US is also dominated by the government securities as

[6] Last available data are from December 2008. Sixty-one institutions participated in the survey.

collateral, before the crisis started, there were also active market segments in agency bonds, mortgage-backed securities, and corporate bonds. Indeed, anecdotal evidence suggests that non-government collateral contributed significantly to the rapid growth of the US repo market before the start of the crisis.

7 Policy implications

Unsecured markets are particularly vulnerable to changes in the perceived creditworthiness of counterparties. In repo transactions, such concerns are mitigated to some extent by the presence of collateral. Yet, our model illustrates how tensions in the unsecured market or the market secured by risky collateral spill over to the market secured by collateral of the highest quality. Moreover, the volatility of the repo rates can be exacerbated by structural characteristics such as the scarcity of securities that are used as collateral.

Central banks are particularly concerned with the well functioning of interbank markets because it is an important element in the transmission of monetary policy, and because persisting tensions may affect the financing conditions faced by non-financial corporations and households. In many countries, central banks have reacted to events by introducing measures to support the interbank market, trying to avoid market-wide liquidity problems turning into solvency problems for individual institutions. The aim of this Section is to examine policy responses implemented since August 2007 that aimed at reducing tensions in interbank markets.

Specifically, we examine how the range of collateral accepted by a central bank affects liquidity conditions of banks and how central banks can help alleviate tensions associated with the scarcity of high-quality collateral. In line the predictions of the model, we present evidence that these measures can be effective in reducing tensions in secured markets. At the same time, they are not designed to resolve problems in the unsecured segment and the associated spill-overs, if those are driven by credit risk concerns.

7.1 Collateral accepted by the Central Bank

Central banks provide liquidity to the banking sector against eligible collateral. The range of acceptable collateral varies across countries. Since the onset of the crisis, however, central banks have generally lowered the minimum credit rating and increased the quantity of lending they provide. For example, the Fed expanded its collateral list for repo operations in March, May and September 2008, in response to severe market tensions. Moreover, it established the Term Auction Facility (TAF) in December 2007. The TAF provides term credit through periodic auctions to a broader range of counterparties and against a broader range of collateral than open market operations. The Fed stressed that "this facility could help ensure that liquidity provisions can be disseminated efficiently even when the unsecured interbank markets are under stress".[7] The ECB headed into the crisis with the broadest list of eligible collateral among its peers, including nonmarketable securities and commercial loans. As a result, the ECB made no changes until mid-October 2008, when it expanded the eligible collateral significantly and lowered the minimum credit rating from A– to BBB–, as the crisis intensified.

What are the implications of a wider range of collateral accepted in central bank's operations according to our model? First, allowing securities other than Treasuries can reduce the volatility of the repo rates backed by Treasuries as it reduces pressure on acquiring Treasury securities and the limits imposed by their fixed supply. Moreover, a Central Bank can offer more liquidity to banks (apply a lower haircut) for investment-grade securities that, as we show, can be subject to illiquidity in private markets. This is because a central bank can raise liquidity at a unit cost and intervene in selected markets, whereas private pricing of liquidity must take into account the opportunity cost of liquidity, which is higher than one due to credit risk. To see this, note that a Central Bank can set $P_1^r(L) = Y(L)$. This makes sure that the Central Bank does not suffer any losses in the low state since the price is set equal to the return on the risky bond. At the same time, it improves the price that a

[7]Press release of the Federal Reserve Board on December 12, 2007.

bank in need of liquidity receives for its risky bond compared to the private market. This is beneficial for borrowers, who would otherwise have to pay a premium for obtaining liquidity in the interbank market to compensate lenders for credit risk.

Moreover, we showed that if there is an unexpected aggregate liquidity shock, funding pressures can appear in all interbank market segments. By providing liquidity, a Central Bank can counter the effects of aggregate shocks and ensure that financial institutions do not sell their assets at distressed prices.

Central Bank intervention has implications for the composition of collateral used by banks in operations with the Central Bank. In times of interbank tensions, banks will bring less liquid securities as collateral since a Central Bank can offer lower haircuts. This is not the case in "good" times, when the level of credit risk is low, and banks can obtain liquidity easily in both secured and unsecured markets since the price of the risky bond is high. In addition, when credit risk premia are low, the sensitivity of the repo rate to developments in the unsecured market is also low. Having a broad list of collateral is a potent tool for a Central Bank to deal with the tensions since banks only make use of less liquid securities in "bad" times. Indeed, evidence from the US suggests that in normal times, a dealer pledging agency debt securities or agency MBS as collateral typically pays only slightly more interest to borrow funds than a dealer pledging Treasury securities. In the course of the financial crisis however these spreads soared to several dozens basis points.[8]

Similarly, Tapking and Weller (2008) document that with the onset of the financial crisis in August 2007, rates in the euro area repo market secured by government bonds, Eurepo, were below the average marginal rates at the ECB auctions where a larger set of collateral is accepted. They also show that banks with worse collateral bid more aggressively at the ECB main refinancing operations. Following an improvement in euro money market conditions since the start of 2009, the share of asset-backed securities, or notes backed by repayments on other debt such as mortgages or credit-card loans declined to about 20 percent of total

[8]See Fleming et al. (2009).

collateral pledged with the ECB in the first quarter of 2009, compared with 28 percent for the whole of 2008.

How effective were changes the to the collateral framework of central banks during the crisis? McAndrews, Asani and Wang (2008) provide evidence that the introduction of the TAF was associated with downward shifts of the Libor by reducing the liquidity risk premium. Christensen, Lopez and Rudebusch (2009) analyze the role of the TAF in reducing the spreads between term Libor rates and the yield on Treasuries of corresponding maturity. They construct a counterfactual path and conclude that in the absence of the TAF, the Libor would have been higher. On the other hand, Taylor and Williams (2009) argue that the TAF had no siginificant impact on interest rate spreads as it did not address the fundamental problem of credit risk on banks' balance sheets.

7.2 Upgrading collateral

If concerns about the creditworthiness of counterparties make it expensive to borrow in the unsecured market, financial institutions try to obtain more funds in the secured market. However, we show that if the underlying collateral is scarce, the repo market rates will be volatile. Measures aimed at increasing the supply of scarce collateral can thus improve the allocation of liquidity in interbank markets.

For example, the Fed introduced the Term Securities Lending Facility (TSLF) in March 2008. It lends Treasury securities to dealers, taking less liquid securities, including agency debt securities and mortgage-backed securities, as collateral. The Treasury securities are allocated to dealers via auctions.[9] The primary dealers then use those Treasury securities to obtain financing in private repo markets. The TSLF thus increases the ability of dealers to obtain financing and decreases their need to sell assets into illiquid markets.

[9]The TSLF is divided into two schedules: Schedule 1 TSLF operations (i.e. auctions for Treasury and agency securities) are separated from Schedule 2 TSLF operations (i.e. Schedule 1 plus other investment grade collateral). Schedule 2 collateral originally included Schedule 1 collateral plus AAA/Aaa-rated non-agency residential MBS, commercial MBS, and agency collateralized mortgage obligations (CMOs). Schedule 2 collateral was expanded to include AAA/Aaa-rated asset-backed securities starting with the May 8, 2008, auction and all investment-grade debt securities starting with the September 17, 2008, auction.

The direct benefits that can be expected from the TSLF are, first, an increase in the supply of Treasury collateral in the private repo market, and, second, a reduction of the supply of less liquid collateral. In line with our model, the overall effect should be an improvement of the liquidity of those markets. It should relieve the downward pressure on the price of risky bonds and relieve that upward pressure on the price of Treasuries.

The TSLF is closely related to the Primary Dealer Credit Facility, which is also available to primary dealers. A key difference is that the PDCF is a standing facility whereas the TSLF is an auction facility. As a standing facility, the PDCF offers the advantage of availability on a continuous basis. It also accepts a broader class of securities as collateral. Whereas the TAF (discussed in the previous section) is only available to depository institutions, the TSLF is available to primary dealers. Both programs address the tensions in interbank markets via different market participants.

Fleming, Hrung and Keane (2009) provide evidence of the impact of the introduction of the TSLF on repo spreads between Treasury collateral and lower quality collateral. They document that the introduction of the TSLF was associated with an increase in Treasury repo rates relative to the fed funds rate. This is consistent with the predictions of our model that reducing the scarcity of high quality collateral should result in higher Treasury repo rates. We showed above that following a negative return realization of the risky bond, the spread between the price of risky and safe bonds increases. The authors document that the introduction of the TSLF narrowed financing spreads during spring 2008, particularly after the first auction. In addition, much of the narrowing seems to come from an increase in Treasury rates rather as opposed to a decrease of the rates for non-Treasury collateral.

8 Conclusion

Despite the presence of collateral, the disruptions in the unsecured interbank market during the 2007-2009 financial crisis have also affected secured markets. This paper presents a model of secured and unsecured interbank lending in the presence of credit risk. Credit risk premia in the unsecured market also affect the price of riskless bonds when they are used to manage banks' liquidity shocks. Moreover, if the collateral in the secured market is subject to aggregate risk, the resulting market risk also impacts the price of riskless bonds.

Going forward, our analysis points to a number of issues for further research. First, the size of the banking sector relative to the amount of collateral matters. We saw that the presence of bonds reduces the amount banks have to borrow in unsecured markets which is vulnerable to credit risk concerns. The positive effect of bonds is stronger when the ratio of banks' balance sheet to the value of bonds is larger. Hence, the interplay between the relative size of banking, financial markets and the economy deserves further attention.

Second, our analysis abstracted from risk sharing concerns. Banks were simply maximizing the total amount of demandable liabilities. Even without risk sharing, we obtain a credit risk premium in unsecured interbank markets. Introducing risk aversion of banks' customers is beyond the scope of this paper and constitutes a fruitful avenue for further research. With respect to the spillover of credit and market risk across interbank markets, we anticipate that risk aversion adds an additional risk premium that would exacerbate the tensions that we identified.

Third, we assumed that the various shocks in our model are uncorrelated. The financial crisis has made painfully clear that in reality, the risk embedded in banks' illiquid assets, their liquidity needs and shocks to collateral values are interlinked. The challenge will therefore be to model and analyze the joint distribution of the risks in banks' balance sheets, especially "at the tail". Banks' risk management practices have to take into account the forces affecting different collateral classes and the market's response in times of stress when liquidity and high quality collateral is scarce.

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Appendix

Proof of Lemma 1

The interbank market is active so that the Lagrange multipliers on the feasibility constraints on L_k are zero. Let μ^k be the Lagrange multiplier on the budget constraint. The first-order condition for a lender w.r.t. L_l is:

$$p^2(1+r) - \mu^l = 0 \tag{30}$$

while the first-order condition for a borrower w.r.t. to L_h is:

$$-p(1+r) + \mu^h = 0. \tag{31}$$

Proof of Lemma 2

Let μ_1^k be the Lagrange multipliers on $0 \leq \gamma_k^1$. The constraint $\gamma_k^1 \leq 1$ cannot be binding since otherwise all available cash at $t = 1$ is reinvested and nothing can be paid or lent out. The first-order condition for a type-k bank w.r.t. to γ_k^1 is:

$$(1-\alpha)(p - \mu^k) + \mu_1^k = 0 \tag{32}$$

Substituting $\mu^h = p(1+r)$ (Lemma 1) yields:

$$(1-\alpha)(-pr) = -\mu_1^h < 0 \tag{33}$$

since left hand side is negative. It cannot be zero since $\alpha = 1$ cannot be optimal. A type-h bank would have to finance its entire need for liquidity by borrowing in the interbank market at a rate $r > 0$ whereas it could just store some liquidity without cost using the short-term asset. Since $-\mu_1^h < 0$ we have $\gamma_h^1 = 0$.
Consider now the case of a lender. Substituting $\mu^l = p^2(1+r)$ (Lemma 1) into (32) yields:

$$(1-\alpha)p(1 - p(1+r)) = -\mu_1^l$$

Again, $\alpha = 1$ cannot be optimal. A type-l bank cannot invest everything into the illiquid asset and still lend in the interbank market. Hence, $\gamma_l^1 = 0$ if and only if $p(1+r) \geq 1$ (we assume that a type-l bank does not reinvest into the liquid asset when the condition holds as an equality).

Proof of Lemma 3

Using the binding budget constraints from the optimization programs (1) and (2) (Lemma 1) to substitute for L_l and L_h in the market clearing condition $\pi_l L_l = \pi_h L_h$ and using $\gamma_k^1 = 0$ (Lemma 2) gives the result.

Proof of Proposition 1

Competition requires that

$$R\alpha + p(1+r)[(1-\alpha) - \lambda_l c_1] - (1-\lambda_l)c_2 = 0$$

and that

$$R\alpha - (1+r)[\lambda_h c_1 - (1-\alpha)] - (1-\lambda_h)c_2 = 0$$

where we have used the results from Lemma 1 and 2. The pay-out by solvent lenders and borrowers must be such that they make zero profits. If they did not, then another bank could offer a slightly higher combination of early and late pay-out $(\hat{c}_1, \hat{c}_2)$ and attract all consumers. Using the result on c_1 from Lemma 3, using one condition to solve for c_2 and substituting back into the other condition gives the desired result.

Proof of Lemma 4

The first-order condition of a borrower with respect to reinvesting into the liquid asset at $t = 1$, γ_h^1, is

$$\big((1-\alpha-\beta) + \beta_h^S \frac{\beta}{P_0} P_1\big)(p - \mu^h) + \mu_1^h = 0$$

where μ^h is the marginal value of liquidity for a borrower (given by Lemma 1) and μ_1^h is the multiplier on the feasibility constraint $\gamma_h^1 \geq 0$. Note that $(1-\alpha-\beta) + \beta_k^S \frac{\beta}{P_0} P_1 > 0$ since we are considering interior portfolio allocations, $1-\alpha-\beta > 0$. Since $\mu^h = p(1+r) > p$, we have that $\mu_1^h > 0$ and thus $\gamma_h^1 = 0$. As in the case without bonds, a borrower does not reinvest into the liquid asset.
From the first-order condition of a lender with respect to bond purchases at $t = 1$, γ_l^2, we have that

$$\big((1-\alpha-\beta) + \beta_l^S \frac{\beta}{P_0} P_1\big)(p\frac{Y}{P_1} - \mu^l) + \mu_2^l = 0 \tag{34}$$

where μ^l is the marginal value of liquidity for a lender (given by Lemma X) and μ_2^l is the multiplier on the feasibility constraint $\gamma_l^2 \geq 0$. Note that the feasibility constraint $\gamma_l^1 + \gamma_l^2 \leq 1$ must be automatically satisfied since otherwise all available cash at $t = 1$ is reinvested and nothing can be paid or lent out. Since $\mu^l = p^2(1+r)$, the first-order condition holds iff

$$\frac{Y}{P_1} \leq p(1+r) \tag{35}$$

The yield of the bond at $t = 1$ must be less or equal to the expected return of unsecured interbank lending (given that the unsecured interbank market is open).
The first-order condition of a borrower with respect to bond purchases at $t = 1$, γ_h^2, is

$$\big((1-\alpha-\beta) + \beta_h^S \frac{\beta}{P_0} P_1\big)(p\frac{Y}{P_1} - \mu^h) + \mu_2^h = 0$$

where μ_2^h is the multiplier on the feasibility constraint $\gamma_h^2 \geq 0$. Due to condition (35), we have that $\mu_2^h > 0$ and hence $\gamma_h^2 = 0$. A borrower also does not reinvest using bonds.

The first-order condition of a borrower with respect to bond sales at $t = 1$, β_h^S, is

$$\frac{\beta}{P_0}\left[p\left(P_1\gamma_h^1 + Y(\gamma_h^2 - 1)\right) - \mu^h P_1\left(\gamma_h^1 + \gamma_h^2 - 1\right)\right] + \mu_3^h - \mu_4^h = 0$$

where μ_3^h and μ_4^h are the Lagrange multipliers on $0 \leq \beta_h^S \leq 1$. Using $\gamma_h^1 = 0$, $\gamma_h^2 = 0$ and $\mu^h = p(1+r)$, we have

$$p\frac{\beta}{P_0}\left[-Y + (1+r)P_1\right] + \mu_3^h - \mu_4^h = 0$$

Due to condition (35), the term is squared brackets is positive. For the condition to hold, it must be that $\mu_4^h > 0$, and hence $\beta_h^S = 1$. The borrower sells all his bonds at $t = 1$.

Proof of Lemma 5

Market clearing for bonds at $t = 1$ requires that:

$$(\pi_l\beta^S + \pi_h)\frac{\beta}{P_0}P_1 = \pi_l\gamma_l^2\left((1-\alpha-\beta) + \beta_l^S\frac{\beta}{P_0}P_1\right)$$

where we have used $\beta_h^S = 1$ and $\gamma_h^2 = 0$. Market clearing therefore requires that $\gamma_l^2 > 0$. Since borrowers sell bonds, lenders must buy them.
Given that $\gamma_l^2 > 0$, and hence $\mu_2^l = 0$, the lender's first-order condition with respect to bond purchases (34) requires that

$$\frac{Y}{P_1} = p(1+r) \tag{36}$$

The yield on safe bonds must be equal to the expected return on risky interbank loans as both markets are open.
The first-order condition of a lender with respect to bond sales at $t = 1$, β_l^S, is

$$\frac{\beta}{P_0}\left[p\left(P_1\gamma_l^1 + Y(\gamma_l^2 - 1)\right) - \mu^l P_1\left(\gamma_l^1 + \gamma_l^2 - 1\right)\right] + \mu_3^l - \mu_4^l = 0$$

where μ_3^l and μ_4^l are the Lagrange multipliers on $0 \leq \beta_l^S \leq 1$. Using (36), the condition becomes

$$p\frac{\beta}{P_0}\left((P_1 - Y)\gamma_l^1\right) + \mu_3^l - \mu_4^l = 0 \tag{37}$$

It cannot be that $P_1 > Y$ since lenders would not want to buy any bonds at $t = 1$. When $P_1 < Y$ then $\mu_3^l > 0$ and hence $\beta_l^S = 0$. If $P_1 = Y$ then we can let $\beta_l^S = 0$ without loss of generality. To see, this set $P_1 = Y$ and $p(1+r) = 1$ (see condition (36)) into the lender's problem at $t = 1$ (equations (12) and (13)):

$$p[R\alpha + (1-\alpha-\beta) + \frac{\beta}{P_0}Y - \lambda_l c_1 - (1-\lambda_l)c_2] \tag{38}$$

where we substituted the budget constraint into the objective function using L_l. The

objective function is independent of β_l^S.
The first-order condition of a lender with respect to reinvesting into the liquid asset at $t = 1$, γ_l^1, is

$$(1 - \alpha - \beta)p(1 - \frac{Y}{P_1}) + \mu_1^l = 0$$

where we used $\beta_l^S = 0$, the lender's marginal value of liquidity $\mu^l = p^2(1 + r)$ and (36). We have ruled out that $P_1 > Y$. When $P_1 = Y$, the lender's problem is independent of γ_l^1 (see (38)) and we can set $\gamma_l^1 = 0$ without loss of generality. When $P_1 < Y$, then $\mu_1^l > 0$ and hence $\gamma_l^1 = 0$.

Proof of Lemma 6

As in the proof of Lemma 3. The extra element is the presence of γ_l^2, the amount of bonds bought by banks with a liquidity surplus. But we can use the condition on market clearing in the bond market (18) to solve for γ_l^2.

Proof of Proposition 3

Analogous to the proof of Proposition 1. Competition requires that

$$R\alpha + \gamma_l^2 \frac{Y}{P_1}(1 - \alpha - \beta) + p(1 + r)[(1 - \gamma_l^2)(1 - \alpha - \beta) - \lambda_l c_1] + \frac{\beta}{P_0} Y - (1 - \lambda_l)c_2 = 0$$

and that

$$R\alpha - (1 + r)[\lambda_h c_1 - (1 - \alpha - \beta) - \frac{\beta}{P_0} P_1] - (1 - \lambda_h)c_2 = 0$$

where we have used the results from Lemma 1, 4 and 5. The amount of bonds purchased γ_l^2 is given by market clearing in the bond market (equation (16), or after simplification, (18)). Using the result on c_1 from Lemma 6, using one condition to solve for c_2 and substituting back into the other condition gives the desired result.

Proof of Lemma 7

The first-order condition with respect to the allocation into the government bond, β, requires that:

$$p[\frac{Y}{P_0} + p(1 + r)(-\pi_l - \pi_h \frac{P_1}{P_0}) - (1 + r)\pi_h(1 - \frac{P_1}{P_0})] = 0$$

Solving the first-order condition for P_0 yields

$$P_0 = \frac{1}{p\pi_l + \pi_h}\left(\frac{Y}{1 + r} + \pi_h(1 - p)P_1\right)$$

Using (17) to substitute for $\frac{Y}{1+r}$ results in $P_0 = P_1 \frac{(1-\pi_h)p+\pi_h}{p\pi_l+\pi_h}$ which gives the desired result since $1 - \pi_h = \pi_l$.