Household Leverage and the Recession

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Abstract

We present a model where households use their houses as ATMs, and we analyze the macroeconomic consequences of a credit crunch triggered by tightening lending standards. We study the aggregate time series and the cross sectional responses. We argue that cross-sectional evidence on leverage, consumption and employment across US counties places strong restrictions on the set of acceptable parameters for the model. Models that fit the cross section display high sensitivity of economic activity to nominal credit shocks.

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Our paper is related to the literature on housing wealth and consumption. Iacoviello and Neri (2010) extend the model of Iacoviello (2005) to study post war US data. They write an economy with two types of households (patient and impatient), collateral constraints on housing wealth, nominal rigidities, and endogenous housing supply. They use the model to analyze post-war data. Collateral constraints are important to generate a positive response of non housing consumption to a shock to housing preferences. Without collateral constraints, a stronger preference for housing would lead to a substitution away from other forms of consumption. The collateral constraints play essentially no role in the monetary transmission mechanism (fig 3 in Iacoviello and Neri (2010)). They use the model to study trends and cycles in the data. They find slower technological progress in housing construction which accounts for much of the trend increase in relative house prices. Detrended real house prices still fluctuate a lot, however. The model attributes most of the run-up in house prices and housing investment from the mid 1998 to 2005 to a shift in preferences towards housing (figure 6 and table 7).[1] Our work differs in two dimensions. First, we focus on the more recent period and we model the cross section of US counties. Second, the most important part of our model is the interaction between the CIA and collateral constraints. The CIA does not appear in other models.

Following Bernanke and Gertler (1989), most macro paper introduce credit constraints at the entrepreneur level (Kiyotaki and Moore (1997), Bernanke, Gertler, and Gilchrist (1999)). In all these models, the availability of credit limits *corporate* investment. As a result, credit constraints affect the economy by affecting the size of the capital stock. These models can therefore be understood without reference to money or nominal credit.

In addition the current recession seems more due to household leverage than to corporate leverage. Corporate balance sheet were very strong before and remained strong well into the recession. Today the corporate sector has low leverage, high productivity and historically high profitability. It seems difficult to argue that corporate sector balance sheets caused the recession, while there is ample evidence suggesting that households' balance sheets contributed significantly.

Recently, Gertler and Karadi (2009) and Gertler and Kiyotaki (2010) study a model where shocks hit the financial intermediation sector. These shocks lead to tighter borrowing constraints for borrowers. We model shocks in a similar way. The difference is that our borrowers are households, not entrepreneurs, and,

[1] They also show that the preferences shifts are partly explained by subprime lending, fees and demographics.

as we have explained, this matters a lot for macro dynamics.

On the empirical side, we rely on the microeconomic evidence in Mian and Sufi (2010a) and Mian and Sufi (2010b) to calibrate the key parameters of our model.

In Section 1 we present the model. In Section 2 we describe the equilibrium. In Section 3 we study the qualitative and theoretical properties of the model. In Section 4 we propose a quantitative calibration and we study the response of the economy to various shocks.

1 Model

We study a closed economy with a continuum of islands that trade with each other. Each island produces tradeable and non-tradeable goods and is populated by a representative household. Means of payment are provided by the government and by private lenders (banks and shadow banks).

Our model can be interpreted as a large country with a collection of regions (e.g., USA), or a monetary union with a collection of states (e.g., EU). The key assumption are that these regions share a common currency, and that agents live and work in only one region.

1.1 Households

The representative household on island i seeks to maximize the following objective function:

$$\sum_{t=0}^{\infty} \beta^t u\left(\bar{c}_{i,t}, l_{i,t}, h_{i,t}\right)$$

where $\bar{c}_{i,t}$ is an aggregate of consumption goods, $l_{i,t}$ is the amount of labor supplied and $h_{i,t}$ is the stock of housing owned by the household. There are two sources of liquidity: money issued by the government, and private credit. We call M_{it} the government-issued cash in the hands of consumers at the beginning of period t, and B_{it} the size of the credit line available from financial firms. Let $Q_{i,t}$ be the price of houses on island i at time t. Housing purchases $Q_{i,t}\left(h_{i,t} - h_{i,t-1}\right)$ occur first in the period, followed by purchases of consumption goods. These purchases are subject to the “credit in advance” constraint:

$$\bar{P}_{i,t}\bar{c}_{i,t} + Q_{i,t}\left(h_{i,t} - h_{i,t-1}\right) \leq M_{i,t} + B_{i,t}, \tag{1}$$

where $\bar{P}_{i,t}$ is the price index for consumption on island i at time t (defined below). Equation (1) means that firms accept to sell goods in exchange for bills printed by the government as well as units of credit backed by banks.[2] We assume that private credit for consumption must be collateralized by housing wealth. The amount of private credit is subject to the collateral constraint:

$$B_{i,t} \leq \theta_{i,t} Q_{i,t} h_{i,t}. \tag{2}$$

The parameter $\theta_{i,t}$ is exogenous and will be the source of shocks to the economy. For now, one can simply think of it as a constant.

Let $W_{i,t}$ denote the nominal wage and let Π_t be the profits paid by private firms. For simplicity, we assume that ownership of private firms (financial and non-financial) is diversified, so that profits are equal across islands. At the end of the period, the liquidity position of the household is therefore[3]

$$X_{it} = \Pi_t + W_{i,t} l_{i,t} + M_{i,t} - \bar{P}_{i,t}\bar{c}_{i,t} - rB_{it} - Q_{i,t}\left(h_{i,t} - h_{i,t-1}\right).$$

Finally, government implements monetary policy by printing new bills at the beginning of time t, and distributing them across islands

$$M_{i,t+1} = X_{i,t} + T_{i,t+1}.$$

The flow budget constraint of the consumer is therefore

$$M_{i,t+1} = \Pi_t + W_{i,t} l_{i,t} + M_{i,t} - \bar{P}_{i,t}\bar{c}_{i,t} - rB_{it} - Q_{i,t}\left(h_{i,t} - h_{i,t-1}\right) + T_{i,t+1}. \tag{3}$$

The total amount printed by the government is simply $T_{t+1} = \int T_{i,t+1}$. The timing of the model is summa-

Table 1: Timing of Households Cash and Credit Flows

	First Half	Second Half
Cash	$M_{it} = X_{it-1} + T_{i,t}$	X_{it}
Credit	$B_{i,t}$	$-(1+r) B_{it}$
Spending	$\bar{P}_{i,t}\bar{c}_{i,t} + Q_{i,t}(h_{i,t} - h_{i,t-1})$	0
Income	0	$\Pi_t + W_{i,t}l_{i,t}$

rized in Table 1.

The first-order conditions for consumption, labor, and housing, are:

$$
\begin{aligned}
\frac{u_{c,it}}{\bar{P}_{i,t}} &= \beta(1+r) E_t \frac{u_{c,it+1}}{\bar{P}_{i,t+1}} + \mu_{i,t} \\
-u_{l,it} &= W_{i,t}\beta E_t \frac{u_{c,it+1}}{\bar{P}_{i,t+1}} \\
u_{h,it} + \mu_{i,t}\theta_{i,t}Q_{i,t} &= \frac{Q_{i,t}}{\bar{P}_{i,t}} u_{c,it} - \beta E_t \frac{Q_{i,t+1}}{\bar{P}_{i,t+1}} u_{c,it+1}
\end{aligned}
$$

where $\mu_{i,t}$ is the multiplier on the borrowing constraint. In the remaining of the paper, we use the following specification for the utility function:

$$
u(\bar{c}_{i,t}, l_{i,t}, h_{i,t}) = \log \bar{c}_{i,t} + \eta \log h_{i,t} - \frac{1}{1+1/\nu} l_{i,t}^{1+\frac{1}{\nu}}.
$$

1.2 Nominal credit

B is nominal credit provided by banks to help consumers purchase goods and services from firms. As in the search literature (#REF), the idea is that consumers are anonymous to firms, but not to banks. Firms therefore cannot trust consumers to repay but they can go after the banks. Banks can keep track of consumers and recuperate a fraction θ of the collateral in case of default.

At the end of the period, the consumer repays $(1+r) B_t$ to the bank, and the bank pays B_t to the firm, and makes a profit equal to $\Pi_t = rB_t$. We assume free entry in the banking sector, thus $r = 0$.[4] Finally, we

[2] An equivalent interpretation of (1) is that houses are purchased with credit, and goods with both cash M_{it} and left-over credit $B_{it} - Q_{i,t}(h_{i,t} - h_{i,t-1})$.

[3] We provide detailed accounting in the appendix.

[4] If there is a nominal cost of creating nominal credit, simply replace $\theta\beta$ by $\theta\beta(1+r)$ in all the equations of the model.

assume that β and θ are low enough for the constraints (1) and (2) to bind in all islands at all times.

1.3 Sticky Wages

So far we have described the program of households as if there were no frictions in the labor market. In the model we assume that nominal wages are sticky. The wage in island i at time t is given by

$$W_{i,t} = (W_{i,t-1})^{\lambda} \left(W_{i,t}^{*}\right)^{1-\lambda} \tag{4}$$

where we define the frictionless nominal wage

$$W_{i,t}^{*} = \frac{-u_{l,it}}{\beta E_t \frac{u_{c,it+1}}{\bar{P}_{i,t+1}}} \tag{5}$$

The parameter λ measures the degree of nominal rigidity. When $\lambda = 1$ wages are fixed, and when $\lambda = 0$ wages are fully flexible.

1.4 House prices

The Euler equation for housing investment pins down the dynamics of $Q_{i,t}$. Using the consumption Euler equation to replace μ_{it}, the housing Euler equation becomes:

$$\frac{\eta}{h_{i,t}} + \left[\frac{u_{c;i,t}}{P_{i,t}} - \beta E_t \frac{u_{c;i,t+1}}{\bar{P}_{i,t+1}}\right] \theta_{it} Q_{i,t} = \frac{Q_{i,t}}{\bar{P}_{i,t}} u_{c;i,t} - \beta E_t \frac{Q_{i,t+1}}{\bar{P}_{i,t+1}} u_{c;i,t+1} \tag{6}$$

We assume that the supply of houses is fixed on each island and equal to h_i. Hence

$$h_{i,t} = h_i \tag{7}$$

together with the evolution of marginal utilities $u_{c;i,t}$ and prices $\bar{P}_{i,t}$ pins down the evolution of house prices.

1.5 Consumption

Household's consumption is an aggregate over the consumption of different varieties of tradeable and non-tradeable goods. We assume that the aggregation function has a constant elasticity of substitution σ between tradeables and non tradeables:

$$\bar{c}_{i,t} = [\omega^{\frac{1}{\sigma}} \left(\bar{c}^{\tau}_{i,t}\right)^{\frac{\sigma-1}{\sigma}} + (1-\omega)^{\frac{1}{\sigma}} \left(c^{n}_{i,t}\right)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}},$$

where $\bar{c}^{\tau}_{i,t}$ is the consumption of the tradeable good, $c^{n}_{i,t}$ is the consumption of the non-tradeable good, and $\omega \in (0,1)$. The tradeable good is itself an aggregate of the goods produced on different islands, with elasticity of substitution γ between goods produced on different islands:

$$\bar{c}^{\tau}_{i,t} = \left(\int_{j} c^{\tau}_{i,t}(j)^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}$$

where j denotes the island where the good is produced. Let $\bar{P}^{\tau}_{t}$ denote the price index for tradeable goods. It is common to all islands since we assume no trade costs, and it given by $\bar{P}^{\tau}_{t} \equiv \left(\int_{i} \left(P^{\tau}_{i,t}\right)^{1-\gamma}\right)^{\frac{1}{1-\gamma}}$, where $P^{\tau}_{i,t}$ denotes the price at which the tradeables produced on island i are sold. Let $P^{n}_{i,t}$ denote the price of non-tradeable goods in island i. The total consumption price index on island i is: $\bar{P}_{i,t} \equiv \left[\omega \left(\bar{P}^{\tau}_{t}\right)^{1-\sigma} + (1-\omega)\left(P^{n}_{i,t}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$. Demand for non-tradeables is:

$$c^{n}_{i,t} = (1-\omega) \left(\frac{P^{n}_{i,t}}{\bar{P}_{i,t}}\right)^{-\sigma} \bar{c}_{i,t} \tag{8}$$

The demand on island i for tradeables produced by island j is:

$$c^{\tau}_{i,t}(j) = \omega \left(\frac{P^{\tau}_{j,t}}{\bar{P}^{\tau}_{t}}\right)^{-\gamma} \left(\frac{\bar{P}^{\tau}_{t}}{\bar{P}_{i,t}}\right)^{-\sigma} \bar{c}_{i,t} \tag{9}$$

1.6 Production

We assume perfect competition in both tradeables and non-tradeables. Each island is inhabited by a continuum of firms that produce a tradeable good, and a continuum of firms that produce a non-tradeable good.

We also assume labor is the only factor and constant returns:

$$y^n_{i,t} = l^n_{i,t} \text{ and } y^\tau_{i,t} = l^\tau_{i,t} \tag{10}$$

Because of perfect competition the price of both tradeable and non-tradeable goods is equal to the nominal wage on the island:

$$P^\tau_{i,t} = P^n_{i,t} = W_{i,t} \tag{11}$$

Market clearing in the tradeable sector requires

$$y^\tau_{i,t} = \int\limits_{j\in[0,1]} c^\tau_{j,t}(i) \tag{12}$$

and in the non tradeable sector it is simply $y^n_{i,t} = c^n_{i,t}$. With free entry in banking and perfect competition and constant returns in goods markets, aggregate profits are zero.[5]

2 Equilibrium

The supply of houses is $h_{i,t} = h_i$. An equilibrium is a collection of prices, $Q_{i,t}$, $P_{i,t}$, $\bar{W}_t$, $W_{i,t}$, and allocations, $B_{i,t}$, $y^n_{i,t}$, $y^\tau_{i,t}$, $l^n_{i,t}$, $l^\tau_{i,t}$, $l_{i,t}$, $c^n_{i,t}$, $c^\tau_{i,t}(j)$, satisfying (1), (2), (3), (4), (5), (6), (7), (8), (9), (10), (11), and (12). To complete the description of the economy, we need to specify the shocks and the government policy.

The shock in our model is a change in the tightness of the borrowing constraint. We assume the following dynamics

$$\ln\theta_{i,t} = (1-\rho)\ln\bar{\theta} + \rho\ln\theta_{i,t-1} + \varepsilon_{i,t} \tag{13}$$

where $\bar{\theta}$ is the (common to all islands) steady state value of the collateral constraint.

Transfers $\{T_{i,t}\}_{i,t}$ and money supplies $\{M_{i,t}\}_{i,t}$ must be consistent with the budget constraints (3). If we aggregate the budget constraints across islands, and we impose the fixed housing restriction, we get

[5] Profits come from lenders and industrial firms: $\Pi_t = \Pi^B_t + \Pi^F_t$. Defining aggregate credit as $B_t = \int_i B_{i,t}di$, banking profits are: $\Pi^B_t = rB_t$. With free entry, we have $r = 0$ as explained above. Non tradeable firms end up with profits equal to zero because of perfect competition and constant returns: $P^n_{i,t}c^n_{i,t} - W_{i,t}l^n_{i,t} = 0$. Similarly, tradeable firms end up with $P^\tau_{i,t}\int c^\tau_{j,t}(i) - W_{i,t}l^\tau_{i,t} = 0$. We therefore always have $\Pi_t = 0$.

$\int X_{i,t} = M_t$ or equivalently, $T_t = M_t - M_{t-1}$. In the aggregate, we can impose $M_t = 1$ and set average transfers to zero. A more delicate issue is how to deal with local money supplies. We assume $T_{i,t} = 0$ for all i and t. In this case, local money follows the process

$$M_{i,t} = M_{i,t-1} + W_{i,t-1} l_{i,t-1} - \bar{P}_{i,t-1} \bar{c}_{i,t-1}. \tag{14}$$

It is useful to analyze dynamics by separating the real and nominal sides of the economy.

2.1 Nominal Credit and Liquidity

If we combine the CIA constraint (1) with the collateral constraint equation (2) we obtain the collateralized-credit-in-advance (CCIA) constraint: $\bar{P}_{i,t}\bar{c}_{i,t} = M_{i,t} + \theta_{i,t} Q_{i,t} h_{i,t}$. We define $x_{i,t}$ as nominal spending in island i at time t, $x_{i,t} \equiv \bar{P}_{i,t}\bar{c}_{i,t}$, and $q_{i,t}$ as the housing wealth to spending ratio, $q_{i,t} \equiv \frac{Q_{i,t} h_i}{\bar{P}_{i,t}\bar{c}_{i,t}}$. The CCIA constraint becomes

$$x_{i,t} = \frac{M_{i,t}}{1 - \theta_{i,t} q_{i,t}} \tag{15}$$

With these new variables, we can rewrite the house price equation (6) as

$$\eta + \beta E_t \left[q_{i,t+1}\right] = \left(1 - \theta_{i,t}\left(1 - \beta E_t \left[\frac{x_{i,t}}{x_{i,t+1}}\right]\right)\right) q_{i,t} \tag{16}$$

Equations (15) and (16) provide a lot of intuition for the model. Given processes for $M_{i,t}$ and $\theta_{i,t}$ we could solve for $x_{i,t}$ and $q_{i,t}$ using (15) and (16). This is what we do in a one-island economy with aggregate money supply M_t controlled by a central bank. Note that $\theta_t q_t$ acts as a shock to velocity in equation (15).

Across islands, however, we do not want to assume that $M_{i,t}$ is exogenous, for at least two reasons. First the central bank does not control the allocation of money across industries or locations within a country, and even less across countries in a monetary union. Second, islands accumulate or decumulate government money depending on the private credit shocks that they experience. In particular, it would never be optimal for a government to reset $M_{i,t} = 1$ at the beginning of each period.

In our benchmark model, we set $T_{it} = 0$. Money then becomes a state variable at the island level. Since $\bar{P}_{i,t}\bar{c}_{i,t} = M_{i,t} + B_{i,t}$, $B_{i,t} = \theta_{i,t} q_{i,t} x_{i,t}$ and $M_{i,t} = (1 - \theta_{i,t} q_{i,t})\, x_{i,t}$, we can write the money accumulation process (14) as:

$$(1-\theta_{i,t}q_{i,t})\,x_{i,t} = W_{i,t-1}l_{i,t-1} - \theta_{i,t-1}q_{i,t-1}x_{i,t-1}. \tag{17}$$

This equation shows that islands accumulate public money when labor income exceeds nominal spending. This will play an important role in shaping the response of the economy to credit shocks.

2.2 Labor Markets and Consumption

Nominal wage setting is given by (4), and labor market clearing in each island implies $l_{i,t} = l^n_{i,t} + l^\tau_{i,t}$. Using $x_{i,t}$, we can rewrite the labor supply (5) as

$$\left(l^n_{i,t} + l^\tau_{i,t}\right)^{\frac{1}{\nu}} = W^*_{i,t}\beta E_t\left[x^{-1}_{i,t+1}\right]. \tag{18}$$

Trade and technology pin down labor demands. For local goods, we have $l^n_{i,t} = c^n_{i,t}$, which we can rewrite as

$$l^n_{i,t} = (1-\omega)\,\frac{x_{i,t}W^{-\sigma}_{i,t}}{\bar{P}^{1-\sigma}_{i,t}}. \tag{19}$$

For traded goods, we have $l^\tau_{i,t} = \int_j c^\tau_{j,t}(i)dj$ which we can rewrite as

$$l^\tau_{i,t} = \omega W^{-\gamma}_{i,t}\left(\bar{P}^\tau_t\right)^{\gamma-\sigma}\int\limits_j \frac{x_{j,t}}{\bar{P}^{1-\sigma}_{j,t}} \tag{20}$$

The price indexes are such that

$$\left(\bar{P}_{i,t}\right)^{1-\sigma} = \omega\left(\bar{P}^\tau_t\right)^{1-\sigma} + (1-\omega)\left(W_{i,t}\right)^{1-\sigma} \tag{21}$$

and

$$\left(\bar{P}^\tau_t\right)^{1-\gamma} = \int\limits_j \left(W_{j,t}\right)^{1-\gamma} \tag{22}$$

In this simplified system, we now have eight equations (4, 16, 17, 18, 19, 20, 21, 22) and eight unknowns $\left\{q_{i,t}, x_{i,t}, l^n_{i,t}, l^\tau_{i,t}, W_{i,t}, W^*_{i,t}, \bar{P}_{i,t}, \bar{P}^\tau_t\right\}$.[6]

[6]Total labor income equals total spending $\int_i W_{i,t}l_{i,t} = \int_i \bar{P}_{i,t}\bar{c}_{i,t}$. See appendix for more details on the accounting.

3 Qualitative Properties of the Model

We now study a few special cases to build some economic intuition on how credit shocks affect our model economy. In particular, we explain the differences between aggregate responses over time and cross-sectional responses between islands.

3.1 One island economy

Before considering the general model with a continuum of island, it is useful to study a representative island model.

Steady State

Let us first consider the steady state. We have $\bar{c} = l$ and the labor-leisure condition is $\bar{c}l^{1/\nu} = \beta$. Therefore $l = \bar{c} = (\beta)^{\frac{\nu}{1+\nu}}$ and the only steady state distortion is the (small) labor income wedge. Equations (16) implies $\bar{q} = \frac{\eta}{(1-\beta)(1-\theta)}$, and (15) implies $x = \frac{M}{1-\theta\bar{q}}$, and the price level must be such that

$$\frac{M}{\bar{P}\bar{c}} = 1 - \bar{\theta}\bar{q} \tag{23}$$

The parameters must be such that $\bar{\theta}\bar{q} < 1$, or $(1-\beta)\left(1-\bar{\theta}\right) > \eta\bar{\theta}$. In particular, β, η and θ must all be small enough.

Dynamics

Let us first consider credit dynamics first. Given processes $\{M_t\}_t$ and $\{\theta_t\}_t$ for aggregate money supply and credit tightness, the system can be solved for $\{x_t, q_t\}_t$ using (15) and (16) without reference to the rest of the model, i.e., independently of technology, nominal rigidity, and labor supply preferences. When $\theta = 0$, the solution is always $x_t = M_t$ as in the standard cash-in-advance model. When $\theta > 0$, shocks to are transmitted by the collateral constraint. In the one island economy, we have $W_t = \bar{P}_t$ and the price indexes equations are trivial. We also have $\bar{c}_t = l_t$. Once we have solved for x_t and q_t we can therefore solve for W_t

and l_t by using the equations:

$$\begin{aligned} W_t l_t &= x_t \\ (l_t)^{\frac{1}{\nu}} &= \beta W_t^* E_t\left[x_{t+1}^{-1}\right] \\ W_t &= W_{t-1}^{\lambda}\left(W_t^*\right)^{1-\lambda} \end{aligned}$$

Note that the labor shares are constant in the one island economy. Since $l_t^n = (1-\omega)\frac{x_t}{P_t}$ and $l_t^\tau = \omega\frac{x_t}{P_t}$, we always have $\frac{l_t^n}{l_t^n + l_t^\tau} = 1-\omega$. We can analyze some special cases.

No nominal rigidity.

When $\lambda = 0$, we have only one equation:

$$(l_t)^{\frac{1+\nu}{\nu}} = \beta E_t\left[\frac{x_t}{x_{t+1}}\right]$$

This reflects the intertemporal labor distortion coming from the CIA constraint. So the model without nominal friction is neutral with respect to permanent nominal credit shocks. It is not super-neutral because ϕ is not constant, then x moves around, and this creates intertemporal disturbances in labor supply.

One time permanent shock.

When M and θ are constant, we get: $q(\theta) = \frac{\eta}{(1-\beta)(1-\theta)}$ and $x(\theta) = \frac{M}{1-\theta q}$. After a permanent shock to the borrowing constraint, if monetary policy is unchanged, the economy evolves along a path with constant nominal spending. If the shock is positive, nominal spending jumps up and remains constant. We see that q is increasing in θ: if credit is easier to obtain, housing value must increase relative to consumption spending because the collateral dimension of housing services makes houses more valuable. Spending must go up because of both θ and q. Going back to q, this means that housing prices must increase a lot so that even though spending goes up, the ratio still goes up.

Following a permanent shock, x is constant and since $W_t l_t = x$ and employment is

$$\ln(l_t) = \frac{(1-\lambda)\nu}{1-\lambda+\nu}\ln(\beta) + \frac{\lambda\nu}{1-\lambda+\nu}\left(\ln(x) - \ln(W_{t-1})\right),$$

while W satisfies

$$(1-\lambda+\nu)\ln W_t = \lambda\nu \ln W_{t-1} + (1-\lambda)\left((1+\nu)\log x - \nu \log \beta\right).$$

Without nominal rigidities -$\lambda = 0$- wages adjust to nominal credit shocks and employment remains constant. In general, the persistence of real effects following a permanent credit shock is $\frac{\lambda\nu}{1-\lambda+\nu}$. It depends on the degree of nominal rigidity and on the elasticity of labor supply. If wages are fixed -$\lambda = 1$- the real impact of *aggregate* nominal credit shocks is permanent. We will show that this result does not hold in the cross-section.

3.2 Cross-sectional responses

Two issues arise at the island level. First, $M_{i,t}$ is endogenous since islands can accumulate more or less public money. Second, $W_{i,t}l_{i,t} \neq \bar{P}_{i,t}c_{it}$ since some goods are traded. The two issues are related by $M_{i,t+1} - M_{i,t} = W_{i,t}l_{i,t} - \bar{P}_{i,t}\bar{c}_{i,t}$. Credit dynamics satisfy (16) and (17). The eight equilibrium conditions have been described earlier.

Steady State.

All islands have same real allocations: $l = \bar{c} = (\beta)^{\frac{\nu}{1+\nu}}$. Since $l^n = (1-\omega)\,\bar{c}$ and $l_t^\tau = \omega\bar{c}$, we always have $\frac{l^n}{l^n+l^\tau} = 1-\omega$. All wages are the same $W_i = \bar{P}$. Therefore all x_i are the same. The CIA constraints determine the required money balances $M_i = (1-\theta_i q_i)\,\bar{P}\bar{c}$. With constant x, we have $q_i = \frac{\eta}{(1-\beta)(1-\theta_i)}$. In the aggregate, we must have, $\int M_i = M$ so the price level must solve

$$\frac{M}{\bar{P}\bar{c}} = \int_i \left(1 - \frac{\eta\theta_i}{(1-\beta)(1-\theta_i)}\right) di. \tag{24}$$

Equation (24) is the generalization of (23) to an economy with heterogeneous nominal credit supplies.

Wages and heterogeneity in employment

With fixed wages we have: $W_{it} = W_{it-1}$. All prices are unchanged. Then $\Delta \ln\left(l_{i,t}^n\right) = \Delta \ln x_{i,t}$ shows that nominal spending moves tradeable employment one for one in the cross section. By contrast, there are no cross sectional differences in tradeable employment: $\Delta \ln\left(l_{i,t}^\tau\right) = \Delta \ln\left(\int_j \frac{x_{j,t}}{\bar{P}_{j,t}^{1-\sigma}}\right)$ is the same in all i.

3.2.1 Log Linear System

We now present log-linear approximations. For any variable z_{it} we write

$$z_{i,t} = \bar{z}\left(1 + \hat{z}_t + \hat{z}_{it}\right),$$

where $\hat{z}_t$ is the solution to the one-island log-linear model, and the total log-change is $d \ln z_{i,t} = \hat{z}_t + \hat{z}_{it}$. The first part of the island-level system deals with trade and labor demand. In the aggregate, we have $P_t = W_t$. Around these aggregate dynamics, we have:

$$\begin{aligned}
\hat{l}^n_{i,t} &= \hat{x}_{i,t} - \sigma \hat{W}_{i,t} - (1-\sigma)\,\hat{\bar{P}}_{i,t}, \\
\hat{\bar{P}}_{i,t} &= (1-\omega)\,\hat{W}_{i,t}, \\
\hat{l}^\tau_{i,t} &= -\gamma \hat{W}_{i,t}.
\end{aligned}$$

So we have $\hat{l}^n_{i,t} = \hat{x}_{i,t} - (1 - \omega(1-\sigma))\,\hat{W}_{i,t}$. Since $\hat{l}_{i,t} = (1-\omega)\,\hat{l}^n_{i,t} + \omega \hat{l}^\tau_{i,t}$, we obtain

$$\hat{l}_{i,t} = (1-\omega)\,\hat{x}_{i,t} - (\omega\gamma + (1-\omega)(1-\omega(1-\sigma)))\,\hat{W}_{i,t}. \tag{25}$$

This equation links island level employment to island level nominal spending on non tradeable goods and island specific wage. Compare to the aggregate economy, employment is less sensitive to (local) spending. The wage elasticity of labor demand depends on both elasticities γ and σ, and on the importance of traded goods ω.

The second part of island level system deals with labor supply and wage dynamics:

$$\begin{aligned}
\hat{W}_{i,t} &= \lambda \hat{W}_{i,t-1} + (1-\lambda)\,\hat{W}^*_{i,t}, \\
\hat{l}_{i,t} &= \nu \left(\hat{W}^*_{i,t} - E_t\left[\hat{x}_{i,t+1}\right]\right).
\end{aligned}$$

Solving for the desired wage, we obtain wages dynamics as a function of total spending:

$$(1-\lambda)\left(\hat{l}_{i,t}+\nu E_t\left[\hat{x}_{i,t+1}\right]\right)=\nu\left(\hat{W}_{i,t}-\lambda\hat{W}_{i,t-1}\right). \tag{26}$$

This equation is relevant only when $\lambda<1$. When $\lambda=1$, wages are fixed and we can ignore (26).

The third and last part of the system describes credit dynamics. In the aggregate, we have $x_t=W_t l_t$. At the island level, we have:

$$\left(1-\bar{\theta}\bar{q}\right)\hat{x}_{i,t}-\bar{\theta}\bar{q}\hat{q}_{i,t}=\hat{W}_{i,t-1}+\hat{l}_{i,t-1}-\bar{\theta}\bar{q}\left(\hat{q}_{i,t-1}+\hat{x}_{i,t-1}\right)+\bar{\theta}\bar{q}\left(\hat{\theta}_{i,t}-\hat{\theta}_{i,t-1}\right), \tag{27}$$

and

$$\beta E_t\left[\hat{q}_{i,t+1}+\bar{\theta}\hat{x}_{i,t+1}\right]=\left(1-(1-\beta)\bar{\theta}\right)\hat{q}_{i,t}+\bar{\theta}\beta\hat{x}_{i,t}-(1-\beta)\bar{\theta}\hat{\theta}_{i,t}. \tag{28}$$

We therefore have a system of four equations (25, 26, 27, 28) in four endogenous unknowns $(\hat{W}_{i,t},\hat{l}_{i,t},\hat{x}_{i,t},\hat{q}_{i,t})$ and one exogenous processes for $\theta_{i,t}$. We calibrate and solve system numerically in Section 4, but much intuition can be gained by considering the special case of fixed wages.

3.2.2 Permanent shocks and fixed (relative) wages

We consider permanent shocks to $\theta_{i,t}$ so after the initial shock $\theta_{i,0}$ at $t=0$, we have $\hat{\theta}_{i,t}=\hat{\theta}_{i,t-1}$ for $t=1,..\infty$ and the credit system (27,28) is simplified. We also assume that relative wages do not change: $\hat{W}_{i,t}=0$.[7] With fixed relative wages we have $\hat{l}_{i,t}=(1-\omega)\hat{x}_{i,t}$, and the money accumulation equation (27) becomes:

$$\left(1-\bar{\theta}\bar{q}\right)\hat{x}_{i,t}-\bar{\theta}\bar{q}\hat{q}_{i,t}=\left(1-\omega-\bar{\theta}\bar{q}\right)\hat{x}_{i,t-1}-\bar{\theta}\bar{q}\hat{q}_{i,t-1}.$$

We are going to 'guess and verify' a solution of the type:

$$\hat{q}_{i,t}=\tilde{q}_i-a\hat{x}_{i,t}. \tag{29}$$

[7] This could be either because wages are rigid in nominal terms, $\lambda=1$, or because nominal wages are sticky in the aggregate and relative wages are fixed across islands. In the first case, we can drop equation (26). In the second case, we are simply saying $W_{it}=W_t$ in all islands. (Empirically, this might be a reasonable approximation to the data. Theoretically, we know that $W_{it}=W_t$ in the long run. See below for a discussion of what happens if relative wages move.

The intuition for why this is a good guess comes from aggregate dynamics and steady state cross section. In the aggregate, we know that permanent shocks to θ lead to constant value for x and q. This is not going to be the case here, so x will move, and q will be affected. In the cross sectional steady state, we have $q_i = \frac{\eta}{(1-\beta)(1-\theta_i)}$ so it is easy to guess that there must be a time invariant component to q.

The money accumulation equation implies

$$\hat{x}_{i,t} = \left(1 - \frac{\omega}{1 - \bar{\theta}\bar{q}\,(1-a)}\right)\hat{x}_{i,t-1}. \tag{30}$$

In the special case $\omega = 0$, we go back to the one island economy with constant x. The house pricing equation becomes

$$\beta\left(\bar{\theta} - a\right)E_t\left[\hat{x}_{i,t+1}\right] + \beta\tilde{q}_i = \left(1 - (1-\beta)\,\bar{\theta}\right)\left(\tilde{q}_i - a\hat{x}_{i,t}\right) + \bar{\theta}\beta\hat{x}_{i,t} - (1-\beta)\,\bar{\theta}\hat{\theta}_i$$

We can now identify the constant terms and the dynamic terms. For the constant term we get $\tilde{q}_i = \frac{\bar{\theta}}{1-\theta}\hat{\theta}_i$. This is what we expected since the long run value for $\hat{q}_i$ implies $d\log q_i = -d\log(1-\theta_i) = \frac{\bar{\theta}}{1-\theta}\hat{\theta}_i$.

For the dynamic terms we get

$$E_t\left[\hat{x}_{it+1}\right] = \left(1 - \frac{a(1-\beta)\left(1-\bar{\theta}\right)}{\beta\left(\bar{\theta} - a\right)}\right)\hat{x}_{it}$$

Using perfect foresight and the law of motion (30), we get an equation for a:

$$\omega\left(\bar{\theta} - a\right)\beta = a(1-\beta)\left(1-\bar{\theta}\right)\left(1 - \bar{\theta}\bar{q}\,(1-a)\right) \tag{31}$$

We can find a solution for a, which validates our initial guess in equation (29). If $\omega = 0$, we have $a = 0$ as in the closed economy. When $\omega > 0$, the LHS of (31) decreases and reaches zero when $a = \bar{\theta}$, while the RHS is zero when $a = 0$ and increases afterward. There is therefore a unique solution $0 < a < \bar{\theta}$. Equation (30) shows that the system is stable and $\lim_{t\to\infty}\hat{x}_{it} = 0$.[8]

[8] Finally, for small values of $\bar{\theta}$ (the relevant empirical case) we have $a \approx \frac{\omega\beta\bar{\theta}}{\omega\beta + (1-\beta)\left(1-\bar{\theta}\right)\left(1-\bar{\theta}\bar{q}\right)}$. Since θ is small, a is also small, and the persistence of nominal spending is approximately

$$1 - \frac{\omega}{1-\bar{\theta}\bar{q}} \approx 1 - \frac{0.22}{1-0.5} \approx 0.56$$

using an annual calibration with low tradeable share.

In the cross section, permanent shocks have temporary consequences because money can flow across islands. The persistence of shocks at the island level does not depend much on the degree of nominal rigidity. This is in sharp contrast with the response of the aggregate economy. The reason is that islands that are hard hit by the nominal credit shock accumulate money balances

$$M_{i,t+1} - M_{i,t} = \bar{x}\left(\hat{l}_{i,t} - \hat{x}_{it}\right) = -\omega\bar{x}\hat{x}_{i,t}.$$

This shows again the role of trade in smoothing the cross-sectional shocks.

The impact response, assuming we start from steady state with $\hat{\theta}_{i,t-1} = 0$, is $\left(1 - \bar{\theta}\bar{q}\right)\hat{x}_{i,0} - \bar{\theta}\bar{q}\hat{q}_{i,0} = \bar{\theta}\bar{q}\hat{\theta}_i$ and since $\hat{q}_{i,0} = \tilde{q}_i - a\hat{x}_{i,0}$ we have

$$\left(1 - (1-a)\,\bar{\theta}\bar{q}\right)\hat{x}_{i,0} = \frac{\bar{q}}{1-\bar{\theta}}\bar{\theta}\hat{\theta}_i.$$

A positive shock to credit increases spending in the island.

Finally we can come back to our assumption of constant wages. If relative wages can move, they will help smooth the transition by making hard hit islands temporarily more competitive. Without this we force all the adjustment through consumption and nominal spending. But the main intuition should not change much. We can see which way wages want to adjust by looking at equation (26). Since $\hat{l}_{i,t} = (1-\omega)\,\hat{x}_{i,t}$ and since $x_{i,t}$ follows an AR(1) process, wages would like to follow an AR(2) process. We thus expect the response of wages to be hump-shaped. Following a negative shock, relative wages fall first, then rise back to one, the long run value. As long as labor supply is somewhat elastic, the response of wages is small, and the dynamics derived under the assumption of fixed wages give a good approximation.

3.3 Comparison of Time Series and Cross-Section

Let us compare the time-series and cross sectional responses of the economy to permanent shocks to credit supply. In the aggregate we have $q(\theta) = \frac{\eta}{(1-\beta)(1-\theta)}$ and $x(\theta) = \frac{M}{1-\theta q}$. Therefore, on impact, we have $d\ln q = \frac{\bar{\theta}}{1-\bar{\theta}}d\ln\theta$ and thus $\frac{\partial\ln(x)}{\partial\ln(\theta)} = \frac{\bar{\theta}\bar{q}}{\left(1-\bar{\theta}\right)\left(1-\bar{\theta}\bar{q}\right)}$. Across islands, relative housing wealth evolves as $d\ln q_{i,t} = \tilde{q}_i - a\hat{x}_{i,t}$. The permanent component, $\tilde{q}_i = \frac{\bar{\theta}}{1-\bar{\theta}}\hat{\theta}_i$, is the same as in the aggregate case. Because of the temporary component, however, the adjustment of relative housing wealth is gradual. Spending reacts according to: $\frac{\partial \ln(x_i)}{\partial \ln(\theta_i)} = \frac{\bar{\theta}\bar{q}}{\left(1-\bar{\theta}\right)\left(1-(1-a)\bar{\theta}\bar{q}\right)}$. The response of local spending to local credit is muted by a. For employment, we have $\frac{\partial \ln(l_{i,0})}{\partial \ln(x_{i,0})} = 1 - \omega$. We summarize the employment responses in Table 2.

Table 3: Elasticities with Fixed Wages and Permanent Credit Shocks

$\lambda = 1,\ \rho = 1$	Aggregate	Across Islands
Spending to Credit	$\frac{\partial \ln(x)}{\partial \ln(\theta)} = \frac{\theta\bar{q}}{\left(1-\bar{\theta}\right)\left(1-\bar{\theta}\bar{q}\right)}$	$\frac{\partial \ln(x_{i,0})}{\partial \ln(\theta_i)} = \frac{\theta\bar{q}}{\left(1-\bar{\theta}\right)\left(1-(1-a)\bar{\theta}\bar{q}\right)}$
Labor to Spending	$\frac{\partial \log(l_0)}{\partial \log(x_0)} = 1$	$\frac{\partial \ln(l_{i,0})}{\partial \ln(x_{i,0})} = 1 - \omega$
Persistence	permanent	temporary

With fixed wages, spending is equal to real consumption. So Table 2 also shows that in the cross section, employment react by ω% less than consumption, while in the time series, it is one for one.

We summarize our results in the following Proposition.

Proposition 1. *Cross sectional responses are muted in three ways relative to time series responses. Upon impact, local spending reacts less to local credit, labor reacts less to spending, and the effects dissipate over time even when the shocks are permanent and wages are fixed.*

–

3.4 Some Impulse Responses

We report some impulse responses to further illustrate the workings of the model. Figure 1 shows impulse responses to a 1% aggregate (common to all islands) drop in θ_t in this economy[9]. W^* drops by about 1.3% while actual wages adjust more gradually due to nominal rigidities. As a result consumption and employment drop by about 0.65%. House prices drop because nominal spending drops and because the drop in θ_t makes houses less useful in undoing the borrowing constraints. The drop in B is therefore (much) larger than the drop in θ and we have an amplification mechanism.

[9]We report the parameter values used in this calculation in Table 4 (column 2) below.

Figure 1: Aggregate Responses

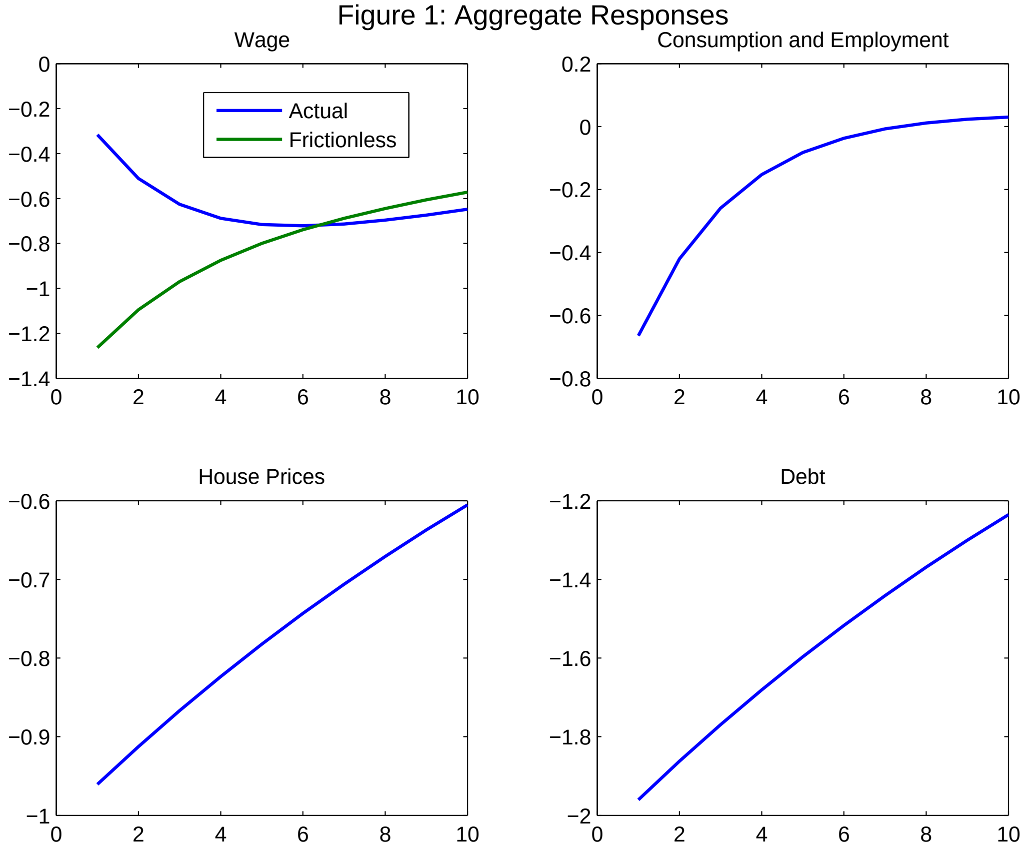
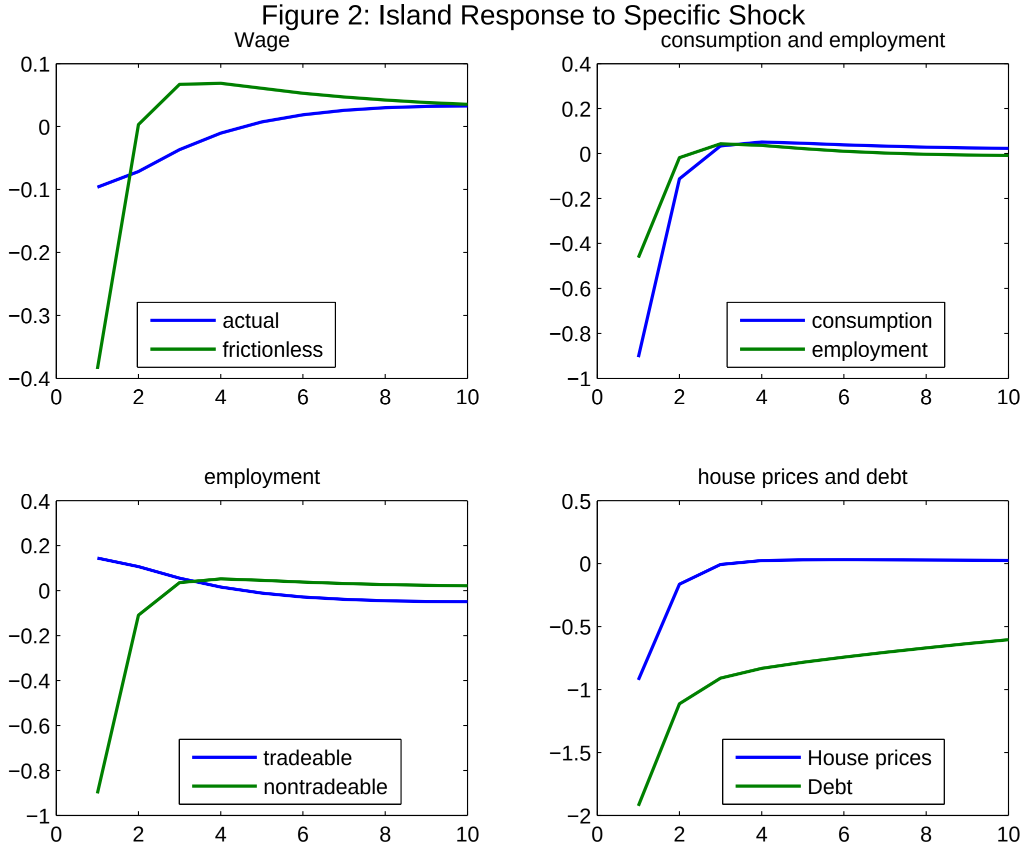


Figure 2 reports similar responses to an island-specific shock, $\theta_{i,t}$, assuming all other islands are at their steady-state values. Consumption responds by more (-0.9% on impact) than employment does (-0.45% on impact) because that wages decrease in the island and hence demand for its tradeables increases. From the results of the previous section, we know that when shocks are permanent and wages rigid, the ratio of change between l and c is $1-\omega$, which is 0.58 for our benchmark value of $\omega = 0.42$. In the actual simulation, the ratio is 0.51, which is close but, as expected, a bit less since wages do adjust.

Figure 2: Island Response to Specific Shock



The response of wages the persistence of all series are a lot smaller than in the aggregate case because of endogenous monetary adjustment as can be seen on Figure 3.

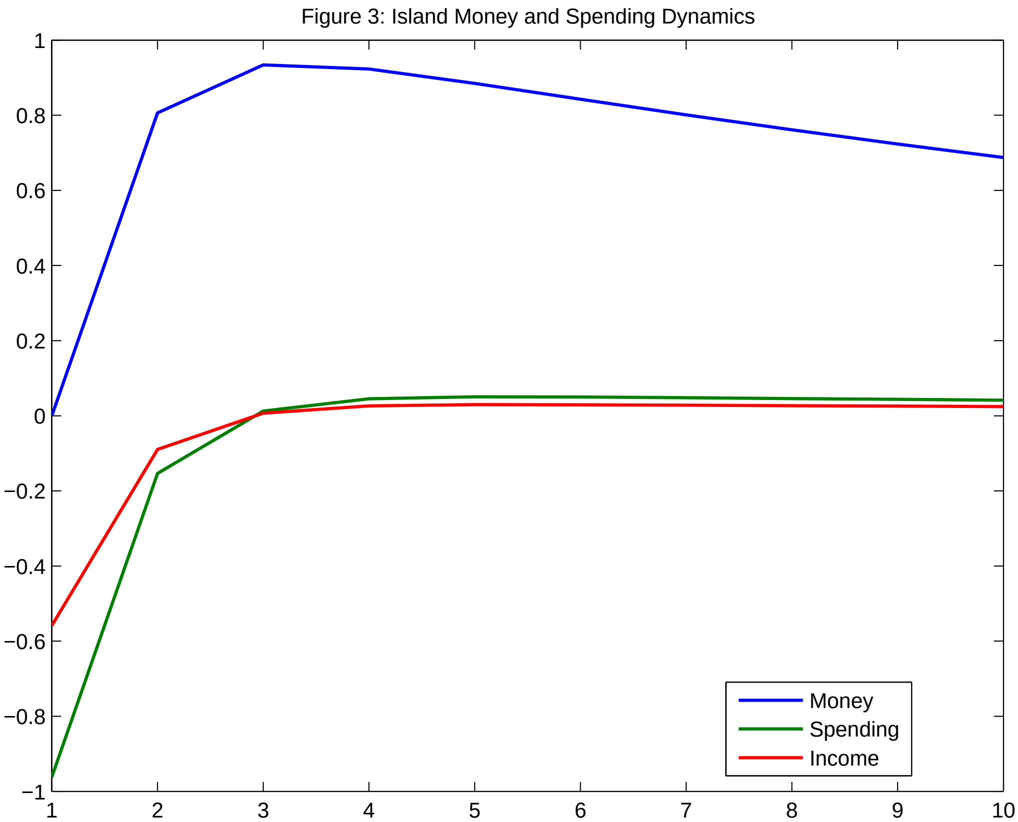


Figure 3 shows the evolution of nominal variables. The fact that consumption drops more than employment implies that the island accumulates public money, M, which rises after the shock. This increase compensates the decline in private money, so that nominal spending reverts to the steady-state in about 5 quarters.

4 Quantitative Results

4.1 Calibration

We assume that a period is one quarter. For the borrowing constraints to bind in equilibrium, households must be sufficiently impatient. We therefore set $\beta = 0.975$. We set $\lambda = 0.75$ implying a median length of fixed wage of 1 year.

Table 5: Flow of Funds Data

($ Trillions)	Model	2002	2007	2009
Annual Consumption	$P\bar{c}$	7.40	9.83	10.09
Value of Housing Stock	Qh	14.90	20.98	16.57
Non Home Debt	.	2.00	2.56	2.48
Home Debt	.	6.01	10.54	10.33
$Qh/P\bar{c}$	q	2.01	2.13	1.64

The key parameter in our model are θ and η, the parameters that determines the share of consumption financed by credit. Table 3 shows that the value of housing stock relative to consumption expenditure, q, is equal to 2 at the annual frequency or 8 at the quarterly frequency. In steady state, we have that $q = \frac{\eta}{(1-\beta-\theta[1-\beta(1+r)])}$, so we choose $\eta = 8\left(1-\beta-\theta\left[1-\beta\left(1+r\right)\right]\right)$.

To pin down the steady-state value of the collateral constraint, θ, we use micro evidence from Mian and Sufi (2010a). Mian and Sufi (2010a) argue that borrowing against the value of home equity accounts for a significant fraction of the rise in US household leverage from 2002 to 2006. They follow from 1997 to 2008 a random sample of 74,000 U.S. homeowners (who owned their homes as of 1997) in 2,300 zip codes located in 68 MSAs. As of 1997, median total debt is \$100,000 of which \$88,000 is home debt (home equity plus mortgages), and the debt to income ratio is 2.5. Total debt grows by 8.6% between 1998 and 2002, and by 34.4% between 2002 and 2006. These changes are driven by home debt growth. The debt to income ratio does not change from 1998 to 2002 and then increases by 0.75.

They argue that there is a causal link from house price growth to borrowing. The critical issue is that house price growth is endogenous. An omitted factor, such as expected income growth, could be driving both house prices and current borrowing (and consumption). To identify a causal link they use instruments for house price growth based on housing supply elasticity at the MSA level.

In their estimates, a \$1 increase in house price causes a \$0.25 increase in home equity debt. This suggests $\bar{\theta} = 0.25$ at annual frequency, or $\bar{\theta} = 0.25/4 = 0.0625$ with quarterly data. To get a sense of what this means note that, in our model, the fraction of consumption that is financed with home-linked credit is $\bar{\theta}\bar{q} = 0.5$ in our benchmark calibration. Between 2002 and 2006 consumption spending went up by about \$2T while

Mian and Sufi (2010a)'s numbers predict that home equity based borrowing due to house price appreciation accounts for \$1.25T of the rise in household debt from 2002 to 2006, which is a bit more than half of the increase in spending.

There are two ways to calibrate the share of tradeables and the elasticity of substitution. We can use the BEA data on Personal Consumption Expenditure. We identify tradeables with "goods" and non-tradeables with "services excl. housing". The share of tradeables shows a trend decline over time and is around 0.42 in 2002. We thus choose $\omega = 0.42$. Finally, we choose an elasticity of substitution between tradeables and non-tradeables, σ, in order to match the comovement of the relative price of tradeables to non-tradeables and the share of tradeables in the data. In the data, there was a substantial decline in the relative price of tradeables and only a modest increase in real tradeables consumption. A value of σ equal to 0.1 fits this evidence best.

Another way to think about ω is to look at cross sectional responses of consumption and employment in Mian and Sufi (2010b). Aggregate auto sales drop by 30% but with a lot of heterogeneity. In safe places (where household leverage was low) they decline by only 20%. In hard-hit places, they decline by 60%, three times more. Moving from one to the other, consumption drops three times more (of course total consumption is less volatile, but it is the ratio that matters here). Looking at employment for the same places, the numbers are -3% and -7%, an increase of 2.33. This suggests $1-\omega = 2.33/3 = 0.777$ or $\omega = 0.222$. The discrepancy probably comes from the distinction between short term and long term elasticities. In the model, with constant returns, workers can shift immediately into the tradeable sector to take advantage of increased demand. This dampens the response of employment relative to consumption. In the data, there are probably decreasing returns in the short run, so this shift is less drastic. A simple way to capture it is to lower ω. Another reason to lower ω is distribution costs.

It is more difficult to pin down the elasticity of substitution between tradeables produced on different islands, γ. In the international trade and macro literature, estimates of trade elasticities range from 0.5 to 4. We consider below a value equal to $\gamma = 1.5$, the typical value used in the international macro literature. It turns out that the value of γ is not very important as long as there is enough wages stickiness.

4.2 Quantitative Experiments

To compare the dynamics of the model economy with the actual behavior of the US economy from 2002 to 2009, we must calibrate the process that governs the evolution of θ_{it} in each island. We would like our model to capture the gradual rise in household leverage from 2002 to 2006 and its subsequent decline. To generate a gradual increase in θ_{it}, we assume $\varepsilon_{it} = \varepsilon_i$ for 16 periods (quarters). This corresponds to the gradual increase in household leverage in the US from 2002 (Q4) to 2006 (Q4). Thereafter we set $\varepsilon_{it} = 0$, so that θ_{it} reverts to its steady-state value. Since Mian and Sufi report the effect of the increase in household leverage from 2002-2006 on the evolution of county-level variables from 2006 (Q4) to 2009 (Q2), we report similar statistics for periods 17 to 28 (10 quarters).

We assume ε_i is uniformly distributed across islands over the interval $[\underline{\varepsilon}, \bar{\varepsilon}]$. These upper bounds are chosen to match two statistics reported by Mian and Sufi (2010b). The first statistics is the mean percentage change in debt-to-income ratio across counties of 35% (0.775/2.211 in Table 1 of their paper). The second is the coefficient of variation of the change in the debt-to-income ratio of 0.68 (0.53/0.775 in Table 1 of their paper). We choose $\underline{\varepsilon}$ and $\bar{\varepsilon}$ to ensure the model matches these statistics exactly.

Table 7: Parametrization

	Data	Model
Mean %Δ debt-to-income, 2002-2006	35%	35%
CV Δ debt-to-income, 2002-2006	0.68	0.68
Nominal Rigidity λ		0.75
Elasticity across Tradeables γ		1.5
Tradeables Share ω	0.22 or 0.42	
Lending Constraint θ	0.0625	0.0625
$\underline{\varepsilon}$		-0.0035
$\bar{\varepsilon}$		0.073

Table 4 summarizes the parameter values and statistics we target. It reports the upper and lower bounds on the distribution of shocks. Since these values, on their own, are not very intuitive, we report the upper and lower bound on the change in debt-to-income across islands during the expansionary period : -6% and + 79%, respectively.

Figure 4: Response of aggregate variables

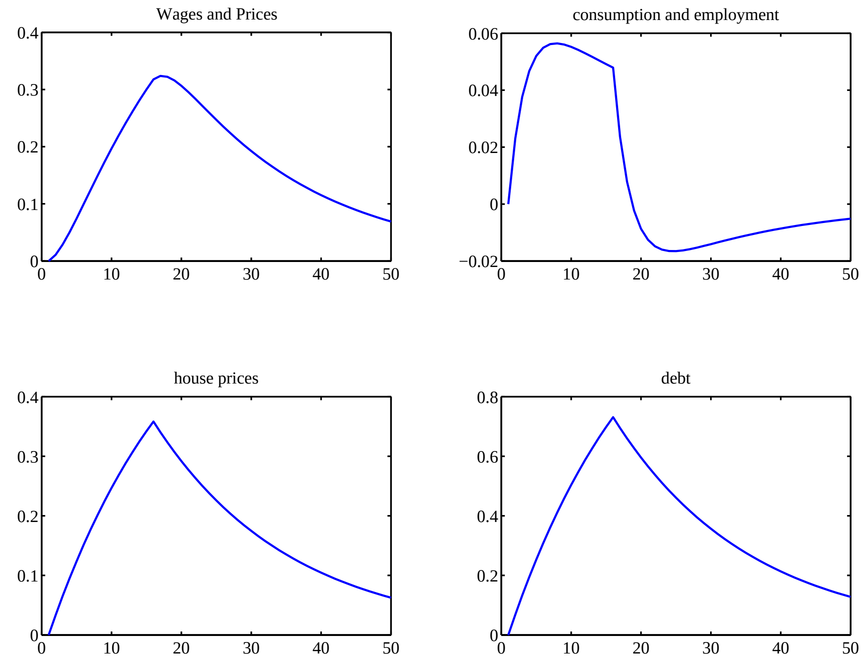


Figure 4 reports the aggregate dynamics. Note from 2002 to 2006 wages and prices increase by about 30%, aggregate consumption/employment increase by about 5%, house prices increase by about 35%, while nominal debt increase by about 70%. After the contraction in credit, wages and prices decline, consumption and employment decrease by about 7%, while debt and house prices decline.

Figure 5: Cross−sectional predictions

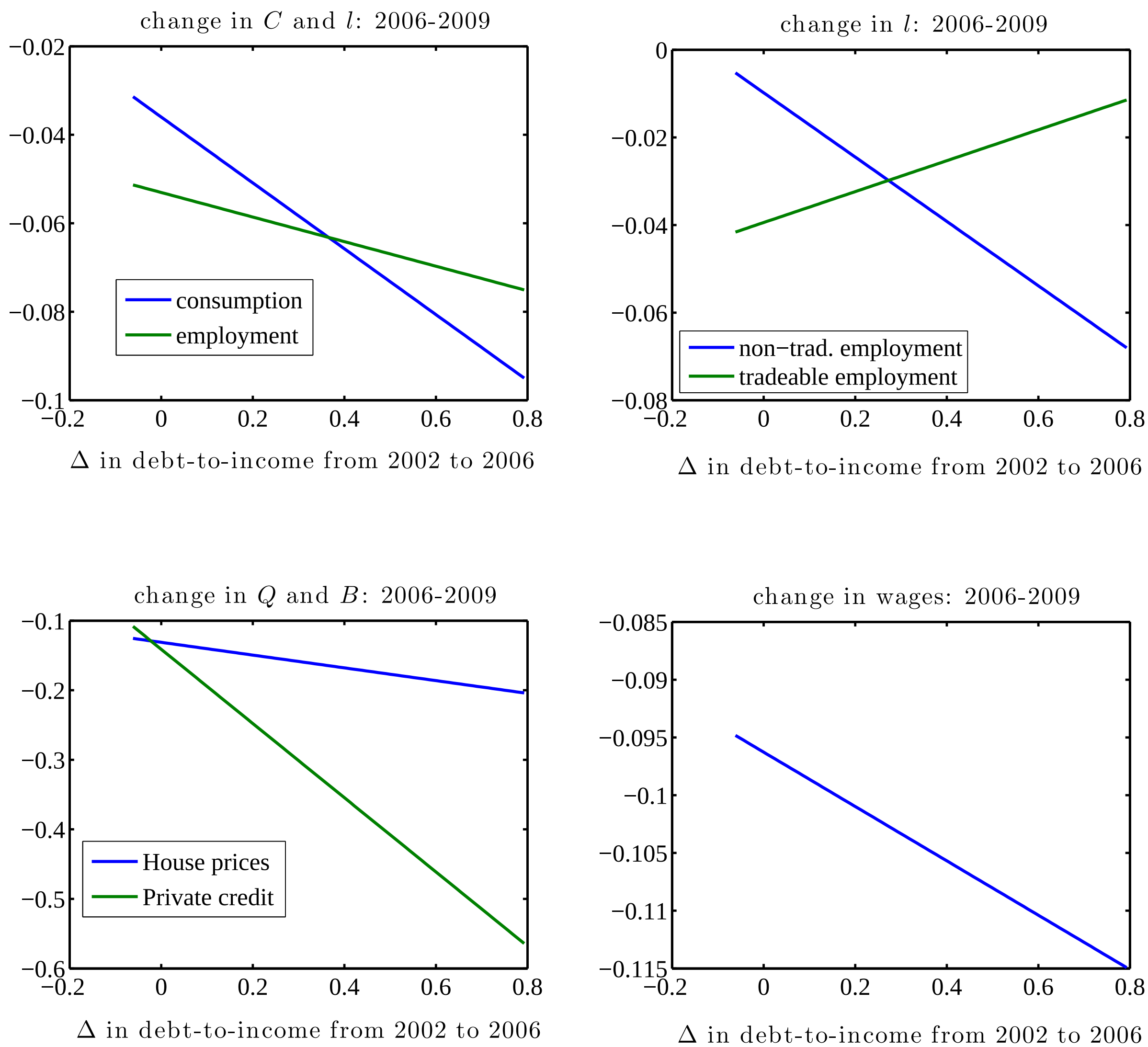


Figure 5 shows the model's cross-section implications: the relationship between each islands' change in debt-to-income ratio from 2002-2006 against the response of equilibrium variables after the tightening of the constraint post-2006. Notice the strong relationship between consumption and change in debt: the islands that have expanded most prior to 2006 experience a consumption drop that is 6.5% greater than that of islands where debt-to-income has expanded little (-3% vs. -9.5%). The response of employment is dampened, however, and the maximum difference in the drop of employment is about 2% (-5% vs. -7%). This weaker response is accounted for by the fact that non-tradeable employment (which strongly declines in islands with large increases in household leverage prior to the crisis) is offset by tradeable employment (which strongly increases in the most leveraged islands). Notice also the model generates a fairly weak differential response of housing prices and (as expected) wages.

Table 9: Quantitative predictions

2007-2009	Benchmark	Economy 2	Flexible wages
aggregate Δ C, %	-6.32%	-13.57%	-0.56%
max-min ΔC_i, %	6.36	12.40	13.45
iqr ΔC_i, %	3.21	6.26	6.79
max-min Δl_i,%	2.37	6.50	3.29
iqr Δl_i, %	1.20	3.28	1.66
max-min ΔQ_i, %	7.84	22.28	35.69
iqr ΔQ_i,%	3.96	11.25	18.02

Table 5 summarizes this discussion by reporting the key statistics of the model (column labeled 'Benchmark' is the model described above). Here we have that aggregate consumption declines by about 6.4%. The max-min drop in consumption across islands is about 6.4% while the inter-quartile range is 3.2%. Employment responses are more similar across islands: the max-min drop in employment is 2.4%, while the inter-quartile range is 1.2%. Finally, the model does not generate much dispersion in the evolution of house prices: the max-min drop is 7.8%, while the inter-quartile range is 3.96%.

The second column of Table 5 reports results from an experiment with stickier wages $(\lambda = 1 - 1/8)$, i.e., in which wages change once every 8 quarters, lower elasticity of substitution across tradeables, $\gamma = 1$, and a lower share of tradeables, $\omega = 0.22$, chosen as described above.

We see that in this economy aggregate consumption drops a lot more (about 13.5%), and the dispersion in the change in consumption (iqr is 6.26%) and labor (iqr is 3.28%) is larger than earlier. Moreover, the model generates also more dispersion in house price changes (iqr 11.25). These results are mostly accounted for by the decrease in the tradeables share in this experiment.

What is the role of sticky wages in this analysis? To answer this question, the third column of Table 5 studies an economy with $\lambda = 0$, $\omega = 0.22$ and $\gamma = 1$. Note consumption in the aggregate is essentially flat (dropping by 0.56% only compared to about 13.5% earlier), but the dispersion of consumption growth across islands does not change much. The model generates more dispersion in wage and price changes, and also in house price changes. Employment dispersion declines by about one-half. This does not fit the data well, since employment dispersion is now much too low relative to consumption dispersion.

5 Conclusion - PRELIMINARY

We view our main contributions as follows. First, using cross-sectional evidence gives us a lot of information to to pin down the key parameters of the model. The cross-sectional evidence of employment and consumption clearly favors model 2.

Second, and perhaps most importantly, our novel calibration strategy allows us to run counter-factual experiments. We can ask what the aggregate response to the crisis would have been for any path of official money M. In our calibration the Fed is passive and model 2 predicts a drop in real (nominal) consumption of 13.5% (16.81%). Instead the actual drops were of 6.3% (5.35%). We can attribute the difference to the expansion of the Fed's balance sheet during the crisis.

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Appendix

Accounting

For each agent (household, firms, banks) we can keep track of two accounts: government currency and private credit account. Let δ_{it} denote the transfers from the cash account to the credit account, and let Π_t^m denote the profits paid in cash. Households reach the end period t with liquidity:

$$X_{it} \equiv M_{i,t} - \left(\bar{P}_{i,t}\bar{c}_{i,t} - B_{i,t}\right) + \Pi_t^m - \delta_{it}.$$

We assume that labor payments and housing transactions are done in credit accounts. Let $W_{i,t}$ denote the nominal wage and let Π_t^c be the profits paid in the credit accounts. At the end of period t, households repay debt to the banks $(1+r)\,B_{i,t}$, and therefore end up with a credit balance:

$$Z_{it} \equiv \Pi_t^c + W_{i,t}l_{i,t} - (1+r)\,B_{it} - Q_{i,t}\left(h_{i,t} - h_{i,t-1}\right) + \delta_{it}.$$

Credit accounts must clear at the end of each period. For all i and t, we must have:

$$Z_{it} = 0.$$

This pins downs δ_{it}, which we can then use to write the the flow budget constraint of the consumer (3). $\Pi_t = \Pi_t^c + \Pi_t^m$ are total profits (dividend payments). The consumer ends the period with cash on hand X_{it} and no credit balance.

First order conditions for households

We write the relevant part of the Lagrangian as

$$
\begin{aligned}
\mathcal{L} &= u\left(\bar{c}_{it}, l_{i,t}, h_{i,t}\right) + \chi_t\left(M_{i,t} + B_{i,t} - \bar{P}_{i,t}\bar{c}_{i,t} - Q_{i,t}\left(h_{i,t} - h_{i,t-1}\right)\right) + \mu_t\left(\theta_{i,t}Q_{i,t}h_{i,t} - B_{i,t}\right) + ... \\
&+ \beta E_t\left[\lambda_{t+1}\left(\Pi_t + W_{i,t}l_{i,t} + M_{i,t} - \bar{P}_{i,t}\bar{c}_{i,t} - rB_{it} - Q_{i,t}\left(h_{i,t} - h_{i,t-1}\right) + T_{i,t} - M_{i,t+1}\right)\right] - \lambda_t M_t ...
\end{aligned}
$$

The FOC are then (i subscripts are omitted for simplicity)

$$
\begin{aligned}
c_t &: \frac{u_{ct}}{P_t} - \chi_t - \beta E_t\left[\lambda_{t+1}\right] = 0 \\
l_t &: \frac{u_{lt}}{W_t} + \beta E_t\left[\lambda_{t+1}\right] = 0 \\
h_t &: \frac{u_{ht}}{Q_t} - \chi_t + \beta E_t\left[\frac{Q_{t+1}}{Q_t}\chi_{t+1}\right] + \mu_t\theta_t - \beta E_t\left[\lambda_{t+1}\right] + \beta^2 E_t\left[\frac{Q_{t+1}}{Q_t}\lambda_{t+2}\right] = 0 \\
M_t &: \chi_t - \lambda_t + \beta E_t\left[\lambda_{t+1}\right] = 0 \\
B_t &: \chi_t - \mu_t - r\beta E_t\left[\lambda_{t+1}\right] = 0
\end{aligned}
$$

which we can rearrange to get

$$
\begin{aligned}
\lambda_t &= \chi_t + \beta E_t\left[\lambda_{t+1}\right] \\
\frac{u_{ct}}{P_t} &= \lambda_t \\
-\frac{u_{lt}}{W_t} &= \beta E_t\left[\frac{u_{ct+1}}{P_{t+1}}\right] \\
\lambda_t &= (1+r)\,\beta E_t\left[\lambda_{t+1}\right] + \mu_t
\end{aligned}
$$

Finally for the house pricing equation, we have

$$
\begin{aligned}
\frac{u_{ht}}{Q_t} + \mu_t\theta_t &= \lambda_t - \beta E_t\left[\lambda_{t+1}\right] - \beta E_t\left[\frac{Q_{t+1}}{Q_t}\lambda_t - \beta E_{t+1}\left[\lambda_{t+2}\right]\right] + \beta E_t\left[\lambda_{t+1}\right] - \beta^2 E_t\left[\frac{Q_{t+1}}{Q_t}\lambda_{t+2}\right] \\
&= \lambda_t - \beta E_t\left[\frac{Q_{t+1}}{Q_t}\lambda_{t+1}\right]
\end{aligned}
$$

Labor income and consumption spending

Total labor income from traded goods

$$\begin{aligned}
\int_i W_{i,t} l_{i,t}^{\tau} &= \omega \int_i W_{i,t} \left(\frac{W_{i,t}}{\bar{P}_t^{\tau}}\right)^{-\gamma} \int_j \left(\frac{\bar{P}_t^{\tau}}{\bar{P}_{j,t}}\right)^{-\sigma} c_{j,t} \\
&= \omega \int_i \left(W_{i,t}\right)^{1-\gamma} \left(\bar{P}_t^{\tau}\right)^{\gamma-1} \int_j \left(\frac{\bar{P}_t^{\tau}}{\bar{P}_{j,t}}\right)^{1-\sigma} \bar{P}_{j,t} c_{j,t} \\
&= \omega \int_j \left(\frac{\bar{P}_t^{\tau}}{\bar{P}_{j,t}}\right)^{1-\sigma} \bar{P}_{j,t} c_{j,t}
\end{aligned}$$

Total labor income from non traded goods is

$$\begin{aligned}
\int_i W_{i,t} l_{i,t}^{n} &= \int_i W_{i,t} \left(1-\omega\right) \left(\frac{W_{i,t}}{\bar{P}_{i,t}}\right)^{-\sigma} c_{i,t} \\
&= \left(1-\omega\right) \int_i \left(\frac{W_{i,t}}{\bar{P}_{i,t}}\right)^{1-\sigma} \bar{P}_{i,t} c_{i,t}
\end{aligned}$$

Therefore total labor income is

$$\int_i W_{i,t} l_{i,t} = \int_i \frac{\omega \left(\bar{P}_t^{\tau}\right)^{1-\sigma} + \left(1-\omega\right)\left(W_{i,t}\right)^{1-\sigma}}{\left(\bar{P}_{i,t}\right)^{1-\sigma}} \bar{P}_{i,t} c_{i,t} = \int_i \bar{P}_{i,t} c_{i,t}$$