CORRELATED LEVERAGE AND ITS RAMIFICATIONS*

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Abstract

This paper develops a theory of the relationship between the leverage ratios of banks and borrowers who take loans to purchase houses. The bank's payoff depends heavily on the value of the house that serves as collateral backing the loan. The analysis is in the context of a two-period model in which the capital structure decisions of banks and borrowers and house prices are endogenized. There are four main results. First, leverage is a "positively correlated" phenomenon in that high leverage among borrowers is positively correlated with high leverage among banks. Both borrower and bank leverage are higher when house prices are higher. Second, higher bank leverage leads to greater volatility of house prices in response to shocks to fundamental house values. Third, a bank's exposure to credit risk depends not only on its own leverage choice but also on the leverage decisions of *other* banks. Fourth, positive fundamental shocks to house prices dilute financial intermediation by reducing banks' pre-lending screening. Although the model is developed in the context of the housing market, it is applicable in any borrower-lender setting in which collateral values depend on the aggregate availability of credit, and credit risk in turn depends significantly on collateral values. Empirical and policy implications of the analysis are drawn out.

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1 Introduction

It is by now well understood that high leverage ratios of banks make the financial system more fragile and increase the likelihood of financial crises (see, for example, Allen and Gale (2008)).[1] Indeed, much of prudential capital regulation of banks is based on this fundamental premise (e.g., Bhattacharya and Thakor (1993), and Freixas and Rochet (1997)). The issues of financial leverage and bank capital have gained special prominence in light of recent events. The subprime lending crisis of 2007-09 is a striking example of the alacrity with which a high-leverage financial system can find itself beset with a crisis that further erodes capital and sets in motion forces that exacerbate the crisis.

However, our knowledge of the dynamics of financial-system leverage is rather limited. We do not know the circumstances under which banks become more highly levered, *outside* of crises periods in which exogenous shocks impose losses on banks, drain capital and cause leverage ratios to spike up. In other words, if more highly levered banks make the financial system more fragile, what causes banks to be so? A related issue that is especially noteworthy in the recent financial crisis is that consumer (borrower) leverage ratios have also increased substantially prior to the crisis (e.g., Gerardi, Lehnert, Sherland, and Willen (2008)), and this may have been a significant contributing factor to the crisis (e.g., Mian and Sufi (2009, 2010)).[2] Was this higher consumer leverage just a coincidence or was it in any way related to the leverage ratios of banks themselves? What are the consequences of this?

In this paper we address these questions by developing a theoretical model that explores the relationship between the leverage decisions of borrowers and banks, in the context of the home mortgage market. We consider a two-period overlapping-generations economy in which first-period homebuyers with limited wealth endowments need bank loans to finance house purchases. Borrow-

[1] There is a vast literature on financial crises that we will not review here. See, for example, Allen and Carletti (2006, 2008), Allen and Gale (1998, 2000a, 2000b), and Boyd, Kwak, and Smith (2005). On the role of bank capital during crises, Berger and Bouwman (2010) document that higher capital allows banks to capture greater market share during crises.

[2] Mian and Sufi (2009, 2010) have documented a substantial increase in borrower leverage during 2002-06 that was correlated with the increase in house prices. Homeowners extracted 25-30% of the increase in home equity values to increase consumption. They document that the increased borrower leverage during 2002-06 significantly contributed to the higher borrower defaults during 2006-08.

ers' leverage decisions are driven by first-period house prices that dictate the amounts they need to borrow. Higher house prices necessitate larger bank loans and thus higher borrower leverage for borrowers with fixed initial wealth endowments. Since house prices in the first period depend on *expected* house prices in the second period, banks (correctly) interpret high first-period house prices as implying a relatively low likelihood of low second-period house prices. This, in turn, lowers their assessment of the probability of default on loans because borrowers repay with proceeds from the sale of their houses to second-period homebuyers. Banks thus keep lower capital in the first period when first-period house prices are higher. This phenomenon, whereby the leverage ratios of borrowers and banks move in unison, is what we call "correlated leverage," and is our first main result.

An essential element of the analysis is that in addition to the bank's first-period capital structure, house prices and the borrower's capital structures in both periods are also endogenously determined. This introduces an interesting source of fragility for the credit market. More highly-levered banks find it costlier to raise fund in the second period conditional on default on first-period deposits. This reduces the aggregate supply of credit for homebuyers in the second period and leads to a more volatile equilibrium house price in the second period. Thus, higher first-period house prices make the banking system and the housing market more vulnerable to negative shocks in the future – shocks that precipitate housing price declines. Note, however, there is nothing within the model to suggest that high leverage ratios for banks and borrowers generate any social inefficiency. This is because there are no social externalities related to house price levels or defaults by banks or borrowers in the model.

The endogeneity of house prices allows us to examine *bank leverage price effects*. Our second main result is that an increase in first-period bank leverage leads to greater second-period house price volatility. There is thus a transmission from the capital structure decisions of banks in the financial sector to house prices in the real sector.

Our third main result is that each bank's first-period credit-risk exposure is increasing in the equilibrium first-period leverage choices of *other* banks. That is, bank leverage generates a form of *interconnectedness* among otherwise-independent banks.

Fourth, we extend the model to show that an increase in house prices leads to "intermediation thinning," whereby banks invest less in screening borrowers when first-period house prices are higher. This, in contrast to our second main result, highlights a *reverse* transmission mechanism, whereby fundamental shocks to house prices in the real sector reverberate in the nature of financial intermediation via the screening decisions of banks.

In addition to these main results, the analysis allows us to examine a host of other issues, such as the manner in which house price *cycles* can emerge from the capital structure adjustments of banks to initial exogenous-shock-driven changes in house prices, how exogenous shocks to bank capital – say due to losses on unrelated investments – can affect house price dynamics, and how ex ante capital requirements compare with ex post capital infusions by the government when banks are hit with a negative shock.

The paper proceeds as follows. Section 2 reviews the related literature. Section 3 develops the model. Section 4 contains the preliminaries for the analysis, including definition of the equilibrium. Section 5 contains the main analysis. Section 6 considers extensions. It explores how house prices induce intermediation thinning, the manner in which bank leverage choices respond to house prices and how this in turn contributes to house price cycles, the impact of exogenous bank capital shocks on house price dynamics, and the regulatory choice between ex ante capital requirements and ex post capital infusions. Section 7 concludes with a discussion of the empirical implications. All proofs are in the Appendix.

2 Related Literature

The purpose of this section is to briefly review various strands of the literature that are related to our work. There is a vast literature on bank capital that we will touch upon only briefly here because its relationship to our work is mostly tangential.[3] For example, there is a significant theoretical literature on bank capital requirements and their effects (e.g., Furlong and Keeley (1990), Hellmann, Murdock, and Stiglitz (2000), Kim and Santomero (1988), Koehn and Santomero (1980), and Thakor (1996)). Empirically, some of the predictions of these models have been tested and the effects of changes to the capital requirements regime on banks' portfolios have been examined (e.g., Bernanke and Lown (1991), and Thakor (1996)). Hancock, Laing, and Wilcox (1995) study

[3]There are also numerous papers on the capital structure decisions of banks (e.g., Inderst and Mueller (2008), and Mehran and Thakor (forthcoming)). However, these too are not directly related to our paper because they do not examine the interaction between banks' and borrowers' leverage dynamics.

the dynamic response to shocks in the capital of U.S. banks and show that these banks adjust their capital ratios faster than they adjust their loan portfolios. See Rochet (2008) for a review. What distinguishes our paper from this literature is that our focus is not on the determination of regulatory capital requirements. What we are interested in is how bank leverage ratios respond to changes in borrowers' leverage ratios when loans are secured by collateral whose future value is dependent on aggregate bank credit supply.

This paper is also related to the literature on real estate and household finance. Stein (1995) theoretically examines how down payment restriction affects both the house prices and the trading volume in the real estate market. Lamont and Stein (1999) provide empirical evidence for the Stein (1995) model and find that house prices in U.S. cities where more homeowners take loans with higher loan-to-value ratios are more sensitive to city-specific shocks. Ortalo-Magné and Rady (2006) extend Stein's (1995) analysis to a life-cycle model of the housing market and demonstrate a correlation between house prices and the incomes of young households that are eager to climb a property ladder but are credit-constrained. Kiyotaki and Moore (1997) develop a dynamic model in which lenders cannot force borrowers to repay unless debt is secured. Consequently, durable assets serve not only as factors of production but also as collateral for loans. The dynamic interaction between credit limits and asset prices leads to temporary shocks to technology or income becoming persistent shocks to asset prices. Bernanke, Gertler, and Gilchrist (1996) provide evidence that at the onset of a recession borrowers facing high agency costs receive a relatively lower share of the credit extended and hence account for a proportionally greater part of the decline in economic activity. Chen (2001) develops a dynamic general equilibrium model which explains why banking crises so often coincide with depressed prices in asset markets. Fostel and Geanakoplos (2008) study how leverage cycles can cause contagion, flight to collateral and issuance rationing in a so-called "anxious economy."[4] Elul (2008) shows how a drop in the value of the underlying collateral in secured borrowing may help stabilize aggregate fluctuations in the housing market.[5] None of them, however, studies the relationship between borrower leverage and bank leverage or bank leverage price effects as we do.

[4] There are also papers that examine frictions and the consequent amplification of shocks. See Cooley, Marimon, and Quadrini (2004), and Kocherlakota (2000).

[5] There are also numerous papers on the Swedish banking crisis that discuss crisis solutions that have some similarity to those we discuss. See, for example, Went (2009).

The following papers are more closely related to ours. In Shleifer and Vishny (1992), collateral value depends on other industry peers' ability to buy the asset.[6] The similarity is that in our model the future house price depends on a future borrower's ability to purchase the house. However, our focus differs substantially in that we link the future borrower's ability to purchase the house to the bank's future ability to lend, and examine how this, in turn depends on the capital structure decisions of banks in the current period. Another related paper is Holmstrom and Tirole (1997) in which the capital of the bank interacts with the capital of the borrower. Moral hazard prevents low-capital borrowers from being able to raise unmonitored finance. Banks can provide monitoring and hence not only extend credit to these borrowers but also enhance their ability to obtain credit from elsewhere. However, banks need to have sufficient capital of their own to have incentives to monitor. Thus, access to credit may depend on capital in both banks and borrowers. However, unlike our paper, Holmstrom and Tirole (1997) take the capital levels of the bank and the borrower as exogenous and do not address why their leverage ratios may be correlated. Moreover, they also do not address the impact of these leverage ratios on the housing market. That is, the focus of their paper is different from ours.

Acharya and Viswanathan (forthcoming) develop a model in which more highly-levered financial institutions (e.g., broker-dealers) are more likely to face difficulty in rolling over their short-term debt contingent upon adverse shocks, and hence delever by selling assets to other lower-leverage firms in the sector when hit with an adverse shock. This delevering leads to greater asset price deterioration, and sometimes drying-up of liquidity. We examine a different set of issues in the context of the housing market in that, unlike Acharya and Viswanathan (forthcoming), we focus on the correlation between borrower and bank leverage and endogenize *both* equity and debt in characterizing bank leverage.

Also related is a paper by Farhi and Tirole (2009) which shows that the private leverage choices of banks may exhibit strategic complementarities through reaction to monetary policy. By contrast, in our model a bank's leverage choice affects other banks through its impact on the underlying

[6] Benmelech and Bergman's (2009) evidence from bankruptcies in the airline industry is consistent with this. See also the literature on "cash-in-the-market" pricing (e.g., Acharya and Yorulmazer (2008a), Allen and Carletti (2008), and Allen and Gale (2005)). Rampini and Viswanathan (2009) develop a model in which a firm's leverage is determined by the tangible assets it can use as collateral for borrowing, similar to the role of houses as collateral in our model.

value of "common collateral" (i.e., houses).[7] In Tsomocos, Bhattacharya, Goodhart, and Sunirand (2007), under-diversified banks may become interconnected through the interbank market. In our model, it is the fact that all banks' loans are backed by collateral from the same asset class (i.e., houses) that engenders interconnectedness among otherwise-independent banks.

3 The Model

3.1 The agents and economic environment

Consider a three-date ($t = 1, 2,$ and 3) economy with universal risk neutrality.[8] There are two goods in the economy, money and houses, where money is the numeraire good and can be stored costlessly over time, and houses are indivisible. There is a continuum of atomistic and identical houses available in the market at $t = 1$ with Lebesgue measure of S. There are no new houses built after $t = 1$. We call the period between $t = 1$ and $t = 2$ the first period, and the period between $t = 2$ and $t = 3$ the second period. We normalize discount rates between dates to zero.

There is also a continuum of atomistic consumers in each period. Consumers within a given period are identical, but they may differ across periods. A consumer in period $i \in \{1, 2\}$ is born at $t = i$ without a house but with a monetary endowment $M_i > 0$, and earns an income X_i at $t = i + 1$. She maximizes her expected utility at $t = i$ given by:

$$U_i = h_i B_i + C_i + \mathbb{E}(C_{i+1}), \tag{1}$$

where h_i is an indicator variable that equals 1 if the consumer owns a house in period i and zero otherwise, $B_i > 0$ is the consumer's utility from home ownership in period i, and C_i and C_{i+1} are, respectively, the consumer's monetary consumptions at $t = i$ and $t = i + 1$. $\mathbb{E}(\cdot)$ is the expectation operator.

The measure of consumers in each period, S_c, exceeds the total housing supply in that period, i.e., $S_c > S$. Each consumer born at date t takes the house price at date t as given and decides whether to buy a house or not. First-period consumers who buy houses at $t = 1$ sell their houses to some second-period consumers at $t = 2$, who in turn sell theirs to some (unmodeled) third-period

[7]With respect to our result on bank interconnectedness, Acharya and Yorulmazer (2008b) show that banks may undertake correlated investments and minimize the impact of information contagion on the expected cost of borrowing.

[8]We add an additional date $t = 0$ in Section 5.3 to analyze bank entry to the industry.

consumers at $t = 3$, and so on.[9] The house price at $t = 3$, $P_3 \geq 0$, is exogenously given but random with probability density function f when viewed at $t = 1$ and 2. At $t = 1$, a (first-period) consumer's utility from home ownership in the first period, B_1, is known to everyone, whereas a (second-period) consumer's utility from home ownership in the second period, B_2, is a random variable that realizes its value at $t = 2$. B_2 takes a high value, B_{2h}, with probability θ, and a low value, B_{2l}, with probability $1 - \theta$, where $B_{2h} > B_{2l} > 0$. This is the only fundamental uncertainty in the housing market. Note that in any given period i, *all* consumers attach the *same* value, B_i, to home ownership in that period. The house prices at $t = 1$ and 2, P_1 and P_2, are endogenously determined by competition among the first-period and second-period consumers for buying the fixed housing supply, S, at $t = 1$ and 2, respectively.[10] Buying and selling houses involve no transaction costs. It is clear that in the absence of wealth and credit constraints, the house price at $t = 2$ will be $P_2 = B_2 + \mathbb{E}(P_3)$, and the house price at $t = 1$ will be $P_1 = B_1 + \mathbb{E}(P_2) = B_1 + \mathbb{E}(B_2 + P_3)$.

However, consumers are wealth constrained, and their monetary endowments are not large enough to finance home purchases at those prices. Specifically,

Assumption 1. *Consumers' wealth endowments at dates $t = 1$ (M_1) and $t = 2$ (M_2) are not large enough to completely finance house purchases in the absence of wealth and credit constraints. That is, $M_1 \in (0, B_1 + \mathbb{E}(B_2 + P_3))$ and $M_2 \in (0, B_2 + \mathbb{E}(P_3))$.*

Consumers can borrow from banks by taking mortgage loans, but cannot directly borrow and lend money to each other or get funding from any other source.[11] There is a continuum of atomistic and ex ante identical banks with a measure of S/N, where N is a positive constant. In period i, each bank takes the size of loans demanded by the consumers (L_i) and the interest rate on loans (R_i) as given, and chooses the number of loans to extend. Bank j extends n_{ij} loans in period i. All

[9] The initial housing stock is owned by a generation of consumers that we do not explicitly analyze in the model. These consumers exit the model at $t = 1$ with either a capital gain or a capital loss, depending on the house price at $t = 1$. Alternatively, we could think of the initial housing stock as having been created by builders who financed the construction with their own equity and who exit the model at $t = 1$ with either a gain or loss on their investment based on the housing price at $t = 1$. In either case, the payoffs of house sellers at $t = 1$ are irrelevant to our analysis.

[10] We assume that housing supply is inelastic and focus on the interaction between housing demand and credit availability. The assumption facilitates the result that housing prices fall as credit constraints worsen. This result also obtains in Stein (1995). Vigdor (2006) shows that this result holds even when housing supply is elastic.

[11] This assumption can be justified on the basis of the specialization of banks as information processors (e.g., Allen (1990), and Ramakrishnan and Thakor (1984)).

loans are for one period: a loan extended at date t ($t = 1$ or 2) must be repaid at date $t+1$. The population of banks across the two periods does not change, so new banks do not enter at $t = 2$.

The consumer can choose whether or not to repay her loan. To minimize the risk of default as well as the loss given default, banks require that each loan be secured by the house purchased using that loan. If a consumer does not repay her loan in full, the bank can seize her house (i.e., foreclose) without any cost and sell it through a foreclosure auction at the prevailing market price of the house.[12] The bank has no legal claim on the borrower's other assets or income, and similarly the borrower has no legal claim on the bank's proceeds from the sale of the foreclosed house.

We now make an assumption about the relationship between the borrower's first-period endowment (M_1) and the first-period utility from home ownership (B_1).

Assumption 2. *The difference between the first-period consumer's monetary endowment and utility from home ownership, $M_1 - B_1$, is strictly positive but not too high. That is, $M_1 - B_1 \in (0, \bar{u})$, where $\bar{u}$ is an upper bound.*

If $M_1 - B_1 \leq 0$, then all first-period consumers *strictly* prefer to purchase a house regardless of the loan terms, and the housing market will never clear. Assuming $M_1 > B_1$ eliminates this possibility. To understand the role played by the upper bound, $\bar{u}$, on $M_1 - B_1$, note that the size and risk of bank loans are determined by the consumers' monetary endowment (the explicit expression for $\bar{u}$ is in the Appendix). If the first-period consumers have sufficiently high monetary endowments, bank loans are relatively small and therefore riskless. The capital structure decision of a bank is trivial in this case. The bank relies only on deposits because it incurs no cost with riskless deposits, whereas equity is costly. We are interested in the more realistic situation in which there is a positive probability that the bank defaults on its deposit obligations and incurs the associated default cost (which arises from an increase in its cost of lending in the second period; see Section 3.2). It is this cost that the bank trades off against the cost of equity in determining its first-period capital structure.

[12]In reality, banks will incur some foreclosure cost and may only get a fraction of the market price of the house in a foreclosure auction. Adding these details into the model does not qualitatively change our results.

3.2 Bank capital structure

Banks extend loans using the funds they raise through equity capital and deposits. Each bank independently chooses the amount of equity and deposits on its balance sheet. We endogenously determine banks' capital structure choices in the first period, but take their optimal capital structure choices as given in the second period.[13] Let E_j and D_j be the amount of equity and deposits per loan, respectively, raised by bank j in the first period. In the first period, deposits are in elastic supply and fully insured, so the deposit interest rate is zero. Raising equity capital is costly. This cost may arise due to capital market frictions of various sorts such as asymmetric information (e.g., Myers and Majluf (1984)) and the transaction costs of raising equity.[14] Specifically, bank j's cost of equity capital in the first period is $\Lambda(n_{1j}E_j)$, an increasing and convex function with $\Lambda(0) = \Lambda'(0) = 0$. Since capital is costly and banks do not have any other investment opportunities, no bank will raise more funds than needed to finance its loan. That is,

$$E_j + D_j = L_1 \ \forall j. \tag{2}$$

A bank defaults when it cannot repay its depositors in full. In this case, deposit insurance covers the shortfall, which allows the bank to continue to operate in the next period. The deposit insurance is fairly priced ($1 for each $1 of expected loss). However, default increases the cost of lending for banks in the subsequent period. Specifically, if bank j does not default in the first period, its faces a cost of $Q(K)$ to raise an amount K of funds (equity and deposits) in the second period under a profit-maximizing capital structure (which we take as given and do not model explicitly),[15]

[13] Our main result on correlated leverage focuses on the first-period bank leverage. Explicitly considering banks' capital structure choices in the second period substantially complicates the mathematical analysis but does not offer additional insights. In a previous version of the paper, we had endogenously solved for the second-period capital structure choices of banks, and encountered results qualitatively similar to those reported here. However, note that taking the bank's second-period capital structure as given does not require it to be the same as the first-period capital structure.

[14] The assumption that bank equity capital is costly is fairly standard. See, for example, Allen, Carletti, and Marquez (forthcoming), and Mehran and Thakor (forthcoming). However, as Mehran and Thakor (forthcoming) show, capital may still have *net* benefits.

[15] The assumption that the cost of lending is independent of the fundamental shock in the housing market (i.e., the realization of B_2) is for simplicity. In reality, a bank's second-period optimal capital structure would be contingent on the realization of B_2, and so is its cost of lending as a result. Our analysis can accommodate this under an alternative specification with the lending cost being $Q_h(K)$ when $B_2 = B_{2h}$, and $Q_l(K)$ when $B_2 = B_{2l}$, where $Q_h(K) \neq Q_l(K)$.

where $Q(K)$ is an increasing and convex function with $Q(0) = Q'(0) = 0$. However, this cost is $Q(K) + \alpha n_{1j}(D_j - P_2)^+ K$ if the bank defaults in the first period, where α is a positive constant and z^+ denotes the maximum of z and 0. Note that $n_{1j}(D_j - P_2)^+$ is bank j's deposit repayment shortfall in the first period. Thus, the bank's second-period *marginal* cost of lending increases by $\alpha n_{1j}(D_j - P_2)^+$ if it defaults in the first period. The rationale for this is that banking defaults make it more difficult for banks to raise equity capital to replace what is wiped out by these losses, so capital raising becomes more costly.[16] Although there is deposit insurance, the bank may also face higher marketing costs to convince depositors to return to a bank in which depositors' claims had to be settled by the deposit insurer.[17] That is, we assume:

Assumption 3. *A bank's default in the first period increases its marginal cost of lending in the second period, but not by too much. That is,* $\alpha \in (0, \alpha_{max})$.

A positive α means that a bank's second-period marginal cost of lending increases when it defaults in the first-period. The upper bound on α, called α_{max}, means that the increase in a bank's second-period lending cost following its first-period default is not too high (see Appendix for the expression for α_{max}). The main reason for making this assumption is to preclude a situation in which the bank's financing friction (a higher second-period marginal cost of funding contingent upon first-period default) by itself becomes the dominant factor in the determination of the second-period house price.[18]

[16] See Acharya, Shin, and Yorulmazer (2009) for an analysis of why banks that have their capital wiped out by losses are unable to raise funds to replace this capital. Others have pointed out that the reduction in bank capital can lead to a decline in lending. Bernanke (1983) shows that financial market disruptions in the U.S. during 1930-33 reduced the effectiveness of the financial sector and increased the cost of credit intermediation. Watanabe (2007) shows that Japanese banks cut back lending after incurring large losses in 1990s. Anari, Kolari, and Mason (2005) and Ashcraft (2006) show that the process of resolving failed banks reduces liquidity and bank lending.

[17] Formally, this can be modeled by introducing an "inconvenience cost" for depositors because they have to wait to be paid by the deposit insurer rather than being able to withdraw their deposits on demand. Introducing such a cost does not affect the analysis.

[18] In addition, if the increase in the second-period lending cost following first-period default exceeds the amount of first-period deposit shortfall, then banks may have a perverse incentive to manipulate the house price by "forgiving" the borrower's indebtedness. Moreover, if banks can not coordinate their actions, multiple equilibria can arise. In one equilibrium, everyone expects house price to be high so banks do not default and their consequent low cost of lending in the second period sustains such high price. In another equilibrium, expectation of low house price causes

Each bank maximizes its expected second-period profit at $t = 2$ by choosing the number of loans to extend (note that banks are assumed to operate under an optimal capital structure in the second period which we do not model). At $t = 1$, each bank chooses the number of loans and the first-period capital structure to maximize the sum of its expected profits in both periods, taking into account the effect of default in the first period on its cost of lending in the second period. In each period, banks take the size of a loan and loan interest rate as given (which are determined under a competitive equilibrium as explained in Section 4.6), since each bank is atomistic.

The sequence of events is summarized in *Figure 1*.

[Figure 1 goes here]

4 The Analysis: Some Preliminaries

This section presents essential preliminaries for an analysis of the model. We begin by providing an example that illustrates the key intuition of the result on correlated leverage. Next, we examine trading in the housing market, describe the bank's expected profits in the two periods, analyze the determination of the bank's first-period capital structure, and analyze equilibrium in the loan market in terms of the number of loans the bank chooses to make. We subsequently define the overall equilibrium.

4.1 An example showing that higher house prices lead to higher consumer leverage and bank leverage

We now provide a simple example to illustrate the equilibrium as well as to serve as a preamble to the core idea of the paper, correlated leverage, which we will analyze more formally in the following subsections. In this example, second-period house prices are perfectly correlated with the second-

banks to default in the first period and their second-period cost of lending to increase, eventually leading to low house price and bank default in the first period. This self-fulfilling bank default may occur even when the housing market receives a positive shock at $t = 2$ (i.e., $B_2 = B_{2h}$) if α is sufficiently large. In this scenario, government policies may be able to credibly alter beliefs and lead to a socially desirable equilibrium. Such considerations may have motivated US government programs such as liquidity assistance to banks and direct support for mortgage markets. We thank Matt Pritsker for suggesting these issues. Imposing an upper bound on α eliminates this possibility.

period value of home ownership, B_2.[19] Hence, the second-period house price P_2 is a binary random variable: with probability θ it is $P_{2h} > 0$ which is assumed to be sufficiently large to cover the contractually stipulated loan repayment, and with probability $1-\theta$ it is P_{2l} which is normalized to zero. We assume: (i) the cost of equity for a bank with equity capital E in the first period is $\Lambda(E) = \lambda E^2/2$, where $\lambda > 0$ is a constant, (ii) each dollar of deposit repayment shortfall imposes a deadweight cost of $\delta > 0$ dollars on the bank, and (iii) the measure of banks equals the measure of houses (i.e., $N = 1$). We normalize the first-period consumers' income to zero (i.e., $X_1 = 0$).

Each consumer's expected utility at $t = 1$ is given by:

$$U_1 = \begin{cases} B_1 + \theta[P_{2h} - R_1 L_1] & \text{buy a house,} \\ M_1 & \text{does not buy a house.} \end{cases} \tag{3}$$

Here B_1 is the value attached to home ownership by first-period consumers, R_1 is the gross interest rate charged by the bank, L_1 is the size of the loan, and M_1 is the consumer's monetary endowment.

In equilibrium, demand must equal supply in the housing market, i.e., the first-period consumers must be indifferent between buying a house and not buying one:

$$M_1 = B_1 + \theta[P_{2h} - R_1 L_1]. \tag{4}$$

We now analyze how banks choose capital structure. Consider bank j. It extends n_{1j} loans at $t = 1$, each of size L_1. It chooses equity per loan, E_j, and deposits per loan, D_j, to maximize the expected net payoff to shareholders at $t = 1$:[20]

$$n_{1j}\{\theta R_1 L_1 - L_1 - \delta[1-\theta]D_j\} - \frac{\lambda[n_{1j}E_j]^2}{2}. \tag{5}$$

Thus, the first-order-condition for a profit-maximizing capital structure for the bank is:

$$\lambda n_{1j} E_j = \delta[1-\theta] \ \forall j. \tag{6}$$

[19] In the more general model, P_2 will be affected by B_2 as well as the cost of credit for second-period homebuyers, which will be affected by first-period loan losses suffered by banks. These complications in the determination of the second-period home price are absent in this example.

[20] To understand (5), note that $\theta R_1 L_1 - L_1 - \delta[1-\theta]D_j$ is the bank's expected profit per loan, which equals the expected loan repayment, $\theta R_1 L_1$, net of the loan extended, L_1, and the expected deadweight cost of default, $\delta[1-\theta]D_j$; $\lambda[n_{1j}E_j]^2/2$ is the cost of equity capital.

Each bank chooses the number of loans it extends (n_{1j}) to maximize its expected profit (5) in that period. Bank j's first-order-condition for a profit-maximizing choice of n_{1j} is:

$$[\theta R_1 - 1]L_1 = \delta[1-\theta][L_1 - E_j] + \lambda n_{1j} E_j^2 \ \forall j, \tag{7}$$

where E_j satisfies (6). Since the measure of banks equals the measure of houses, each bank extends one loan in a symmetric equilibrium:

$$n_{1j} = 1 \ \forall j. \tag{8}$$

From equations (4), (6), (7) and (8), we can solve for the equilibrium:

$$L_1 = P_1 - M_1 = \frac{\theta P_{2h} + B_1 - M_1}{1 + \delta[1-\theta]}, \tag{9}$$

$$E_j = \frac{\delta[1-\theta]}{\lambda} \ \forall j, \tag{10}$$

$$R_1 = \frac{1 + \delta[1-\theta]}{\theta}. \tag{11}$$

Each consumer's leverage is:

$$\frac{P_1 - M_1}{M_1} = \frac{\theta P_{2h} + B_1 - M_1}{M_1\{1 + \delta[1-\theta]\}}, \tag{12}$$

and each bank's leverage is:[21]

$$\frac{D_j}{L_1} = 1 - \frac{\delta[1-\theta]\{1 + \delta[1-\theta]\}}{\lambda[\theta P_{2h} + B_1 - M_1]} \ \forall j. \tag{13}$$

It is clear from (12) and (13) that both consumer leverage and bank leverage are increasing in θ, the probability of a higher future house price. Thus, bank leverage is high precisely when consumer leverage is high. This is the *correlated leverage* result that we pursue in this paper. A greater probability of a higher future house price makes home ownership more attractive, causing an increase in current house price that requires homebuyers to borrow more. At the same time, banks increase leverage because they perceive a lower probability $(1-\theta)$ of loan default.

The bank leverage result does *not* rely on any specific assumptions about the nature of financing costs, i.e., the convex cost of equity and the linear cost of debt. To see this, suppose the cost of debt is also quadratic so that bank j incurs a cost of $\delta D_j^2/2$ when it defaults on its debt D_j. Since

[21] Note that bank leverage is defined as debt over total assets, whereas consumer leverage is defined as debt over equity. These are essentially equivalent definitions from the standpoint of our model.

the probability of default is $1-\theta$, the bank's expected cost of debt is $\delta[1-\theta]D_j^2/2$. By equating the marginal costs of equity and debt, the bank chooses a debt-equity ratio of $\lambda/\{\delta[1-\theta]\}$, which is increasing in θ (and independent of the loan size). In this example, we assume a linear cost of debt to simplify, and a convex cost of equity to obtain a unique profit-maximizing bank capital structure. In what follows, we analyze correlated leverage more formally. In particular, we endogenize the cost of default to the bank through its adverse impact on the bank's subsequent lending. This analysis also illuminates another key issue that we examine in this paper: *the effect of bank leverage on house prices.*

4.2 Housing market and the determination of equilibrium house prices

Our goal in this subsection is to examine how the equilibrium house prices are determined in the first and the second periods. House prices are determined by competition among consumers to buy a limited supply of houses in period i:

$$\textit{House Supply}_i = S. \tag{14}$$

A consumer who purchases a house at $t = i$ uses her endowment M_i as down payment and borrows the following loan amount from a bank:[22]

$$L_i = P_i - M_i. \tag{15}$$

Let $R_i > 1$ be the gross interest rate charged by the bank (which will be endogenously determined later) in period i. This rate is independent of the identity of the consumer or the bank because of our assumption of identical consumers and identical banks. The expected utility of a consumer from buying a house at $t = i$ consists of the utility from home ownership in period i (B_i), her income at $t = i+1$ (X_i), and the expected gain from house price appreciation in period i after paying off the bank loan (which is the maximum of zero and the excess of the house price at $t = i+1$, P_{i+1}, over the repayment to the bank, R_iL_i):

$$U_i^b = B_i + X_i + \mathbb{E}((P_{i+1} - R_iL_i)^+). \tag{16}$$

[22]Note that the loan size, L_i, cannot be negative in equilibrium because Assumption 1 rules out $P_i \leq M_i$.

The expected utility of a consumer who does not buy a house equals her endowment (M_i) plus her income (X_i):

$$U_i^{nb} = M_i + X_i. \tag{17}$$

The demand of houses at $t = i$ is:

$$House\ Demand_i \begin{cases} = 0 & \text{if } U_i^b < U_i^{nb}, \\ \in [0, S_c] & \text{if } U_i^b = U_i^{nb}, \\ = S_c & \text{if } U_i^b > U_i^{nb}. \end{cases} \tag{18}$$

In equilibrium, we need to have $House\ Demand_i = House\ Supply_i$, so consumers are indifferent between purchasing a house and not purchasing one:

$$M_i = B_i + \mathbb{E}((P_{i+1} - R_i L_i)^+). \tag{19}$$

This equation is the consumer's indifference condition that determines the equilibrium house price P_i taking the interest rate R_i as given. Note that the left-hand-side (LHS) of (19) is the monetary consumption that the consumer gives up in buying a house, and its right-hand-side (RHS) consists of the utility from home ownership in period i and the expected gain from house price appreciation in period i after paying off the bank loan. The competition in the housing market uniquely determines the loan repayment amount $R_i L_i$, but not the house price $P_i = L_i + M_i$, which also depends on the interest rate R_i charged by the banks. An increase in R_i dampens the demand for houses, so housing market equilibrium is restored with a lower P_i.

4.3 The bank's expected profits

In this subsection, we describe the bank's expected profits in the first and second periods, as a prelude to examining the bank's optimal capital structure in the first period.

4.3.1 The first-period expected profit

Bank j's first-period realized profit (depending on the realization of P_2) is given by:[23]

$$\pi_{1j} = n_{1j}[(\min(P_2, R_1L_1) - D_j)^+ - \mathbb{E}((D_j - \min(P_2, R_1L_1))^+) - E_j] - \Lambda(n_{1j}E_j). \tag{20}$$

There are two terms in π_{1j}. The first term equals the number of loans the bank chooses to make, n_{1j}, multiplied by the repayment from the consumer per loan, $\min(P_2, R_1L_1)$, net of the deposit payment, D_j, the cost of deposit insurance, $\mathbb{E}((D_j - \min(P_2, R_1L_1))^+)$, and the equity capital raised, E_j. Note that deposit insurance makes the depositors' claim riskless, and since the riskless rate is zero, the bank's promised repayment to depositors is equal to the amount of deposits raised (D_j). The consumer's repayment on each loan equals $\min(P_2, R_1L_1)$ because the consumer, when facing a choice of paying her loan obligation (R_1L_1) or forfeiting her house (worth P_2), chooses the option that costs her the least.[24] The second term in π_{1j} is the deduction for the cost of equity capital ($\Lambda(n_{1j}E_j)$). Taking the expected value of π_{1j} and simplifying, we get the bank's first-period expected profit:[25]

$$\mathbb{E}(\pi_{1j}) = n_{1j}[\mathbb{E}(\min(P_2, R_1L_1)) - L_1] - \Lambda(n_{1j}E_j). \tag{21}$$

4.3.2 The second-period expected profit

The second-period outcomes are contingent on the realization of B_2. We introduce subscript $k \in \{h, l\}$ to denote the state in which $B_2 = B_{2k}$. When $B_2 = B_{2k}$, L_{2k}, R_{2k}, n_{2jk}, and $\mathbb{E}(\pi_{2jk})$ are the second-period loan size, loan interest rate, the number of loans extended by bank j, and bank j's expected second-period profit, respectively. We can write $\mathbb{E}(\pi_{2jk})$ as:

$$\mathbb{E}(\pi_{2jk}) = n_{2jk}\mathbb{E}(\min(P_3, R_{2k}L_{2k})) - n_{2jk}L_{2k} - Q(n_{2jk}L_{2k}) - \alpha n_{1j}(D_j - P_{2k})^+ n_{2jk}L_{2k}. \tag{22}$$

The expected profit equals the repayment from borrowers, $n_{2jk}\mathbb{E}(\min(P_3, R_{2k}L_{2k}))$, net of the funds raised, $n_{2jk}L_{2k}$, and the cost of funds. The cost of funds includes $Q(n_{2jk}L_{2k})$ plus the increase in

[23] The size of each loan is determined by the amount of money that a homebuyer needs to borrow and is not under a bank's control. A bank can, however, determine the amount it lends by choosing the number of loans (which can be any positive, possibly non-integral number). If a borrower's loan is financed by multiple banks, a bank's fraction of the loan repayment equals the fraction of the borrower loan that the bank provided.

[24] A consumer who chooses to repay the loan may sell the house to do so.

[25] The simplification uses the following facts: $z^+ - (-z)^+ = z$ with $z = \min(P_2, R_1L_1) - D_j$, $D_j + E_j = L_1$, and $R_1L_1 \geq D_j$.

the lending cost due to the first-period default, given by a fraction $\alpha n_{1j}(D_{1j} - P_{2k})^+$ of the second-period loan amount $(n_{2jk}L_{2k})$. Viewed at $t = 1$, the bank's expected second-period profit is:

$$\mathbb{E}(\pi_{2j}) = \theta\mathbb{E}(\pi_{2jh}) + [1 - \theta]\mathbb{E}(\pi_{2jl}). \tag{23}$$

4.4 Bank capital structure in the first period

As indicated earlier, we will endogenously solve for the bank's first-period capital structure, while taking its second-period optimal capital structure as given (note that the second-period capital structure is reflected in the cost function $Q(\cdot)$; see footnote 14). The bank's capital structure choice in the first period affects its cost of lending in the second period, so the bank takes second-period cash flows into account when choosing its first-period capital structure at $t = 1$. The first-order-condition for a capital structure that maximizes the bank's total expected profits in the two periods, $\mathbb{E}(\pi_{1j}) + \mathbb{E}(\pi_{2j})$, is:[26]

$$n_{1j}\Lambda'(n_{1j}E_j) + \frac{\mathrm{d}\mathbb{E}(\pi_{2j})}{\mathrm{d}D_j} = 0. \tag{24}$$

If the bank's first-period capital structure does not impact its second-period payoffs ($\alpha = 0$), then the bank finances its first-period loan entirely with deposits to avoid raising costly equity capital. However, since $\alpha > 0$ by Assumption 3, the bank's default on its first-period deposit obligations increases its second-period cost of lending and this represents an indirect cost of using excessive deposits. The optimal capital structure trades off the direct cost of equity against the indirect cost of deposits.

Note that $\mathrm{d}\mathbb{E}(\pi_{2j})/\mathrm{d}D_j = \partial\mathbb{E}(\pi_{2j})/\partial D_j$, because each bank is atomistic and its capital structure choice in the first period will not affect P_2 or L_2, but may impact the number of loans it extends in the second period. However, bank's second-period choices maximize its expected profit in that period, so the Envelope Theorem applies. From (23), we have $\partial\mathbb{E}(\pi_{2j})/\partial D_j =$

[26]Since P_2 has a discrete probability distribution, the first-order-condition may not hold if $D_j = P_{2l}$ or $D_j = P_{2h}$. The first possibility is ruled out by Assumption 2 and the second possibility requires that the loan be risky and yet completely financed by deposits, which is not optimal (see (25)).

$-\alpha n_{1j}\mathbb{E}(n_{2j}L_2\mathbf{1}_{\{P_2<D_j\}})$, where $\mathbf{1}_A$ equals 1 if A is true and 0 otherwise. Substituting it into (24), we get:[27]

$$\Lambda'(n_{1j}E_j) = \alpha\mathbb{E}(n_{2j}L_2\mathbf{1}_{\{P_2<D_j\}}). \tag{25}$$

Note that in choosing its optimal capital structure, each bank takes the terms of the loan and the house price distribution as given.

4.5 Loan market

We now solve for equilibrium in the loan market, which allows us to solve for n_{ij}, the number of loans made in equilibrium by bank j in period i. The aggregate demand for loans is the measure of consumers who purchase houses:

$$\textit{Loan Demand}_i = \textit{House Demand}_i. \tag{26}$$

Consumers' demand for houses depends both on the loan interest rate and the house price. When the housing market is in equilibrium, competition among consumers ensures that the equilibrium house price adjusts so that the measure of consumers who purchase houses equals the supply of houses S and the housing market clears. Thus, the aggregate demand for bank loans equals the measure of houses when the housing market is in equilibrium. The aggregate supply of loans in period i is given by:

$$\textit{Loan Supply}_i = \int_{S/N} n_{ij}dj. \tag{27}$$

Bank j's first-order-condition for a choice of n_{2j}, the number of loans it extends in the second-period, that maximizes its expected second-period profit (23), is:

$$\mathbb{E}(\min(P_3, R_2L_2)) - L_2 = L_2[Q'(n_{2j}L_2) + \alpha n_{1j}(D_j - P_2)^+]. \tag{28}$$

The left-hand-side (LHS) of the equation is the expected repayment by the borrower on the marginal loan net of the loan amount, and the right-hand-side (RHS) is the marginal cost of lending for the bank. The convex cost Q results in a marginal cost of lending for the bank that is increasing in the number of second-period loans, n_{2j}. An increase in the second-period loan interest rate R_2 or a

[27] Our assumptions on $\Lambda(\cdot)$ guarantee interior solutions to (25): at $E_j = 0$ we have LHS $= 0 \leq$ RHS for (25), whereas at $D_j = 0$ we have LHS $> 0 =$ RHS. Thus, $E_j \geq 0$ and $D_j > 0$.

decrease in the second-period loan size L_2 raises the bank's marginal return from each loan above the marginal cost of the loan and causes banks to lend more until the marginal return and marginal cost become equal. Thus, each bank's supply of loans is increasing in R_2 and decreasing in L_2, as is the aggregate supply of loans.

Bank j's first-order-condition for a choice of the number of first-period loans, n_{1j}, to maximize the sum of its first-period and second-period expected profits, $\mathbb{E}(\pi_{1j}) + \mathbb{E}(\pi_{2j})$, is:

$$\mathbb{E}(\min(P_2, R_1 L_1)) = L_1 + E_j \Lambda'(n_{1j} E_j) - \frac{\mathrm{d}\mathbb{E}(\pi_{2j})}{\mathrm{d}n_{1j}}. \tag{29}$$

Note $\mathrm{d}\mathbb{E}(\pi_{2j})/\mathrm{d}n_{1j} = \partial\mathbb{E}(\pi_{2j})/\partial n_{1j} = -\mathbb{E}(n_{2j}\alpha(D_j - P_2)^+ L_2)$. Substituting it into (29), we get:

$$\mathbb{E}(\min(P_2, R_1 L_1)) = L_1 + E_j \Lambda'(n_{1j} E_j) + \alpha\mathbb{E}(n_{2j}[D_j - P_2]^+ L_2). \tag{30}$$

Equilibrium in the loan market requires that the loan interest rate is such that *Loan Demand*$_i$ equals *Loan Supply*$_i$ in period i. The reason for this is the standard market-clearing argument for equilibrium.[28] Equating the equilibrium demand for loans S to the supply (27), in a symmetric equilibrium with identical banks, we must have:

$$n_{ij} = N \ \forall i, j. \tag{31}$$

[28]To see this, first consider an interest rate R_i such that *Loan Supply*$_i$ exceeds the *Loan Demand*$_i$. This means that there is a positive measure of banks that are extending less than their profit-maximizing number of loans. This cannot be an equilibrium because these banks can increase their expected profits by charging a rate marginally lower than R_i and thereby attracting increased demand for their loans by consumers. Now, consider an interest rate R_i such that *Loan Supply*$_i$ is less than the *Loan Demand*$_i$. This means that there are some consumers who do not get loans even though they strictly prefer to get loans at the prevailing interest rate. This cannot be an equilibrium either because a bank can tap part of the unsatisfied demand to increase its expected profit by extending more loans at an interest rate marginally higher than R_i. In what follows, our definition of equilibrium includes incentive-compatibility conditions for banks' choices of the number of loans to extend and capital structure, but not for the loan interest rate which is determined by the market-clearing condition above. Nonetheless, *no* bank wishes to deviate from the interest rate at which loan demand and supply are equated. At this rate, each bank extends its profit-maximizing number of loans and its expected profit is positive because the bank has the option of earning zero profit by not extending any loans. Now, if a bank charges a rate higher than R_i, there will be no demand for its loans and its profit will drop to zero. If it charges a rate less than R_i, the marginal revenue from each loan will decline, so the bank will not only optimally reduce the number of loans it extends but also earn lower expected profits on the remaining loans. So, it is incentive compatible for each bank to charge R_i.

Thus, the number of loans made by each bank equals N, where N is the ratio of the measure of consumers (S) over the measure of banks (S/N).

4.6 The Equilibrium

We now define the equilibrium involving house prices in both periods, loan sizes and interest rates, consumer leverage, and the bank's first-period capital structure choice.

Definition of Equilibrium: A competitive rational expectations equilibrium consists of the house prices (P_1, P_{2h}, P_{2l}), loan sizes (L_1, L_{2h}, L_{2l}), the amount of equity capital per loan in the first period (E_j), the amount of bank deposits per loan in the first period (D_j), and the interest rates (R_1, R_{2h}, R_{2l}) in each state (first period or second period with realization of B_{2h} or B_{2l}) such that:

1. Each consumer chooses whether to buy a house (expected utility in (16)) or not (expected utility in (17)) to maximize her expected utility, taking the house price and the loan interest rate as given. The consumer's choice to buy a house in any given period determines the consumer's capital structure in that period, given the loan size and the consumer's endowment.

2. Each bank, indexed j, chooses the number of loans (n_{2jk}) it extends in the second period to maximize its expected profit (21) conditional on realization of B_{2k}, taking the loan interest rate (R_{2k}) and the loan size (L_{2k}) as given. In the first period, each bank j chooses the number of first-period loans (n_{1j}) to make and its capital structure (E_j and D_j), subject to (2), to maximize the total expected profits from both periods.

3. In each period, banks and consumers form (rational) expectations about the future house price and the actions of other banks and consumers that are consistent with the equilibrium actions of banks and consumers and the expected future house price.

4. The loan size and each consumer's monetary endowment are just sufficient to allow a consumer to buy a house (see (15)).

5. House demand (18) equals house supply (14), and loan demand (26) equals loan supply (27).

The equilibrium house prices P_i, loan interest rates R_i, and banks' first-period equity and deposits, E_j and D_j, are jointly determined by the consumer's indifference condition (19), the bank's optimal capital structure condition (25), the bank's optimal loan amount conditions ((28) and (30)), and the loan market clearing condition (31).

5 Analysis of the Equilibrium in the Loan and Housing Markets

In this section we analyze the equilibrium defined in the previous section and derive three of our four main results. In the usual dynamic programming manner, we use backward induction and begin with the second period first.

5.1 Equilibrium in the second period

The main variables of interest are the second-period equilibrium house price and the impact of the equilibrium house price on the credit risk exposures of banks with respect to their first-period loans. Our first result deals with the relationship between the equilibrium house price and the value of the fundamental shock, B_2.

5.1.1 Shock to fundamentals and the second-period house price

Lemma 1. *The equilibrium house price at $t = 2$, P_2, is increasing in the second-period consumer's utility from home ownership in the second period, B_2.*

As the consumer's utility from home ownership, B_2, increases, buying houses becomes more attractive if house price and interest rates do not change. The demand for houses exceeds the fixed supply of houses. Equilibrium is restored with an increase in the second-period house price and the loan interest rate.

5.1.2 Bank leverage and the second-period house price

Next, we examine how the first-period leverage choices of banks affect the second-period house price. The following result is a comparative statics analysis of the equilibrium about the consequence of higher bank leverage in the first period for the second-period house price.

Proposition 1. *Suppose the housing market experiences a negative fundamental shock ($B_2 = B_{2l}$) at $t = 2$. The higher the equilibrium bank leverage chosen at $t = 1$, the greater is the decline in the second-period house price P_{2l}. Conditional on a positive fundamental shock ($B_2 = B_{2h}$), however,* *the house price P_{2h} is unaffected by the bank leverage chosen at $t = 1$. Thus, higher bank leverage leads to higher volatility in the equilibrium second-period house price.*

Note that the bank's leverage choice at $t = 1$ was based on the probability distribution of B_2. Proposition 1 deals with how the equilibrium house price at $t = 2$ responds, conditional on this earlier leverage choice and the actual realization of the fundamental shock, B_2. The intuition for this result is that the higher a bank's first-period leverage, the higher is the volatility of its profit at $t = 2$. To see why this happens, note that higher leverage causes a bank's first-period profit to increase conditional on a positive shock ($B_2 = B_{2h}$) because higher leverage replaces equity with deposits which are ex post cheaper in the event of a positive fundamental shock. However, higher leverage also causes the bank to default by a greater amount when the housing market suffers a negative fundamental shock ($B_2 = B_{2l}$) at $t = 2$. That is, higher bank leverage amplifies the adverse impact of the negative fundamental shock in the housing market and increases the amount of bank default on deposits. As a result, the lending cost of the bank in the second period increases, leading to a higher second-period loan interest rate. But this lowers the second-generation consumers' demand for loans to finance house purchases, causing the equilibrium house price P_{2l} at $t = 2$ to decline further in order to clear the housing market. We have thus established the second of the four main results mentioned in the Introduction – higher bank leverage causes higher volatility in the second-period house price.

5.1.3 The effect of the leverage choices of other banks on a bank's credit risk exposure

Next, we consider an extension of Proposition 1. Bank j's expected default cost at $t = 2$ from default on its deposits D_j in the first period, defined as its "credit exposure," is:

$$\mathbb{E}(\alpha n_{1j}(D_j - P_2)^+ n_{2j} L_2). \tag{32}$$

Note that bank j's credit exposure depends on the house price P_2 at $t = 2$, which, in turn, depends on the leverage choices of all other banks in the first period (see Proposition 1). The fact that all the banks' loans are backed by the same collateral (i.e., houses) engenders interconnectedness between otherwise-independent banks. Summarizing, we have:

Proposition 2. *For any given θ, as we compare across different equilibria corresponding to different sets of exogenous parameter values, each bank's credit risk exposure at $t = 2$ is increasing in the equilibrium leverage ratios of other banks.*

This result highlights the role of bank leverage in contributing to interconnectedness between banks. It is important to note that this result holds even when we hold θ fixed, so the focus here is on variations in equilibrium leverage caused by other parameters, e.g., cost of equity. As the proposition indicates, when an equilibrium involves higher bank leverage than another equilibrium, the second-period house price declines more in response to a negative shock in the higher-leverage equilibrium. This adversely affects the value of collateral at *all* banks, so each bank's credit risk exposure is higher when the equilibrium leverage choices of all the other banks are higher. Interconnectedness is generated by the impact of bank leverage on the value of *common* collateral. This establishes the third of the four main results discussed in the Introduction.

5.2 Equilibrium in the first period

In the proceeding analysis, we took as given the first-period variables. We now examine the endogenous determination of these variables and hence the relationship between house prices, consumer leverage, and bank leverage in the first period.

Proposition 3. *In the first period, consumer leverage L_1/M_1, bank leverage D_1/L_1, and house price P_1 are all increasing in θ, the probability of a positive fundamental shock to the value attached to home ownership by second-period homebuyers.*

This is the correlated leverage result, the first of the four main results discussed in the Introduction. It asserts that as the probability of a high future value of home ownership increases, both borrowers and banks become more highly levered. The intuition is as follows. The demand for houses depends on a comparison that the consumer makes between the benefits of home ownership – the utility associated with home ownership and the expected house price appreciation during the period of ownership – with the price he pays for the house. Consider an increase in θ, the probability of a high value of second-period home ownership at $t = 2$. *Ceteris paribus* it makes house price appreciation more likely, causing aggregate housing demand to increase at $t = 1$. A market-clearing equilibrium is restored when aggregate housing demand is lowered via two channels: banks increase the interest rate they charge on loans in response to the increased demand for loans, and consumers compete more aggressively with each other to buy houses and bid up the price of houses. The higher first-period house price causes the borrower, who has a fixed initial wealth endowment, to ask for a bigger bank loan, which leads to higher borrower leverage. Moreover, since an increase in θ diminishes the probability $(1-\theta)$ of a decline in the house price at $t=2$, the bank's credit risk declines because the borrower's loan repayment is predicated on the future value of the house as collateral. This reduces the marginal benefit of equity capital to the bank as a cushion to absorb credit risk, so the bank keeps lower capital precisely when borrowers are more highly leveraged, generating correlated leverage.

The first-period increase in bank leverage makes bank payoffs more volatile – greater profits when the housing market experiences a positive shock and larger defaults on deposits when the housing market experiences a negative shock – but risk-neutral bank shareholders are unconcerned about this. The increased magnitude of bank defaults conditional on a negative shock to the housing market increases the bank's second-period lending cost. This elevated cost causes banks to charge higher loan interest rates, which leads to lower consumer demand for houses. Consequently, the house price P_{2l} falls further, and the feedback from a low P_{2l} to a still lower P_{2l} via increased bank costs exacerbates the effect of a negative shock on P_2.

Note that when θ increases, two effects are generated. First, as discussed above, banks increase their leverage and P_{2l} falls. Second, a higher θ means a lower probability $(1-\theta)$ of P_{2l} being realized. These two effects have opposing influences on the first-period price P_1. As long as the bank's lending cost in the second period is not too sensitive to first-period bank failures (which is guaranteed by Assumption 3), the decrease in P_{2l} due to the feedback effect of bank leverage is small, so the direct effect of a higher θ dominates, and the first-period price P_1 goes up when θ increases.

5.3 Bank profits and competition: events at $t=0$

In examining the equilibrium at $t=1$ and $t=2$, we took as given the measure of banks in the industry. We now characterize the competitive equilibrium in the banking industry at $t=0$ and endogenize the measure of banks. Banks compete in our model by choosing the interest rate they charge on loans, the number of loans they extend to consumers, and their capital structure. Despite this, lending generates positive expected equilibrium profits. To see this, note that the expected profit of each bank in the second period is determined using (22) and (28) as:[29]

$$\mathbb{E}(\pi_2) = NL_2Q'(NL_2) - Q(NL_2) > 0. \tag{33}$$

The expected bank profit is positive because Q is a convex function. Banks extend loans to the point at which the marginal revenue from loans equals the marginal cost of extending loans; this marginal cost exceeds the average cost of extending loans because the cost function Q is convex. However, because all loans are identical, the marginal revenue from loans equals the average revenue, and the point at which the bank ceases lending (or the number of loans it chooses to make) in equilibrium is such that its average revenue exceeds its average cost, resulting in positive expected profit. In effect, the increasing marginal cost of equity capital softens competition so banks do not compete away all profits. This result is consistent with Gertner (1985) who shows that when firms compete by simultaneously choosing prices and quantities and there are increasing marginal costs, the symmetric Nash equilibrium involves positive profits. The expected profit of each bank in the first period is determined using (21) and (30) as:

$$\mathbb{E}(\pi_1) = NE\Lambda'(NE) - \Lambda(NE) + \alpha N^2[1-\theta][D - P_{2l}]L_{2l} > 0, \tag{34}$$

which is also positive.

An obvious question is: what specifically restrains the individual bank from increasing its lending when it is earning positive expected profit? There are two factors. The first is the increasing marginal cost of equity (convex Λ) discussed earlier. The second is the banks' concern that making more loans increases the magnitude of possible losses in the first period, leading to a higher cost of funds in the second period and lower second-period profits.[30]

These positive expected bank profits will attract competing banks to enter the industry. Suppose now that this entry decision is made at $t = 0$ and entails a fixed cost of entry, $F > 0$. We would intuitively expect that enough new banks will enter the industry to ensure that each bank's expected

[29] We drop the subscript j (which is used to indicate bank j) through out this subsection to simplify notation without adding confusion.

[30] While this second factor contributes to positive expected bank profits, it is not necessary. If $\alpha = 0$, the second concern is absent, and yet banks earn positive expected profits due to the convexity of the marginal cost of equity for each bank.

profit net of the entry cost F is zero. Assume that there is a large measure of identical potential banks, each of whom faces the payoffs described for banks in Section 3. We now add the following requirement to the two-period equilibrium defined in the previous subsection: *A bank enters the industry at $t = 0$ if the expected profit net of the entry cost F is positive, and does not enter at $t = 0$ if the expected profit net of F is negative.*

With this extended version of the model, we can now endogenously determine the equilibrium measure of banks in the industry such that each bank earns zero expected profit in equilibrium net of the entry cost as long as each bank's expected post-entry profit (ignoring F) is decreasing in the number of banks in the industry.

Lemma 2. *Each bank's expected profit is declining in the measure of banks entering the industry if $\alpha = 0$.*

Each bank's post-entry expected profit is decreasing in the number of competing banks if the second-period cost of funds for the bank does not depend on the first period outcome, i.e., $\alpha = 0$. The intuition is that as the measure of competing banks increases, each bank makes fewer loans and hence needs to raise less equity capital. Since the cost of equity capital is convex, a smaller amount of equity capital means that there is a decline in the difference between the average and the marginal costs of equity capital. The expected bank profit in (33), depends on this difference, and declines as the measure of banks increases. By continuity the result also holds for sufficiently small positive values of α.

As long as the entry cost F is not so high that it dissuades bank entry altogether, Lemma 2 ensures that the zero profit condition, under which the expected profit of each bank net of F equals zero, uniquely determines the equilibrium measure of banks in the industry as that at which the marginal bank is indifferent between entering and not entering.

6 Extensions

We now extend the analysis of our base model and examine various ramifications. First, we analyze how house prices affect the *depth* of the financial intermediation services provided by banks. Second, we analyze the feedback effects that arise between housing market shocks and bank leverage. Shocks to house value fundamentals impact bank leverage choices, and these leverage choices affect how much equilibrium house prices respond to shocks to fundamental values in those periods. Third, we also discuss how exogenous shocks to bank capital can impact house price dynamics. Finally, we examine the regulatory tradeoff between ex ante capital requirements and ex post capital infusions.

6.1 Intermediation thinning

One traditional intermediation function served by the bank is to screen and discover the borrower's ability to generate income to repay the loan. Note that in our previous analysis, we have sidestepped this intermediation role played by the bank by assuming that there is no uncertainty about the borrower's income from any source other than the proceeds from the sale of the house in the second period to repay the loan. We now extend our model to study the bank's intermediation role by adding the following structure.

1. For a fraction $q \in (0, 1)$ of the first-generation consumers (high-type borrowers), the income X_1 at $t = 2$ equals X (high income), where $X > 0$ is a constant, while for the remaining $1 - q$ fraction (low-type borrowers), the income X_1 equals 0 (low income). It is common knowledge that there is a fraction q of the high-type borrowers and that the income levels are X and 0, but no one can distinguish between high-type and low-type borrowers at $t = 1$. Further, consumer types are independent of each other. To focus on bank screening in the first period, we assume that for each second-generation borrower the income at $t = 3$ is nonstochastic and equals $X_2 \geq 0$.

2. If a first-generation borrower does not repay her loan in full at $t = 2$, the lending bank can claim a fraction μ of her income, where $\mu \in (0, 1)$ is a constant.

3. Banks specialize in pre-lending screening of the borrower's income-generation ability at $t = 1$. The screening yields a signal σ with two possible values: a high value σ_X and a low value σ_0. Let:

   $$\Pr(\sigma = \sigma_X | X_1 = X) = \Pr(\sigma = \sigma_0 | X_1 = 0) = \xi, \tag{35}$$

   where $\xi \in [1/2, 1]$ is the precision of bank screening with $\xi = 1/2$ representing an uninformative signal and $\xi = 1$ representing a perfectly revealing signal. Each bank can independently choose the precision ξ of screening by investing $c(\xi)$ in a screening technology. We assume

$c(\xi)$ is an increasing convex function of ξ with $c'(1/2) = 0$ and $\lim_{\xi \to 1} c'(\xi) = \infty$, so that banks will choose screening precision ξ between 0.5 and 1. Let $\eta \equiv q\xi/\{q\xi + [1-q][1-\xi]\}$ represent the posterior probability that a high-signal borrower is of the high type. This probability is increasing in the precision ξ of the signal.

Banks classify borrowers into two groups: the high-signal group and the low-signal group depending on the outcome of screening; banks may choose different loan provisions and/or loan terms across these two groups of borrowers. High-signal borrowers are less likely than low-signal borrowers to default on loans and impose deadweight costs of default on banks. Thus, lending to high-signal borrowers creates more surplus than lending to low-signal borrowers, so banks prefer to lend to high-signal borrowers.

We assume that the measure of high-signal borrowers exceeds the supply of houses. A sufficient condition for this is that the ratio of the supply of houses to the measure of borrowers, S/S_c, does not exceed the minimum of q and 0.5. The banks lend only to high-signal borrowers in this case as we now explain. Since there are more high-signal borrowers than there are houses, not all high-signal borrowers can purchase houses, which means that high-signal borrowers must be indifferent in equilibrium between purchasing a house and not purchasing.[31] A bank will make a loan to a low-signal borrower instead of a high-signal borrower only if doing so does not decrease its profit. However, a loan to a low-signal borrower has higher expected deadweight costs, so the low-signal borrowers must expect to absorb greater deadweight costs than do high-signal borrowers. Since both high-signal and low-signal borrowers derive the same benefit from home ownership, this implies that low-signal borrowers must strictly prefer to not purchase houses if high-signal borrowers are indifferent between purchasing and not purchasing.[32]

[31] If this were not the case and high-signal borrowers strictly preferred to purchase houses at the equilibrium price, there will be excess demand for houses and high-signal borrowers will bid up the house price. If, on the other hand, high-signal borrowers strictly prefer not to purchase houses at the equilibrium price, lack of any demand for houses will cause the house price to decline. None of these outcomes can be an equilibrium.

[32] Another way of observing this is to compare the prices offered for houses by the high-signal and the low-signal borrowers. The price offered by the high-signal borrowers, as given by equation (A20) in the Appendix, is $P_1 = \theta P_{2h} + [1-\theta]P_{2l} + B_1 - \alpha\mathbb{E}(n_{2j}[D_j - P_2 - \mu\eta X]^+ L_2)$, where $\eta \equiv q\xi/\{q\xi + [1-q][1-\xi]\}$ is the posterior probability of a high-type borrower conditional on a high signal. The price offered by the low-signal borrowers will be obtained by replacing η with the posterior probability of a high-type borrower conditional on a low signal. Since this posterior

The average loan repayment received by a bank will be a weighted average of the repayment from high-type and low-type borrowers. When house prices are high, both types repay the loan in full. When houses prices are low, low-type borrowers sell their houses to repay P_{2l} while high-type borrowers repay $P_{2l}+\mu X$. As in the previous analysis, the average repayment received by the banks will be insufficient to pay depositors in full conditional on low house prices: $P_{2l} + \mu\eta X < D_j \ \forall j$.[33]

We now examine the bank's choice of screening precision. Bank j's expected profit in the first period, given by (21) in the main model without screening, now becomes:

$$\mathbb{E}(\pi_{1j}) = n_{1j}\{\theta R_1 L_1 + [1-\theta][P_{2l} + \mu\eta X]\} - n_{1j}L_1 - \Lambda(n_{1j}E_j) - c(\xi). \tag{36}$$

The first term, $n_{1j}\{\theta R_1 L_1 + [1-\theta][P_{2l} + \mu\eta X]\}$, is the expected repayment from the borrowers. This consists of the promised amount $R_1 L_1$ conditional on high house prices (probability θ), plus a term that includes the low house price P_{2l} plus a fraction μ of the borrower's income X if she is high-type (probability η) conditional on a low house price (probability $1-\theta$). The second term, $n_{1j}L_1$, represents the loan amount, the third term represents the cost of equity capital, $\Lambda(n_{1j}E_j)$, and the last term represents the cost of screening, $c(\xi)$. Bank j's expected profit in the second period, given by (22) in the main model, now becomes:

$$\pi_{2j} = n_{2j}\mathbb{E}(\min(P_3, R_2 L_2)) - n_{2j}L_2 - Q(n_{2j}L_2) - \alpha n_{1j}(D_j - P_2 - \mu\eta X)^+ n_{2j}L_2. \tag{37}$$

In addition to determining the number of loans to extend and the capital structure, each bank now also chooses the precision with which to screen potential borrowers. We provide a comparative statics result pertaining to the equilibrium in this model.

Proposition 4. *Compare two equilibria with different exogenous parameter values. Then, the precision ξ with which banks screen is higher in the equilibrium in which the probability of a high fundamental home-ownership-value shock, θ, is lower, the sensitivity of the second-period lending cost to first-period default, α, is higher, the fraction of a borrower's income that the bank can claim,*

probability is lower than η, the price offered by the low-signal borrowers is lower because it accounts for the bank's higher expected cost of default on deposits.

[33] If this condition holds without screening, then this must also hold in equilibrium. To see why, suppose banks do not default on deposits in equilibrium. Then, a marginal reduction in screening reduces screening costs but has no effect on expected costs of default on deposits. This means the banks can increase expected profit by decreasing their investment in screening. However, this cannot be true in an equilibrium.

μ, is higher, and the difference between the incomes of the high-type and the low-type borrowers, X, is higher.

The first result is that a higher expected future house price (larger θ) dilutes the bank's incentive to screen the borrower's income-generation ability. The intuition is as follows. Note that the bank's credit risk depends on both the value of the house (collateral) and the fraction μ of the borrower's income available for loan repayment. Thus, a higher expected future house price diminishes the bank's reliance on borrower income in collecting the loan repayment, which in turn dilutes the bank's pre-lending screening incentive. That is, intermediation "thins" as house prices rise. If default on deposits is less costly (α is low), the deadweight costs of risky loans is low as banks can require a higher payment when house prices are high to make up for the lower payment when house prices are low. In this case, banks are less concerned about risk and thus screen less. Finally, the objective of screening is to distinguish between high-type and low-type borrowers because they earn different incomes and the bank captures a fraction μ of that income. If the difference between the incomes of the two groups of borrowers is higher or if banks can capture a greater fraction of these incomes, then identifying high-type borrowers is more valuable, leading banks to invest more in screening.

6.2 Bank leverage and house price cycles

Consider an extension of our model's two-period structure to one with many more than two periods. In each period the consumer's utility from home ownership evolves stochastically, generating either a positive or a negative shock for the housing market. The shocks in all the periods are identical and independently distributed, where in each period the probability is θ for a positive shock and $1-\theta$ for a negative shock. However, initially no one knows the exact value of θ, except its prior probability distribution which is common knowledge. Banks and consumers update their beliefs about θ over time as they observe shocks in different periods. This specification is consistent with the empirical evidence that house price appreciation is highly positively autocorrelated (e.g., Ashcraft, Goldsmith-Pinkham, and Vickery (2009), Case and Shiller (1987, 1989), and Goetzmann, Peng, and Yen (2009)).[34]

[34] Goetzmann, Peng, and Yen (2009) examine how forecasts of future increase in house price based on past price trends affect both the demand and supply of mortgages prior to the subprime crisis. In particular, past home price

In the first period, house price, bank leverage and consumer leverage are all determined based on agents' prior beliefs about θ. If a positive shock is realized in the second period, then the second-period house price increases above the first-period price. Moreover, banks and consumers revise upward their beliefs about θ, and both increase their leverage ratios in the second period. Thus, house price and bank leverage move up in unison, until a negative shock is realized in the housing market. When a negative shock hits, the house price drops in that period, and banks and consumers revise downward their beliefs about θ, leading them to decrease their leverage ratios.

One interesting result is that the longer the housing market boom survives via a sequence of positive shocks, the larger is the adverse impact of a negative shock when it occurs. This is because banks attain very high leverage during the boom, and this high leverage magnifies the impact of even a small negative shock, resulting in a sharp decline in the house price.

By contrast, suppose there is a sequence of negative shocks in the initial periods, so the house price keeps declining initially. Banks keep revising downward their beliefs about θ, and as a result bank leverage also keeps declining. The impact of a future negative shock is thus smaller than that of previous negative shocks, because banks have low leverage. If a positive shock occurs after those initial negative shocks, the house price increase will be modest because banks' beliefs about θ are relatively low due to the initial negative shocks and banks also have relatively low leverage ratios.

In summary, we have the following two observations. First, if a large number of positive shocks occur initially, bank leverage rises to relatively high levels, and this may lead to a seemingly dramatic house price reversal once a negative shock occurs. The longer the boom in the housing market, the larger is the house price response to a negative shock. Second, if a large number of negative shocks occur initially, bank leverage declines, and the house price reversal in response to a positive shock is rather modest. That is, the reaction of house prices to shocks depends on previous shocks and may be asymmetric, and this is because house price dynamics are affected by bank leverage dynamics and vice versa.

increases are associated with higher demand for loans to purchase homes that is manifested in higher subprime applications and loan-to-value ratios.

6.3 Impact of exogenous bank capital shocks

Consider a situation in which banks receive an exogenous shock that depletes their capital. This may be due, for example, to losses on assets other than home loans, or due to trading losses. This increases the cost of equity capital for banks and increases the cost of credit for borrowers, thereby reducing housing demand. Alternatively, banks may be unable to replace the lost equity capital so they reduce credit supply. In both cases, the first-period house price declines. Moreover, the negative shock to bank capital drives up bank leverage.

Thus, an exogenous negative shock to bank capital has two adverse effects on house prices. First-period house prices fall due to diminished bank credit supply, and the higher bank leverage also causes the volatility of the second-period house price to go up.

Similarly, a positive exogenous shock to bank capital – via profits on other investments – causes a decrease in the bank's cost of equity capital or simply increases the available supply of loanable funds. The bank responds by increasing credit supply, and this leads to a higher P_1. But since this increase in P_1 comes from cheaper credit rather than an expectation of a higher future house price, there is not an increase in bank leverage induced by house price dynamics. In fact, bank leverage declines due to the positive exogenous shock to bank capital. Consequently, the volatility of the second-period house price does *not* go up as P_1 increases.

We can now summarize the contrast between the result of our main model and this alternative scenario. In our main model, the exogenous shock is to the value second-period homebuyers are expected to attach to home ownership. When this shock is positive, it causes the first-period house price to increase, which then causes both bank and borrower leverage to rise. Thus, when the direction of causality runs from house price shocks to bank leverage, we find that higher initial house prices are correlated with: (i) higher bank leverage, (ii) higher borrower leverage, and (iii) higher second-period house price volatility. When the direction of causality runs from shocks to bank capital to house prices, higher initial house prices are correlated with: (i) lower bank leverage, (ii) higher borrower leverage, and (iii) lower second-period house price volatility. In this latter case, exogenous shocks to bank capital can induce price cycles in the housing market. A sufficiently high positive shock can even induce a house price bubble. Once the bubble has set in, then of course house prices can also affect bank leverage as banks may choose to reduce capital levels – by paying higher dividends – in response to an expectation of higher future house prices. The correlated leverage story we have told in this paper will then commence.

6.4 Ex ante capital regulation and ex post capital infusion

Since bank leverage affects the housing market, it is natural to ask how regulatory interventions that affect bank capital may influence house prices. We now discuss, within the context of our model, the consequences of ex ante bank capital regulation (at $t = 1$) and ex post capital infusion (at $t = 2$) by a regulator. Recall that bank equity capital acts as a cushion against default when the housing market experiences a negative shock ($B_2 = B_{2l}$) at $t = 2$. But when the shock is positive ($B_2 = B_{2h}$), the bank would have been better off if it had chosen a higher leverage ratio at $t = 1$. Ex ante each bank makes its own capital structure decision by trading off the benefit of equity capital against its cost to maximize its own expected profit, and it assumes that its own capital structure decision has no impact on the house price. But we know that collectively the first-period leverage choices of banks do affect the second-period house price.

It should be noted that despite the effect of bank leverage on house prices, there is *no* rationale for regulatory intervention because there is *no* inefficiency *per se* due to the price impact of high bank leverage. So assume now that the bank regulator is concerned with house price dynamics and wishes to influence the dynamics by affecting banks' leverage choices. We ask: what is the economic consequence of the regulator imposing a capital requirement on banks? If this capital requirement is binding, then our earlier results show that it will help to elevate the expected equilibrium house price at $t = 2$ via an increase in bank capital levels. However, imposing a capital requirement higher than the bank's privately-optimal choice of equity capital reduces the bank's ex ante expected profit, which in turn will make the first-period loan more costly for the first-generation homebuyers. This will reduce the first-generation consumers' demand for houses, causing the house price at $t = 1$, P_1, to decline. Thus, a binding capital requirement is likely to reduce the house price initially, but will also lower the likelihood of a further housing price decline. Whether a capital requirement is adopted depends on the objective of the regulator with respect to house price dynamics.

An alternative to capital requirement is for the regulator to infuse capital into the banks with a non-zero probability at $t = 2$ when the housing market is hit by a negative shock. A capital infusion reduces the banks' reliance on the capital market to raise (now very costly) equity to finance the second-period loan, which then helps to arrest any house price decline at $t = 2$. In this sense, an ex post capital infusion is similar to a capital requirement. However, these two methods of intervention may have very different *ex ante* consequences. To see that, note if the first-period homebuyers anticipate the government's capital infusion will occur ex post with a positive probability in the adverse state, they will attach a lower probability to a housing price decline in the second period. This will cause housing demand to increase in the first period, and as a result the equilibrium house price at $t = 1$, P_1, will increase. As for banks, the possibility of an ex post capital infusion reduces the marginal value of bank capital ex ante, and hence banks will choose higher leverage in the first period. However, this will increase the amount by which banks default on deposits if the housing market is hit by a negative shock at $t = 2$ and the government does not provide an ex post capital infusion. In turn, this increases the attractiveness of an ex post capital infusion from the standpoint of the government. Thus, an ex post government capital infusion could become a self-fulfilling prophecy, contributing to an increase in banking system leverage and fragility.

7 Empirical Implications and Conclusion

In this paper we have shown that the leverage decisions of borrowers and banks may move in unison – banks choose to become more highly levered when their borrowers are more highly levered. This finding is obtained in a setting in which bank loans are secured and borrowers' repayment of bank loans depends primarily on the stochastic value of the collateral backing the loan. Moreover, the probability distribution of the value of the collateral is affected by the aggregate lending behavior of banks, which in turn is dependent on their earlier capital structure decisions. Markets such as the one examined in this paper are also characterized by a link between bank leverage ratios and house prices – higher leverage ratios chosen by banks at a given point in time tend to increase the volatility of future house prices. That is, banks' capital structure decisions in the financial market have ramifications for equilibrium prices in the housing market. Further, bank leverage generates a form of interconnectedness among otherwise-independent banks in that each bank's credit risk exposure is increasing in the equilibrium leverage ratios of *other* banks. And there is also a transmission mechanism whereby house price dynamics impact the depth/quality of financial intermediation services provided by banks. Bull housing markets tend to dilute banks' screening incentives.

What empirical implications can we draw from this analysis? First, the main prediction of the model is that we should find in the data that high house prices, high borrower leverage and high bank leverage occur together. The recent home mortgage crisis is an example of this. Second, there will be a positive correlation between aggregate bank leverage in a given period and *subsequent* house price volatility. That is, house price volatility in any given period will be decreasing in *appropriately-lagged* bank capital. Third, because banks wish to operate with less capital during such periods, regulatory capital requirements will tend to be more binding for banks that engage in more secured lending and when the price of collateral is higher for borrowers. Finally, the dynamics of bank leverage can generate house price cycles. When there are only exogenous shocks to house prices, a sufficiently long sequence of positive house price shocks drive up bank leverage to such high levels that even a small number of subsequent negative shocks can precipitate a large reversal in house prices. When bank capital is also subject to exogenous shocks from sources other than house prices, we see that positive exogenous shocks to bank capital lead to higher initial house prices and lower future house price volatility. In both cases, the dynamics of bank leverage will have observable effects on the dynamics of house prices.

Appendix

Explicit Expressions Corresponding to Assumptions 2 and 3:

Assumption 2

$$\bar{u} \equiv \theta[P_{2h}^* - P_{2l}^*], \tag{A1}$$

where $[P_{2k}^* - M_2]\{1 + Q'(N[P_{2k}^* - M_2])\} = \mathbb{E}(P_3) + B_{2k} - M_2$, $k \in \{h, l\}$.

Further discussion of Assumption 2: Note that $M_1 - B_1$ is an upper bound on a first-period homebuyer's expected home equity, and $\bar{u}$ as defined in (A1) is the expected increase in home value if the second-period house price is high (P_{2h}^*) rather than low (P_{2l}^*), where P_{2h}^* and P_{2l}^* are defined above as if there were no default in the first period (by setting $\alpha = 0$ in (A5)). The inequality, $M_1 - B_1 < \bar{u}$, implies that homeowners' expected equity is less than the expected upside in house price. This is possible only if the house price upon a negative shock in the housing market is insufficient to cover the loan repayment obligation. Thus, bank loans must be risky. Finally, it is optimal for banks to accept some risk of default on deposits to trade off against the costly equity capital. The following proof formally shows that Assumption 2 is a sufficient condition for the following inequality:

$$P_{2l} < D_j \leq R_1 L_1 \leq P_{2h}, \tag{A2}$$

which says: (i) when the housing market receives a positive shock ($B_2 = B_{2h}$), the second-period house price is sufficiently high so that first-period homebuyers will repay their loans ($P_{2h} > R_1 L_1$), whereas (ii) when the housing market receives a negative shock ($B_2 = B_{2l}$), the second-period house price is sufficiently low so that a first-period homebuyer will default on her loan, and the bank will seize the house but the market value of the house is insufficient to pay off all the deposits ($P_{2l} < D_j \leq R_1 L_1$).

Proof: Substituting (15), (19) and (31) into (30), we have:

$$\begin{aligned} P_1 &= \mathbb{E}(P_2) + B_1 - E_j \Lambda'(N E_j) - \alpha N \mathbb{E}((D_j - P_2)^+ L_2) \\ &= \mathbb{E}(P_2) + B_1 - \alpha N \mathbb{E}(L_2 \mathbf{1}_{\{P_2 < D_j\}}) E_j - \alpha N \mathbb{E}((D_j - P_2)^+ L_2) \\ &= \mathbb{E}(P_2) + B_1 - \alpha N \mathbb{E}(L_2 [P_1 - M_1 - P_2] \mathbf{1}_{\{P_2 < D_j\}}), \end{aligned} \tag{A3}$$

where the second equality follows from (25).

Suppose bank deposits are not risky, so $D_j \leq P_{2l}$. Then, (A5) shows that $P_{2l} = L_{2l} + M_2 = \mathbb{E}(P_3) + B_{2l} - L_{2l} Q'(N L_{2l})$, and a higher value $P_{2h} > P_{2l}$ solves (A5) for the case when $B_2 = B_{2h}$. So, $D_j \leq P_2$. Substituting this into (A3), we get $P_1 = \mathbb{E}(P_2) + B_1$; substituting it into (25), we get $E_j = 0$. Substituting $D_j \leq P_2$ into (A5) shows that $P_{2h} = P_{2h}^*$ and $P_{2l} = P_{2l}^*$, where P_{2k}^* is defined in Assumption 2. Substituting $D_j \leq P_2$ and $E_j = 0$ into (30), we get $\mathbb{E}(\min(P_2, R_1 L_1)) = L_1 \leq P_{2l}$, which implies $R_1 L_1 \leq P_2$. Substituting this condition into (19), we get $M_1 = B_1 + \mathbb{E}(P_2) - R_1 L_1 \geq B_1 + \mathbb{E}(P_2) - P_{2l}$. Rearranging terms, we get $M_1 - B_1 \geq \theta[P_{2h} - P_{2l}] = \theta[P^*_{2h} - P^*_{2l}]$, which contradicts Assumption 2. Thus, we must have $D_j > P_{2l}$. The remaining inequalities in (A2) are obvious. □

Assumption 3

$$\alpha_{max} \equiv \frac{1}{N[\mathbb{E}(P_3) + B_{2l} - M_2]}. \tag{A4}$$

Proof of Lemma 1: Substituting equilibrium conditions (15), (19) and (31) into (28), we have:

$$[1 + \alpha N(D_j - P_2)^+]P_2 = \mathbb{E}(P_3) + B_2 + \alpha N(D_j - P_2)^+ M_2 - L_2 Q'(NL_2). \tag{A5}$$

Totally differentiating (A5) with respect to B_2 yields:

$$\begin{aligned} &[1 + \alpha N(D_j - P_2)^+ - \alpha N P_2 \times \mathbf{1}_{\{P_2 < D_j\}}]\frac{\mathrm{d}P_2}{\mathrm{d}B_2} \\ &= 1 - \alpha N M_2 \times \mathbf{1}_{\{P_2 < D_j\}}\frac{\mathrm{d}P_2}{\mathrm{d}B_2} - [Q'(NL_2) + NL_2 Q''(NL_2)]\frac{\mathrm{d}L_2}{\mathrm{d}B_2}. \end{aligned} \tag{A6}$$

The above equation simplifies to:

$$[1 + \alpha N(D_j - P_2)^+ - \alpha N \times \mathbf{1}_{\{P_2 < D_j\}} L_2 + Q'(NL_2) + NL_2 Q''(NL_2)]\frac{\mathrm{d}P_2}{\mathrm{d}B_2} = 1. \tag{A7}$$

The coefficient of $\mathrm{d}P_2/\mathrm{d}B_2$ is positive because $\alpha N L_2 < 1$ by Assumption 3. This shows $\mathrm{d}P_2/\mathrm{d}B_2 > 0$. □

Proof of Proposition 1: The desired results are comparative statics of symmetric equilibria, so we shall replace individual bank choices in the equilibrium conditions with the equilibrium choices (by removing subscript j in equilibrium equations). Consider the realization of B_{2l} at $t = 2$. Totally differentiating (A5) with respect to D_1 yields:

$$\{1 + \alpha N[D_1 - P_{2l}] - [P_{2l} - M_2]\alpha N\}\frac{\mathrm{d}P_{2l}}{\mathrm{d}D_1} = -\alpha N[P_{2l} - M_2] - [Q'(NL_{2l}) + NL_{2l}Q''(NL_{2l})]\frac{\mathrm{d}L_{2l}}{\mathrm{d}D_1}. \tag{A8}$$

But $\mathrm{d}L_{2l}/\mathrm{d}D_1 = \mathrm{d}P_{2l}/\mathrm{d}D_1$, so the above equation simplifies to:

$$\{1 + \alpha N[D_1 - P_{2l}] - \alpha N[P_{2l} - M_2] + [Q'(NL_{2l}) + NL_{2l}Q''(NL_{2l})]\}\frac{\mathrm{d}P_{2l}}{\mathrm{d}D_1} = -\alpha N[P_{2l} - M_2]. \tag{A9}$$

Note that the coefficient of $\mathrm{d}P_{2l}/\mathrm{d}D_1$ on the left-hand-side (LHS) is positive because $\alpha N[P_{2l} - M_2] \leq \alpha N[\mathbb{E}(P_3) - M_2] < 1$ (Assumption 3), whereas the right-hand-side (RHS) is negative. Thus, we must have $\mathrm{d}P_{2l}/\mathrm{d}D_1 < 0$. When $B_2 = B_{2h}$, banks do not default in the first period (see Assumption 2), so P_{2h} is not affected by the first-period capital structure and hence is not a function of D_1, i.e., $\mathrm{d}P_{2h}/\mathrm{d}D_1 = 0$. Thus, $P_{2h} - P_{2l}$ is increasing in D_1, and this proves the proposition. □

Proof of Proposition 2: As the first-period leverage ratio D_1/L_1 increases for all banks, the probability distribution of the second-period price P_2 shifts to the left (from Proposition 1), and the consequently lower P_2 increases bank j's credit exposure as defined in (32). □

Proof of Proposition 3: The equilibrium is characterized by (2), (15), (25), (31), (A3) and (A5), and is symmetric, so we shall remove the subscripts j from all equilibrium equations when performing comparative statics analysis.

Incorporating $P_{2l} < D_1 \leq P_{2h}$ from (A2) into (A3) and totally differentiating with respect to θ yield:

$$\begin{aligned}\frac{\mathrm{d}P_1}{\mathrm{d}\theta} &= P_{2h} - P_{2l} + \theta\frac{\mathrm{d}P_{2h}}{\mathrm{d}\theta} + [1-\theta]\frac{\mathrm{d}P_{2l}}{\mathrm{d}\theta} + \alpha N[P_{2l} - M_2][P_1 - M_1 - P_{2l}] \\ &\quad - \alpha N[P_{2l} - M_2][1-\theta]\left[\frac{\mathrm{d}P_1}{\mathrm{d}\theta} - \frac{\mathrm{d}P_{2l}}{\mathrm{d}\theta}\right] - \alpha N[1-\theta][P_1 - M_1 - P_{2l}]\frac{\mathrm{d}P_{2l}}{\mathrm{d}\theta}. \end{aligned} \tag{A10}$$

P_{2h} is independent of θ because P_{2h} is determined by (A5) which is independent of the first-period outcomes as $(D_1 - P_{2h})^+ = 0$ from (A2). Substituting $\mathrm{d}P_{2h}/\mathrm{d}\theta = 0$, the above equation simplifies to:

$$\begin{aligned}\{1 + \alpha N[P_{2l} - M_2][1-\theta]\}\frac{\mathrm{d}P_1}{\mathrm{d}\theta} &= [1-\theta]\{1 + \alpha N[P_{2l} - M_2] - \alpha N[P_1 - M_1 - P_{2l}]\}\frac{\mathrm{d}P_{2l}}{\mathrm{d}\theta} \\ &\quad + P_{2h} - P_{2l} + \alpha N[P_{2l} - M_2][P_1 - M_1 - P_{2l}]. \end{aligned} \tag{A11}$$

Consider the realization of B_{2l} at $t = 2$. Totally differentiating (A5) with respect to θ yields:

$$\{1 + \alpha N[D_1 - P_{2l}] - \alpha N[P_{2l} - M_2] + Q'(NL_{2l}) + NL_{2l}Q''(NL_{2l})\}\frac{\mathrm{d}P_{2l}}{\mathrm{d}\theta} = -\alpha N[P_{2l} - M_2]\frac{\mathrm{d}D_1}{\mathrm{d}\theta}. \tag{A12}$$

Substituting (31) into (25) and totally differentiating with respect to θ, we get:

$$N\Lambda''(NE_1)\frac{\mathrm{d}E_1}{\mathrm{d}\theta} = -\alpha N[P_{2l} - M_2] + \alpha[1-\theta]N\frac{\mathrm{d}P_{2l}}{\mathrm{d}\theta}. \tag{A13}$$

Substituting (2) into the above equation yields:

$$\frac{\mathrm{d}D_1}{\mathrm{d}\theta} = \frac{\mathrm{d}P_1}{\mathrm{d}\theta} + \frac{\alpha[P_{2l} - M_2]}{\Lambda''(NE_1)} - \frac{\alpha[1-\theta]}{\Lambda''(NE_1)}\frac{\mathrm{d}P_{2l}}{\mathrm{d}\theta}. \tag{A14}$$

Substituting this into (A12) yields:

$$\begin{aligned}&\left\{1 + \alpha N[D_1 - P_{2l}] - \alpha N[P_{2l} - M_2] + Q'(NL_{2l}) + NL_{2l}Q''(NL_{2l}) - \frac{\alpha^2 N[1-\theta][P_{2l} - M_2]}{\Lambda''(NE_1)}\right\}\frac{\mathrm{d}P_{2l}}{\mathrm{d}\theta} \\ &= -\alpha N[P_{2l} - M_2]\frac{\mathrm{d}P_1}{\mathrm{d}\theta} - \frac{\alpha^2 N[P_{2l} - M_2]^2}{\Lambda''(NE_1)}. \end{aligned} \tag{A15}$$

We prove by contradiction that $\mathrm{d}P_1/\mathrm{d}\theta > 0$. Suppose $\mathrm{d}P_1/\mathrm{d}\theta \leq 0$. Then, substituting $\mathrm{d}P_1/\mathrm{d}\theta = 0$ into (A11) provides an upper bound on $\mathrm{d}P_{2l}/\mathrm{d}\theta$, while substituting $\mathrm{d}P_1/\mathrm{d}\theta = 0$ into (A15) provides a lower bound on $\mathrm{d}P_{2l}/\mathrm{d}\theta$. For sufficiently small values of α, the lower bound exceeds the upper bound (for $\alpha = 0$, the lower bound is 0, while the upper bound is negative) which yields a contradiction. Thus, P_1 must be increasing in θ for sufficiently small α (as long as $\alpha < \alpha_{max}$, the upper bound imposed in Assumption 3). Then, (A15) shows that $\mathrm{d}P_{2l}/\mathrm{d}\theta < 0$. Substituting this into (A13) and (A14) shows that D_1 is increasing in θ while E_1 is decreasing in θ, so bank leverage D_1/L_1 is increasing in θ. □

Proof of Lemma 2: The measure S/N of banks increases as N declines. From (25), NE is independent of N (and equals zero) if $\alpha = 0$ so $\mathbb{E}(\pi_1)$ in (34) is independent of N. Substituting $\alpha = 0$ into (A5), we get,

$$[1 + Q'(NL_2)]L_2 = \mathbb{E}(P_3) + B_2 - M_2. \tag{A16}$$

The right-hand-side (RHS) of the above equation is independent of N. If NL_2 is weakly decreasing in N, then L_2 and consequently the left-hand-side (LHS) of the above equation are strictly decreasing in N, a contradiction. So NL_2 must increase with N. This means that $\mathbb{E}(\pi_2)$ in (33) is increasing in N and decreasing in the measure S/N of banks. □

Proof of Proposition 4: The equilibrium conditions incorporate heterogeneity among consumers by allowing each homeowner's loan repayment to be contingent on her income. Equation (19) for equilibrium in the housing market becomes:

$$M_1 = B_1 - \mu\eta X + \mathbb{E}((P_2 + \mu\eta X - R_1 L_1)^+), \tag{A17}$$

equation (25) for the optimal capital structure becomes:

$$\Lambda'(n_{1j}E_j) = \alpha\mathbb{E}(n_{2j}L_2\mathbf{1}_{\{P_2+\mu\eta_j X<D_j\}}), \tag{A18}$$

and equation (30) for the optimal number of loans becomes:

$$\mathbb{E}(\min(P_2 + \mu\eta_j X, R_1 L_1)) = L_1 + E_j\Lambda'(n_{1j}E_j) + \alpha\mathbb{E}(n_{2j}[D_j - P_2 - \mu\eta_j X]^+ L_2). \tag{A19}$$

Substituting (A17) and (15) into the above equation, we get:

$$P_1 = \theta P_{2h} + [1-\theta]P_{2l} + B_1 - \alpha\mathbb{E}(n_{2j}[D_j - P_2 - \mu\eta_j X]^+ L_2). \tag{A20}$$

In the above equations, $\eta_j \equiv q\xi_j/\{q\xi_j + [1-q][1-\xi_j]\}$, where ξ_j is the screening effort chosen by bank j to maximize the sum of expected first-period profit and second-period profit given by (36) and (37), respectively. Further, the Envelope Theorem application shows that $\mathrm{d}\mathbb{E}(\pi_{1j}+\pi_{2j})/\mathrm{d}\xi_j = \partial\mathbb{E}(\pi_{1j}+\pi_{2j})/\partial\xi_j$, because bank j chooses other choice variables to maximize its expected profits and aggregate variables (P_i and R_i) are not affected by bank j's choice of ξ_j. Then, the equilibrium first-order-condition for a bank's choice of screening effort ξ ($\xi_j = \xi\ \forall j$ in a symmetric equilibrium) is:

$$N\mu X[1-\theta][1+\alpha NL_{2l}][\mathrm{d}\eta/\mathrm{d}\xi] = c'(\xi). \tag{A21}$$

The net marginal benefit of an increase in ξ equals $N\mu X[1-\theta][1+\alpha NL_{2l}][\mathrm{d}\eta/\mathrm{d}\xi] - c'(\xi)$ and is increasing in α, μ, and X but decreasing in θ, so the optimal ξ is increasing in α, μ, and X but decreasing in θ. □

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Figure 1: **Sequence of Events**

$t=0$

- Banks decide whether to enter the industry.

$t=1$

- Each atomistic first-period consumer chooses whether to buy a house or not. Consumers attach value to home ownership and maximize expected utility: $U_1 = h_1 B_1 + C_1 + E(C_2)$.
- First-period house price, P_1, is endogenously determined.
- Atomistic banks compete to lend to first-period homebuyers. All loans are secured by houses.
- Each bank chooses its first-period capital structure, in the face of complete deposit insurance, costly bank equity and the potential impact of the first-period default to its cost of lending in the second period.

$t=2$

- First-period homebuyers sell their houses.
- Each second-period consumer chooses whether to buy a house or not.
- Second-period house price, P_2, is endogenously determined by second-period housing demand, housing supply and credit supply.
- Bank loans to first-period homebuyers are settled.
- Each bank experiences loss/profit and consequent shock to its second-period lending cost.
- Banks compete to lend to second-period homebuyers.

$t=3$

- Second-period homebuyers sell their houses.
- Banks' loans to second-period homebuyers are settled.