Federal Reserve Bank of New York
Staff Reports

Do Stock Price Bubbles Influence Corporate Investment?

Simon Gilchrist
Charles P. Himmelberg
Gur Huberman

Staff Report no. 177
February 2004

Do Stock Price Bubbles Influence Corporate Investment?
Simon Gilchrist, Charles P. Himmelberg, and Gur Huberman
Federal Reserve Bank of New York Staff Reports, no. 177
February 2004
JEL classification: E22, G31, G32, D92

Abstract

Building on recent developments in behavioral asset pricing, we develop a model in which an increase in the dispersion of investor beliefs under short-selling constraints predicts a "bubble," or a rise in a stock's price above its fundamental value. Our model predicts that managers respond to bubbles by issuing new equity and increasing capital expenditures. We test these predictions, as well as others, using the variance of analysts' earnings forecasts—a proxy for the dispersion of investor beliefs—to identify the bubble component in Tobin's Q.

When comparing firms traded on the New York Stock Exchange with those traded on NASDAQ, we find that our model successfully captures key features of the technology boom of the 1990s. We obtain further evidence supporting our model by using a panel-data VAR framework. We find that orthogonalized shocks to dispersion have positive and statistically significant effects on Tobin's Q, net equity issuance, and real investment—results that are consistent with the model's predictions.

Key words: investment, stock market, bubble, dispersion

Gilchrist: Department of Economics, Boston University (e-mail: sgilchri@bu.edu). Himmelberg: Research and Market Analysis Group, Federal Reserve Bank of New York (e-mail: charles.himmelberg@ny.frb.org). Huberman: Graduate School of Business, Columbia University (e-mail: gh16@columbia.edu).

The authors are grateful to Anna Scherbina for providing data, and they thank Andy Abel, Ignazio Angeloni, Brian Chernoff, Jim Mahoney, David Marshall, Asani Sarkar, Jeremy Stein, and seminar participants at the Federal Reserve Bank of New York, the Federal Reserve System meetings, the Federal Reserve Bank of San Francisco, Colorado University, Columbia University, and the JME/Gerzensee Conference on Behavioral Macroeconomics for helpful comments and suggestions. The authors are also grateful to Brian Chernoff for excellent research assistance. The views expressed in the paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System.

1 Introduction

Research on asset prices increasingly challenges the view that asset prices equal fundamental value. Asset pricing bubbles arise most obviously if all agents are assumed to be systematically biased in their beliefs. But price departures from fundamentals can arise even when beliefs are, on average, unbiased. If pessimists are constrained in their ability to short, then prices disproportionately reflect beliefs of optimists, and thus rise above their fundamental value.[1] Miller (1977) formalizes this intuition in a model with exogenously heterogeneous beliefs and short-sale constraints. His model has subsequently been extended and enhanced in a variety of interesting directions. Scheinkman and Xiong (2003), for example, consider a model in which overconfidence leads to endogenous heterogeneous beliefs, and even causes speculative investors to pay prices that exceed their own valuations.[2]

In this paper, we ask: how would rational managers behave in a Miller-style model of stock pricing?[3] That is, in the presence of bubbles generated by heterogeneous beliefs and constraints on short sales, would firms exploit mispricing by issuing more equity? Would the bubble survive? Would real investment decisions be distorted?

We consider the simplest possible model of exogenously heterogeneous beliefs and short-sale constraints. We begin with the observation that firms, unlike other agents, are completely unconstrained in their ability to sell short – they can simply issue new shares. The cost of issuing is the equity dilution, which is proportional to fundamental value. Hence, if managers have unbiased beliefs, they will issue new shares at the inflated market price. Somewhat counter-intuitively, however, this supply of new

[1] Such models generally assume that *average* beliefs are unbiased. Of course, this not preclude the possibility or even the likelihood that average beliefs are biased, too. Such biases in average beliefs would simply provide a second source of bubbles.

[2] See also the short reviews of related models in Scheinkman and Xiong (2003) and Allen, Morris, and Shin (2003). For surveys of behavioral asset pricing models more generally, see Barberis and Thaler (2002), Hirshleifer (2001) and Shleifer (2000).

[3] Stein (1996) explores rational capital budgeting in the presence of "irrational" market prices. In this class of problems, our paper considers the special case when market pricing "irrationalities" are generated by heterogeneous beliefs and short-sale constraints (as in Miller, 1977).

shares is not sufficient to drive the market price back to its fundamental value.

Because the firm is a monopolist in the supply of its own shares, the firm issues only to the point where the marginal revenue from issuance equals the marginal cost of dilution. This occurs where price is above fundamental value. The resulting arbitrage effect is therefore only partial, and the Miller result survives. The cost of capital is reduced, and the firm over-invests.

To test these predictions, we borrow an empirical idea from Diether, Malloy and Scherbina (2002).[4] Most stocks are tracked by more than one analyst, and these analysts rarely agree on their forecast of a firm's future earnings. Diether et al. propose using the variance of analysts' earnings forecasts as a proxy for the dispersion of shareholder opinion about a firm's fundamental value. They show that high-dispersion stocks have abnormally low future returns, consistent with the view that such firms are overvalued and slowly mean-revert to their fundamental value.

Extending this approach, we first consider the time series evidence regarding dispersion of analysts forecasts for the corporate sector and show that they comove with Tobin's Q, net new share issuance, and real investment. NASDAQ firms, in particular, experienced a run-up in dispersion during the late 1990s through 2001 that was accompanied by higher values of Tobin's Q, an increase in new share issues, and higher levels of real investment. This pattern is consistent with the predictions of our model.

We further investigate the predictions of our model by estimating the effect of changes in dispersion on investment, Tobin's Q and net equity issuance within a VAR framework. The available time series is short, so instead of using aggregate data, we exploit the longitudinal dimension of our data by estimating a panel data VAR. To control for the fact that changes in dispersion might be correlated with investment opportunities, we consider the effect of shocks to dispersion that are orthogonal to

[4]See also Park (2001).

innovations in the marginal product of capital.[5] Conditional on fixed time and firm effects, the impulse response functions of the estimated model show that dispersion shocks give rise to higher values of Tobin's Q, higher equity issuance, and higher real investment. This pattern is uniformly consistent with the predictions of our model.

Finally, we also compute variance decompositions to assess the quantitative importance of dispersion shocks. As a fraction of the explainable variation in the data, we find that dispersion shocks have a large impact on equity issuance, a modest impact on Tobin's Q, and a relative small impact on real investment. In our model, large bubbles do not necessarily imply large investment distortions. In our empirical findings, this is in fact the case.

Recent research in finance provides additional empirical support for our model assumptions. Most notably, Diether, Malloy, and Scherbina (2002) find that high dispersion forecasts low future returns. A portfolio of stocks in the highest quintile of dispersion underperforms a portfolio of stocks in the lowest quintile of dispersion by 9.48% percent per year. Chen, Hong and Stein (2002) report related evidence. Instead of using data on analysts' forecasts, they define a measure of "breadth" based on the number of funds prevented from taking a short position due to legal constraints. They find that "short-constrained" stocks have low future returns. Additional evidence on the price effects of short-sale constraints is provided by Lamont and Jones (2002). They show that stocks that were expensive to short during the 1920s and 30s delivered lower returns than other stocks. Using more recent data, Ofek and Richardson (2003) report evidence showing that the collapse of the internet bubble coincided with a sudden supply of new shares created by the expiration of lock-up restrictions. Finally, D'Avolio's (2003) detailed description of the market for borrowed stock provides extensive direct evidence showing that short selling is costly.

Polk and Sapienza (2002) is the only other paper of which we are aware that at-

[5] If disagreement increased by shocks to the investment opportunity set, then disagreement would contain information about investment opportunities, and could thus explain the pattern observed in aggregate means.

tempts to measure the distortionary effect of stock price bubbles on real investment. They argue that new equity issues, discretionary earnings accruals, and lagged returns can be used as proxies for bubbles. Using Tobin's Q to control for investment opportunities, they find that, consistent with their predictions, these bubble proxies enter positively and statistically significant in a regression for investment.[6] Our paper expands upon their results. We would argue, however, that our dispersion measure is less prone to alternative explanations.

Several other empirical papers are related in various ways. Motivated in part by the possibility of bubbles in stock prices, Mϕrck, Shleifer and Vishny (1991) and Blanchard, Rhee and Summers (1993) compare the responsiveness of investment to Tobin's Q and fundamentals and broadly conclude that investment is driven primarily by fundamentals.[7] Chirinko (1996) and Chirinko and Schaller (2001) implement similar tests by including both fundamental and market Q measures, but conclude instead that the evidence favors the existence of bubbles. By estimating Q equations for investment, Erickson and Whited (1999) and Bond and Cummins (2000) both document measurement error in Tobin's Q, and speculate that stock price bubbles are a likely source.[8]

In the remainder of the paper, we begin by exploring the implications for firm behavior of a simple equilibrium model of heterogeneous investor beliefs under short-selling constraints. In section 3, we briefly describe the data and econometric approach, followed by a description of our empirical results. Section 4 concludes.

[6] Polk and Sapienza (2002) also point out that abnormally high investment levels may be caused in part by stock bubbles, in which case they should predict low subsequent returns. This is indeed what they find.

[7] Baker, Stein, and Wurgler (2002) similarly ask whether some firms are intrinsically more dependent on equity for their external financing, and thus more sensitive to stock prices.

[8] Less closely related to ours are papers that examine the behavioral biases of *executives* rather than market prices, and explores the potential impact on corporate investment decisions. Heaton (1999) develops a model in which CEOs are both overconfident and overoptimistic. Malmendier and Tate (2002) use the timing of stock option exercise to measure overconfidence. Bertrand and Schoar (2002) report evidence that CEOs appear to have managerial "styles" that accompany them when they change jobs. By contrast with these papers, we assume managers have rational (unbiased) expectations.

2 A Model of Real Investment, Equity Issuance, and Bubbles

In this section, we develop a simple model of firm behavior when investors with heterogeneous beliefs face short-selling constraints in the equity market. First, we aggregate heterogenous portfolio demands to derive the inverse demand function for new shares. Demand for new shares is increasing in the degree of dispersion in beliefs. Second, we consider the share issuance and real investment decisions of a bubble-aware rational manager who seeks to maximize the objective value of the firm. Optimal behavior implies that the bubble persists in equilibrium and that the user-cost-of-capital is lower than would be the case in the absence of short-sales constraints. We then characterize the effect of an increase in dispersion on the equilibrium values of equity issuance, real investment, and the stock price bubble; we also characterize the effect of dispersion on the equilibrium value of Tobin's Q.

Within the context of our model we establish the following three results. An increase in dispersion leads to an increase in the equilibrium price bubble and an increase in net equity issuance. As a result, the user cost of capital falls and investment increases. Finally, the rise in investment combined with the increase in the bubble imply a higher equilibrium value of Tobin's Q. We conclude by discussing the model's implications for empirical work.

2.1 The Demand for New Share Issues

Let investor valuations be denoted by vV, where V is the true value of the firm, and $v \in [0, \infty]$ is a random variable that measures idiosyncratic variation in investors beliefs. Let P denote the market value (price) of the firm.[9] We assume the investor's portfolio demand for a firm's shares (i.e., the fraction of the investor's wealth invested

[9] Investors and managers may disagree about the value of the firm, but for simplicity, we assume they all use the same discount rate.

in the firm) is given by[10]

$$\omega_v = \gamma \left(vV - P \right). \tag{1}$$

Multiplying ω_v by investor wealth and dividing by the market value of the firm translates the investor's demand from a portfolio share to a proportional claim on the firm's equity value, $n_v = \omega_v W/P$. Without loss of generality, we can normalize investor wealth and the total mass of investors to equal one. Thus, the investor's demand for shares is given by $n_v = \gamma \left(vB^{-1} - 1 \right)$, where $B = P/V$. We refer to B – the extent to which price deviates from fundamental value – as the "bubble."

Under short-selling constraints, the only investors who take positions in the stock are those for whom $vV \geq P$, or $v \geq B$. Hence, assuming v has the distribution function $F(v)$, the aggregate demand for shares is

$$n^d (B; \sigma) = \gamma \int_B^\infty \left(vB^{-1} - 1 \right) dF(v). \tag{2}$$

To characterize the demand function we assume that v is log-normally distributed with $\ln v \sim N \left(-0.5\sigma^2, \sigma^2 \right)$, so that $E(v) = 1$. This normalization says that average beliefs are unbiased. It also implies that net demand for shares is zero when $P = V$ and short-sale constraints are not binding. Let ϕ and Φ denote the p.d.f and c.d.f of the standard normal distribution respectively, and b denote a normalized log transformation of B

$$b \equiv \frac{\ln B + 0.5\sigma^2}{\sigma}. \tag{3}$$

Using properties of the log-normal distribution, equation 2 can be expressed as

$$n^d (B; \sigma) = \gamma \left(1 - \Phi(b) \right) \left[\frac{h(b)}{h(b - \sigma)} - 1 \right] \tag{4}$$

[10] This functional form for portfolio demand can be derived from a model in which investors have CARA utility and returns are normally distributed.

where $h(b)$ denotes the hazard rate for the standard normal distribution:

$$h(b) \equiv \frac{\phi(b)}{1-\Phi(b)}.$$

The first term in equation 4 measures the mass of market participants as a function of the bubble B. The second term in equation 4, $\frac{h(b)}{h(b-\sigma)} = \frac{E(v|v>B)}{B}$, measures the average valuation conditional on market participation.[11] Because the hazard rate is strictly increasing, the ratio $h(b)/h(b-\sigma)$ is greater than one, hence market demand is strictly positive for $B > 0$. As the bubble increases, market participation falls but valuation conditional on participation rises. On net, an increase in price reduces demand.

Denote the fraction of total shares supplied to the public by n. By inverting the function $n = n^d(B;\sigma)$ for B, we define the inverse demand function, $B(n;\sigma)$, which gives the price relative to fundamentals as a function of dispersion and the number of shares issued. In the appendix we show that $\lim_{n\to\infty} B(n,\sigma) = 0$ and $\lim_{n\to 0} B(n,\sigma) = \infty$. We also show that

$$B_n = \frac{-B^2}{\gamma(1-\Phi(b-\sigma))} < 0. \tag{5}$$

and

$$B_\sigma = Bh(b-\sigma) > 0, \tag{6}$$

where B_n and B_σ denote the partial derivatives. These results establish that the inverse demand curve is downward sloping in the size of the equity issue, and that it shifts outward in response to an increase in dispersion.

The derivatives in equations 5 and 6 lead to simple expressions for the respective

[11] To obtain equation 4 we note $(1-\Phi(B-\sigma) = E(v|v > B)\Pr(v > B)$ so that equation 2 may be written as

$$n^d(B;\sigma) = \gamma\left[(1-\Phi(b-\sigma))B^{-1} - (1-\Phi(b))\right]$$

(see Johnson, Kotz and Balikrishnan (1994)). Equation 3 may be equivalently expressed as $B = \phi(b-\sigma)/\phi(b)$. Inserting this expression into $n^d(B;\sigma)$ yields the result.

demand elasticities. In particular, the inverse-price elasticity of demand $\eta_n \equiv -\frac{\partial \ln B}{\partial \ln n}$ is

$$\eta_n = 1 - \frac{h(b-\sigma)}{h(b)}. \tag{7}$$

The ratio $h(b-\sigma)/h(b)$ is bounded between zero and one. Hence, the inverse-demand curve is inelastic over its entire range.

The semi-elasticity of the price (bubble) to dispersion, $\eta_\sigma \equiv \frac{\partial \ln B}{\partial \sigma}$, is

$$\eta_\sigma = h(b-\sigma).$$

The shift in demand caused by an increase in dispersion depends on the degree of truncation, and hence the hazard rate of the normal distribution evaluated at the bubble. To understand the implications of such a demand shift for investment, we now turn to the firm problem.

2.2 Equity Issuance and the Equilibrium Price Bubble

Given the inverse-demand function $B(n;\sigma)$, we can now formally consider the firm's problem. Let the expected value of installed capital, K, be given by $V(K) = E[\Pi(K,\theta)] + (1-\delta)K$ where θ represents a shock to the profitability of capital. To install new capital, the firm incurs an adjustment cost $\frac{1}{2}\psi K^2$. We assume managers recognize mispricing and choose K to maximize the true value of the firm from the perspective of old shareholders.[12] Managers can finance this investment using risk-free debt at the rate r, or they can issue new equity by selling a fraction n of the firm's equity. They can invest the proceeds in K, or pay them out as a dividend to the old shareholders.[13] The market value of equity is given by $B(n;\sigma)V(K)$,

[12] For example, managers might own a stake in the firm for incentive reasons, in which case their incentives are to act on behalf of old rather than new shareholders.

[13] We assume investors have biased expectations over the future value of operating assets but not cash. Therefore, to maximize the value of their equity share, managers prefer to invest the proceeds of equity issuance in capital rather than hold cash or pay dividends.

so proceeds from new equity issues are given by the discounted value of the new shareholders' claim, or

$$X = \frac{1}{1+r} nB(n;\sigma) V(K). \tag{8}$$

Thus the firm's optimization problem is:

$$\max_{I,X,n} -K - \frac{1}{2}\psi K^2 + X + (1-n)\frac{1}{1+r} V(K) \tag{9}$$

subject to equation (4). Note that the future value of the firm in equation (9) is multiplied by $1 - n$ to reflect the dilution of old shareholders.

The firm is a monopolist in the supply of its own shares, so issue size depends on the elasticity of the demand curve as well as the marginal (dilution) cost of issuing. As a monopolist, the firm sets price above marginal cost. Thus a key feature of our model is that the firm never issues enough shares to drive the bubble down to its fundamental value. That is, equilibrium features $B > 1$. To see this logic formally, the first-order condition for equity issuance derived from equation (9) is:

$$B(n;\sigma) - 1 + nB_n(n;\sigma) = 0. \tag{10}$$

Applying the result that the inverse demand curve is downward sloping ($B_n < 0$), it follows that the bubble satisfies $B > 1$ when the firm is issuing new shares ($n > 0$), and $B < 1$ when the firm is repurchasing ($n < 0$).

Applying equation 7, the equilibrium price satisfies[14]

$$B = \frac{h(b)}{h(b-\sigma)}. \tag{11}$$

Equation 11 defines a unique mapping $B(\sigma)$, that is, for any $\sigma > 0$ there is a unique equilibrium price B.[15] Given the equilibrium price $B(\sigma)$, the equilibrium value of equity issuance is determined by

$$n(\sigma) = \gamma (1 - \Phi(b)) (B(\sigma) - 1). \tag{12}$$

For $\sigma = 0.5$, $\gamma = 1$, the equilibrium is depicted in figure 1. Figure 1 plots the market demand curve and the marginal revenue curve for new equity issuance. Equilibrium equity issuance is denoted by n^*. For these parameter values the bubble is sizable – on the order of 50%.

We now consider the effect of an increase in dispersion on the equilibrium bubble B and equity issuance n. Totally differentiating equation 11 yields

$$\frac{dB}{d\sigma} = \frac{B\left(b\left[h(b-\sigma) + \sigma - h(b)\right)\right] + \sigma\left[h(b) - b\right])}{\left(\sigma + \left[h(b-\sigma) + \sigma - h(b)\right]\right)} > 0. \tag{13}$$

Thus an increase in dispersion causes an increase in the equilibrium size of the bubble. We further establish that

$$\frac{d \ln B}{d\sigma} < h(b-\sigma) = \eta_\sigma. \tag{14}$$

[14] From the monopolist's viewpoint, the bubble is analogous to the ratio of price to marginal cost, where the marginal cost of new share issues is unity. The equilibrium bubble in equation 10 can analogously be expressed as a relationship between the markup and the inverse demand elasticity:

$$B = \frac{1}{1 - \eta_b}.$$

[15] Equation 11 implies that the equilibrium value $B(\sigma)$ is independent of other model parameters, notably the demand parameter γ. Thus, a monopolist facing a demand curve of the form specified in equation 4 chooses a constant markup that only depends on demand characteristics through σ, the degree of consumer heterogeneity. This result can be applied to a variety of consumer settings characterized by a log-normal distribution of underlying demand characteristics.

In words, the equilibrium response of the bubble to an increase in dispersion is less than the implied elasticity obtained from the demand curve.[16] Intuitively, a firm issues new equity in response to an increase in dispersion, partially offsetting the effect of a rise in σ on price. To formally see the effect of an increase in dispersion on equity issuance, we totally differentiate equation 12 to obtain

$$\frac{dn}{d\sigma} = \gamma \frac{[(1 - \Phi(b - \sigma))]}{B} \left(h(b - \sigma) - \frac{d \ln B}{d\sigma} \right) > 0. \tag{15}$$

In Figure 2, we depict the effect of an increase in σ from 0.5 to 0.7. An increase in dispersion represents an outward shift in the market demand for shares and an increase in the equilibrium value of the bubble. It also increases the fraction of equity issued (from n^* to n^{**}). As shown in equation 12, equity issuance $n^d(B, \sigma)$ depends on both the average net-revenue per share $(B - 1)$ and the percentage of market participants $(1 - \Phi(b))$. In our model, the rise in revenue per participant increases enough to offset drop in market participation, and an increase in dispersion causes an increase in share issuance.

2.3 Dispersion and Investment

A convenient feature of our model formulation is that it allows us to consider the equilibrium behavior of share issuance and stock pricing without directly considering the firm's investment decision. As we now show, in equilibrium, an increase in dispersion leads to a lower cost of capital and an increase in investment.

The first-order condition with respect to capital (from the firm's problem in equations 8 and 9) implies the (modified) Q equation

$$1 + \psi K = \frac{1 + n(B - 1)}{1 + r} V_k. \tag{16}$$

[16] See the appendix for details of the derivation of equations 13 and 14.

For the case where there is no bubble ($B \equiv 1$), this reduces to $1 + \psi K = \frac{1}{1+r} V_k$. This is the usual first-order condition for investment, which says that the firm invests up to the point where the marginal cost of investment, $1 + \psi K$, equals the discounted expected marginal value of capital, $\frac{1}{1+r} V_k$ (or marginal Q).

To see the effect of the bubble on investment, we first consider the case of no adjustment costs ($\psi = 0$). Substituting for V_k, we can approximate equation (16) as

$$E[\Pi_k] \simeq r + \delta - n(B-1), \tag{17}$$

where the approximation used is valid when $n(B-1)$ is small.[17] This expression allows us to interpret the effect of bubbles in terms of a modification to the Jorgensonian cost of capital, which is defined as the right side of equation (17). When $n(B-1)$ is zero, that is, when there is no bubble, or when there is a bubble but the firm does not issue, the approximation is exact and we have the familiar optimality condition for capital which sets the marginal profitability of capital equal to its user cost. That is, $E[\Pi_k] = r + \delta$. If, however, the bubble is positive *and* the firm actively exploits the bubble by issuing shares, then this has the effect of reducing the cost of capital by $n(B-1)$.

The reduction in the cost of capital depends not only on the size of the bubble but also on the response of new share issues. For example, if the demand curve were relatively steep so that a sizeable bubble could be eliminated by a small share issue, the financing benefits of such a bubble (and hence the effect on the cost of capital) would be negligible. Firms would have little incentive to exploit such a highly elastic bubble. Large bubbles could theoretically persist in equilibrium yet have only a trivial impact on investment.[18]

[17] This logic still holds without the approximation, but a linear approximation simplifies the intuition.

[18] According to our model, two firms with the same level of mispricing i.e. $(B(\sigma))$ may face substantially different effective user costs of capital to the extent that share issuance differs across firms in response to the mispricing. Such heterogeneity can be formally justified by allowing for

Now consider the effect of an increase in dispersion. An increase in dispersion increases the size of the bubble and increases new share issuance. Hence, from equation (17), the user cost of capital falls, which leads to an increase in investment. Equation (17) shows that the amount by which a positive stock price bubble reduces the cost of capital depends on the extent to which new equity is issued.

In the case of positive adjustment costs we also obtain the result that investment increases in response to an exogenous increase in dispersion, with the magnitude of the effect depending on the willingness of the firm to issue new equity in equilibrium. With positive adjustment costs, $\psi > 0$. Substituting for the definition of V_k, we can write the first-order condition for capital in equation (16) as

$$\frac{\Pi_k + 1 - \delta}{1 + \psi K} = \frac{1 + r}{1 + n\,(B - 1)}. \tag{18}$$

Assuming that the marginal profit of capital (Π_k) is weakly decreasing in K, it follows immediately that the right side of this equation is monotonically decreasing in K. Again, an increase in dispersion causes equilibrium values of B and n to increase, which leads to a rise in investment.

2.4 Dispersion, stock prices and Tobin's Q

Finally, we consider the relationship between dispersion, investment and *measured* Q in equilibrium. This analysis is of interest because most empirical work on investment uses measured Q rather than marginal Q (V_k) as a conditioning variable when analyzing the effect of bubbles on investment.[19] Because our model generates an equilibrium relation among investment, Tobin's Q, bubbles, and share issuance, it has important implications for alternative testing strategies based on estimates of modified Q equations.

differences in the demand parameter γ across firms.

[19]We are grateful to Andy Abel for bringing these points to our attention and for particularly lucid comments.

For simplicity, we assume that $\Pi(K,\theta)$ is linearly homogeneous in K. We define Tobin's Q as the ratio of the market value of equity to the replacement value of capital:

$$Q \equiv \frac{1}{1+r}\left[\frac{BV}{K}\right]. \tag{19}$$

If $\Pi(K,\theta)$ is homogeneous of degree one, then $V_k = V/K$. To see that Q is increasing in σ, it suffices to note that because V is homogeneous of degree one in K, the ratio V/K is independent of K. Hence, Q depends on σ only through B, which is increasing in σ. Applying equation 19 to the first-order condition for investment (equation 16), we obtain the relationship between in investment (K), the bubble, and **measured** Tobin's Q:

$$Q = \left(\frac{B}{1+n(B-1)}\right)(1+\psi K). \tag{20}$$

If adjustment costs and the stock price bubble were both zero, then we would obtain the result that $Q = 1$. Otherwise, since the first term on the right side of equation (20) is increasing in B, either bubbles *or* adjustment costs (or both) are sufficient to imply $Q > 1$.

Conditional on investment, equation 20 implies a positive relationship between dispersion and Q. Thus, a regression of investment on Q and dispersion is unlikely to yield a positive coefficient on dispersion. In effect, because the firm does not fully offset the effect of dispersion on price through new equity issuance, the response of Q to dispersion overstates the effect on investment.

To summarize the results in this section, heterogeneous beliefs and short-selling constraints can generate bubbles. When the distribution of investor valuations is log-normal, increases in dispersion increase both the size of the bubble and the amount of new equity issued. This lowers the cost of capital and therefore stimulates investment. The bubble alone is not sufficient for determining the magnitude of this distortion. Rather, it is the interaction between the bubble and the fraction of new equity issued that matters. Finally, we showed that the equilibrium value of Tobin's Q (that is, average *measured* Q as opposed to average *true* Q) is increasing in not only the rate of investment but also in the size of the bubble. Thus, our results provide a measure of support for the common practice of using Tobin's Q (or market-to-book ratios) as indirect measures of stock price bubbles.

3 Empirical Analysis

To evaluate the empirical predictions of the model we focus on the relationship between investment, Tobin's Q, net equity issuance, and our proxy for the dispersion of beliefs. We first compare trends in dispersion, new equity issues, Tobin's Q and investment over the period 1986-2000.[20] We divide firms into those listed on the New York Stock Exchange vs Nasdaq. The stock price movements of the latter were arguably more likely to have been driven by bubbles than the former. We then consider a more detailed analysis of the data at the firm-level.

The model in the previous section highlights the difficult identification issues presented by the Q framework. Specifically, because net equity issuance and Tobin's Q *both* respond endogenously to dispersion, one cannot use Tobin's Q to control for investment opportunities to conclude that the effect of new equity issues as a bubble-driven. Our empirical strategy is motivated by this identification problem. We pursue two ideas. First, we use the variance of analysts' earnings forecast as an indicator of bubbles. In contrast to variables like equity issuance and lagged stock returns, there is no obvious reason why this measure would be correlated with investment opportunities. Second, we use recursively ordered VARs to further isolate and identify the exogenous component of this variable. This approach is a (minimally) structural attempt to improve identification.

We assemble annual, firm-level data from two sources. We use Compustat for data on sales, capital expenditures, net equity issuance, total assets, total liabilities,

[20] This time frame is set by data availability.

preferred equity, and property, plant and equipment. These variables are merged with data on analysts' earnings forecasts from IBES.[21] The variables that we construct for our analysis are the rate of investment, I_t/K_t, net new equity issuance as a fraction of total equity, neq_t, Tobin's Q ratio, Q_t, dispersion of analysts forecasts, d_t, and the marginal product of capital, mpk_t. The appendix provides a more complete description of the variables and the sample construction.

3.1 The 1990's boom: Nasdaq vs NYSE

Figure 3 plots the time-series averages of dispersion, Tobins' Q, the sales to capital ratio, the investment rate and net equity issuance for the sub-samples of firms listed on Nasdaq versus NYSE over the period 1990-2002.[22] For comparison's sake, we also plot the Nasdaq vs NYSE stock price indices as well.

Nasdaq firms experienced a steady increase in dispersion relative to NYSE firms over the period 1990-2001, followed by a slight decline in 2002.[23] NASDAQ firms also experienced a steady increase in their investment rate relative to NYSE firms over most of this period. Tobin's Q and net equity issuance also diverge for Nasdaq vs NYSE during this time period, with Nasdaq firms showing a sharp increase in both Tobins' Q and net equity issuance during the later part of the boom. This sharp increase coincides with a rise in the growth rate of dispersion for the 1998-2001 period. Although timing between these variables is not exact, the latter part of the 1990's is characterized by sharp increases in dispersion, Tobin's Q, net equity issuance, and investment for Nasdaq firms relative to NYSE firms. These patterns

[21]These data are used in Deither, Malloy and Scherbina (2002) and were kindly provided by Anna Scherbina.

[22]With the exception of the net-equity issuance, we report the mean of the log of all variables for each sub-sample. For all variables, we trim outliers using a 1% cutoff rule applied to the combined NYSE and NASDAQ sample.

[23]Because of reporting issues with IBES vs Compustat, we lose approximately 20% of our observations in the last year of the sample. Thus the mean dispersion estimates for 2002 may not be entirely representative. Consistent with the idea that increases in dispersion contributed to the stock market boom, using medians rather than means, we see a sharper reduction in dispersion in the last year of our sample.

are broadly consistent with our model's predictions.

The divergence in investment rates between Nasdaq and NYSE firms is difficult to justify based on investment fundamentals alone (as measured by the sales to capital ratio). In fact, during the early sample period, there is little difference between the marginal product of capital for NYSE versus Nasdaq firms. Then in 1999, MPK for Nasdaq firms begins to collapse while dispersion, Tobin's Q, new equity issuance, and investment all continue to rise. This is all consistent with the bubble view. To provide additional insight we now consider an empirical analysis based on the microeconometric data.

3.2 Panel Data VAR Analysis

We start with a three variable VAR system, estimated in logs, that includes the marginal product of capital, dispersion and investment. To allow for the possibility that dispersion may contain information about current investment opportunities, we consider the effect of an innovation to dispersion that is uncorrelated with the innovation to MPK.[24] Hence, when computing impulse responses, we use a Choleski decomposition using the ordering mpk_t, d_t, I_t/K_t.[25]

Table 1 reports the coefficient values of this three variable VAR system. Table 1

[24] Dispersion would contain information about investment opportunities if shocks to fundamentals trigger disagreement among analysts.

[25] Formally, we estimate the model $\mathbf{y}_{it} = \mathbf{A}\mathbf{y}_{it-1} + \mathbf{f}_i + \mathbf{e}_t + \mathbf{v}_{it}$, where $\mathbf{y}_{it} = \{mpk_{it}, d_{it}, I_{it}/K_{it}\}'$, $\mathbf{A}$ is a 3×3 matrix of coefficients, $\mathbf{f}_i$ is a vector of fixed firm effects, and $\mathbf{e}_t$ is a vector of common time shocks. We estimate the model following the procedure described in Arellano and Bover (1995). Our ordering for the three-variable case implies that the vector of residuals $\mathbf{v}_{it}$ is related to a set of mutually orthogonal *structural* shocks $\boldsymbol{\eta}_{it} = \left\{\eta_{it}^{mpk}, \eta_{it}^{d}, \eta_{it}^{I/K}\right\}'$ according to the following recursive structure:

$$\begin{aligned} v_{it}^{mpk} &= \eta_{it}^{mpk} \\ v_{it}^{d} &= \rho_{iq}\eta_{it}^{mpk} + \eta_{it}^{d} \\ v_{it}^{I/K} &= \rho_{dq}\eta_{it}^{mpk} + \rho_{di}\eta_{jt}^{d} + \eta_{jt}^{I/K}. \end{aligned}$$

also reports the t-statistics for the coefficients.[26] Consistent with a key implication of our model, we observe a statistically significant positive link between dispersion and investment, controlling for the marginal product of capital. The marginal product of capital is also highly significant in the investment equation, as we would expect. We also see a positive relationship between dispersion and mpk, a finding which suggests that our orthogonalization scheme will be helpful when identifying increases in dispersion that are not related to fundamentals.

Table 1
Estimates of Three-Variable VAR

	$\ln mpk_t$	$\ln d_t$	$\ln (I/K)_t$
$\ln mpk_{t-1}$	0.933	0.436	0.459
	(30.408)	(10.920)	(9.523)
$\ln mpk_{t-2}$	−0.093	−0.229	−0.308
	(4.117)	(7.267)	(8.647)
$\ln d_{t-1}$	0.044	0.531	0.091
	(3.996)	(27.754)	(4.322)
$\ln d_{t-2}$	0.029	0.121	0.097
	(4.871)	(10.582)	(7.948)
$\ln (I/K)_{t-1}$	−0.164	−0.080	0.459
	(13.763)	(4.416)	(22.042)
$\ln (I/K)_{t-2}$	0.052	0.087	0.134
	(5.972)	(6.416)	(8.266)

Notes: Robust t-statistics appear in parentheses.
Sample contains 18421 firm-year observations.

Figure 4 reports the impulse response functions from this three variable VAR. We report the effects of shocks to mpk_t which we interpret as a shock to the fundamental investment opportunities of the firm, and we report the effects of a shock to dispersion, which, within the context of our model leads to an increase in the bubble (price relative to fundamentals).

The effect of a one-standard deviation shock to mpk_t is reported in the first row of Figure 4. The immediate effect of the shock is to increase both mpk_t and invest-

[26]Because we use a GMM procedure to control for fixed effects, we do not report R^2 statistics since they are not particularly informative in the context of instrumental variables estimation.

ment by approximately the same magnitude (0.2), following which both variables return to steady-state at approximately the same rate. This finding implies a unit elasticity between investment and the marginal product of capital following a shock to fundamentals.

The effect of a one standard deviation shock to dispersion is reported in the second row of Figure 4. Consistent with our model, an innovation to dispersion leads to a pronounced increase in investment. The peak response of investment is on the order of 0.1 percent and occurs in the year following the shock. The increase in dispersion also causes a rise in mpk_t but the magnitude is relatively small. Using unit elasticity as a reasonable measure of how investment should respond to fundamentals, these results imply that most of the increase in investment following a shock to dispersion can be attributed to changes in dispersion that are orthogonal to future mpk.[27]

To examine the empirical link between dispersion, Tobin's Q and net equity issuance, we augment the three variable VAR by adding Tobin's Q and net equity issuance. For parsimony, we focus on the impulse response functions rather than coefficient values.[28] We again consider innovations based on a Cholesky decomposition using the following ordering: $[mpk_t, d_t, I/K_t, Q_t, neq_t]$. The results are reported in Figure 5.

The impulse response to a one standard deviation shock to mpk_t is reported in the first row of Figure 5. Adding the additional variables does not change the basic relationship between fundamentals and investment that we observed in Figure 4. A

[27] If we interpret approximately unit elasticity response of of investment to the innovation in mpk as providing a reasonable measure of how investment responds to fundamentals, then we would attribute 1/3 (0.03 out of 0.1) of the rise in investment to fundamentals following a shock to dispersion. The remaining 2/3 response (0.07 out of 0.1) would be attributable to movements in dispersion not linked to fundamentals.

[28] Our model suggests that in a regression of investment on Tobin's Q and dispersion, we should find a negative effect of dispersion on investment. Adding Tobin's Q to the investment equation reduces the coefficient on dispersion but they remain positive. Because such regressions do not control for the contemporaneous correlations however, we do not necessarily interpret this as a rejection of the model. Rather, it highlights the need for additional identification through the choleski decomposition.

shock to mpk_t leads to a modest rise in Tobin's Q and a small increase in equity issuance upon impact of the shock. Both of these responses are consistent with the notion that Tobin's Q and equity issuance respond endogenously to fundamental investment opportunities.

The response of investment and fundamentals to an innovation in dispersion is also similar to the results obtained using the three variable VAR system albeit slightly weaker. Investment responds with some lag and shows a peak response on the order of 0.08. The increase in mpk_t is again positive but relatively small in magnitude – on the order of 0.04. Again, using unit elasticity as a benchmark, this finding suggests that slightly less than half of the response of investment to the dispersion shock can be explained by the response of fundamentals, the other half is attributable to a non-fundamental component and is therefore consistent with the notion that bubbles drive investment.

The innovation to dispersion leads to an increase in Tobin's Q and a rise in equity issuance – both of these responses are consistent with the model's predictions. They are also large in magnitude relative to the investment response. Following a shock to mpk_t, the peak increase in Tobin's Q is one third the size of the peak increase in investment. In contrast, following a shock to dispersion, the peak increase in Tobin's Q is nearly the same size as the increase in investment. Our model implies that in the absence of bubbles, investment is a sufficient statistic for Tobin's Q regardless of the source of the shock. In the presence of bubbles, Tobin's Q should reflect both the increase in investment and the increase in the bubble however (see equation 20). This additional impact on Q through the bubble, controlling for investment implies that Q_t should respond more to dispersion shocks, controlling for investment. Our model thus rationalizes the finding that $\Delta \ln Q_t / \Delta \ln (I_t/K_t)$ is larger in response to shocks to dispersion relative to shocks to mpk_t.

In both the three variable and the five variable VAR results, innovations in dispersion cause increases investment, Tobin's Q, and net equity issuance that are consistent with our model predictions. Identification is somewhat complicated by the fact that mpk_t tends to respond positively to increases in dispersion, but the response is relatively weak, suggesting that most of the movement in investment, Q and net equity issuance following a shock to dispersion can be attributed to non-fundamental components, i.e. bubbles.

To assess the quantitative importance of these results, we compute a variance decomposition of the five variable VAR. The variance decomposition is based on the ordering specified above. We report results at the 10 year horizon, though similar results are obtained at shorter horizons. Because we control for time dummies and fixed effects in our panel-data framework, these variance decompositions provide information about the within-firm variation only, and hence do not measure the importance of bubbles in the aggregate.

Table 2
Variance Decomposition at 10-Year Horizon

	Fraction of Total Variance Explained				
Shocks	$\ln mpk$	$\ln d$	$\ln Q$	$\ln neq$	$\ln (I/K)$
$\ln mpk$	0.869	0.068	0.153	0.043	0.480
$\ln d$	0.015	0.897	0.015	0.059	0.014
$\ln Q$	0.002	0.012	0.727	0.003	0.075
$\ln neq$	0.002	0.000	0.083	0.884	0.015
$\ln (I/K)$	0.111	0.023	0.021	0.010	0.416

Table 2 reveals that most of the variation in each variable is determined by its own shock. The exception is investment, for which fundamentals play the dominant role. Dispersion explains only a small fraction of the total variance of investment. When compared to the fraction explained by Tobin's Q (7.5 percent), this number is reasonably large however. Dispersion also explains 1.5 percent of the variation in mpk and Tobin's Q. Interestingly, dispersion accounts for more of the variance of net equity issuance (6 percent) than any other variable besides net equity issuance itself. In the absence of mispricing, the firm is indifferent between equity issuance and other forms of finance. It is therefore perhaps not surprising that dispersion would account for a reasonable fraction of the variation in share issuance.

The variance decompositions suggest that dispersion only accounts for a small fraction of investment. This finding is not surprising for several reasons. First, as mentioned above, our panel data estimates do not identify the macro variation in the bubble component.[29] Second, analysts are reasonably informed agents. Dispersion in analysts forecasts is therefore likely to understate the true amount of disagreement in the market place. Finally, the model itself implies that the effect of bubbles on investment will be limited, owing to the fact that the firm is unwilling to fully exploit the bubble in equilibrium.

4 Conclusion

We have developed a model in which shocks to the dispersion of investor opinion cause stock prices to rise above their fundamental values. In the equilibrium response to such shocks, the model predicts a rise in Tobin's Q, net new share issues, and real investment. We test these predictions using the variance of analysts' earnings forecasts to proxy for shocks to the dispersion of investor beliefs. This proxy effectively allows us to identify a portion of the "bubble" component in Tobin's Q. Using a recursive ordering of a panel data VAR for identification, we find that shocks to dispersion have positive and statistically significant effects on Tobin's Q, net equity issuance, and real investment, all of which are consistent with the predictions of the model.

Our variance decompositions show that conditional on fixed time and firm effects, the percentage of the "within" variation in real investment and Tobin's Q that can be explained by dispersion shocks is relatively small – about 1.5 percent in each case. For equity issuance, however, the fraction is considerably larger – about 6 percent. This contrast is interesting because it suggests that distortions in financial prices imply larger distortions for financial decisions than for real activity.

[29] Our aggregate plots, though anecdotal, suggest that the distortion caused by dispersion could be more substantial than our panel data estimates suggest.

Substantial room for future research remains. Our model uses the simplest possible depiction of the asset pricing equilibrium that can be used to generate pricing in the spirit of Miller (1977). Extending our model in the direction of more modern treatments of the asset pricing equilibrium like Scheinkman and Xiong (2003), for example, would obviously be desirable. Embedding such extensions in a dynamic model of the real side would provide a quantitative framework suitable for structural estimation and testing.

5 Bibliography

Allen, Franklin, Stephen Morris, and Hyun Song Shin (2003), "Beauty Contests and Iterated Expectations in Asset Markets," working paper, Yale University.

Arellano, Manuel and Olympia Bover (1995), "Another Look at the Instrumental Variable Estimation of Error Component Models," *Journal of Econometrics* 68, 29-51.

Baker, Malcolm, Jeremy Stein, and Jeffrey Wurgler (2003), "When Does the Market Matter? Stock Prices and the Investment of Equity Dependent Firms," *Quarterly Journal of Economics* 118, 969-1006.

Bertrand and Schoar (2002), "Managing with Style: The Effect of Managers on Firm Policies," working paper, MIT.

Blanchard, Olivier J., Changyong Rhee, and Lawrence Summers (1993), "The Stock Market, Profit and Investment," Quarterly Journal of Economics 108(1), 115-36.

Bond, Stephen R. and Jason G. Cummins (2000), "Noisy Share Prices and the Q Model of Investment," Working Paper no. W01/22, Institute for Fiscal Studies, London, (http://www.ifs.org.uk/workingpapers/wp0122.pdf).

Chen, Joseph, Harrison Hong, and Jeremy C. Stein (2002), "Breadth of Ownership and Stock Returns," *Journal of Financial Economics* 66, 171-205.

Chirinko, Robert S. (1996), "Bubbles, Fundamentals, and Investment: A Multiple Equation Testing Strategy," *Journal of Monetary Economics* 38(1), 47-76.

Chirinko, Robert S. and Huntley Schaller (2001), "Business Fixed Investment and 'Bubbles': The Japanese Case," *American Economic Review* 91(3), 663-80.

D'Avolio, Gene D. (2002), "The Market for Borrowing Stock," *Journal of Financial Economics* 66, 271-306.

Duffie, Darrell, Nicolae Garleanu and Lasse Pedersen (2001), "Securities lending, shorting, and pricing," *Journal of Financial Economics* 66, 307-39.

Erickson, Timothy and Toni Whited (1999), "Measurement Error and the relation between Investment and q," *Journal of Political Economy* 108, 1027-1057.

Gilchrist, Simon, and Charles P. Himmelberg (1998), "Investment, Fundamentals, and Finance," In Ben Bernanke and Julio J. Rotemberg, eds., *NBER Macroeconomics Annual* (Cambridge: MIT Press).

Heaton, J. B. (1999), "Managerial optimism and corporate finance," University of Chicago working paper.

Hirshleifer, David (2001), "Investor Psychology and Asset Pricing," *Journal of Finance* 56, 1533-1597.

Hong, Harrison, and Jeremy C. Stein (2003), "Differences of Opinion, Short-Sales Constraints and Market Crashes," *Review of Financial Studies* 16, 487-525.

Johnson, Norman L., Samuel Kotz, and N. Balakrishnan, Continous Univariate Distributions, Volume 1, Second Edition, New York: John Wiley & Sons, 1994.

Jones, Charles, and Owen Lamont (2002), "Short-sale Constraints and Stock Returns," *Journal of Financial Economics* 66, 207–39.

Lakonishok, Joseph, Andrei Shleifer, and Robert V. Vishny (1994), "Contrarian Investment, Extrapolation and Risk," *Journal of Finance*, Vol. 49, pp. 1541-1578.

Malmendier, Ulrike, and Geoffrey Tate (2001), "CEO Overconfidence and Corporate Investment," Harvard University Working Paper.

Mϕrck, Randall, Andrei Shleifer, and Robert W. Vishny (1990), "The Stock Market and Investment: Is the Market a Sideshow?" *Brookings Papers on Economic Activity* 0(2), 157-202.

Miller, Edward M. (1977), "Risk, Uncertainty, and Divergence of Opinion," *Journal of Finance* 32, 1151-1168.

Mϕrck, Randall, Andrei Shleifer, and Robert W. Vishny (1990), "Do Managerial Objectives Drive Bad Acquisitions?" *Journal of Finance* 45(1), 31-48.

Ofek, Eli and Matthew Richardson (2003), "DotCom Mania: The Rise and Fall of Internet Stock Prices." *Journal of Finance* 58, 1113–37.

Park, Cheolbeom (2001), "Stock returns and the Dispersion in Earnings Forecasts," working paper no. 0117, department of economics, National University of Singapore ; http://www.fas.nus.edu.sg/ecs/pub/wp/wp0117.pdf

Polk, Christopher, and Paola Sapienza (2002), "The Real Effects of Investor Sentiment," Working Paper

Scheikman, Jose and Wei Xiong (2003), "Overconfidence and Bubbles," *The Journal of Political Economy* 111,1183-1219.

Shleifer, Andrei (2000), *Inefficient Markets: An Introduction to Behavioral Finance*, Oxford: Oxford University Press.

Shleifer, Andrei, and Robert V. Vishny (1997), "The Limits of Arbitrage," *Journal of Finance* 52, 35-55.

Stein, Jeremy C. (1996), "Rational capital budgeting in an irrational world," *Journal of Business* 69, 429-455.

A Appendix: Model Results

A.1 Properties of the Inverse Demand Curve

We first use equation 4 combined with equation 3 to establish the limiting behavior of the demand curve. We then compute and sign the derivatives B_n and B_σ. It is easier to work with the following version of equation 4

$$n^d(B,\sigma) = \gamma\left[(1-\Phi(b-\sigma))\,B^{-1} - (1-\Phi(b))\right]. \tag{21}$$

Since $\lim_{B\to 0} b = -\infty$ and $\lim_{B\to\infty} b = \infty$

$$\lim_{B\to 0}\gamma\left[(1-\Phi(b-\sigma))\,B^{-1} - (1-\Phi(b))\right] = \lim_{B\to 0}\gamma B^{-1} = \infty \tag{22}$$

To compute the upper limit, we note that $\frac{\partial b}{\partial B} = \frac{1}{\sigma B}$ and apply l'Hopital's rule to obtain

$$\lim_{B\to\infty}\gamma\left[(1-\Phi(b-\sigma))\,B^{-1} - (1-\Phi(b))\right] = \lim_{B\to\infty} -\gamma\frac{\phi(b-\sigma)}{\sigma B} = 0. \tag{23}$$

Thus we have that $\lim_{B\to 0} n^d(B,\sigma) = \infty$ and $\lim_{B\to\infty} n^d(B,\sigma) = 0$. We now compute the partial derivative of n with respect to B, recognizing that b is a function of B:

$$\frac{\partial n}{\partial B} = \gamma\left[-\left[\phi(b-\sigma)B^{-1} - \phi(b)\right]\frac{\partial b}{\partial B} - \left[1-\Phi(b-\sigma)\right]\frac{1}{B^2}\right].$$

By the properties of the lognormal, $\phi(b-\sigma) = \phi(b)B$. Hence the first term in this expression is zero, so that

$$\frac{\partial n}{\partial B} = -\gamma\left[1-\Phi(b-\sigma)\right]B^{-2} < 0. \tag{24}$$

The limiting conditions in equations 22 and 23 and the derivative in equation 24 establish that the market demand curve in equation 4 is invertible with

$$B_n = \frac{-B^2}{\gamma\left[1-\Phi(b-\sigma)\right]} < 0.$$

To compute $B_\sigma = \frac{\partial B}{\partial \sigma}$, we totally differentiate equation (21), holding n fixed:

$$\begin{aligned} 0 &= \gamma\left[-\left(\phi(b-\sigma)B^{-1} - \phi(b)\right)\frac{\partial b}{\partial \sigma} + \phi(b-\sigma))\frac{1}{B}\right]\partial\sigma + \frac{\partial n}{\partial B}\partial B \\ &= \gamma\frac{\phi(b-\sigma)}{B}\partial\sigma + \frac{\partial n}{\partial B}\partial B. \end{aligned}$$

Solving for $\frac{\partial B}{\partial \sigma}$ we obtain:

$$B_\sigma = -\gamma\phi(b)B_n.$$

Substituting in our expression for B_n, we obtain

$$B_\sigma = Bh(b-\sigma) > 0.$$

A.2 Derivation of the Equilbrium Price

To show that a unique solution to equation 11 exists, we use equation 3 to express equation 11 as an equation in b:

$$\exp(\sigma b - 0.5\sigma^2) = \frac{h(b)}{h(b-\sigma)}.$$

The left-hand-side of this equation is strictly positive and monotonically increasing in b. The hazard rate for the standard normal $h(b)$ is monotonically increasing so that $\frac{h(b)}{h(b-\sigma)} > 1$. It is also straightforward to show that $\lim_{b\to-\infty}\frac{h(b)}{h(b-\sigma)} = \infty$, $\lim_{b\to-\infty}\frac{h(b)}{h(b-\sigma)} = 0$ The derivative of the right-hand-side of equation 11 satisfies

$$\begin{aligned}\frac{\partial}{\partial b}\left(\frac{h(b)}{h(b-\sigma)}\right) &= \frac{h(b)}{h(b-\sigma)}\left(\frac{h'(b)}{h(b)} - \frac{h'(b-\sigma)}{h(b-\sigma)}\right)\\ &= \frac{h(b)}{h(b-\sigma)}\left(h\left(b\right) - h\left(b-\sigma\right) - \sigma\right) < 0,\end{aligned}$$

Log-concavity of $h\left(x\right)$ implies that $\left(h\left(b\right) - h\left(b-\sigma\right) - \sigma\right) < 0$ which establishes the inequality (see the appendix of Gilchrist and Williams, 2001). These results are sufficient to guarantee a unique equilibrium value of b and hence B for equation 11. The uniqueness of the equilibrium value $n(\sigma,\gamma)$ for equity issuance, equation 12, follows directly from these results.

A.3 The Effect of an Increase in Dispersion on Price and Equity Issuance

Before analyzing the equilibrium response of B and n to an increase in dispersion, we first establish that equilibrium occurs at $b > \sigma$. To do so we note that at $b = \sigma$ we have

$$\begin{aligned}\frac{h(b)}{h(b-\sigma)} - B \quad \mid \quad_{b=\sigma} &= \exp(-0.5\sigma^2)\left[\frac{\exp(0.5\sigma^2)}{2(1-\Phi(\sigma))} - 1\right]\\ &> 0 \quad for \quad \sigma > 0.\end{aligned}$$

To obtain the inequality, we note that at $\sigma = 0$, the term in brackets on the right-hand-side of this expression is identically zero. This term is also strictly increasing in σ and therefore positive for $\sigma > 0$. Thus at $b = \sigma$ we have $\frac{h(b)}{h(b-\sigma)} > B$. Since $h(b)/h(b-\sigma)$ is decreasing in b and $B = \exp(\frac{b-0.5\sigma^2}{\sigma})$ is increasing in b, the equilibrium must therefore occur at $b > \sigma$.

We now totally differentiate equation (11) to obtain $\frac{dB}{d\sigma}$. For short-hand notation, let $g(B, \sigma) = h(b)/h(b-\sigma)$ so that equilibrium implies $B = g$. It is straightforward to show that

$$\frac{dB}{d\sigma} = \frac{\left[g_b \frac{\partial b}{\partial \sigma} + g_\sigma\right]}{\left[1 - g_b \frac{\partial b}{\partial B}\right]}.$$

where

$$\begin{aligned} g_b &= g\left[h(b) - h(b-\sigma) - \sigma\right] < 0 \\ g_\sigma &= g[h(b-\sigma) - (b-\sigma)] > 0 \end{aligned}$$

and

$$\begin{aligned} \frac{\partial b}{\partial \sigma} &= \frac{-(b-\sigma)}{\sigma} \\ \frac{\partial b}{\partial B} &= \frac{1}{B\sigma}. \end{aligned}$$

Inserting these expressions and simplifying, we obtain

$$\begin{aligned} \frac{dB}{d\sigma} &= \frac{\left[\frac{-(b-\sigma)}{\sigma} g\left[h(b) - h(b-\sigma) - \sigma\right] + g[h(b-\sigma) - (b-\sigma)]\right]}{\left[1 - g\left[h(b) - h(b-\sigma) - \sigma\right] \frac{1}{B\sigma}\right]} \\ &= \frac{B\left[-(b-\sigma)\left[h(b) - b - (h(b-\sigma) - (b-\sigma)\right] + \sigma[h(b-\sigma) - (b-\sigma)]\right]}{\left[\sigma - \left[h(b) - h(b-\sigma) - \sigma\right]\right]}. \end{aligned}$$

Simplifying we have:

$$\frac{dB}{d\sigma} = \frac{B\left(b\left[h(b-\sigma) + \sigma - h(b)\right)\right] + \sigma\left[h(b) - b\right])}{\left(\sigma + \left[h(b-\sigma) + \sigma - h(b)\right]\right)} > 0. \tag{25}$$

Again, log-concavity of the hazard rate implies that $h(b-\sigma) + \sigma - h(b) > 0$ so that all the terms in square brackets in equation 25 are positive which establishes the inequality.

To show $\frac{dB/B}{d\sigma} < h(b-\sigma)$ we note that $\left[1 - g_b \frac{\partial b}{\partial B}\right] > 1$ so that

$$\begin{aligned} \frac{dB/B}{d\sigma} &= \frac{\left[\frac{g_b}{g}\frac{\partial b}{\partial \sigma} + \frac{g_\sigma}{g}\right]}{\left[1 - g_b \frac{\partial b}{\partial B}\right]} \\ &< \frac{g_b}{g}\frac{\partial b}{\partial \sigma} + \frac{g_\sigma}{g} \end{aligned}$$

where

$$\begin{aligned} \frac{g_b}{g}\frac{\partial b}{\partial \sigma} + \frac{g_\sigma}{g} &= [h(b-\sigma) - h(b) + \sigma]\left(\frac{b-\sigma}{\sigma}\right) + [h(b-\sigma) - (b-\sigma)] \\ &= h(b-\sigma) - [h(b) - h(b-\sigma)]\left(\frac{b-\sigma}{\sigma}\right) \end{aligned}$$

As established above, equilibrium requires $b > \sigma$. The term in brackets is positive implying,

$$\frac{g_b}{g}\frac{\partial b}{\partial \sigma} + \frac{g_\sigma}{g} < h(b-\sigma)$$

therefore

$$\frac{dB/B}{d\sigma} < h(b-\sigma).$$

To compute $\frac{dn}{d\sigma}$, we use equation 21 and equation 11 to provide an alternative expression for equity issuance in equilibrium:

$$n = \gamma\left[\left(1 - \Phi(b-\sigma)\right)B^{-1} - \left(1 - \Phi(b)\right)\right]. \tag{26}$$

Totally differentiating equation 26 with respect to σ we obtain

$$\frac{dn}{d\sigma} = -\gamma\left[\phi(b-\sigma)B^{-1} - \phi(b)\right]\frac{db}{d\sigma} + \gamma\left(1 - \Phi(b-\sigma)\right)B^{-2}\frac{dB}{d\sigma} + \frac{\gamma\phi(b-\sigma)}{B}.$$

The term in brackets is identically zero. Rearranging yields:

$$\frac{dn}{d\sigma} = \gamma\frac{[(1 - \Phi(b-\sigma))]}{B}\left(h(b-\sigma) - \frac{dB/B}{d\sigma}\right). \tag{27}$$

B Appendix: Data construction

We assemble annual, firm-level data from two sources. Data on sales, capital expenditures, net cash flows from equity issuance, total assets, total liabilities, preferred equity, and property, plant and equipment are obtained from Standard & Poors Compustat. These variables are merged with a custom data extract on analysts forecasts provided by IBES to Deither, Malloy and Scherbina (2002) (and kindly provided to us by Anna Scherbina). In contrast to the usual IBES data, the data used in Deither et al. do not suffer from measurement errors caused by the truncation of significant digits (see their paper for further details). Variable definitions are as follows.

- *Investment* (I_t/K_t) is the ratio of capital expenditures to beginning-of-period net book value of property, plant and equipment.

- *Marginal profit of capital* (mpk_t, or "MPK") is the logarithm of a standardized ratio of sales divided by lagged book value of property, plant and equipment (end-of-fiscal-year values). Before taking logs, the sales-to-capital is divided by the industry average ratio (computed on a sample trimmed at the one percent tails), and then multiplied by 0.2. This standardization accomodates cross-industry differences in the fixed capital share of production, and reduces the chance of misclassifying ratios in low-capital industries as "outliers." In steady state, MPK should equal the long-run cost of capital, $r + \delta$. Normalizing the scaled ratio by $r + \delta = 0.2$ thus centers the sample average of MPK at a reasonable value, but obviously has no effect on the statisticaly properties of our estimates. For details, see Gilchrist and Himmelberg (1998).

- *Dispersion* (d_t) is the logarithm of the fiscal year average of the monthly standard deviation of analysts' forecasts of earnings per share, times the number of shares, divided by the book value of total assets. That is,

  $$d_t = \log\left(\frac{\sum_{j=1}^{12} N_{t-j}\sigma_{t-j}/12}{Total\ Assets}\right),$$

  where N_{t-j} is the number of shares outstanding, and σ_{t-j} is the variance of earnings forecasts for all analysts making verified forecasts for the month. These data are taken from the IBES summary tape. "Verified" means that IBES has confirmed the forecast is not stale.

- *Net equity issuance* (neq_t) is cash from new share issues minus cash used for share repurchases during the fiscal year divided by the beginning-of-period market value of equity (and multiplied by 100).

- *Tobin's Q*, (Q_t) is the market value of equity plus the book value of preferred equity plus the book value of total liabilities divided by beginning-of-period book value of total assets.

The variables I_t/K_t, Q_t, d_t, and mpk_t are set to missing if their values are below zero or higher than their 99th percentile; neq_t is trimmed at the 1st and 99th percentiles. Trimming reduces the impact of extreme values which are common for ratios in firm panels drawn from accounting data. The use of logs (where possible) also mitigates this problem. We drop observations for which the lag between consecutive fiscal-year-ends is not exactly 12 months. (The month in which the fiscal years sometimes changes for such reasons as mergers or restructurings.) Our final sample size is constrained by the availability of IBES data. From 1979 through 1985, sample size rises from 54 to 95. (Excluding these early years from our sample does not change our results). In 1986, the sample size rises sharply to 1185 firms, and then increases more or less steadily to 1771 firms in year 2000. In total, our sample contains 22522 non-missing firm-year observations, of which 18421 have non-missing values for the first two lags, too. Table A reports summary statistics on the full sample.

Table A: Summary Statistics

				Percentiles				
Variable		Mean	S.D.	Min	25th	50th	75th	Max
$\ln mpk_t$	22522	-1.786	0.712	-6.086	-2.181	-1.809	-1.426	3.249
$\ln d_t$	22522	0.383	1.370	-5.495	-0.538	0.280	1.231	7.065
$\ln (I_t/K_t)$	22522	-1.341	0.912	-7.131	-1.897	-1.365	-0.800	4.922
$\ln Q_t$	22522	0.542	0.585	-1.548	0.154	0.417	0.836	4.694
$\ln neq_t$	22522	0.012	0.137	-1.188	-0.008	0.000	0.000	8.068

Finally, the IBES data provided by Deither et al. effectively end in 2000. To construct aggregate means through 2002, we instead use standard IBES data. Comparing overlapping data in the pre-2001 period shows that annual means are not sensitive to the truncation issues.

Figure 1: Equilibrium share price (B) and share issuance (n).

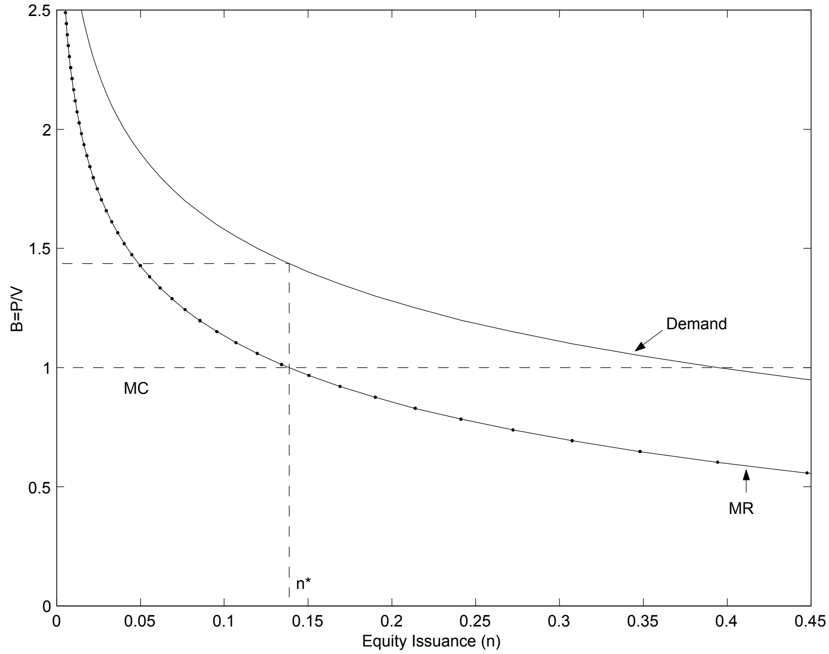

Figure 2: The effect of an increase in dispersion.

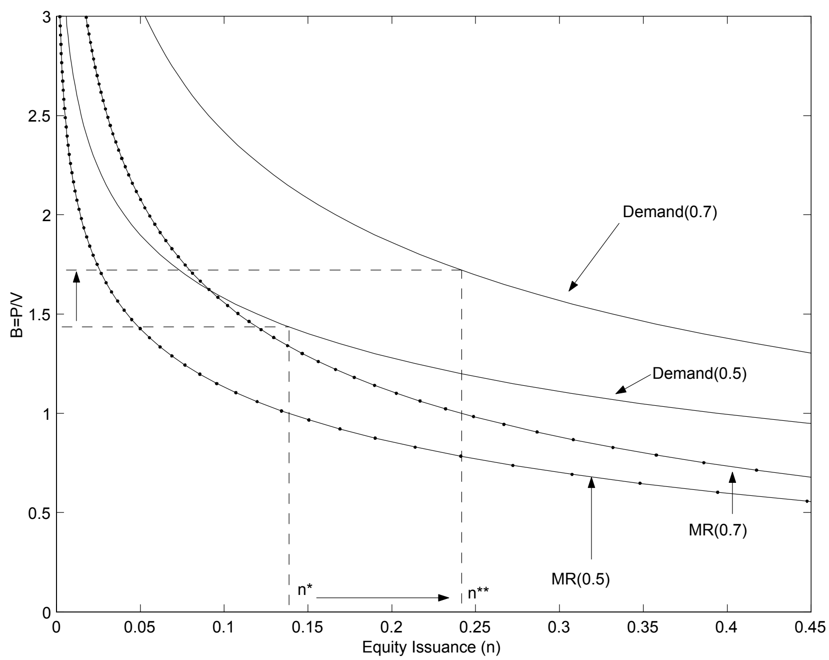

Figure 3

Comparison of Nasdaq vs. NYSE firms for the time period 1990-2002. Figure (a) plots the stock market index. Figures (b)-(f) plot the log of the (trimmed) sample means in each year, normalized to one in 1990.

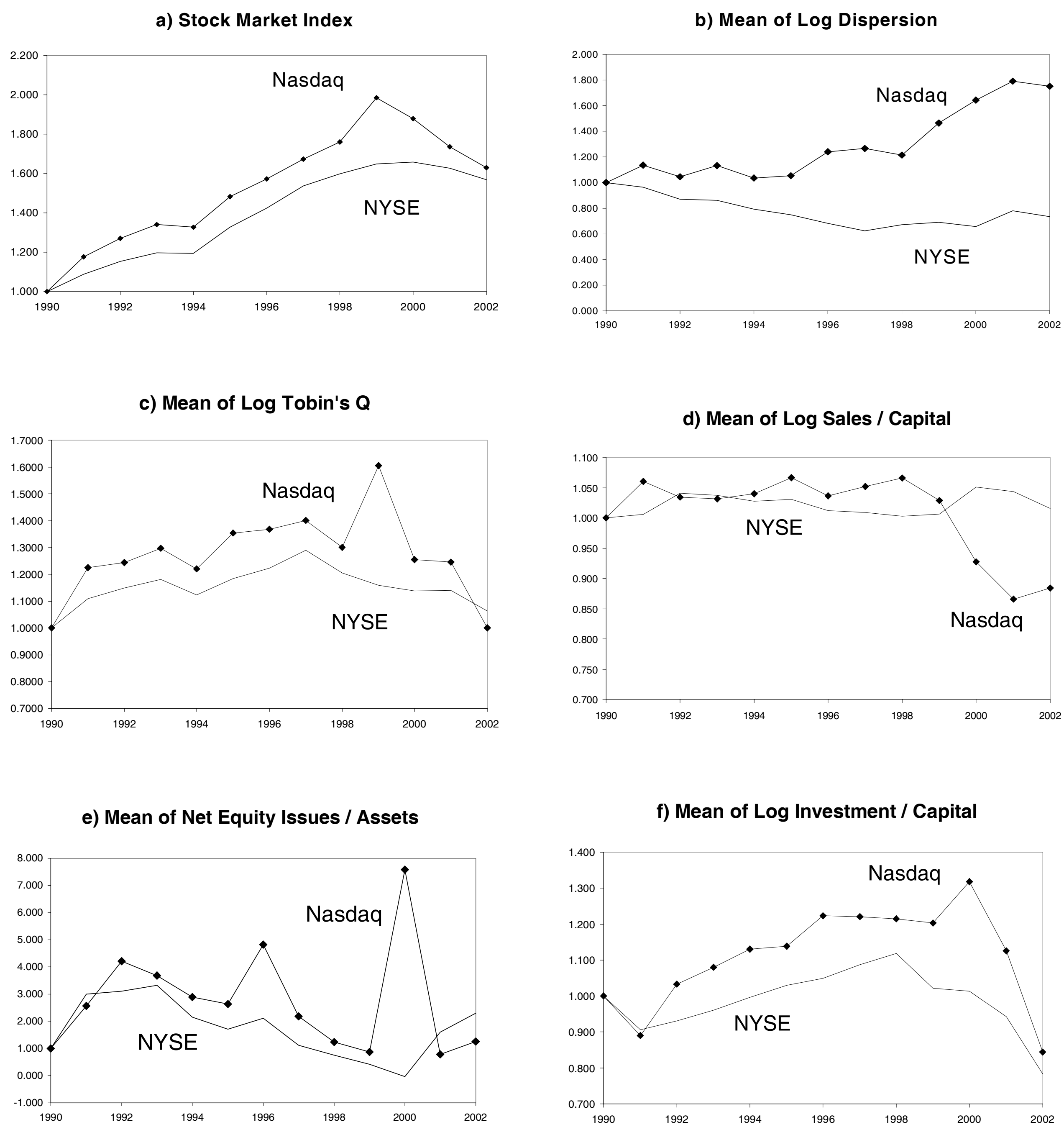

Figure 4

Vector-autoregressions for 3-variable model. Column headings indicate response variables; row headings indicate shocks. Horizontal axis shows 10-year response interval (not labeled).

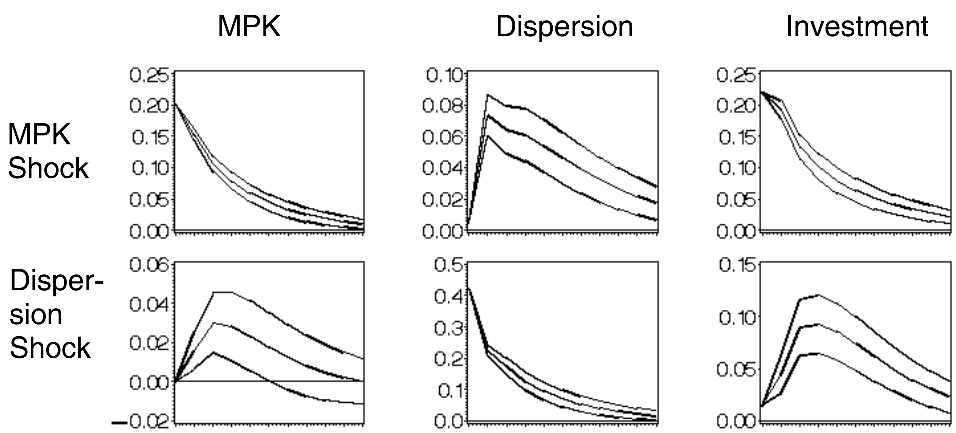

Figure 5

Vector-autoregressions for 5-variable model. Column headings indicate response variables; row headings indicate shocks. Horizontal axis shows 10-year response interval (not labeled).