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The Topology of Interbank Payment Flows

Kimmo Soramäki
Morten L. Bech
Jeffrey Arnold
Robert J. Glass
Walter E. Beyeler

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Abstract

We explore the network topology of the interbank payments transferred between commercial banks over the Fedwire® Funds Service. We find that the network is compact despite low connectivity. The network includes a tightly connected core of money-center banks to which all other banks connect. The degree distribution is scale-free over a substantial range. We find that the properties of the network changed considerably in the immediate aftermath of the attacks of September 11, 2001.

Key words: network, topology, interbank, payment, Fedwire, September 11, 2001

Soramäki: Helsinki University of Technology. Bech: Federal Reserve Bank of New York. Arnold: Federal Reserve Bank of New York. Glass: Sandia National Laboratories. Beyeler: Sandia National Laboratories. Address correspondence to Morten L. Bech (e-mail: morten.bech@ny.frb.org). The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System.

I. INTRODUCTION

At the apex of the U.S. financial system is a network of interconnected financial markets by which domestic and international financial institutions allocate capital and manage their risk exposures. The events of September 11, 2001 and, to a lesser extent, the North American blackout of August 14, 2003 underscored that these markets are vulnerable to wide-scale disruptions. The inability of one of these markets to operate normally can have wide-ranging effects not only for the financial system but potentially for the economy as a whole. Currently, the financial industry and regulators are devoting considerable resources to strengthen the resiliency of the U.S. financial system [35, 37]. Critical to the smooth functioning of these markets are a number of wholesale payments systems and financial infrastructures that facilitate clearing and settlement.

Despite their importance, little empirical research on the impact of disruptions these systems and infrastructures is available. One branch of the literature has focused on simulating the effects from a default of a major participant [4, 10, 16, 25]. Another branch has done detailed case studies of disruptions to the U.S. financial system, e.g. the 1987 stock market crash and the events following September 11th [11, 20, 28, 30].

However, the payment system can be treated as a specific example of a complex network. In recent years, the physics community has made significant progress towards understanding the structure and functioning of complex networks [5, 7, 18, 32, 39]. The literature has focused on characterizing the structure of networked systems and how the properties of the observed topologies relates to stability, resiliency and efficiency in case of pertubations and disturbances [5, 15].

A few recent papers [12, 22, 26] have started to describe the actual topologies observed in the financial system using this methodology. This paper adds to this literature by describing the network topology of the interbank payment flows in the U.S. Moreover, we add new insight to the response of complex networks to pertubations by analyzing the effects of September 11th on the network.

The paper is organized as follows. Section II introduces network theory and the main concepts applied. Section III describes the data. Section IV presents a visualization of the data. Section V defines and discusses the topological characteristics of the interbank payment flows. Section VI looks at the impact of September 11th on these characteristics. Section VII concludes.

II. NETWORKS

A *network* consists of two types of elements, *nodes* and the connections between them, *links*. Links can have *weights* attached to them representing the importance of the relationship between nodes. Links can be either *undirected* or *directed*. A link from a node to itself is called a *loop*. The *neighbors* of a node are all the nodes to which it has a link. The *predecessors* of a node are the nodes that have a link to the node and the *successors* are the nodes that have a link from the node. A walk is a sequence of nodes in which each node is linked to the next. A walk is a *path* if all its nodes are distinct. The length of a path is measured by the number links. If the start node and the end node of a path are one and the same, then it forms a *cycle*.

A *complete* network is a network where all nodes have a link to each other. A *tree* is a network in which any two nodes are connected by exactly one path. In a *star* network all nodes connects to a central node called the *hub*. A *component* of a network is a subset of nodes in a network such that any two nodes can be joined by a path. A *connected* network consists of a single component, while a *disconnected* network is made up of two or more components. These concepts are illustrated in Fig. 1a.

There are two classes of network formation models some times referred to as equilibrium and non-equilibrium models [18]. Equilibrium models have a fixed set of nodes with randomly chosen pairs of nodes connected by links. Erdös and Rényi [19] proposed a basic model of a network with n nodes, where each pair of nodes is connected by a link with probability p. This type of network is commonly referred to as a *classical random* network. Non-equilibrium network models grow a network by successively adding nodes and setting probabilities for links forming between the new nodes and existing nodes and between already existing nodes. Many of these models, notably the Barabasi and Albert (BA) [6] model, are based on preferential attachment. Preferential attachment assigns a probability of a link forming with a node that is increasing with the number of links of the node. Both the classical random network and the BA network will be used as benchmarks in the following sections.

III. INTERBANK PAYMENTS

We use transaction data from the Fedwire® Funds Service (Fedwire) service to create the interbank payment network. Fedwire is a real-time gross settlement (RTGS) system, operated by the Federal Reserve System, in which more than $9,500$ participants initiate funds transfers that are immediate, final, and irrevocable when processed. Participants use Fedwire to process large-value, time-critical payments, such as payments for the settlement of interbank purchases and sales of federal funds; the purchase, sale, and financing of securities transactions; the disbursement or repayment of loans; and the settlement of real estate transactions. Fedwire is used also for the settlement of ancillary payment systems such as automated clearing houses (ACH) and other large
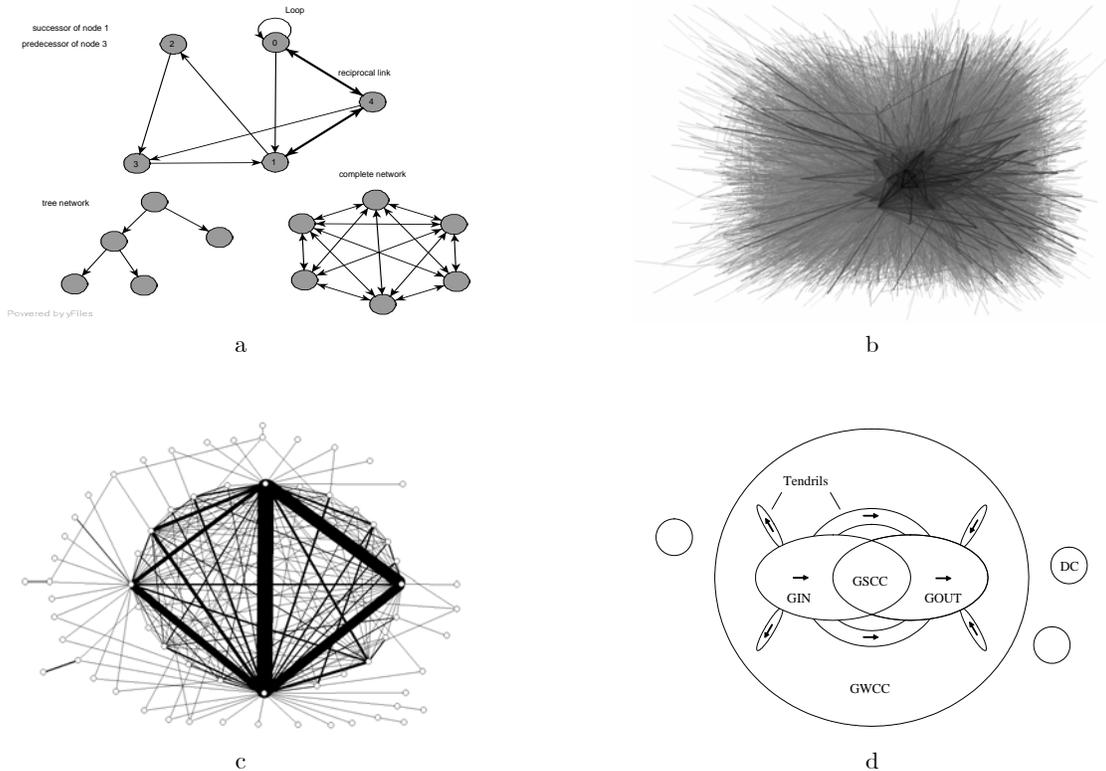
FIG. 1: a) Simple networks and some commonly used terms. b) Fedwire interbank payment network. First day of sample. 6600 nodes and 70.000 links. c) Core of Fedwire Interbank Payment Network, largest links equivalent to 75 percent of daily value transferred. d) Network Components. GWCC = giant weakly connected component, DC = disconnected component, GSCC = giant strongly connected component, GIN = giant in component, GOUT = giant out component.

value payment systems e.g. CHIPS and CLS.[1]

We analyze at the network of the actual payments flows transferred over Fedwire. We choose to model the payment flows as a directed network and establish a link from the sender of payment to the receiver of payment on the basis of payments sent. While Fedwire participants include a variety of entities, including government agencies, we consider only the subset of payments between commercial banks. Thus, commercial banks constitute the nodes in the network, and a directed link from one bank to another is present in a day if at least one transaction debits the account of former and credits the account of the latter.[2] Additional transactions between any two banks add to the associated link weights in terms of value and volume (number) of payments settled. The network could be defined in alternative ways. From a technical perspective, Fedwire is a star network where all participants are linked to a central hub, i.e., the Fed-

eral Reserve, via a proprietary telecommunications network. From a payment processing perspective Fedwire is a complete network as all nodes (particpants) are linked in the sense that they can send and receive payments from each other. However, these representations do not represent the actual behavior of participants and the flow of liquidity in the system.

We consider data for the first quarter of 2004. Each day is modeled as a separate network, for an ensemble of 62 daily networks. The average daily value of funds transfers between commercial banks amounts to $1.3 trillion and the daily number of payments to $345,000$. On the peak activity day, $644,000$ payments worth more than $1.6 trillion were processed. The average value per payment is $3 million, but the distribution is highly right-skewed with a median payment of only $30,000$. Both the daily value and volume of payments show periodicity around the first and last days of the month as well as on mid month settlement days for fixed income securities (see Fig. 2).

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[1] The Clearing House Inter-bank Payment System (CHIPS) and Continuous Linked Settlement (CLS).

[2] We use master-accounts, and exclude loops that result from banks making transfers across their own sub-accounts.

IV. VISUALIZING THE NETWORK

An intuitive way to analyze a network is to draw it as a graph, as in Fig. 1a. The interbank payment network on the first day of our sample is illustrated in Fig. 1b. The figure includes over $6,600$ nodes and more than $70,000$ links. Each link is shaded by the associated weight, with darker shades indicating higher values. Despite the appearance of giant fur ball, the graph suggests the existence of a small group of banks connected by high value links. To gain a clearer picture of this group, we graph a subset of our network in Fig. 1c where we focus on high value links. This graph shows the undirected links that comprise 75 percent of the value transferred. This network consists of only 66 nodes and 181 links. The prominent feature of this network is that 25 nodes form a densely connected sub-graph, or clique, to which the remaining nodes connect. In other words, we find that only a small number of banks and the links between them constitute the majority of all payments sent over the network.

V. TOPOLOGY CHARACTERISTICS

The large number of nodes and links makes detailed analysis of the structure by visualization difficult and comparisons across time and between networks almost infeasible. The complexity of our network leads us to consider statistical measures. In this section, we describe a series of commonly used statistical measures of topological characteristics for the interbank payment network.

A. Components

A starting point for the quantitative analysis of a network is to partition the set of nodes into components according to how they connect with other nodes. Dorogovtsev et al [17] divide a network into a single *giant weakly connected component* (GWCC) and a set of *disconnected components* (DCs). The GWCC is the largest component of the network in which all nodes connect to each other via undirected paths. The DCs are smaller components for which the same is true. In empirical studies the GWCC is found to be several orders of magnitude larger then any of the DCs [13]. This is also the case here.

The GWCC consists of a *giant strongly connected component* (GSCC), a *giant out-component* (GOUT), a *giant in-component* (GIN) and *tendrils*. The GSCC comprises all nodes that can reach each other through a directed path. A node is in the GOUT if it has a path from the GSCC but not to the GSCC. In contrast, a node is in GIN if it has a path to the GSCC but not from it. Tendrils are nodes that have no directed path to or from the GSCC. They have a path to the GOUT and/or the GIN (see Fig. 1d).

Over our sample period a total of $7,584$ different banks are part of the network. We find that the network's GWCC is composed of on average $6,490 \pm 83$ (mean $\pm$ standard deviation over the 62 days) banks. On 36 of the 62 days we also find a small number of DCs consisting of between two to eight banks. Over the sample period $6,854$ different banks were part of the GSCC. Of these, $2,578$ were present in the network on all days. The GSCC contains 78 percent of the nodes in the GWCC on average whereas the GIN, GOUT and tendrils contain 8, 12 and 2 percent, respectively. In terms of value transferred, 90 percent occur within the GSCC and 7 percent is from GIN to GSCC. We will focus on the GSCC component in the analysis below.

B. Size, connectivity and reciprocity

Definition The most basic properties of a network are the number of nodes n and the number of links m. The number of nodes defines the *size* of the network while the number of links relative to the number of possible links defines the *connectivity* of a network. The connectivity (p) is the unconditional probability that two nodes share a link. For a directed network, the connectivity is $p = \frac{m}{n(n-1)}$. It ranges from $\frac{1}{n}$ for a tree network to 1 for a complete network. Reciprocity is the fraction of links for which there is a link in the opposite direction in the network.

Discussion The average size of the daily network was $5,086 \pm 128$ nodes. Almost $710,000$ different links were found between banks over the sample period, with only $11,000$ of them present on all days. On average the network had $76,614 \pm 6,151$ directed links. In comparison, a complete network of similar size has over 25 million links. The connectivity is only 3 ± 0.1 per mil. In other words, the interbank payment network is extermely sparse as 99.7 % of the potential links are not used on any given day. The reciprocity averages 22 ± 0.3 percent. Hence, less than a quarter of the relationships that exist between banks had payments going in both directions.

The number of nodes and links are almost perfectly correlated across days and are both highly correlated with the value and volume of payments settled (see Tab. I). In particular, the size and number of links spike on the high value and volume days identified in section III (see Fig. 2). Interestingly, the connectivity is also highly correlated with value and volume. On high payment activity days, the network not only grows but it also becomes denser as the level of interactions between banks increases faster than size. However, the same is not true for reciprocity. Reciprocity is uncorrelated with value and volume and connectivity. The extent to which the relationships between banks are bilateral does not appear to depend on either the overall level of payment activity or the connectivity between banks.
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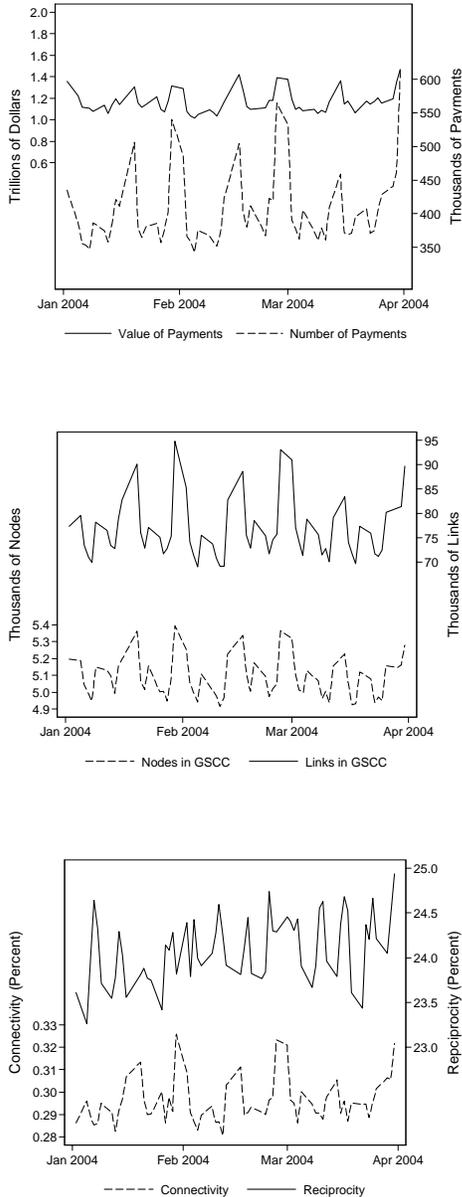
FIG. 2: Top: Value and volume of interbank payments transferred over Fedwire in fourth quarter 2004. Middle: The number of nodes and links in giant strongly connected component (GSCC). Bottom: Connectivity and reciprocity in GSCC.

The periodicity in the size and number links suggests that it might be insightful to model high payment activity days separately. For simplicity, we ignore this and treat the networks as coming from the same data generating process.

ρ_{xy}	Value	Volume	n	m	p	r
Value						
Volume	0.88					
n	0.73	0.84				
m	0.78	0.92	0.96			
p	0.76	0.91	0.80	0.93		
r	0.06	0.09	-0.32	-0.18	-0.003	

TABLE I: Correlations of basic network properties. n = size, m = number of links, p = connectivity, r = reciprocity.

C.   Distance and diameter

Definition The *distance* from node i to node j (d_{ij}) is the length of the shortest path between the two nodes. If node i has a link to node j, then $d_{ij} = 1$. The average distance from a node to any other node, commonly referred to as the *average path length* of a node, is $\ell_i = \frac{1}{n-1} \sum_{j \neq i} d_{ij}$. The average path length of a network is defined as $\langle \ell \rangle = \frac{1}{n} \sum_i \ell_i$. In a classical random network $\langle \ell \rangle \approx \frac{\ln(n)}{\ln(p \cdot n)}$. The *eccentricity* of a node (ε_i) is the maximum distance to any other node in the network, i.e., $\varepsilon_i = \max_j d_{ij}$. The *diameter* of a network (D) is the maximum eccentricity (or distance) across all nodes, i.e., $D = \max_i e_i$. Goh et al. [23] defines the *mass distance function* as the fraction of nodes within a certain distance of a node. At the network level it is defined as $M(x) = \frac{1}{n(n-1)} \sum_i \sum_{i \neq j} 1(d_{ij} \leq x)$, where $1(\cdot)$ is the indicator function taking the value one if true and zero otherwise.

Discussion The average path length is $\langle \ell \rangle = 2.6 \pm 0.2$ across our sample. In comparison, the average path length of a same size classical random network is 3.2. The mean eccentricity is $\langle \varepsilon \rangle = 4.7 \pm 0.33$, and the diameter ranges between 6 and 7 across days (see Tab. II). The interbank payment network exhibits the *small-world* phenomenon common for many complex networks. Informally, small world means that any node can be reached from any other node in only a few steps. A short path length to any other node is a common property for all nodes in the network as illustrated by the mass distance function is shown in Tab. II. It shows that it is possible for a sparse network with low connectivity to be extremely compact. Although few nodes directly connect, 41 percent are within two links, and 95 percent are within three links from each other. This further reflects the notion that the interbank payment network is comprised of a core of hubs with whom smaller banks interact.

D.   Degree distribution

Definition Two important characteristics of a node in a directed network are the number of links that originate from the node and the number of links that terminate

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	Mean	Median	Min.	Max.	SD		Mean	Median	Min	Max	SD
Payments						Distance Measures					
Volume (,000)	436	411	371	644	60.3	$\langle\ell\rangle$	2.62	2.63	2.56	2.66	0.02
Value ($tr)	1.30	1.27	1.13	1.64	0.11	$\langle\varepsilon\rangle$	4.67	4.63	4.18	5.74	0.33
Average ($mn)	3.01	3.06	2.48	3.35	0.20	D	6.6	7	6	7	0.5
Components						$M(2)$ (%)	41.6	41.3	38.9	47.3	2.0
GWCC	6,460	6,484	6,355	6,729	83	$M(3)$ (%)	95.9	95.8	95.1	97.1	0.5
DC	2	2	0	8	2	$M(4)$ (%)	99.9	99.9	99.8	100	0.0
GSCC (n)	5,086	5,066	4,914	5,395	123	Clustering					
GIN	527	528	404	645	49	$\langle C\rangle$	0.53	0.53	0.51	0.55	0.01
GOUT	774	782	595	916	67	Degree Distribution					
Tendrils	103	103	88	116	7	$\langle k\rangle$	15.2	14.8	13.9	17.6	0.8
Connectivity and Reciprocity						Max k^{out}	1,922	1,913	1,772	2,269	121
m	76,614	75,397	69,077	94,819	6,151	Max k^{in}	2,097	2,070	1,939	2,394	115
p (%)	0.3	0.29	0.28	0.33	0.01	$\hat{\gamma}^{out}_{\mathrm{MLE}}$	2.11	2.11	2.09	2.14	0.01
r (%)	21.5	21.5	21	23	0.03	$\hat{\gamma}^{in}_{\mathrm{MLE}}$	2.15	2.15	2.15	2.18	0.01

TABLE II: Turnover, component and network statistics for the Fedwire interbank payment network, fourth quarter 2004. $tr = $trillion, $mn = $million, GWCC = giant weakly connected component, GSCC = giant strongly connected component, GIN = giant in component, GOUT = giant out component, DC = Disconnected component. All network statistics are calculated for GSCC. n = size, m = number of links, p = connectivity, r = reciprocity, $\langle\ell\rangle$ = average path length, $\langle\varepsilon\rangle$ = average eccentricity, D = diameter, $M(x)$ = mass distance function, $\langle C\rangle$ = clustering coefficient, $\langle k\rangle$ = average degree, k^{in} = in-degree, k^{out} = out-degree, γ = power law coefficient.

at the node. These two quantities are referred to as the *out-degree* (k_i^{out}) and *in-degree* of a node (k_i^{in}), respectively. The average degree of a node in a network is the number of links divided by the number of nodes, i.e., $\langle k\rangle = \frac{1}{n}\sum k_i^{out} = \frac{1}{n}\sum k_i^{in} = \frac{m}{n}$. Networks are often categorized by their degree distributions, $P(k_i = x)$. The degree distribution of a classical random network is a Poisson distribution. Many real networks have fat-tailed degree distributions and a large number have been found to follow the power law $P(k_i = x) \backsim k_i^{-\gamma}$ in the tail. A power-law distribution is also sometimes called a *scale-free* distribution and networks with such a degree distribution are referred to as scale-free networks.[3]

Discussion The average degree in the network is $\langle k\rangle = 15.2 \pm 0.8$. However, most banks have only a few connections and a small number of "hubs" have thousands of links to other banks. Almost half of all banks have 4 or fewer outgoing links and 15 percent have only a single outgoing link. The banks with the largest in- and out-degrees averaged $2,097\pm115$ and $1,922\pm121$ links respectively. The out-degree distribution of the interbank payment network is shown in Fig. 3a together with the degree distribution for a classical random network with

the same connectivity. For degrees greater than 10, the distribution follows a power law. The maximum likelihood estimate of the coefficient is $\hat{\gamma} = 2.11 \pm 0.01$ with a standard error of $\hat{\sigma}_\gamma = 0.03\pm0.001$. Appendix I describes the estimation procedure. The correlation between in and out degrees across nodes is 0.97 and the in-degree distribution looks similar to the out-degree distribution. It has a lower power law coefficient of $\hat{\gamma} = 2.15 \pm 0.01$ with the same standard error. The degree distributions are comparable to those of the Japanese interbank payment system (BOJ-NET) reported in Inaoka et al [26].[4] A degree distribution following a power law distribution with a coefficient between 2.1 and 2.3, is not unique to payment systems, and appears to be a common feature in complex networks.[5] Albert et al [3], and Crucitti et al. [15] find that scale-free networks are robust to random failures, but vulnerable to targeted attacks.

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[3] The term "scale-free" refers to the fact the the distribution remains unchanged within a multiplicative factor under a rescaling of the random variable, i.e., $P(bk) = aP(k)$ (Newman [32] p. 186). For example, if a bank with four links is twice as likely as one with eight links then a bank with eight links should also be twice as likely as a bank with 16 links and so on.

[4] Inaoka et al [26] and Boss et al. [12], both find evidence of scale-free distributions in their tails of their networks. Inaoka et al. report a degree distribution with a power law tail of 2.3 for k > 20 and Boss et al. report a coefficient of 3.1 for the out-degree and 1.7 for in-degree k>40.

[5] Power law degree distributions have been observed in a wide variety of systems, including the phone call network [1], the metabolic network of E. coli bacteria [27], the movie actor collaboration network [6], and the World Wide Web [2]. Although the observation of power-law behavior in complex networks is recent, the phenomena was observed by as long ago as 1896, by Vilfredo Pareto in the distribution of income [34].

a

b

c

d
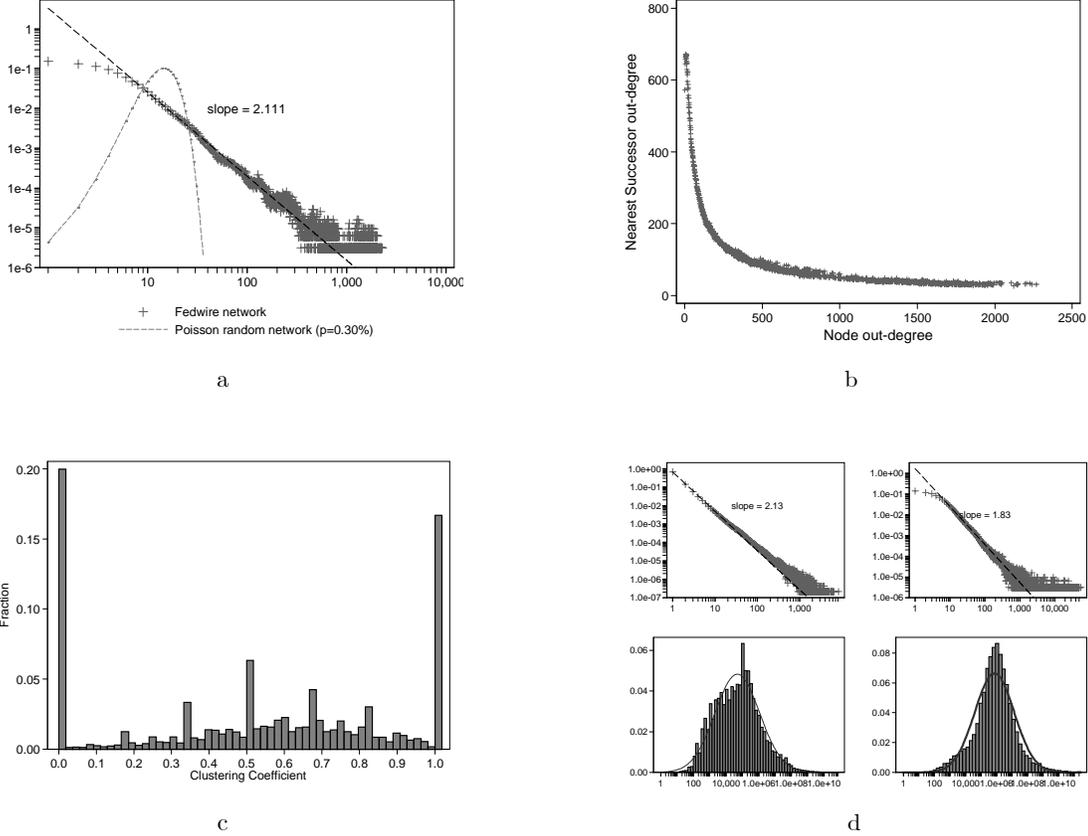
FIG. 3: a) Out-degree Distribution. b) Average nearest successor out-degree as a funtion of out-degree. c) Distribution of clustering coefficient d) Top left hand corner: Volume link weight distribution. Top right hand corner: Volume out-strength distribution. Bottom left hand corner: Value link weight distribution. Bottom right hand corner: Value out-strength distribution.

E. Degree correlations

Definition The probability that a node connects to another node may depend on the characteristics of the respective nodes. One characteristic of a node that is endogenous to the structure of the network is its degree. In an *uncorrelated* network there is no dependence between the degree of a node and the degrees of its neighbors. A network is said to be *assortative* if nodes with a given degree are more likely to have links with nodes of similar degree. A network is said to be *disassortative* if the opposite is true, i.e., nodes with low degrees are more likely to be connected to nodes with high degrees, and vice versa (e.g. Catanzaro et al [14]). The sign of the Pearson correlation coefficient between the degree of each nodes at the end of a link shows the direction of the dependency: zero for uncorrelated networks, positive for assortative networks and negative for dissassortative networks. Another method is to compute the average degree of the nearest neighbors of a node as a function of the node degree. This is known as the average nearest neighbor

degree (ANND) function, $\langle k_{nn} \rangle (k)$.[6]

Discussion Both the correlation coefficients and the average nearest neighbor degree functions show that the interbank payment network is disassortative. For example, the correlation of out degrees is -0.31 and the ANND defined for the out-degree of successors is inversely related to the out degree of a node as shown in Fig. 3b. Using other combinations of neighbors and degree direction, we get similar coefficients. Disassortivity is more common among technological and biological networks, as opposed to social networks which tend to exhibit assori-

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[6] In general, the correlations as well as the ANND function can be defined for any combination of in and out degree of nodes. Moreover, the ANND function can be defined for any set of nearest neighbors e.g. predessors or successors (see Serrano et al [36]). Let $P(k', k)$ denote the probability of two nodes, with degrees k' and k, being joined by a link. The conditonal probalbility of a node with degree k being joined to a node of degree k' node is given by $P(k'|k) = P(k', k)/P(k)$ where $P(k)$ is the degree distribution. The average nearest neighbor degree (ANND) function is given by $\langle k_{nn} \rangle (k) = \sum_{k'} k' P(k'|k)$.

tative properties [32]. Moreover, Newman [31] shows that assortative networks percolate more easily than disassortative networks and that they are more robust to node removal.

F. Clustering coefficient

Definition Another common correlation between nodes is the probability that two nodes which are the neighbors of the same node, themselves share a link. This is equivalent to the observation that two people, each of whom is your friend, are likely to be friends with each other. One way of measuring the tendency to cluster is the ratio of the actual number of directed links between the neighbors of a node $(m_{nn,i})$ over the number of potential links among them

$$C_i = \frac{m_{nn,i}}{k_i(k_i - 1)} \qquad (1)$$

The clustering coefficient for the entire network, $\langle C \rangle$, is given by the average of all individual coefficients [38]. A tree network has a clustering coefficient of zero, and a complete network a coefficient of one. In a classical random network, the clustering coefficient is the unconditional probability of connection, i.e., $\langle C \rangle = p$.

Discussion The average clustering coefficient for the networks calculated for the successors of node is 0.53 ± 0.01 suggestive of cliquishness in the interbank payments. As such the observed clustering coefficient of the network is 90 times greater than the clustering coefficient of a comparable random network. However, the clustering coefficient for the network as a whole conceals the fact that the clustering across nodes is highly disperse, as illustrated in Fig. 3c. More than 35 percent of the nodes have either a coefficient of zero or one. This is largely the result of low degree nodes.[7] Ignoring nodes with a degree smaller than three increases the average clustering coefficient to 0.62. A high level of clustering is observed in many other real world networks [38].

G. Link Weights and Node Strength

Definition Weights (w_{ij}) are assigned to the links in a network to show the importance of each link. Barrat et al. [8] define *strength* (s_i) of a node as the sum of the weights of all the links attached to it, i.e., $s_i = \sum_j w_{ij}$, and suggest calculating the average strength as a function

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[7] In a scale-free network low degree nodes are plentiful as discussed above. If a node has a degree of one, then the clustering coefficient is by definition zero. Moreover, all nodes in a triplet, which consists of nodes with degree two, have a clusterings coefficient of one.
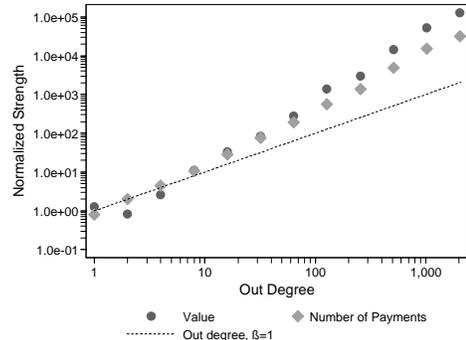
FIG. 4: Node out-strength as a function of Degree. Strength measures are normalized by dividing the strength by the average link weight $\langle w_{ij} \rangle$. The strength functions can be approximated by power law relationships $s(k) \sim k^\beta$ with $\hat{\beta}_{\mathrm{vol}} = 1.2$, $\hat{\sigma}_{\beta_{\mathrm{vol}}} = 0.001$ and $\hat{\beta}_{\mathrm{val}} = 1.9$, $\hat{\sigma}_{\beta_{\mathrm{val}}} = 0.001$, respectively. The dotted line represents $\beta = 1$. Similar relationshiops were observed between in-strength and in-degree.

of the degree of a node, $s(k)$, in order to investigate the relationship between these two node characteristics. For a directed network, strength can be defined over both the incoming and outgoing links.

Discussion The average weight per link in terms of value and volume are 15.2 ± 0.8 million and 5.2 ± 0.3 payments, respectively. The distribution of link weights follow a power law when weighted by the volume of payments but when weighted by the value of payments, the distributions of the link weights is closer to a lognormal distributions. The same is true for the distribution of node strengths (see Fig. 3d).

For both weight measures, strength increases faster than the degree of a node. Like Barrat et al. [8], we see a power-law relationship between (out) strength and the degree of a node, $s(k) \sim k^\beta$. The coefficient is $\hat{\beta}_{\mathrm{volume}} = 1.2$ when volume is used as weight and it is $\hat{\beta}_{\mathrm{value}} = 1.9$ when value is used (see Fig. 4). More connected nodes transact a higher value and volume of payments than would be suggested by their degree alone. For example, if a bank has twice as many out links as another bank, it would be expected to send 2.4 times the number of payments, and 3.8 times the value of payments.

VI. THE IMPACT OF SEPTEMBER 11TH ON NETWORK TOPOLOGY

The terrorist attacks of September 11th, 2001 disrupted the financial systems of the United States, including the interbank payment system. The attacks affected the structure of the interbank payment network in two ways. First, the massive damage to property
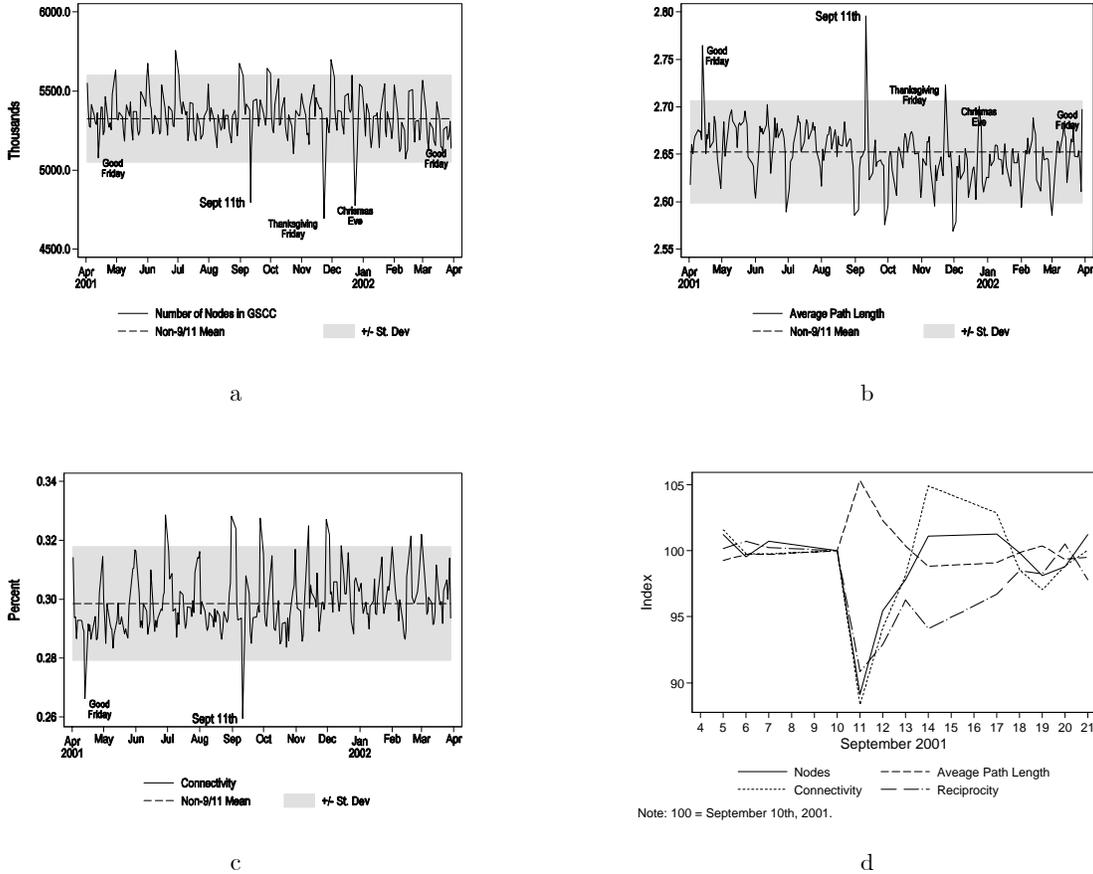
FIG. 5: a) Number of nodes, n, in giant stronly connected component (GSCC) April 2001 - April 2002. b) Average path length, $\langle \ell \rangle$, April 2001 - April 2002 c) Connectivity, p, in GSCC, April 2001 - April 2002 d) Size, average path length, connectivity and reciprocity Sept. 4 - 21, 2001, Sept. 10, 2001 = 100

and communications systems in lower Manhattan made it more difficult, and in some cases impossible, for many banks to execute payments to one another [21, 28], i.e., some nodes were removed from the system or had their strength reduced. Second, the failure of some banks to make payments disrupted the payment coordination by which banks use incoming payments to fund their own transfers to other banks. Once a number of banks began to be short of incoming payments, others became more reluctant to send out payments themselves, i.e., links were either removed or had their weight reduced [21, 30].

Both effects reduced the circulation of funds and collectively banks were growing short of liquidity. The Federal Reserve recognized this trend toward illiquidity and provided liquidity through the discount window and open market operations in unprecedented amounts in the following week [21]. The impact on the interbank payment network is summarized in Table III and Figure 5. The number of nodes in the giant weakly connected component decreased by 5 percent from 6,755 to 6,466, while the number of nodes in the giant strongly connected component (GSCC) decreased by 10 percent, from 5,325 to

4,795. The relative size of the GSCC on September 11th was 6 percentage points lower than typical, as non-offsetting payment flows placed more nodes in the giant in- and giant out-components. The number of nodes in the GSCC did not return to its normal level until September 14th.

The connectivity on September 11th dropped from 3.0 per mil to 2.6 per mil. After two days of low connectivity, the connectivity shot up to over 3 per mil from the 14th to 17th. This "overshooting" is likely the result of banks settling payments they had delayed in the days after September 11th.

As the connectivity decreased and key nodes were removed from the system, the average distance between nodes increased. The mass distance function for September 11th was well below its normal range. Thus both the average path length and the average eccentricity rose (see Table III). The local structure of the network was also disrupted. Reciprocity fell from 24 percent to 22 percent, and the clustering coefficient from 0.52 to 0.47. These results are indicative of the breakdown in the coordination between banks found in [30].

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	Non 9/11		9/11-2001
	mean	Std. Dev.	
n	5,325	137	4,795
m	84,786	6,638	59,640
p (%)	0.30	0.01	0.26
r (%)	24.3	0.4	22.3
$\langle k \rangle$	15.9	0.8	12.4
$\langle \ell \rangle$	2.65	0.03	2.80
$\langle \varepsilon \rangle$	4.84	0.30	5.28
$\langle C \rangle$	0.52	0.012	0.465
$M(2)$ (%)	39.3	2.1	29.3
$M(3)$ (%)	95.3	0.6	91.3
$M(4)$ (%)	99.8	0.0	99.5

TABLE III: Network Statistics for giant strongly connected component, n = size, m = number of links, p = connectivity, r = reciprocity, $\langle k \rangle$ = average degree $\langle \ell \rangle$ = average path length, $\langle \varepsilon \rangle$ = average eccentricity, $\langle C \rangle$ = clustering coefficient. Non 9/11-2001 = Sept. 4 - Sept. 21, 2001.

The size of the September 11th network is similar to the Friday after Thanksgiving and Christmas Eve (see Fig. 5a). In terms of connectivity and average path length the September 11th network is more similar to Good Friday of 2001, when the New York Stock Exchange was closed. However, none of these semi holidays can capture all he changes to the topology that occurred on September 11th.

VII. CONCLUSION

In this paper, we analyzed the topology of the 62 daily networks formed by the payment flows between commercial banks over Fedwire. These networks share many of the characteristics commonly found in other empirical complex networks, such as a scale-free degree distribution, high clustering coefficient and the small world phenomenon. The network is disassortative like many other technological networks. We also found that, apart from a few holidays, the statistics characterizing the network are quite similar from day to day. Moreover, we found that the topology of the network was significantly altered by the attacks of September 11th, 2001. The number of nodes and links in the network and its connectivity was reduced, while the average path length between nodes was significantly increased.

Because scale-free networks are found in many areas, the performance of such networks under ordinary and disrupted conditions is receiving increasing attention. Static scale-free networks, for example, have been found to preserve their connectivity under random node removal yet to be vulnerable to disconnection following removal of high-degree nodes [3]. The vulnerability of a particular network depends both on its structure and on the mechanisms of contagion. As these differ across networks, we cannot extrapolate this conclusion to payment networks. In the case of a payment system, understanding the dynamics of the liquidity flows is essential for assessing network robustness. A question for further research is how the degree distribution and other topological measures relate to contagion of disturbances.

Finally, many of the network statistics appear to vary periodically and abruptly. An interesting question is whether the networks of daily payments naturally form distinct clusters, and what processes create such distinctions. The scale-free daily network is constructed from individual payments accumulated over the day. Analysis of the intra-day payment data will give us insights into how the network forms and on the regularity of this process.

I. MAXIMUM LIKELIHOOD ESTIMATION OF THE POWER LAW EXPONENT

In recent years, a significant amount of research has focused on showing that the distribution of many physical and social phenomena follow a power-law, i.e., $P(k) \sim k^{-\gamma}$. Maximum likelihood estimators for slope coefficient γ for the discrete and continuous case are derived in Goldstein et al. [24] and Newman [33], respectively. Since, many empirical distributions are found only to follow a power law in the right hand tail of the distribution, i.e., $P(k) \sim k^{-\gamma}$ for $k > a$, we derive the maximum likelihood estimator for a truncated power law distribution. The probability distribution of a truncated random variable is given by

$$P(k|k > a) = \frac{P(k)}{1 - P(k \leq a)}$$

In the discrete case the power law distribution is given by

$$P(k) = \frac{k^{-\gamma}}{\zeta(\gamma)}$$

where $\zeta(\gamma)$ is the Riemann Zeta function. Hence, we have

$$P(k|k > a) = \frac{\frac{k^{-\gamma}}{\zeta(\gamma)}}{1 - \sum_{s=1}^{a} \frac{s^{-\gamma}}{\zeta(\gamma)}} = \frac{k^{-\gamma}}{\zeta(\gamma, a)}$$

where $\zeta(\gamma, a)$ is the Hurwitz Zeta function. Analogous to Goldstein et al [24] we have that maximum likelihood estimator, $\hat{\gamma}_{\mathrm{MLE}}$, equates the negative logarithmic derivative of the Hurwitz Zeta function with the average logarithm of the data in the sample, i.e.,

$$\frac{\zeta'(\gamma, a)}{\zeta(\gamma, a)} = \frac{1}{n} \sum_{i} \log(x_i)$$

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The equation can be solved numerically for $\hat{\gamma}_{\mathrm{MLE}}$. An estimate of the standard error of the maximum likelihood estimator can be computed by evaluating the second derivative of the log-likelihood function at $\hat{\gamma}_{\mathrm{MLE}}$

$$\hat{\sigma}_{\mathrm{MLE}} = \frac{1}{\sqrt{n}} \cdot \left(\frac{\zeta(\hat{\gamma}_{\mathrm{MLE}}, a)^2}{\zeta''(\hat{\gamma}_{\mathrm{MLE}}, a)\zeta(\hat{\gamma}_{\mathrm{MLE}}, a) - \zeta'(\hat{\gamma}_{\mathrm{MLE}}, a)^2} \right)^{\frac{1}{2}}$$

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