Federal Reserve Bank of New York
Staff Reports

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Staff Report No. 628
August 2013
Revised May 2014

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JEL classification: E31, E52

Abstract

Most macroeconomic models for monetary policy analysis are approximated around a zero inflation steady state, but most central banks target an inflation rate of about 2 percent. Many economists have recently proposed even higher inflation targets to reduce the incidence of the zero lower bound constraint on monetary policy. In this survey, we show that the conduct of monetary policy should be analyzed by appropriately accounting for the positive trend inflation targeted by policymakers. We first review empirical research on the evolution and dynamics of U.S. trend inflation and some proposed new measures to assess the volatility and persistence of trend-based inflation gaps. We then construct a Generalized New Keynesian model that accounts for a positive trend inflation. In this model an increase in trend inflation is associated with a more volatile and unstable economy and tends to destabilize inflation expectations. This analysis offers a note of caution regarding recent proposals to address the existing zero lower bound problem by raising the long-run inflation target.

Key words: trend inflation, monetary policy, inflation target, inflation persistence, New Keynesian model

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1 Introduction

The notion of price stability as defined in modern monetary theory and central banking practice today is typically associated with a moderate rate of price inflation. For example, during the most recent period of relatively stable inflation, from 1990 to 2008, average inflation (as measured by the consumer price index) was about 2.8 percent in the U.S., 2.2 percent in Germany, and 4 percent in the OECD countries. Countries whose central banks officially adopt an "inflation targeting" regime typically target inflation at about 2 percent. Recent communication by the Federal Open Market Committee (FOMC) states that a 2 percent annualized increase in the price deflator for personal consumer expenditures is the rate of inflation "most consistent over the longer run with the Federal Reserve's statutory mandate."[1] The recent crisis has led some researchers to advocate a higher target rate of inflation even in normal times to allow more room for monetary policy to react to deflationary shocks (e.g., Blanchard et al., 2010; Williams, 2009; and Ball, 2013). In particular, the proposal in Blanchard et al. (2010) to adopt a 4 percent inflation target in the U.S. to address the current crisis situation, in which monetary policy is constrained by the zero lower bound, stimulated a lively debate in both policy and academic circles.[2] In various speeches, however, Federal Reserve Chairman Bernanke has argued against such proposals. In his 2010 Jackson Hole speech, for example, he observed that, "Inflation expectations appear reasonably well-anchored, and both inflation expectations and actual inflation remain within a range consistent with price stability. In this context, raising the inflation objective would likely entail much greater costs than benefits. Inflation would be higher and probably more volatile under such a policy, undermining confidence and the ability of firms and households to make longer-term plans, while squandering the Fed's hard-won inflation credibility. Inflation expectations would also likely become significantly less stable, and risk premiums in asset markets—including inflation risk premiums—would rise."[3]

In light of this debate, it is worth investigatng the implications of a higher target rate of inflation for the conduct of monetary policy in normal times. In particular, what are the implications of different underlying rates of inflation - we call these inflation "trends" - for the volatility and persistence of inflation and for the dynamics and volatility of the economy as a whole? And what are their implications for anchoring inflation expectations?[4]

Although estimating the rate of trend inflation is the subject of a large time series literature, there is no comprehensive discussion of the implications of different trend inflation rates for the conduct of monetary policy. Workhorse monetary models most

[1] FOMC Statement of Longer-Run Goals and Policy Strategy, released for the first time by the Federal Reserve in January 2012 and last amended in January 2014: http://www.federalreserve.gov/monetarypolicy/files/FOMC_LongerRunGoals.pdf

[2] E.g., see the recent policy commentary by Paul Krugman, 29 April 2012, in the *New York Times* (http://www.nytimes.com/2012/04/29/magazine/chairman-bernanke-should-listen-to-professor-bernanke.html?pagewanted=all&_moc.semityn.www) and by Brad DeLong in *The Economist* :http://www.economist.com/debate/overview/203.

[3] Chairman Ben S. Bernanke, "The Economic Outlook and Monetary Policy," Remarks at the Federal Reserve Bank of Kansas City Economic Symposium, Jackson Hole, Wyoming, August 27, 2010, http://www.federalreserve.gov/newsevents/speech/bernanke20100827a.pdf

[4] Throughout this paper we use the terms trend inflation, steady state inflation, and inflation target interchangeably.

often assume zero inflation in the steady state, or they mute the theoretical relevance of a positive average inflation rate by imposing an appropriate indexation to trend inflation.

Recent work has started to investigate the implications of modeling trend inflation and its evolution from an empirical, theoretical, and policy perspective. Trend inflation is tied to the behavior of monetary policy, while short-run fluctuations of inflation around trend reflect price setting dynamics, external shocks, and monetary policy. Trend inflation itself may have interesting dynamics. Policymakers may have targets that change over time - for example when they implement gradual disinflations (or reflations) or learn about the structure of the economy over time. Or policymakers may have implicit inflation targets, which agents have to learn over time. Understanding the dynamics of the short- and long-run components of inflation helps to estimate the persistence of inflation and has significant implications for the design of monetary policy; for example, monetary policy rules that are optimal in forward-looking models deliver bad outcomes in models that feature intrinsic persistence. Furthermore, in standard models, the dynamics of the economy are affected by the rate of inflation that characterizes the long-run equilibrium. Hence, the assumptions one makes about trend inflation have important implications for the cyclical properties of the economy, affecting the policy trade-offs and the conduct of monetary policy.

To review these implications, we first discuss empirical research on the evolution of the dynamics and persistence of inflation. Next, we discuss what an appropriate accounting of positive trend inflation implies for the theory and practice of monetary policy. Finally, we address the current policy debate by considering the circumstances under which it may be desirable to have higher trend inflation.

We start by discussing whether the volatility and persistence of U.S. inflation have evolved over time. In section 2 we review reduced form methods to characterize the evolving inflation dynamics and then discuss the consequences of allowing for a time-varying trend inflation on the assessment of inflation persistence in a structural Phillips curve relationship.

In section 3 we consider the implications of a positive inflation target for the dynamics and volatility of the economy and for anchoring inflation expectations. We chose the New Keynesian model for this analysis because it is the baseline framework of modern monetary policy literature. It therefore allows both a unified treatment of the issues we intend to address and a direct comparison with other results in the literature that do not take trend inflation into account. As in standard versions of this model, we build the supply side on the Calvo price-setting framework. We start by reviewing the implications of a *low* trend inflation rate for the dynamic properties of the model economy. In particular, we discuss its implications both for the dynamic response to shocks and for the determinacy and E-stability of the rational expectations equilibrium. We discuss results in the literature that show that, everything else equal, higher trend inflation increases the cost of price dispersion, the volatility of the economy, the likelihood of sunspot fluctuations and unstable expectations dynamics under learning.

We also briefly consider whether these implications hold under other time-dependent models of price adjustment. We do not cover the literature on state-dependent prices, which has played a less central role in the monetary policy debate. Indeed our focus on the implications of a low rate of trend inflation makes it reasonable to assume that trend inflation does not affect the frequency of price adjustments.[5] Where appropriate, however, we mention results that obtain in our framework but may not hold under state-dependent pricing.

The end of section 3 covers normative issues. In discussing optimal stabilization policy conditional on a chosen target, we illustrate the sensitivity of optimal policy analysis to the accounting for trend inflation. We also touch briefly on research on the optimal inflation rate.

Finally, in section 4 we directly address the current debate – Would it be desirable for policymakers to target a higher long-run rate of inflation in the current situation in which the zero lower bound (ZLB) continues to constrain the nominal interest rate? Here we address two somewhat separate issues.

The first involves whether the presence of a ZLB alters traditional considerations about the optimal long-run inflation. The rationale for a higher target is to allow policymakers more room for lowering the real rate of interest in case of deflationary shocks, making the ZLB less likely to be reached. Our analysis shows, however, that a higher inflation target increases the cost of price dispersion and makes it more difficult for monetary policy to stabilize inflation around that target. Hence, in answering the question of whether a higher inflation target can mitigate the zero bound constraint, one needs to balance the costs of higher inflation in normal times with the benefit of reducing the likelihood of hitting the ZLB constraint.

Second, even if a higher inflation target can in principle mitigate the zero bound constraint, are there alternative monetary policy strategies that can help exiting the ZLB without the drawbacks of a permanently higher target?

2 Trend inflation, inflation persistence, and the Phillips curve

In this section we discuss empirical evidence on the low frequency component of inflation. We show that a distinction between inflation and the inflation gap (the latter being the difference between inflation and its trend) matters for estimating the dynamics of inflation and inflation persistence in structural models.

The extent of inflation persistence is important for the design of appropriate monetary policy. Knowing the time it takes for inflation to approach a new equilibrium after a shock is crucial for determining how to adjust monetary policy tools to reach desired objectives. But shifts in monetary policy also affect the fundamentals that drive inflation and therefore alter its dynamic properties: one should therefore determine, for example, whether persistence is intrinsic to the inflation process or is instead the result of how monetary policy is conducted. The latter would be the case if persistence is primarily detected in the trend component of inflation, which is typically associated with the long-run objective of the policymakers.

The debate on inflation persistence is part of the general debate on whether the relatively stable inflation that characterized the so-called Great Moderation period (1985

[5]Recent contributions also suggest that the degree of state-dependence is not very high for changes in the rate of inflation when average inflation is moderate, as currently observed in advanced economies. Hence the Calvo model provides a relatively good approximation in this case (see Alvarez et al., 2011; Costain and Nakov 2011a,b; Woodford, 2009b; and Kehoe and Midrigan, 2012).

until the Great Recession) was due to lower volatility of the shocks (better luck) or less persistence in the effects of the shocks, which could be partly attributed to better policy.

Evidence on the topic remains controversial. In a recent, comprehensive, survey Fuhrer (2011) discusses the large body of research on inflation persistence, which has been conducted with an array of statistical methods on the most common measures of U.S. inflation. The surveyed research investigated the occurrence of structural breaks and whether these breaks were likely to have been associated with a change in the systematic behavior of monetary policy or rather with changes in the persistence of the shocks. Most relevant to the present work are the analyses that examine the evolution of persistence in connection with the evolution of the trend component of inflation. We highlight how changes in inflation persistence can manifest themselves as a change in the volatility of the innovations to the trend component (e.g. Stock and Watson, 2007) or as a decline in the inflation gap persistence (Cogley et al., 2010).

We then investigate the presence of intrinsic persistence in a structural model of inflation dynamics that allows for positive trend inflation. To do so we introduce a variation of the New Keynesian Phillips Curve (NKPC), obtained by approximating firms' optimal pricing conditions around a steady state with positive (and possibly time varying) inflation. We call this variation a Generalized New Keynesian Phillips Curve, or GNKPC. Relative to the standard formulations of the NKPC, this generalized model features dynamics of inflation that are more forward-looking, and changes in trend inflation alter the trade-off between inflation and real activity. The GNKPC is the backbone of the general equilibrium model that we use for the policy analysis in section 3.

At the end of this section, we present a way to estimate the GNKPC and discuss its implications for the assessment of the intrinsic persistence in the inflation process (Cogley and Sbordone, 2008; Barnes et al. 2009). A caveat to the empirical literature on the NKPC, however, is the weak identification of the parameters (Kleibergen and Mavroedis, 2009 and Mavroedis et al., 2014).

2.1 Inflation persistence in reduced form frameworks

Most measures of inflation in the U.S. and other advanced economies were significantly higher from the mid-'70s to the early '80s than in the previous and following years, which raises the issue of accounting for possible structural changes when estimating inflation dynamics over those periods. Several research contributions show that the analysis of persistence is sensitive to whether one accounts for variation in the autoregression coefficients when fitting a univariate process to the inflation series. Levin and Piger (2004), for example, analyze inflation dynamics in 12 industrial countries for the period 1984-2003 and allow for possible structural breaks at unknown dates. They find strong evidence of a structural break in the intercept of the autoregressive equations but little evidence of a break in any of the autoregressive coefficients. Allowing for a break in the mean, the sum of the autoregressive coefficients reveals very little persistence in most of the inflation series, leading the authors to conclude that high persistence is not an intrinsic feature of inflation in industrial economies.[6]

Pivetta and Reis (2007) analyze U.S. inflation persistence and find no evidence of

[6] They report the sum of the autoregressive coefficients to be less than 0.7, and they can reject the null hypothesis of a unit root at the 95 percent confidence level.

significant changes over time. They allow for a unit root in inflation and consider several measures of persistence in an effort to distinguish between changes in volatility and changes in persistence. They conclude that persistence has not changed over the past three decades, and attribute their findings to a proper accounting for structural breaks; a break is a way to account for shifting trend inflation.

Stock and Watson (2007) provide evidence that the dynamics of inflation have been largely dominated by the trend component. Their analysis focuses on the forecastability of inflation. They use a split-sample analysis, in which the split is set around the beginning of the second term of Volcker's chairmanship at the Federal Reserve (1984). They observe that on the one hand inflation has become *more* predictable because its innovation variance is smaller, and on the other hand it has also become *less* predictable because future inflation is less closely correlated with current inflation and other predictors. Changes in the forecastability are affected by changes in the volatility and persistence of inflation. To identify the nature of the changes in inflation dynamics as well as the timing of these changes, Stock and Watson (2007) estimate a univariate time-varying trend-cycle model with stochastic volatility;[7] in doing so they found large variations in the standard deviations of the permanent innovations $\sigma_{\varepsilon,t}$ (reported in Figure 1). The period from the 1970 through 1983 was a period of high volatility. The previous period, from the mid-'50s through the late '60s, as well as the '84-'90 period, show moderate volatility; and after the mid-'90, $\sigma_{\varepsilon,t}$ fell to very low levels. By contrast, the variance of the transitory component ($\sigma_{\eta,t}$, in Figure 2) remained largely unchanged.

Stock and Watson's analysis suggests that the inference about inflation persistence may be quite different when conducted on the inflation gap, measured as deviation of inflation from a time-varying trend. However, in their setting, inflation innovations are serially uncorrelated, which makes the model unsuitable to investigate persistence in the inflation gap.

A number of recent contributions have analyzed inflation dynamics in a multivariate framework, aiming at relating shifts in the dynamics of inflation to the evolution of monetary policy. A seminal work is the contribution of Cogley and Sargent (2001), who introduced Bayesian vector autoregression models with drifting coefficients for the study of the joint dynamics of inflation, unemployment, and the short-term nominal interest rate. They estimate the trend component of inflation and find that it appears to bear most of the responsibility for the rise and fall of U.S. inflation in the post-World War II period and for movements in inflation persistence. The results of this paper and its companion (Cogley and Sargent, 2005), which also accounts for stochastic volatility, contrast with other results in the structural VAR literature. For example, in modeling discrete shifts in regimes, Sims and Zha (2006) found that the historical patterns emphasized by Cogley and Sargent can be generated by "stable monetary

[7] In this UC-SV model, inflation is the sum of two components, a permanent stochastic trend component τ_t and a serially uncorrelated transitory component η_t . Specifically: $\pi_t = \tau_t + \eta_t$, and $\tau_t = \tau_{t-1} + \varepsilon_t$, where the processes η_t and ε_t are respectively $\eta_t = \sigma_{\eta,t}\zeta_{\eta,t}$, and $\varepsilon_t = \sigma_{\varepsilon,t}\zeta_{\varepsilon,t}$. The stochastic volatilities evolve as driftless geometric random walks: $ln\ \sigma_{i,t} = ln\ \sigma_{i,t-1} + \nu_{i,t}$ for $i = \eta, \varepsilon$. Furthermore, $\zeta_t = (\zeta_{\eta,t}, \zeta_{\varepsilon,t})$ is iid $N(0, \gamma I_2)$, $\nu_t = (\nu_{\eta,t}, \nu_{\varepsilon,t})$ is iid $N(0, \gamma I_2)$, and ζ_t and ν_t are independently distributed. γ is a scale parameter that controls the smoothness of the stochastic volatility process. The figures reported here show the smoothed estimates of the standard deviation of the permanent and transitory components of inflation ($\sigma_{\varepsilon,t}$ and $\sigma_{\eta,t}$, respectively), measured by the GDP deflator, computed by Markov Chain Monte Carlo (MCMC).

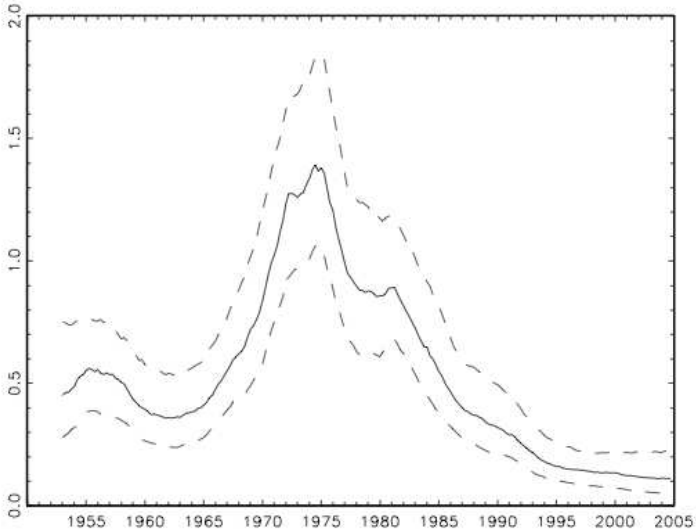


Figure 1: Stock and Watson (2007, p.18) – Figure 2(a): Estimated $\sigma_{\varepsilon,t}$

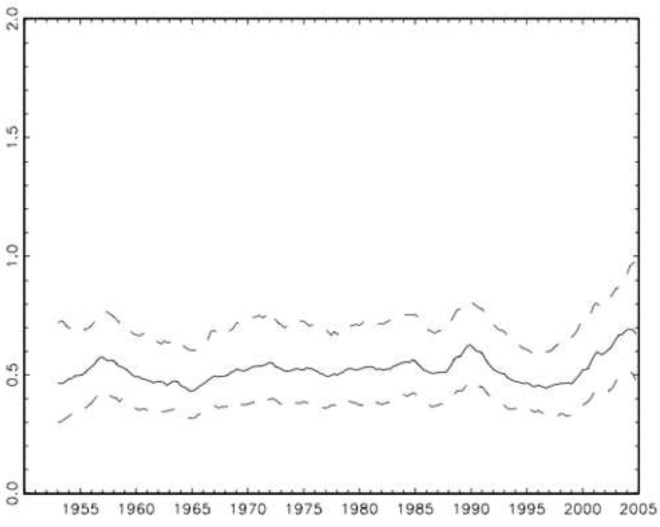


Figure 2: Stock and Watson (2007, p.18) – Figure 2(b): Estimated $\sigma_{\eta,t}$

policy reactions to a changing array of major disturbances" (p. 77). Primiceri (2006) also tends toward this interpretation, arguing that the high volatility of the shocks that characterized the 1970s and early 1980s seems a more likely candidate to explain the peaks in inflation in those periods.

Although the significance of the decline in persistence and its attribution to monetary policy remain debated, undoubtedly more attention is now being paid to modeling the dynamics of trend inflation and hence of the inflation gap. Indeed Cogley et al. (2010) extend the work of Cogley and Sargent (2001, 2005), focusing explicitly on the inflation gap. They provide a new measure of inflation persistence based on predictability and detect a significant decline in inflation gap persistence in the post-Volcker era (post-1987). They also find that inflation innovations account for a small fraction of the unconditional variance of inflation, implying that most of the volatility is in the trend component of inflation. This result is consistent with the univariate analysis of Stock and Watson (2007).

We now formally introduce the notion of trend inflation and its estimation in the context of a Bayesian VAR model, as in Cogley and Sargent (2001). This allows us to illustrate the dynamics of U.S. trend inflation and evaluate the persistence measure just described. It also sets the stage for the discussion of inflation persistence in the context of the structural price setting model that we present in the second half of this section.

A VAR model with time-varying coefficients can be written as follows:[8]

$$\mathbf{x}_t = \mathbf{X}_t' \boldsymbol{\vartheta}_t + \boldsymbol{\varepsilon}_{\mathbf{x}t}, \tag{1}$$

where $\mathbf{x}_t$ is a $N \times 1$ vector of endogenous variables, $\mathbf{X}_t' = \mathbf{I}_N \otimes \begin{bmatrix} 1 & \mathbf{x}_{t-l}' \end{bmatrix}$ (where $\mathbf{x}_{t-l}'$ represents lagged values of $\mathbf{x}_t$), and $\boldsymbol{\vartheta}_t$ denotes a vector of time-varying conditional mean parameters. $\boldsymbol{\vartheta}_t$ is assumed to evolve as a driftless random walk:[9]

$$\boldsymbol{\vartheta}_t = \boldsymbol{\vartheta}_{t-1} + \mathbf{v}_t, \tag{2}$$

where the innovation $\mathbf{v}_t$ is normally distributed, with mean $\mathbf{0}$ and variance $\boldsymbol{\Omega}$. Stochastic volatility in the VAR innovations $\boldsymbol{\varepsilon}_{\mathbf{x}t}$ adds further dynamics to the model.[10] We assume that $\boldsymbol{\varepsilon}_{\mathbf{x}t}$ can be expressed as:

$$\boldsymbol{\varepsilon}_{\mathbf{x}t} = \mathbf{V}_t^{1/2} \boldsymbol{\xi}_t,$$

where $\boldsymbol{\xi}_t$ is a standard normal vector, which we assume to be independent of parameter innovations $\mathbf{v}_t$ ($E(\boldsymbol{\xi}_t \mathbf{v}_s) = 0$, for all t, s). $\mathbf{V}_t$ is modeled as a multivariate stochastic volatility process:

$$\mathbf{V}_t = \mathbf{B}^{-1} \mathbf{H}_t \mathbf{B}^{-1\prime}, \tag{3}$$

where $\mathbf{H}_t$ is diagonal and $\mathbf{B}$ is lower triangular. To represent permanent shifts in innovation variance, the diagonal elements of $\mathbf{H}_t$ are assumed to be independent, univariate stochastic volatilities that evolve as driftless geometric random walks:

$$\ln h_{it} = \ln h_{it-1} + \sigma_i \eta_{it}. \tag{4}$$

[8]This description is based on Cogey and Sargent (2005). The model specification estimated below follows Cogley and Sbordone (2008). More detail on the models and their estimation can be found in those papers.

[9]This is also subject to a reflecting barrier that guarantees non-explosive roots for the VAR at every date.

[10]Cogley et al. (2010) extend this framework to include stochastic volatility also in the parameter innovations $\mathbf{v}_t$.

The innovations η_{it} have a standard normal distribution, are independently distributed, and are assumed independent of innovations $\mathbf{v}_t$ and $\boldsymbol{\xi}_t$.[11] The vector $\mathbf{x}_t$ includes a measure of inflation; it typically also includes a measure of real activity and a policy variable. In our reported estimates of (1), the vector $\mathbf{x}_t$ includes inflation, output growth, a measure of marginal costs, and the short term interest rate, variables that comove with inflation in the structural model that we analyze later.[12]

We define trend inflation in terms of the infinite horizon forecast, following Beveridge and Nelson (1981); that is, as the level to which inflation is expected to settle after short-run fluctuations die out: $\bar{\pi}_t = \lim_{j\to\infty} E_t\pi_{t+j}$. To compute that measure, we use the companion-form notation of (1):

$$\mathbf{z}_t = \boldsymbol{\mu}_t + \mathbf{A}_t\mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_{\mathbf{z}t},$$

where the vector $\mathbf{z}_t = (\mathbf{x}_t, \mathbf{x}_{t-1}, ..., \mathbf{x}_{t-p+1})'$, the matrix $\mathbf{A}_t$ refers to the autoregressive parameters in $\boldsymbol{\vartheta}_t$, and the vector $\boldsymbol{\mu}_t$ includes the intercepts. We approximate $\bar{\pi}_t$ by calculating a local-to-date t estimate of mean inflation from the VAR:

$$\bar{\pi}_t = \mathbf{e}'_\pi(\mathbf{I} - \mathbf{A}_t)^{-1}\boldsymbol{\mu}_t, \tag{5}$$

where $\mathbf{e}'_\pi$ is a selection vector that picks up inflation from the vector $\mathbf{z}_t$. This definition implies that, to a first-order approximation, inflation evolves as a driftless random walk.

Figure 3 reports the estimate of trend inflation obtained from this model with quarterly U.S. data from 1960:Q1 through 2012:Q4. The thin black line in the figure is actual inflation, and the thick line is the median estimate of trend inflation at each date (all expressed at annual rates). The estimates are conditioned on data through the end of the sample. As the figure shows, trend inflation rose from slightly above 2 percent in the early 1960s to around 5 percent in the 1970s, then fell to just about 2 percent at the end of the sample. This path is also largely consistent with long-term inflation expectations derived from survey data. For example, the correlation between our estimate of trend inflation and the 10-year inflation expectations from the Survey of Professional Forecasters (computed from 1981) is 0.96. A time-varying inflation trend implies that the inflation gap, measured as the deviation of inflation from the time-varying trend, is quite different from deviations of inflation from a constant mean.[13] In particular, the persistence of these two series is different: in the figure, long runs appear at the beginning, middle, and end of the sample, when inflation does not cross the mean line; inflation crosses instead the trend line more often, especially after the Volcker disinflation.

The figure also shows, however, that there is a lot of uncertainty about the level of trend inflation at any given date, as shown by the marginal 90 percent credible sets at each date (displayed as dotted lines in the figure).[14]

[11] The factorization in (3) and the log specification in (4) guarantee that $\mathbf{V}_t$ is positive definite, while the free parameters in $\mathbf{B}$ allow for time-varying correlation among the VAR innovations $\boldsymbol{\varepsilon}_{\mathbf{x}t}$.

[12] This is the specification in Cogley and Sbordone (2008), where inflation is computed from the implicit deflator of GDP for the nonfarm business sector; output growth is the rate of growth of GDP in the nonfarm business sector; marginal costs are approximated by unit labor costs for the nonfarm business sector; and the short term interest rate is the effective federal funds rate. The calibration of the priors for the VAR parameters and the estimation procedure also follow those of the original paper, which contains further details.

[13] As we will discuss later, deviations from a constant mean (reflecting the assumption of a constant

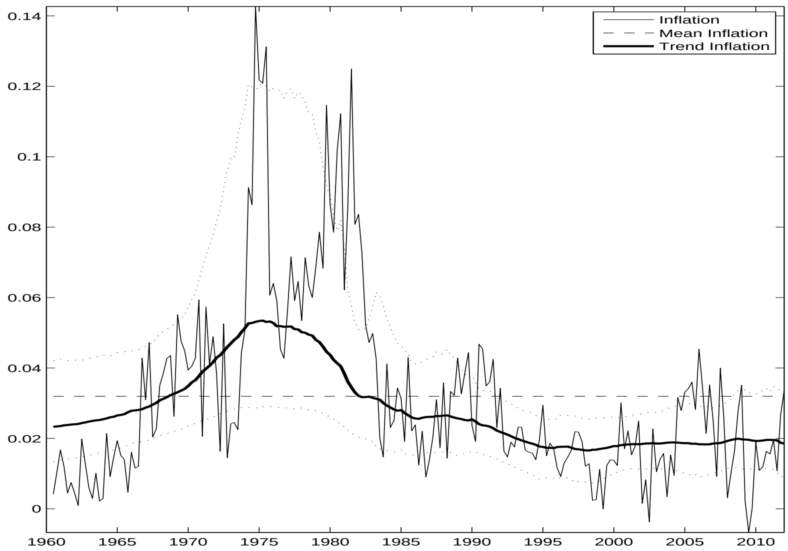


Figure 3: Inflation, Mean Inflation and Trend Inflation

Table 1 summarizes the autocorrelation of the inflation gap. The first row refers to the inflation gap computed as the deviation of inflation from the mean; the second row refers to the inflation gap measured by subtracting the median estimate of trend inflation from actual inflation. The autocorrelation of the mean-based gap hovers around 0.7-0.8, both for the whole sample and for the two subsamples. In contrast, the persistence of the trend-based gap, while still elevated for the period before the Volcker disinflation, drops substantially afterward, to around 0.35.[15]

Table 1: Autocorrelation of Inflation Gaps

	1960-2012	1960-1983	1984-2012
Mean-Based Gap	0.875	0.855	0.718
Trend-Based Gap	0.727	0.785	0.352

To further analyze the decline in the persistence of the trend-based gap, we use the statistical measure introduced by Cogley et al. (2010). They compare the fraction of

steady-state rate of inflation) are typically analyzed in conventional versions of the NKPC.

[14]A credible set is a Bayesian analog to a confidence interval. The marginal credible sets portray uncertainty about the location of $\bar{\pi}_t$ at a given date. The estimates of the structural parameters we present later take this uncertainty into account because they are based on the entire posterior sample for trend inflation, not just on the mean or median path.

[15]These results are consistent with Kozicki and Tinsley (2002), who were among the first to point out the importance of shifts in trend inflation to assess the persistence of inflation. Using the sum of the autoregressive coefficients of an AR(4) model fit to inflation data for the 1962-2001 period and for various subsamples, they found that persistence in the trend-based gap is generally lower and that it declined in the recent subsamples.

the total variation in the inflation gap due to shocks inherited from the past with the fraction due to those that will occur in the future. Specifically, the following forecasting model for the vector of gap variables is defined:

$$(\mathbf{z}_{t+1} - \overline{\mathbf{z}}_t) = \mathbf{A}_t(\mathbf{z}_t - \overline{\mathbf{z}}_t) + \boldsymbol{\varepsilon}_{z,t+1},$$

(where $\overline{\mathbf{z}}_t$ is approximated by $(\mathbf{I} - \mathbf{A}_t)^{-1}\boldsymbol{\mu}_t$), and the j-period-ahead forecasts of the gap variables are approximated by $\mathbf{A}_t^j\, \widehat{\mathbf{z}}_t$, where $\widehat{\mathbf{z}}_t = \mathbf{z}_t - \overline{\mathbf{z}}_t$.[16] The forecast-error variance of $\widehat{\mathbf{z}}_{t+j}$ is then approximated by:

$$var_t(\widehat{\mathbf{z}}_{t+j}) \approx \Sigma_{h=0}^{j-1} \left(\mathbf{A}_t^h\right) var\left(\boldsymbol{\varepsilon}_{z,t+1}\right) \left(\mathbf{A}_t^h\right)'. \tag{6}$$

The unconditional variance of $\widehat{\mathbf{z}}_{t+1}$ is approximated by taking the limit of the conditional variance as the forecast horizon j increases:

$$var(\widehat{\mathbf{z}}_{t+1}) \approx \Sigma_{h=0}^{\infty} \left(\mathbf{A}_t^h\right) var\left(\boldsymbol{\varepsilon}_{z,t+1}\right) \left(\mathbf{A}_t^h\right)'.$$

By virtue of the anticipated-utility approximation, this is also the unconditional variance of $\widehat{\mathbf{z}}_{t+s}$ for $s > 1$. Inflation persistence at any given date t is then defined as the fraction of the total variance of the inflation gap due to shocks inherited from the past. The rationale is that, if past shocks die out fast, persistence is weak, and this measure will converge rapidly to zero. If past shocks instead explain a high proportion of the variation of future inflation gaps, then the persistence of the inflation gap is high. Cogley et al. (2010) call this measure of persistence R_{jt}^2 (being akin to an R-square statistic for j-step ahead forecasts). They compute it as 1 minus the fraction of total variation due to future shocks, where the latter can be expressed as the ratio of conditional to unconditional variance (since future shocks account for the forecast error):

$$R_{jt}^2 = 1 - \frac{var_t(\mathbf{e}_\pi' \widehat{\mathbf{z}}_{t+j})}{var(\mathbf{e}_\pi' \widehat{\mathbf{z}}_{t+j})} \approx 1 - \frac{\mathbf{e}_\pi' \Sigma_{h=0}^{j-1} \left(\mathbf{A}_t^h\right) var\left(\boldsymbol{\varepsilon}_{z,t+1}\right) \left(\mathbf{A}_t^h\right)' \mathbf{e}_\pi}{\mathbf{e}_\pi' \Sigma_{h=0}^{\infty} \left(\mathbf{A}_t^h\right) var\left(\boldsymbol{\varepsilon}_{z,t+1}\right) \left(\mathbf{A}_t^h\right)' \mathbf{e}_\pi}. \tag{7}$$

The R_{jt}^2 statistic (by definition between 0 and 1) converges to 0 as the forecast horizon j lengthens, and its degree of convergence indicates the degree of persistence. Cogley et al. (2010) emphasize that this statistic is more informative than typical statistics used to summarize persistence in VARs, such as the largest autoregressive root in $\mathbf{A}_t$; the reason is that it retains all information in $\mathbf{A}_t$ while still being sensitive to changes in the conditional variance $\mathbf{V}_t$. For example, they note, this measure of persistence would decline if the composition of the structural shocks change toward a predominance of those shocks for which the impulse response declines more rapidly.

Computing such a measure on our estimated VAR, we find that, as in Cogley et al. (2010), inflation innovations account for a small fraction of the unconditional variance

[16] We have used the anticipated-utility approximation to obtain $E_t \overline{z}_{t+j} = \overline{z}_t$. This approximation, introduced in Kreps (1998) as a way to model bounded rationality, is standard in the macro-learning literature. It implies that agents treat their current estimate of drifting parameters as if they were known with certainty. Cogley and Sargent (2008) compare the decision rules under this approximation with exact Bayesian decision rules and demonstrate that the approximation is good as long as precautionary motives are not too strong.

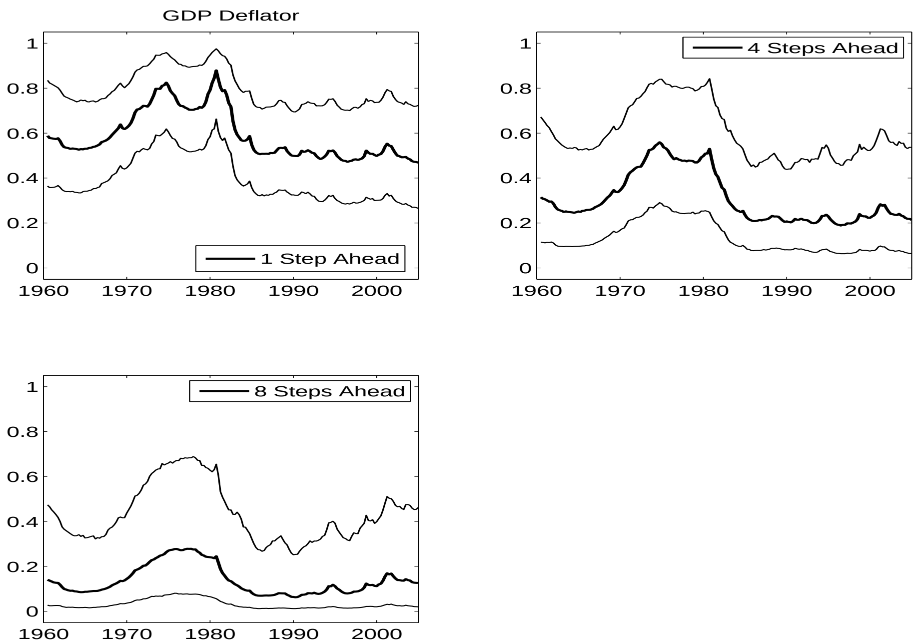


Figure 4: R^2_{jt} statistics

of inflation, implying that most of the volatility is in the trend component of inflation. Furthermore, the R^2_{jt} statistics for one, four, and eight quarters ahead, reported in the three panels of figure 4, suggest overall that inflation gap persistence was higher in the Great Inflation period and lower after the mid-'80s.[17]

To illustrate whether the decline in persistence is statistically significant, we again follow Cogley et al. (2010) by considering the joint posterior distribution of the one-step ahead statistic R^2_{1t} across two pairs of years: 1960 and 1980, and 1980 and 2012.[18] We find that for both pairs the draws from the posterior distribution fall mostly on one side of the 45-degree line; in the upper graph of Figure 5, which compares 1980 and 2012, the draws lie below the line ($R^2_{1,1980} > R^2_{1,2012}$), indicating with high probability that inflation persistence went down from 1980 to 2012. In contrast, the draws from the pairs 1960 and 1980 lie mostly above the 45-degree line ($R^2_{1,1980} > R^2_{1,1960}$), implying that inflation persistence likely went up from 1960 to 1980. The evidence we obtain is not as sharp as that reported in Cogley et al. (2010), where almost the entire distribution of the pairs collapses away from the 45 degree line; nonetheless it clearly indicates a change in persistence across periods, validating the inference from the serial correlation

[17] The figure reports the posterior median and the interquartile range for R^2_{jt} at each date t, for $j = 1, 4, 8$ quarters ahead. By construction, the statistic declines as the length of the forecast period increases.

[18] We captur the points where persistence has likely changed by using the beginning and the end of the overall sample period (in our case, 1960 and 2012) as well as the middle, which represents the high point of the statistic and coincides with the eve of the Volcker disinflation.

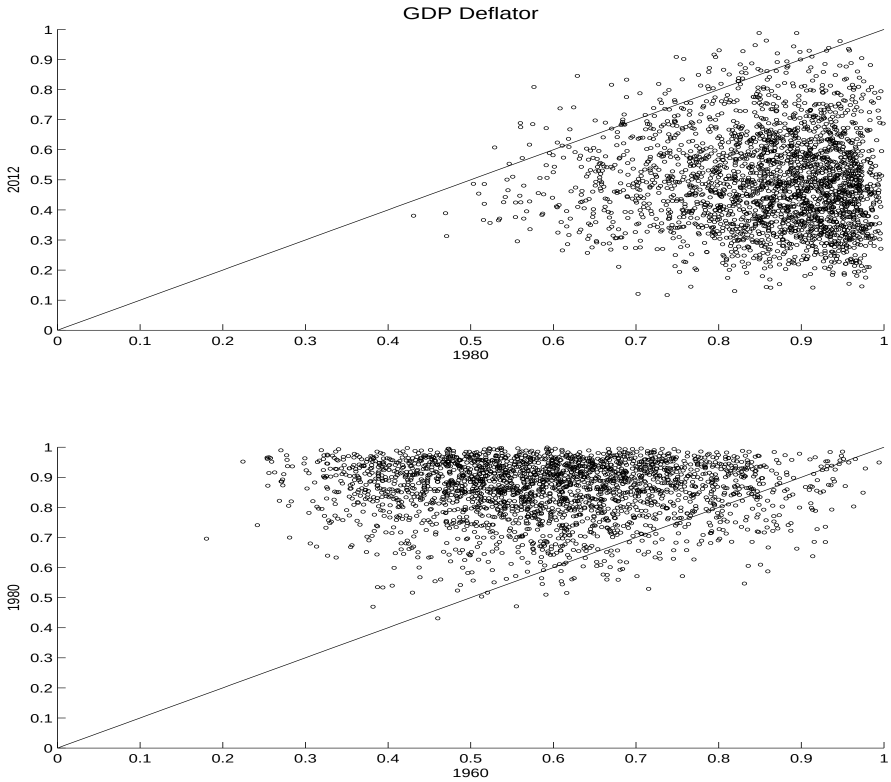


Figure 5: Joint distribution of R_1^2 statistics: 1960-80 and 1980-2012

statistics reported in the table.

2.2 Persistence in structural models

The debate on inflation persistence affects the formulation of structural models in at least two distinct ways. The first is the form of the aggregate supply or Phillips curve relationship. Some argue that standard forward-looking new Keynesian curves fail to account for inflation persistence and typically need a backward component to fit the data. The literature offers different ways of delivering hybrid curves. One can, for example, introduce ad hoc indexation assumptions (e.g., Gali and Gertler, 1999); replace the standard pricing mechanism with state-contingent pricing (e.g., Dotsey et al., 1999; Wolman, 1999); or depart from the rational expectations assumption (e.g., Erceg and Levin, 2003; Milani, 2005, 2007). The previous discussion suggests, however, that a forward looking model may indeed account well for inflation dynamics once the Phillips curve is formulated in terms of the *trend-based* inflation gap rather than the mean-based gap; this is because the persistence of the inflation gap, as we discussed, is significantly lower than that of overall inflation and has also likely declined over time.

To investigate inflation persistence in a structural model, we now introduce an NKPC that accounts for trend inflation. We build this relationship from firms' optimal price-setting behavior, discuss briefly the important features of the model, and show an assessment of its empirical fit. The microfoundations of our GNKPC are those of the Calvo-Yun model,[19] in which price-setting firms face random intervals between price adjustments. This model is the central feature of the small general equilibrium model that we build in section 3 to analyze the implications of positive trend inflation for the evaluation of monetary policy.

2.2.1 The Calvo-Yun price setting model

In each period t, a final good, Y_t, is produced by perfectly competitive firms, which combine a continuum of intermediate inputs, $Y_{i,t}$, $i \in [0,1]$, via the technology:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{8}$$

where $\varepsilon > 1$ is the elasticity of substitution among intermediate inputs. Profit maximization and the zero profit condition imply that the price index associated with the final good Y_t is a CES aggregate of the prices of the intermediate inputs $P_{i,t}$:

$$P_t = \left[\int_0^1 P_{i,t}^{1-\varepsilon} di \right]^{1/(1-\varepsilon)}, \tag{9}$$

and the demand schedule for intermediate input $Y_{i,t}$ is:

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon} Y_t. \tag{10}$$

Intermediate inputs are produced by a continuum of firms with a simple linear technology in labor, which is the only input of production:

$$Y_{i,t} = A_t N_{i,t}, \tag{11}$$

where A_t is a stationary process for aggregate technology.[20] Due to the assumption of constant returns to scale technology and assuming that nominal wages are set in perfectly competitive markets, the real marginal costs of firm i, $MC_{i,t}$, depend only on aggregate variables and thus are the same across firms:

$$MC_{i,t} = MC_t = \frac{W_t}{A_t P_t}. \tag{12}$$

Intermediate firms' price-setting problem - Imperfect substitutability generates market power for intermediate goods producers, which are thus price-setters. We assume random intervals between price resets: in each period a firm can re-optimize its nominal price with fixed probability $1-\theta$, while with probability θ it maintains the price

[19] The model was introduced by Calvo (1983); the discrete time version we use here was first introduced by Yun (1996).

[20] The online appendix presents the more general case of decreasing returns to labor.

charged in the previous period.[21] The problem of firm i, which sets its price at time t, is to choose $P^*_{i,t}$ to maximize expected profits:

$$E_t \sum_{j=0}^{\infty} \theta^j \mathcal{D}_{t,t+j} \left[\frac{P^*_{i,t}}{P_{t+j}} Y_{i,t+j} - TC_{t+j} \left(Y_{i,t+j} \right) \right], \tag{13}$$

subject to the demand function (10). $\mathcal{D}_{t,t+j}$ is a stochastic discount factor and $TC_{t+j} \left(Y_{i,t+j} \right) = \frac{W_{t+j}}{P_{t+j}} \frac{Y_{i,t+j}}{A_{t+j}}$ is the total cost function. Letting $p^*_{i,t}$ denote the relative price of the optimizing firm at t ($p^*_{i,t} = P^*_{i,t}/P_t$), the first order condition of this problem can be written as:

$$p^*_{i,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{j=0}^{\infty} \theta^j \mathcal{D}_{t,t+j} Y_{t+j} \Pi^{\varepsilon}_{t,t+j} MC_{t+j}}{E_t \sum_{j=0}^{\infty} \theta^j \mathcal{D}_{t,t+j} Y_{t+j} \Pi^{\varepsilon-1}_{t,t+j}} \tag{14}$$

where $\Pi_{t,t+j}$ denotes the *cumulative gross inflation rate* over j periods:

$$\Pi_{t,t+j} = \begin{cases} 1 & \text{for } j = 0 \\ \left(\frac{P_{t+1}}{P_t} \right) \times \cdots \times \left(\frac{P_{t+j}}{P_{t+j-1}} \right) & \text{for } j = 1, 2, \cdots . \end{cases} .$$

In what follows, we denote the gross inflation rate by $\pi_t = \frac{P_t}{P_{t-1}}$.

Note that future expected inflation rates enter both the numerator and the denominator in (14) and thus affect the relative weights on future variables. The numerator is the present discounted value of future marginal costs. Forward-looking firms know that they may be stuck with the price set at t and that inflation will therefore erode their markup over time; hence they use future expected inflation rates to discount future marginal costs. The higher these future expected inflation rates, the higher the relative weight on expected future marginal costs; firms become effectively more forward-looking, giving more weight to future than to present economic conditions.

Note also that equation (14) in a steady state with constant inflation is:

$$p^*_i = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{j=0}^{\infty} \left(\beta \theta \bar{\pi}^{\varepsilon} \right)^j MC}{\sum_{j=0}^{\infty} \left(\beta \theta \bar{\pi}^{\varepsilon-1} \right)^j}, \tag{15}$$

where p^*_i is the steady-state value of the relative price $p^*_{i,t}$, $\bar{\pi}$ is steady-state (trend) inflation, β is the steady-state value of the stochastic discount factor $\mathcal{D}_{t,t+j}$, and MC is the steady-state value of real marginal cost. Hence, the model constrains the feasible inflation rate in the steady state: if steady-state inflation is positive (i.e., $\bar{\pi} > 1$), the convergence of the sum in (15) requires that $\beta \theta \bar{\pi}^{\varepsilon-1} < 1$ and $\beta \theta \bar{\pi}^{\varepsilon} < 1$. This implies upper bounds on trend inflation: $\bar{\pi} < (1/\theta\beta)^{1/(\varepsilon-1)}$ and $\bar{\pi} < (1/\theta\beta)^{\frac{1}{\varepsilon}}$.[22]

The aggregate price level evolves according to:

$$P_t = \left[\int_0^1 P_{i,t}{}^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} = \left[\theta P^{1-\varepsilon}_{t-1} + (1 - \theta) P^{*1-\varepsilon}_{i,t} \right]^{\frac{1}{1-\varepsilon}} . \tag{16}$$

[21] In the baseline model we do not assume indexation of prices to past inflation and/or to trend inflation, an assumption we find counterfactual. We discuss indexation in Section 3.6.1.

[22] For somewhat standard calibration values ($\theta = 0.75$, $\beta = 0.99$, $\varepsilon = 10$), those bounds would be 14.1% and 12.6% annual rates, respectively.

2.2.2 The GNKPC

Most studies at this point take a log-linear approximation of the firms' equilibrium conditions and the aggregate price relation around a steady state characterized by zero inflation. They obtain an expression of the type:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa\ \widehat{mc}_t, \tag{17}$$

where hatted variables denote log-deviations from steady state values (for any variable $x_t : \hat{x}_t = \ln(x_t/\bar{x})$) and the coefficient of the marginal cost κ is a combination of the parameters governing the price-setting problem: $\kappa = \frac{(1-\theta)(1-\theta\beta)}{\theta}$. Here we depart from this practice and instead log-linearize the equilibrium conditions around a steady state characterized by a shifting trend inflation and, with the usual manipulations, derive a version of the NKPC that can be written as:[23]

$$\begin{aligned}\hat{\pi}_t \quad = \quad & \kappa(\boldsymbol{\psi}, \bar{\pi}_t)\ \widehat{mc}_t + b_1(\boldsymbol{\psi}, \bar{\pi}_t) E_t \hat{\pi}_{t+1} + b_2(\boldsymbol{\psi}, \bar{\pi}_t) E_t \sum_{j=2}^{\infty} \varphi(\boldsymbol{\psi}, \bar{\pi}_t)^{j-1} \hat{\pi}_{t+j} \\ & + b_3(\boldsymbol{\psi}, \bar{\pi}_t) E_t \sum_{j=0}^{\infty} \varphi(\boldsymbol{\psi}, \bar{\pi}_t)^{j} \widehat{\mathcal{D}}_{t+j,t+j+1} + u_t.\end{aligned} \tag{18}$$

This equation differs from conventional versions of the *NKPC* in two respects. First, all the coefficients are non-linear functions of trend inflation $\bar{\pi}_t$ and the parameters of the pricing model, which we collected in a vector denoted by $\boldsymbol{\psi}$; even though these parameters are assumed to be constant, the coefficients of equation (18) drift when trend inflation drifts. In particular, higher trend inflation implies a lower weight on current marginal cost (in the plane $(\hat{\pi}_t, \widehat{mc}_t)$, the short-run NKPC flattens) and a greater weight on expected future inflation. Second, a number of additional variables appear on the right-hand side of (18).[24] These include higher-order leads of expected inflation and terms involving the discount factor $\widehat{\mathcal{D}}_t$. Excluding these variables when estimating traditional Calvo equations would result in omitted-variable bias on the estimated coefficients, if the omitted terms are correlated with the regressors. The standard NKPC emerges as a special case when steady-state inflation is zero ($\bar{\pi}_t = 1$ for all t). In those cases, $b_2(\boldsymbol{\psi},\bar{\pi}_t) = b_3(\boldsymbol{\psi},\bar{\pi}_t) = 0$, while the other coefficients collapse to those of the standard model (17).

Furthermore, from (15) and (16) evaluated in steady state, one derives a restriction between trend inflation and steady-state marginal cost:

$$\left[\frac{1-\theta\bar{\pi}_t^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \left[\frac{1-\theta\ \beta\bar{\pi}_t^{\varepsilon}}{1-\theta\ \beta\bar{\pi}_t^{\varepsilon-1}}\right] = \frac{\varepsilon}{\varepsilon-1}\overline{mc}_t. \tag{19}$$

2.2.3 Estimation of the GNKPC

To investigate the importance of trend inflation for assessing inflation persistence, Cogley and Sbordone (2008) formulate and apply to the data a slightly more general form of

[23] Following the derivation in Cogley and Sbordone (2008), we use the anticipated-utility approximation to evaluate multi-step expectations when parameters drift.

[24] We also included the error term u_t to account for the fact that this equation is an approximation and to allow for other possible mis-specifications. In the estimation, we assume that u_t is a white noise process.

the GNKPC (18). Specifically, they assume that firms that do not optimally reset prices nonetheless change their price by indexing it to past inflation.[25] Prices that are *not* set optimally thus evolve according to $P_{i,t} = \pi_{t-1}^{\varrho} P_{i,t-1}$,where $\varrho \in [0,1]$ measures the degree of indexation. In this more general formulation, the coefficients of (18) depend also on the parameter ϱ, which is included in the vector $\boldsymbol{\psi}$.[26] Importantly, the more general formulation introduces a backward-looking term in the GNKPC and therefore allows testing for the presence of "intrinsic persistence" in inflation dynamics.

The estimation method exploits a set of cross-equation restrictions between the parameters of the Calvo model and those of a reduced-form vector autoregression with drifting parameters, where the latter takes the form discussed in Section 2.1. The intuition is that, if inflation is determined according to the GNKPC, the VAR should also satisfy a collection of nonlinear cross-equation restrictions; these are embedded in (18), which relates the cyclical components of inflation and marginal cost, and in (19), which constrains the evolution of their steady-state values. These relations involve non-linear combinations of the underlying parameters of the Calvo model, collected in the vector $\boldsymbol{\psi}$, which is now defined as $\boldsymbol{\psi} = [\theta,\varepsilon,\ \varrho]'$, where ϱ indicates the degree of indexation.

Following Cogley and Sbordone (2008), we perform the estimation of $\boldsymbol{\psi}$ with a two-step procedure. First, to estimate trend inflation, we fit to the data an unrestricted reduced-form VAR of the type described in Section 2.1. Then, conditional on those estimates, we estimate the vector of parameters $\boldsymbol{\psi}$ as:

$$\widehat{\boldsymbol{\psi}} = \arg\min \mathcal{F}\left(\widehat{\boldsymbol{\mu}}_t, \hat{\mathbf{A}}_t, \boldsymbol{\psi}\right)' \mathcal{F}\left(\widehat{\boldsymbol{\mu}}_t, \hat{\mathbf{A}}_t, \boldsymbol{\psi}\right). \tag{20}$$

where the function $\mathcal{F}\left(\widehat{\boldsymbol{\mu}}_t, \hat{\mathbf{A}}_t, \boldsymbol{\psi}\right)$ embeds the cross equation restrictions.[27]

Table 2 summarizes the second-stage estimates. Because the distributions are non-normal, we report the median and 90 percent confidence intervals. As in Cogley and Sbordone (2008), the estimates accord well with microeconomic evidence and are reasonably precise.

Table 2: Estimates of the Structural Parameters

	θ	ϱ	ε
Median	0.593	0	9.32
90 percent Confidence Interval	(0.49,0.66)	(0,0.17)	(7.66,11.00)

In particular, there is no evidence of a significant backward component, as discussed in Cogley and Sbordone (2008). We interpret this result as (1) being due to the proper accounting, in this equation, of the dynamics of trend inflation; and (2) as a cautionary

[25] A "hybrid" New Keynesian Phillips Curve, including lagged inflation, was formulated by Galí and Gertler (1999) by assuming the presence of rule-of-thumb firms. A similar hybrid curve is more frequently obtained in the literature by assuming indexation, as in Christiano et al. (2005).

[26] The model estimated in Cogley and Sbordone (2008) is more general than (18) in a few more respects: it assumes a concave production function, allows for output growth, and allows for the presence of strategic complementarity in price setting. The estimates we report below are obtained for this more general formulation.

[27] Note that here hats on vectors $\widehat{\psi}$, $\widehat{\boldsymbol{\mu}}_t$ and matrices $\hat{\mathbf{A}}_t$ indicate that they are estimated values. A full description of the estimation method appears in Cogley and Sbordone (2008).

note against monetary policy analyses that rely on NKPC formulations with important backward-looking components.[28] We return to this issue in Section 3.4.[29,30]

Trend inflation also matters in its effect on the coefficients of the GNKPC, which become time varying. We can evaluate these coefficients by combining the estimates in Table 2 with the estimated trend inflation, illustrating the extent to which, as we noted above, the relationship is more forward-looking than standard NKPCs. Figure 6 portrays as solid lines the coefficients κ_t, b_{1t}, b_{2t} and b_{3t}, computed as in (18): their time variations reflect time variation in $\overline{\pi}_t$. The dashed lines report the value of the same coefficients that would obtain under a standard approximation of the Phillips curve, i.e. assuming zero trend inflation. The figure shows that the weight on current marginal cost (upper left graph) varies inversely with trend inflation, while the forward-looking coefficients b_{it} $(i = 1, 2, 3)$ are all positively related to trend inflation and larger than in the standard approximation case. In particular, the weight on next-period inflation expectations is always bigger than 1, and higher-order expectations matter. Relative to the conventional approximation, current costs matter less, while anticipation matters more.

3 Trend inflation and monetary policy

We embed the GNKPC in a simple general equilibrium model to illustrate the implications of a positive trend inflation for the effects of monetary policy. To keep the model simple, we assume that trend inflation is positive but constant and is set exogenously at $\bar{\pi}$. We analyze how a positive steady-state inflation alters both the dynamics of the standard New Keynesian model and the trade-offs confronted by policymakers. We show in particular that in this Generalized New Keynesian model (GNK) higher trend inflation is associated with a more volatile and unstable economy and tends to destabilize inflation expectations.

Price dispersion plays a crucial role in the GNK model. We start by analyzing the steady-state property of the model, showing that without full indexation, positive steady-state inflation implies a violation of the natural rate hypothesis: because price dispersion increases with trend inflation, the model in steady state exhibits a negative relationship between long-run output and inflation. Next, we show how this long-run Phillips curve relationship importantly affects the Taylor principle, reducing the region of the parameter space that induces a unique rational expectations equilibrium. This in turn affects the ability of monetary policy to anchor inflation expectations.

[28] This is the same interpretation of Ireland (2007), one of the first papers modeling and estimating a process for trend inflation in a small-scale DSGE model. He finds that a backward-looking term in the Phillips curve is not only unnecessary for explaining the post-war U.S. data but actually leads to a significant deterioration in the model's statistical fit.

[29] Barnes et al. (2009) challenges this conclusion arguing that it results from considering too simple a model of indexation. Under the assumption that non-optimally reset prices are indexed to a weighted average of aggregate inflation of the past two periods, they obtain a GNKPC with two lags of inflation whose coefficients are estimated to be statistically significant, albeit quite small.

[30] A similar result is obtained by Coenen et al. (2007) who estimate a generalized Calvo model by allowing the probability of a price revision to depend on the length of time that the existing contract has been in effect. They find that the estimated degree of backward-looking behavior is zero. However, for such result to hold in U.S. data, there must be a proper account of a shift in trend inflation in the early 1990s.

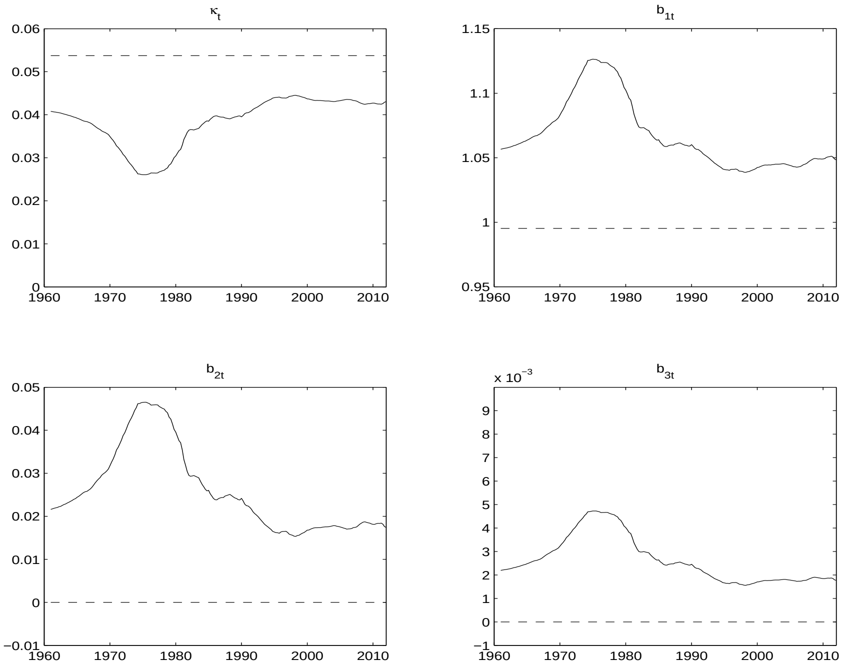


Figure 6: *GNKPC* coefficients

Further, we analyze how trend inflation affects the dynamic response of the endogenous variables to shocks. Higher trend inflation tends to increase the persistence of macroeconomics variables and, for realistic calibrations, to increase their volatility. This result is related to the fact that superneutrality does not hold in the GNK model: changes in trend inflation affect the steady state, which in turn affects the log-linear dynamics of the model, simply because it changes the point around which the model is log-linearly approximated. The extent of this effect obviously depends on how much non-linearities matter: we show that it is particularly important for the standard New Keynesian model because this model is very nonlinear around the zero-inflation steady state (see Figure 9), which is the point around which it is commonly log-linearized in the literature. Finally, we look at the normative implications of positive trend inflation by analyzing how it influences the trade-off confronted by monetary policy and the features of the optimal stabilization policy. We show that positive trend inflation breaks the so-called "divine coincidence" property of the standard NK model.

3.1 The baseline Generalized New Keynesian (GNK) model

Following Ascari and Ropele (2009), we begin building our baseline GNK model by introducing the aggregate demand side and a monetary policy rule. We then complete the description of the supply side of the model introduced in Section 2.2 by relating marginal costs to output. In this part we introduce a measure of price dispersion and show how it increases marginal costs and reduces the output produced per unit of inputs. The cost of price dispersion in turn carries implications for the long-run relationship between inflation and output. Finally we present a log-linear approximation of the model around a steady state characterized by positive inflation, comparing it with a model approximated around a zero steady-state inflation.[31]

The aggregate demand side. The demand side of the model features a representative household, which maximizes an intertemporal utility function, separable in consumption (C) and labor (N):

$$E_t \sum_{j=0}^{\infty} \beta^j \left[\frac{C_{t+j}^{1-\sigma}}{1-\sigma} - d_n e^{\varsigma_t} \frac{N_{t+j}^{1+\varphi}}{1+\varphi} \right], \tag{21}$$

subject to the period-by-period budget constraint:

$$P_t C_t + (1+i_t)^{-1} B_t = W_t N_t + D_t + B_{t-1}. \tag{22}$$

Here, i_t is the nominal interest rate, B_t is holdings of a one-period bond, W_t is the nominal wage rate, and D_t is profits (distributed dividends); ς_t is a labor supply shock, σ is the intertemporal elasticity of substitution in consumption, and φ is the Frish elasticity of labor supply. The household optimization problem yields the following first-order conditions:

$$\text{Euler equation}: \quad \frac{1}{C_t^{\sigma}} = \beta E_t \left[\left(\frac{P_t}{P_{t+1}} \right) (1+i_t) \left(\frac{1}{C_{t+1}^{\sigma}} \right) \right], \tag{23}$$

$$\text{Labor supply equation}: \quad w_t \equiv \frac{W_t}{P_t} = d_n e^{\varsigma_t} N_t^{\varphi} C_t^{\sigma}. \tag{24}$$

The policy rule. We assume a very simple Taylor rule of the form:

$$\left(\frac{1+i_t}{1+\bar{\imath}} \right) = \left(\frac{\pi_t}{\bar{\pi}} \right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}} \right)^{\phi_Y} e^{v_t}, \tag{25}$$

where $\bar{Y}$ is steady-state output; v_t is a monetary policy shock; and ϕ_π, ϕ_Y are non-negative parameters.

Recursive formulation of the optimal price-setting equation. The joint dynamics of the optimal reset price and inflation can be compactly described by rewriting the first-order condition for the optimal price (14) as follows:

$$p_{i,t}^{*} = \frac{\varepsilon}{\varepsilon - 1} \frac{\psi_t}{\phi_t}, \tag{26}$$

where ψ_t and ϕ_t are auxiliary variables that allow one to rewrite in recursive formulation the infinite sums that appear on the numerator and denominator of (14):[32]

$$\psi_t \equiv MC_t Y_t^{1-\sigma} + \theta \beta E_t \left[\pi_{t+1}^{\varepsilon} \psi_{t+1} \right], \tag{27}$$

[31] Our baseline GNK model with positive trend inflation is essentially the same as that of Ascari and Ropele (2009), augmented to include technology and labor supply shocks and abstracting from indexation. See the online appendix for the derivation.

[32] Here we substituted the stochastic discount factor from the Euler equation, i.e., $\mathcal{D}_{t,t+1} = \beta C_{t+1}^{-\sigma} / C_t^{-\sigma}$ and $C_t = Y_t$.

and

$$\phi_t \equiv Y_t^{1-\sigma} + \theta\beta E_t \left[\pi_{t+1}^{\varepsilon-1}\phi_{t+1}\right]. \tag{28}$$

These variables can be interpreted as the present discounted value of marginal costs and marginal revenues (for a unit change in the optimal reset price), respectively. The different exponents on future expected inflation rates in (27) and (28) reflect the different elasticities of marginal costs and of marginal revenues to a change in the relative price in (13).[33]

We should also note that equation (16) implies:

$$p_{i,t}^* = \left(\frac{1-\theta\pi_t^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{1-\varepsilon}}. \tag{30}$$

The role of price dispersion. Price dispersion affects the relationship between aggregate employment and aggregate output. From the individual firms' production functions, aggregating over i, we derive the aggregate labor demand as:

$$N_t = \int_0^1 N_{i,t}di = \int_0^1 \frac{Y_{i,t}}{A_t}di = \frac{Y_t}{A_t}\int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon} di. \tag{31}$$

Denoting by s_t the following measure of price dispersion:

$$s_t = \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon} di, \tag{32}$$

we express aggregate output as:

$$Y_t = \frac{A_t}{s_t}N_t. \tag{33}$$

Schmitt-Grohé and Uribe (2007) show that $s_t \geq 1$, and it is equal to 1 only if all the prices are the same, that is, only if there is no price dispersion. Eq. (33) shows that s_t is a convenient variable to express the resource *cost* of price dispersion in this model:[34] the higher the dispersion of relative prices, the higher is s_t, hence the higher is the labor input needed to produce a given amount of aggregate output. Thus, it immediately follows that, for any given level of output, price dispersion increases the equilibrium real wage (see (24)) and hence the marginal cost of the firms (see (12)). Furthermore, from

[33] This can be easily seen by rewriting (14) as

$$E_t \sum_{j=0}^{\infty} \theta^j \mathcal{D}_{t,t+j} Y_{t+j} \left(\frac{P_{i,t}^*}{P_t}\Pi_{t,t+j}^{\varepsilon-1} - \frac{\varepsilon}{\varepsilon-1}\Pi_{t,t+j}^{\varepsilon} MC_{t+j}\right) = 0 \tag{29}$$

Future inflation changes the relative price of the firms that cannot modify their price. This change affects future marginal costs and marginal revenues in a different way, and the former (ε) relatively more than the latter ($\varepsilon - 1$).

[34] For brevity, we will also refer to s as price dispersion because it is a model-consistent index of price dispersion.

this one can show that price dispersion is an inertial variable, which evolves as:[35]

$$s_t = (1-\theta)\left(p^*_{i,t}\right)^{-\varepsilon} + \theta\pi_t^{\varepsilon} s_{t-1}. \tag{34}$$

The complete non-linear model. In this simple baseline model, there is no capital and no fiscal spending; hence the aggregate resource constraint is simply given by $Y_t = C_t$. Using the latter to substitute for consumption, the non-linear model is described by equations (12), (23), (24), (25), (26), (27), (28), (30), (33) and (34), which determine the evolution of the variables MC_t, Y_t, w_t, i_t, p^*_{it}, ϕ_t, ψ_t, π_t, N_t, s_t.

3.1.1 The cost of price dispersion and the long-run implications of trend inflation

Price dispersion is probably the pivotal characteristic of price staggering models, as it determines the costs of inflation. In an economy with moderate trend inflation, price dispersion is therefore an important first-order variable that matters when studying the long-run behavior, the short-run dynamics, and the welfare properties of the class of NK models we are analyzing.

To illustrate the cost of price dispersion is in our benchmark model, we first derive the steady-state price dispersion from (34), using expression (30) for the optimal price $p^*_{i,t}$, evaluated in steady state:

$$s = \frac{1-\theta}{1-\theta\bar{\pi}^{\varepsilon}}\left(\frac{1-\theta\bar{\pi}^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{35}$$

This expression shows that in a steady state with positive inflation, price dispersion increases in θ, ε, and $\bar{\pi}$. Quite intuitively, the higher the average duration of prices, the higher is price dispersion: if prices were completely flexible ($\theta = 0$), there would be no price dispersion ($s = 1$). The cost of price dispersion is also increasing in ε because the larger the elasticity of substitution among goods, the larger is the inefficient allocation among firms due to the distortion in relative prices. Finally, trend inflation increases price dispersion by causing a greater difference between the price set by the resetting firms and the average price level.

As (33) shows, price dispersion is akin to a negative aggregate productivity shock (recall that $s_t \geq 1$), since it increases the amount of labor required to produce a given level of output. Hence, we follow Damjanovic and Nolan (2010) and measure the cost of price dispersion by defining the variable $\tilde{A}_t = \frac{A_t}{s_t}$ as a measure of "effective" aggregate productivity and mapping a percentage *increase* in s into an equivalent percentage

[35] The expression is obtained by observing that s_t can be written as

$$s_t = (1-\theta)\left[\frac{P^*_{i,t}}{P_t}\right]^{-\varepsilon} + \theta\pi_t^{\varepsilon}\times$$
$$\times\left\{(1-\theta)\left[\frac{P^*_{i,t-1}}{P_{t-1}}\right]^{-\varepsilon} + \theta^2(1-\theta)\left[\frac{P^*_{i,t-2}}{P_{t-1}}\right]^{-\varepsilon} + \cdots\right\},$$

where the expression in the curly brackets is exactly the definition of s_{t-1}.

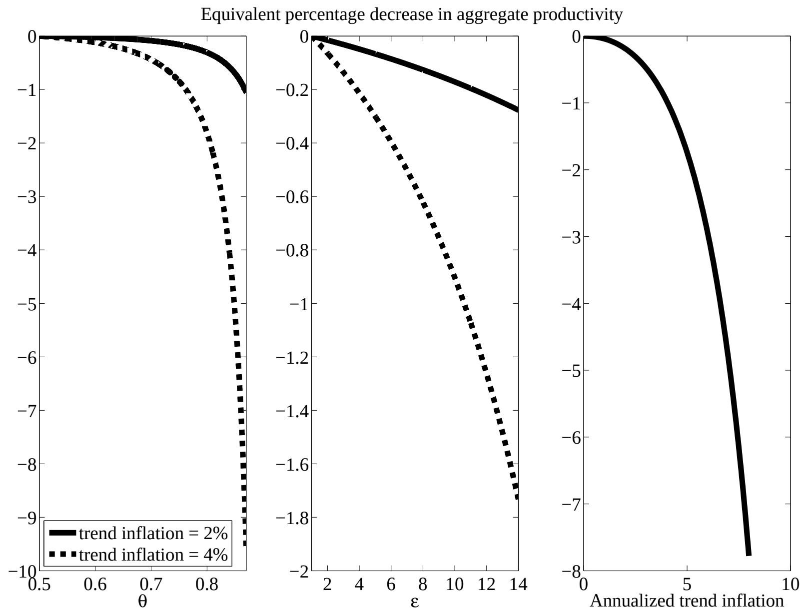


Figure 7: The cost of price dispersion

decrease in aggregate productivity, $\tilde{A}_t$. Figure 7 shows this decrease in aggregate productivity as a function of θ, ε, and $\bar{\pi}$.[36] Aggregate productivity is rapidly decreasing with trend inflation and with values of the price stickiness parameter that are above 0.75. Moreover, higher trend inflation makes the cost of price dispersion more sensitive to the value of the structural parameters θ and ε.[37]

Damjanovic and Nolan (2010) provide an indirect estimate of the productivity loss by mapping the price dispersion measure s_t into the coefficient of variation of prices; they show that if, for example, the coefficient of variation is between 10 and 30 percent, as suggested by some empirical evidence, then the productivity loss ranges from 0.8 to 3.3 percent (see Damjanovic and Nolan (2010) and the references therein).

A loss in aggregate productivity in turn translates into an output loss, as shown in Figure 8. Panel (a) in the figure displays steady-state output as a function of trend inflation. In this stylized New Keynesian model, long-run superneutrality breaks down, and a negative long-run relationship emerges between inflation and output (King and Wolman, 1996; Ascari, 2004; Yun, 2005).[38] To understand this result, it is useful to

[36] Unless otherwise stated, we use fairly standard parameter values in the numerical computations, setting $\beta = 0.99$, $\sigma = 1$, $\varepsilon = 10$, $\theta = 0.75$, and $\varphi = 1$.

[37] Decreasing returns to labor (i.e., a firm technology $Y_{i,t} = A_t N_{i,t}^{1-\alpha}$) would make price dispersion more sensitive to the rate of trend inflation; hence decreasing returns would deliver a larger equivalent change in aggregate productivity (see online appendix). For example, assum $\alpha = 0.3$; then, if annual trend inflation goes from 0 to 4%, the cost of price dispersion increases by almost 9%, a change corresponding to a 5.8% negative shock to aggregate productivity.

[38] See Karanassou et al. (2005) for an empirical appraisal of the long-run Phillips curve, or Beyer and Farmer (2007) for a positive empirical relationship between inflation and unemployment. Vaona (2012) finds empirical support for a negative relationship between inflation and output growth.

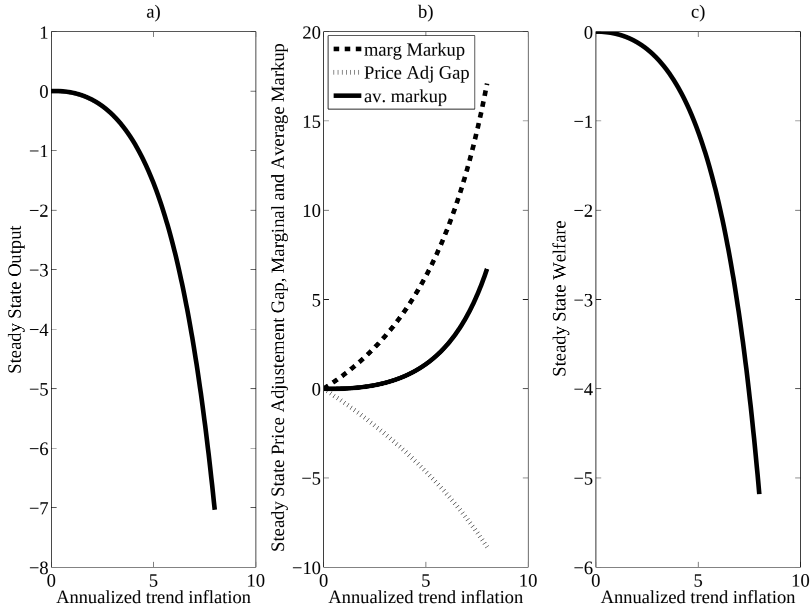


Figure 8: Trend inflation and steady state variables. Variables expressed as percentage deviation from the zero inflation steady state.

use expressions (12), (24), and (33), evaluated in steady state, to write the steady-state output level as:

$$Y = \left(\frac{A^{\varphi+1}}{d_n} \frac{1}{s^{\varphi} \frac{1}{MC}} \right)^{\frac{1}{\varphi+\sigma}} = \left(\frac{A^{\varphi+1}}{d_n} \frac{1}{s^{\varphi} \mu} \right)^{\frac{1}{\varphi+\sigma}}, \tag{36}$$

where A is the steady-state level of technology, and $\mu \equiv \frac{1}{MC}$ is the steady-state (real) markup. As the equation shows, the steady-state level of output depends upon price dispersion and the average markup, which are in turn functions of trend inflation. The negative effect of trend inflation on output through price dispersion is quite powerful: panel (a) in Figure 8 mostly reflects the right-hand panel of Figure 7. The average markup also increases with trend inflation (see panel (b) in Figure 8). The intuition for this result is best seen through the following decomposition of the markup (see King and Wolman, 1996):

$$\mu = \left(\frac{P}{P_i^*} \right) \left(\frac{p_i^*}{MC} \right),$$

where $\frac{P}{P_i^*}$ is a *price-adjustment gap;* and $\frac{p_i^*}{MC}$ is a *marginal markup*, which is the ratio of the newly adjusted price to marginal cost. The first term decreases in inflation (see (30)) because inflation erodes the relative price of firms that reset in past periods. Hence, the higher the current inflation rate, the more the newly set price is (mechanically) away from the average price level. The *marginal markup* instead increases with trend inflation, as can be seen from expression (15):

$$\frac{p_i^*}{MC} = \frac{\varepsilon}{\varepsilon - 1} \frac{1 - \beta\theta\bar{\pi}^{\varepsilon-1}}{1 - \beta\theta\bar{\pi}^{\varepsilon}}, \tag{37}$$

because of the forward-looking behavior of firms and the interactions of future expected inflation rates with the monopolistic competition framework. As we discussed in Section 3.1, future inflation rates affect marginal costs relatively more (ε) than marginal revenues ($\varepsilon - 1$) (see (14)). Thus, the overall effect of trend inflation on the average markup depends on the elasticity of demand ε, a key parameter of the model. The *marginal markup* term in general dominates the *price-adjustment gap* term[39] (Figure 8, panel (b)); therefore higher trend inflation yields a larger average markup, hence a larger monopolistic distortion and a larger negative effect on steady-state output.

Another element that affects the marginal markup and partially offsets the effect of trend inflation is discounting. The more firms discount the future, the less they are going to worry about the erosion of future markups: this implies a lower *marginal markup*[40], a lower aggregate markup, and a higher steady-state output.

Finally, Panel (c) in Figure 8 shows that steady-state welfare (computed by plugging the steady-state values in the utility function (21)) is also strongly decreasing with trend inflation in this simple NK model, both because trend inflation reduces steady-state output and hence consumption and because it increases the amount of labor needed to produce a given amount of output. We will analyze further these implications when we discuss normative issues in Section 3.4.

3.1.2 The log-linearized GNK model

Much of the success of the standard NK model is due to the elegance of condensing a quite sophisticated microfounded model into three simple linear equations, which are typically derived as log-linear approximations of the equilibrium conditions around a steady state with zero inflation. The first two are the Euler equation (23) and the Taylor rule (25). For the Euler equation, we impose the resource constraint $Y = C$ to have:

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \sigma^{-1}\left[\hat{\imath}_t - E_t\hat{\pi}_{t+1}\right]. \tag{38}$$

For the Taylor rule, we have:

$$\hat{\imath}_t = \phi_\pi\hat{\pi}_t + \phi_Y\hat{Y}_t + v_t. \tag{39}$$

The third equation is an NKPC that links inflation and output obtained by log-linearizing the supply side equations: this equation is obtained by combining the *NKPC* (17) with the relationship between marginal costs and output:[41]

$$\hat{\pi}_t = \lambda\hat{Y}_t + \beta E_t\hat{\pi}_{t+1} + \kappa\left(\varsigma_t - (\varphi + 1)\hat{A}_t\right), \tag{40}$$

[39] The price-adjustment gap is actually stronger for extremely low values of trend inflation, so that the average markup first decreases slightly and then increases as trend inflation increases (see King and Wolman, 1996).

[40] It is easy to show that the marginal markup increases with β if $\bar{\pi} > 1$ (see the online appendix).

[41] The derivation is obtained substituting in (17) a log-linear approximation of the marginal cost expression (12), the labor supply equation (24), and the aggregate production function (33), with $\hat{s}_t = 0$.

where the slope is given by $\lambda \equiv \kappa(\sigma + \varphi)$.

When the approximation is around a steady state characterized by positive inflation, the first two log-linearized equations still hold, but getting an expression corresponding to (40) is more cumbersome. We could proceed as above, starting from the GNKPC (18), with $\overline{\pi}_t$ replaced by $\overline{\pi}$, and substituting in it the log-linear expression of marginal costs as function of output. However, it is simpler to log-linearize instead a recursive formulation of the GNKPC, which can be obtained from expressions (26)-(28) of Section 3.1. To do so, we combine (26) and (30) to obtain an expression for ϕ_t, which we substitute in (28). This gives a Phillips curve that comprises two equations describing respectively the dynamics of inflation and the evolution of the present discounted value of future marginal costs ψ_t. We then log-linearize these two equations around a trend inflation $\overline{\pi}$ to obtain:

$$\hat{\pi}_t = \kappa(\overline{\pi})\,\widehat{mc}_t + b_1(\overline{\pi})\,E_t\hat{\pi}_{t+1} + b_2(\overline{\pi})\left((1-\sigma)\,\hat{Y}_t - E_t\hat{\psi}_{t+1}\right) \tag{41}$$

and

$$\hat{\psi}_t = (1-\theta\beta\overline{\pi}^{\varepsilon})\left(\widehat{mc}_t + (1-\sigma)\,\hat{Y}_t\right) + (\theta\beta\overline{\pi}^{\varepsilon})\,E_t\left(\hat{\psi}_{t+1} + \varepsilon\hat{\pi}_{t+1}\right), \tag{42}$$

where $\kappa(\overline{\pi}) \equiv \frac{(1-\theta\overline{\pi}^{\varepsilon-1})(1-\theta\beta\overline{\pi}^{\varepsilon})}{\theta\overline{\pi}^{\varepsilon-1}}$, $b_1(\overline{\pi}) \equiv \beta\left[1+\varepsilon\left(1-\theta\overline{\pi}^{\varepsilon-1}\right)(\overline{\pi}-1)\right]$, and $b_2(\overline{\pi}) \equiv \beta\left(1-\theta\overline{\pi}^{\varepsilon-1}\right)(1-\overline{\pi})$.[42] Finally, to express the Phillips curve as an inflation-output relationship we substitute out the marginal cost, accounting for the price dispersion term in the aggregate production function: $\hat{Y}_t = \hat{A}_t + \hat{N}_t - \hat{s}_t$. Equations (41) and (42) then become, respectively:

$$\begin{aligned}\hat{\pi}_t = {} & \lambda(\overline{\pi})\,\hat{Y}_t + b_1(\overline{\pi})\,E_t\hat{\pi}_{t+1} + \kappa(\overline{\pi})\left(\varphi\hat{s}_t + \varsigma_t - (\varphi+1)\,\hat{A}_t\right) \\ & + b_2(\overline{\pi})\left(\hat{Y}_t(1-\sigma) - E_t\hat{\psi}_{t+1}\right),\end{aligned} \tag{43}$$

and

$$\hat{\psi}_t = (1-\theta\beta\overline{\pi}^{\varepsilon})\left(\varphi\hat{s}_t + (\varphi+1)\left(\hat{Y}_t - \hat{A}_t\right) + \varsigma_t\right) + \theta\beta\overline{\pi}^{\varepsilon}E_t\left(\hat{\psi}_{t+1} + \varepsilon\hat{\pi}_{t+1}\right), \tag{44}$$

where $\lambda(\overline{\pi}) \equiv \kappa(\overline{\pi})(\sigma+\varphi)$, and the dynamic behavior of $\hat{s}_t$ is derived by log-linearizing (34):[43]

$$\hat{s}_t = \left[\frac{\varepsilon\theta\overline{\pi}^{\varepsilon-1}}{1-\theta\overline{\pi}^{\varepsilon-1}}(\overline{\pi}-1)\right]\hat{\pi}_t + \left[\theta\overline{\pi}^{\varepsilon}\right]\hat{s}_{t-1}. \tag{45}$$

The log-linearized GNK model is thus composed of the Euler equation (38); the policy rule (39); and three equations for the supply side, (43), (44), (45). These equations describe the evolution of the variables $\hat{\pi}_t$, $\hat{\psi}_t$, $\hat{Y}_t$, $\hat{s}_t$, and $\hat{\imath}_t$.

Two things should be noted here. First, the supply side block of the GNK model now includes three dynamic equations – (43), (44) and (45) – rather than one, (40), as in the approximation around zero steady-state inflation. Setting $\overline{\pi} = 1$ in these three equations yields the standard NKPC (40). Second, $\hat{s}_t$ is a backward-looking variable,

[42] This formulation is expression (18) written in recursive form. For ease of notation we have suppressed the dependence of coefficients on the vector of structural parameters, retaining only their dependence upon trend inflation. Details of the derivation are in the online appendix.

[43] We use (30) to substitute for $p^*_{i,t}$ in (34).

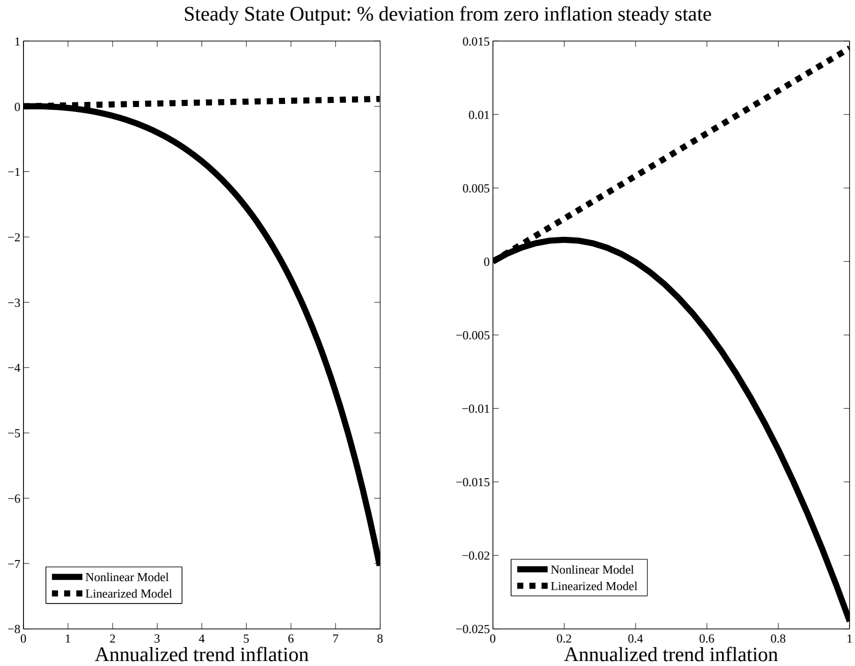


Figure 9: Relation between steady state inflation and output in the model linearized around a zero steady-state inflation and in the nonlinear model. The right-hand panel shows values for trend inflation rates lower than 1%.

so its inclusion adds inertia to the adjustment of inflation; moreover, trend inflation increases the weight on $\hat{s}_{t-1}$ yielding, *ceteris paribus,* a more persistent adjustment path for $\hat{s}_t$. The dynamics of the GNK model is thus richer; and as we stressed in Section 2.2.2, important coefficients depend on the level of trend inflation.

3.1.3 The zero inflation steady-state approximation

Despite the high costs it imposes on output and welfare (discussed in Section 3.1.1), price dispersion is generally considered a term of second-order importance in linearized models because the vast majority of the literature uses models approximated around a zero inflation steady state. Two main reasons lie behind this choice: the first is that zero is the optimal long-run inflation for many specifications of the NK framework (see Goodfriend and King, 2001; Woodford, 2003; and the discussion in Section 3.5). The second, as shown above, is analytical convenience: the resulting model is very simple and tractable.

Zero steady-state inflation, however, does not coincide with the concept of price stability held by central bankers, who generally target a positive inflation rate.

From a theoretical perspective, assuming zero inflation in the steady state eliminates some interesting effects that derive from the interaction of trend inflation, relative prices, and the monopolistic competition framework. In Section 3.1.1, we distinguished three such effects: price dispersion, marginal markup, and discounting. When the model is log-linearized around a zero inflation steady state, the price dispersion effect is completely nullified at first order – price dispersion is irrelevant both in the steady state and in the model dynamics. Setting $\bar{\pi} = 1$ into (45) yields:

$$\hat{s}_t = \theta \hat{s}_{t-1}. \tag{46}$$

Small perturbations have a zero first-order impact on $\hat{s}_t$, because the model is log-linearized exactly around the point at which price dispersion is at its minimum. The marginal markup effect is also muted and would appear only in a second-order approximation. Discounting interacts with trend inflation in the direction opposite to that of the other two effects. The NKPC (40) in the long-run implies:

$$\hat{Y} = \frac{1-\beta}{\lambda} \hat{\pi}, \tag{47}$$

where $\frac{1-\beta}{\lambda}$ represents the long-run elasticity of output to inflation in a *zero steady-state inflation*. From expression (47) one concludes that discounting makes the long-run relationship between inflation and output positively sloped. Figure 9 shows, however, that this conclusion is an artifact of the log-linearization around zero steady-state inflation. The figure plots the non-linear long-run Phillips curve, expression (36), together with the long-run relationship implied by expression (47).[44] The relationship between the two curves is clear: the "long-run Phillips curve" implied by the log-linearized model is the tangent to the non-linear long-run Phillips curve at the zero inflation steady state.[45] More generally, the slope of the long-run Phillips curve depends upon the inflation rate that characterizes the steady state around which the model is approximated. The figure shows that, except at extremely low rates of inflation, the discounting effect in the non-linear model is weak and easily dominated by the effects of price dispersion and marginal markup. At zero inflation, however, these two effects are absent at first order; hence, the time discounting generates a positive inflation-output relationship, with coefficient $\frac{1-\beta}{\lambda}$. The long-run elasticity of output to inflation (the slope of the curve) shapes the determinacy region, as we will see in the next section (see (48) and (49)).

Three main implications follow from this analysis. First, the fact that the markup and price dispersion effects cancel out under the zero inflation steady-state assumption makes the model behave as if there is only one representative firm; hence, price dispersion is no longer an issue. Indeed, the model is, to the first order, observationally equivalent to the Rotemberg (1982) model, which assumes convex costs of price adjustment with no staggering or price dispersion.[46] The zero inflation steady-state assumption is therefore not just a convenient simplification; rather, it washes out some implications of the microfoundations of the Calvo model.

[44] The graph on the right magnifies the version on the left to emphasize the behavior at very low rates of inflation. A similar figure appears in Yun (2005), Levin and Yun (2007), and Ascari and Merkl (2009).

[45] The use of hatted variables in a long-run relationship may require clarification. When a model is approximated, the long-run relationships implied by the log-linearized model are nothing else than the log-linear approximation of the original long-run relationships around the point of the approximation. In the literature, the log-linearized model is sometimes used to look at permanent changes, but that can lead to erroneous findings when the non-linearities of the original model are strong enough (see Ascari and Merkl, 2009).

[46] In this model, firms change prices in every period and in the same way (see the discussion in Section 3.6.2).

Second, the standard NKPC, eqs. (17) or (40), is notable for the absence of a role for the parameter ε. This stands in contrast with our emphasis in Section 3.1 on the effects of the interaction between trend inflation and the monopolistic competition framework, which naturally depends on the demand elasticity ε. This elasticity determines the relative weight of future inflation in the present discounted value of marginal costs and marginal revenues (see (27), (28), and footnote 33). Around zero inflation, these interesting effects are of second-order importance.[47]

Third, Figure 9 makes clear that policy inference under the zero inflation steady-state approximation becomes increasingly problematic with higher steady state inflation. Of course, the extent to which a linear approximation worsens as one moves away from the approximation point depends on how nonlinear the original model is near that point. The standard NK model is unfortunately especially nonlinear around the zero inflation steady state; moving the approximation point slightly away from there delivers a log-linearized model with quite different long-run and dynamic properties. In particular, the explicit accounting for a positive steady state inflation in the GNK model changes the results of the standard NK model regarding the determinacy of the rational expectation equilibrium, the dynamic response to shocks, and the design of optimal policy. We turn to these changes next.

3.2 Trend inflation and the anchoring of expectations

Many economists have advocated an increase in central banks' inflation targets to respond to the current economic recession; others, including Federal Reserve Chairman Bernanke, counterargue that such a move could destabilize inflation expectations. Within our framework we can address the extent to which a higher inflation target makes the anchoring of inflation expectations more difficult for the central bank.

3.2.1 Rational expectations and determinacy

As we discussed, the standard log-linear NK model is composed of the Euler equation (38), the NKPC (40), and the policy rule (39).[48] With this specification, one can show that a necessary and sufficient condition for the rational expectations equilibrium (REE) to be unique is:

$$\phi_\pi + \frac{(1-\beta)}{\lambda}\phi_Y > 1, \tag{48}$$

with both ϕ_π and ϕ_Y positive. As stressed by Bullard and Mitra (2002) and Woodford (2001; and 2003, Ch. 4.2.2), among others, this condition is a generalization of the Taylor (1993) principle according to which the nominal interest rate should rise more than proportionally with respect to inflation, i.e., $\phi_\pi > 1$. Condition (48) states that in an NK model, the Taylor principle should be amended according to the long-run properties of the model: the nominal interest rate should rise by more than the long-run

[47] This point can be seen by comparing κ and $\kappa(\bar{\pi})$. However, simple modifications of the pricing model, such as assuming firm-specific inputs or a more general demand aggregator, introduce alternative channels through which the elasticity of demand can affect the slope coefficient in otherwise standard NK models.

[48] Since this section analyzes the indeterminacy properties of the model, we can abstract from the shocks.

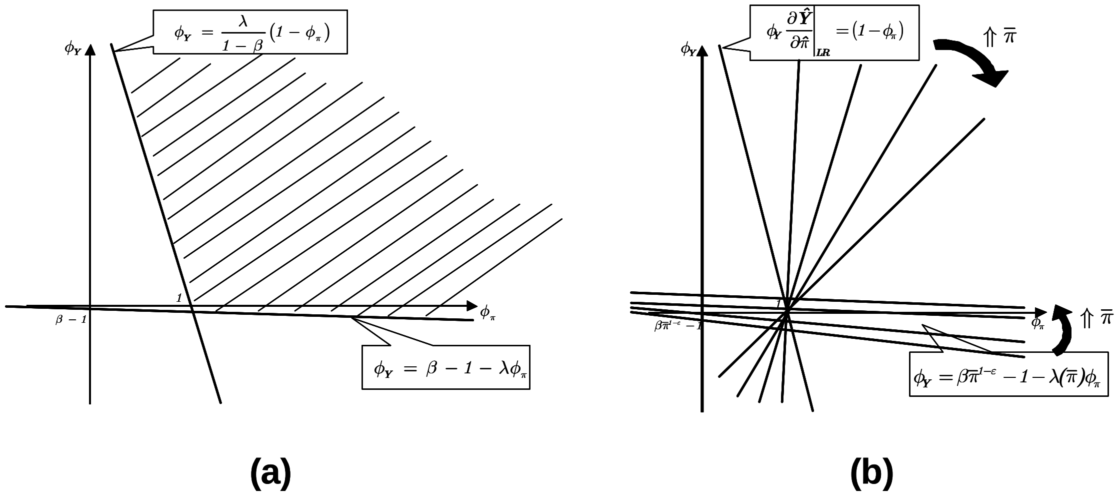


Figure 10: The determinacy region: (a) the zero inflation steady state case; (b) the positive trend inflation case

increase in inflation. As we saw above, $(1-\beta)/\lambda$ represents the long-run elasticity of output to inflation in this model (see eq. (47)).[49] In other words, the Taylor principle should be understood as saying that:

$$\left.\frac{\partial \hat{\imath}}{\partial \hat{\pi}}\right|_{LR} = \phi_\pi + \phi_Y \left.\frac{\partial \hat{Y}}{\partial \hat{\pi}}\right|_{LR} > 1, \tag{49}$$

where LR stands for long run. The intuition is straightforward: a sunspot increase in inflation requires a more than proportional increase in the nominal interest rate in order to drive up the real interest rate. This in turns reduces output via the Euler equation, curbing the initial rise in inflation via the NKPC.[50]

Figure 10 shows the determinacy region in the space of the policy parameters (ϕ_π, ϕ_Y). Panel (a) portrays this region in the zero inflation steady state case as a shaded area. The figure also shows another condition that needs to be satisfied for determinacy: $\phi_Y + \lambda\phi_\pi > \beta - 1$.[51]

The case of positive trend inflation is portrayed in panel (b). In this case, as we saw in Section 3.1.2, the log-linearized model comprises additional equations describing the

[49] Hence, the left-hand side of (48) "represents the long-run increase in the nominal interest rate prescribed [...] for each unit of permanent increase in the inflation rate" (Woodford, 2003, p. 254). Therefore, "The Taylor principle continues to be a crucial condition for determinacy, once understood to refer to the *cumulative* response to a *permanent* inflation increase" (Woodford, 2003, p. 256, italics in the original). For this reason, this generalized version of the Taylor principle is sometimes referred to in the literature as the long-run Taylor principle.

[50] An intuitive analytical explanation is provided again by Woodford (2003). Indeed, (38), (39), and the standard NKPC continue to be satisfied if inflation, output, and interest rates are increased at all dates by constant factors satisfying (49) with equality. This means that a real eigenvalue of 1 corresponds to that equality.

[51] This condition is often neglected in the literature because, in the case of a zero inflation steady state, it is always satisfied in the positive orthant of the space (ϕ_π, ϕ_Y). We show below that this condition becomes important under positive trend inflation.

dynamics of price dispersion $\hat{s}$ and of the variable $\hat{\psi}$; most importantly, the parameters of these equations are generally a function of trend inflation. As a consequence, the determinacy region is affected by the particular value of trend inflation. Despite the higher-order dynamics, Ascari and Ropele (2009) show that, under some simplifying assumptions, it is possible to derive the determinacy conditions analytically and show that the generalized Taylor principle (49) still holds.[52] In this case the term $\left.\frac{\partial \hat{Y}}{\partial \hat{\pi}}\right|_{LR}$ decreases as trend inflation increases, becoming increasingly negative at lower values of trend inflation. The line that defines the generalized Taylor principle (49) therefore rotates clockwise, shrinking the determinacy region, as Figure 10(b) shows. Note also that the second determinacy condition now moves the bottom line upward in the diagram, so that it eventually enters the positive orthant.[53] As a consequence of the joint movement of the two lines, the minimum level of ϕ_π necessary to induce a unique REE increases with trend inflation. Again the intuition is straightforward. Since trend inflation makes price-setting firms more forward-looking, the inflation rate becomes less sensitive to current economic conditions (the short-run GNKPC flattens); hence, monetary policy should respond more strongly to inflation to induce a reduction in output that achieves a given change in inflation.

Simulating the determinacy region for the benchmark calibration of the model (see footnote 36) yields the results shown in Figure 11. In the baseline GNK model, the determinacy region shrinks very rapidly with trend inflation, requiring a weaker policy response to output and a stronger response to inflation to guarantee determinacy as trend inflation increases. The cross in Figure 11 marks the classical values of the Taylor rule parameters, i.e., $\phi_\pi = 1.5$ and $\phi_Y = 0.5/4$, as in Taylor (1993). These values would result in an indeterminate REE for trend inflation greater than 4 percent.[54]

Two general policy implications emerge from this analysis. First, a higher trend inflation tends to destabilize inflation expectations. Second, in a higher inflation targeting regime, monetary policy should respond more to deviations of inflation from target and less to output gaps. Ascari and Ropele (2009) show that these two messages are qualitatively robust across different parameterizations and different types of Taylor rules (backward-looking, forward-looking, and inertial policy). However, in the case of positive steady-state inflation, inertial policies tend to stabilize expectations. Similarly, Taylor-type rules that respond to output growth rather than to the output gap widen the determinacy region, thus making it easier for policy to induce a unique REE (Coibion and Gorodnichenko, 2011).

Coibion and Gorodnichenko (2011) provide an interesting empirical application of these results. They reinterpret the economic instability of the period before the U.S. Great Moderation as resulting from a combination of high trend inflation and a mild policy response. A mainstream interpretation of the volatility of the Great Inflation

[52] The simplifying assumptions we use are log preference in consumption ($\sigma = 1$) and indivisible labor ($\varphi = 0$).

[53] When $\sigma = 1$ and $\varphi = 0$, a necessary condition for this second condition to hold is: $\phi_Y + \lambda(\bar{\pi})\phi_\pi > \beta\bar{\pi}^{1-\varepsilon} - 1$ (see Figure 10). See Ascari and Ropele (2009) for details.

[54] For trend inflation at the high values of 6 and 8 percent, a small region of instability also appears above the indeterminacy region. For our calibration, this region is extremely small for 6 percent and gets bigger as trend inflation increases. However, these values for trend inflation are out of the range of the estimates in Figure 3. Thus, Figure 11 plots only the determinacy regions of the parameter space so as to focus the discussion on determinacy and to put all the cases in a single figure.

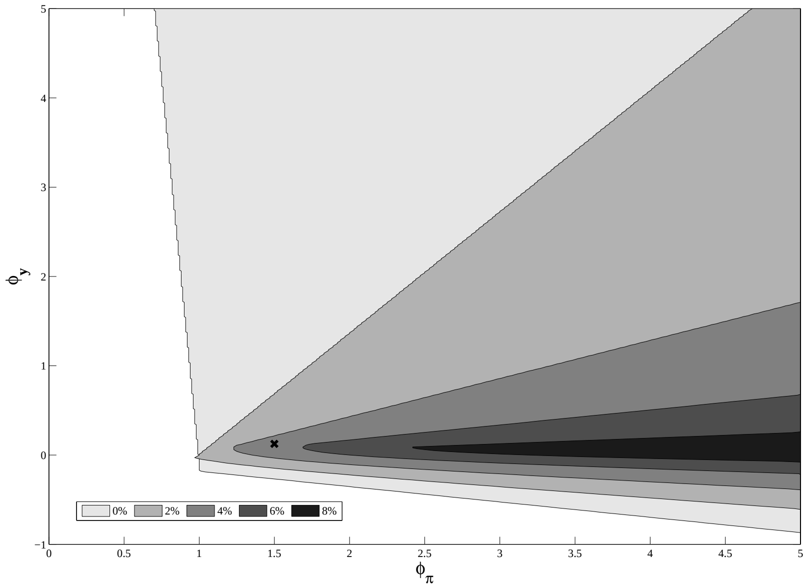


Figure 11: The determinacy region and trend inflation

period is that high inflation was due to the failure of monetary policy to satisfy the Taylor principle (Clarida et al., 2000). By contrast, post-Volker monetary policy responded strongly to inflation (the policy parameter ϕ_π is estimated to be substantially bigger than 1), bringing expectations back in line with a unique stable REE. In this interpretation, the "conquest of American inflation" after the '80s is due to a switch from "bad" to "good" policy. However, Coibion and Gorodnichenko (2011) identify another possible dimension of the bad vs good policy argument: the substantial reduction in the inflation target. To show that, they first estimate a Taylor rule using real-time U.S. data based on Federal Reserve staff forecasts for two sub-samples: 1969-1978 and 1983-2002.[55]

They then conduct some counterfactual experiments on the estimated policy coefficients of each of the two sub-samples and on the rate of trend inflation, using two values that correspond to the average inflation in the two periods (6 percent and 3 percent, respectively).[56] In line with Clarida et al. (2000), they find that the economy was very likely to have been in an indeterminate region, according to the pre-'79 estimates, while the opposite was true according to the post-'82 estimates. However, contrary to Clarida et al. (2000), they also show that replacing the policy rule coefficients from the pre-'79 estimates with those from the post-'82 estimates without at the same time reducing the

[55] They omit the period from 1979 to 1982, during which the FOMC officially abandoned interest rate targeting in favor of monetary aggregate targeting. In addition to a response to inflation and the output gap, their estimated Taylor rule features interest rate inertia and a response to output growth. The forecasts appeared in the Greenbook, the briefing on economic and financial developments prepared by the staff of the Federal Reserve Board for each meeting of the FOMC.

[56] For each sample period, they make 10,000 draws from the distribution of the estimated parameters of the Taylor rule and assess the fraction of draws that yield a determinate REE at 3% and 6% trend inflation.

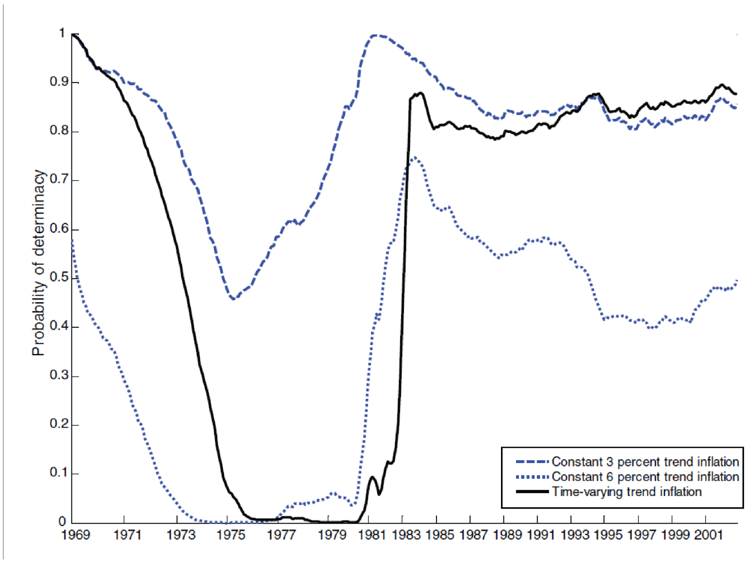


Figure 12: Coibion and Gorodnichenko (2011) Figure 4, p. 363: Probability of Determinacy Using Time-Varying Response Function by the Federal Reserve

target inflation rate would not have been sufficient to rule out indeterminacy.[57]

To dig further into this result, Coibion and Gorodnichenko (2011) estimate a version of their Taylor rule with time-varying parameters and recover a time-varying measure of trend inflation from the intercept of the rule. Moreover, from the estimated time series of the Taylor rule parameters and the estimated measure of trend inflation, they construct time series of the probability of determinacy for the U.S. economy given the estimated distribution of the Taylor rule parameters.

Figure 12 shows the implied evolution of these probabilities and nicely visualizes the two main messages of their work.[58] First, the U.S. economy was very likely to be subject to indeterminacy from the early '70s until the start of the Volcker disinflation. Second, the reduction in trend inflation was an important component of the change in the probability of a unique REE. The two dashed lines are far apart, meaning that an average inflation of 3 percent rather than 6 percent matters in determining the likelihood of a determinate REE. Had policy switched to the "good" post-'82 reaction coefficient without reducing average inflation to 3 percent, the U.S. economy would have remained at the edge of the indeterminacy region.

[57] In particular, it would have increased the likelihood of a determinate REE, but it would have still left a large uncertainty because only 60 percent of draws from the distribution of estimated parameters from the Taylor rule predicted determinacy.

[58] In Figure 12, the dashed (dotted) line assumes a constant rate of trend inflation of 3 percent (6 percent), while the solid line uses the time-varying estimated measure of trend inflation. See Coibion and Gorodnichenko (2011) for details.

3.2.2 Learning and expectations

Learning provides a natural framework to study the concern that a higher inflation target could unanchor inflation expectations.[59] Kobayashi and Muto (2013), Ascari and Florio (2012), and Kurozumi (2013b) extend the analysis of Ascari and Ropele (2009) by introducing adaptive learning à la Evans and Honkapohja (2001) in the baseline GNK model with trend inflation.[60] As expressed in the learning literature, convergence of the agents' perceived law of motion to the actual law of motion of the economy means that the REE is "learnable" or "E-stable," in that expectations under learning converge asymptotically to the REE. , Kobayashi and Muto (2013) and Ascari and Florio (2012) investigate the E-stability regions - the regions of the parameter space (ϕ_π, ϕ_Y) of a given Taylor rule where the REE is E-stable - in the manner of Bullard and Mitra (2002). Their main result is that higher trend inflation generally makes the REE more difficult to learn, reduces the E-stability region, and requires a stronger policy response to inflation and a weaker response to output in order to guarantee E-stability.[61]

Trend inflation therefore has an effect on the E-stability region under adaptive learning that is similar to its effect on the determinacy region under rational expectations; this similarity supports the claim that a higher inflation target could unanchor expectations.[62] Moreover, Ascari and Florio (2012) show that, conditional on E-stability, higher trend inflation reduces the speed of convergence of expectations to the REE. Hence, a higher inflation target destabilizes expectations both asymptotically (E-stability) and in the transition phase.

Branch and Evans (2011) and Cogley et al. (2010) follow a different approach to the relationship between learning and the inflation target. They both study the dynamics of the model when there is imperfect information about a change in the central bank's long-run inflation target. Branch and Evans (2011) show how an increase in the inflation target could generate near-random-walk beliefs and temporarily unstable dynamics due to self-fulfilling paths. The assumption that agents form expectations using adaptive learning with constant gain (that is, that agents discount past data) is key for the result, and it can be justified by a prior on a possible structural change of parameters. In this way, the model can generate significant and persistent departures from rational expectations.

The mechanics of this result is the following. After an increase in the inflation target, inflation expectations increase less than proportionally because of the adaptive learning

[59] A similar point has been made by Federal Reserve Chairman Bernanke: "What is the right conceptual framework for thinking about inflation expectations in the current context? [...] Although variations in the extent to which inflation expectations are anchored are not easily handled in a traditional rational expectations framework, they seem to fit quite naturally into the burgeoning literature on learning in macroeconomics. [...] In a learning context, the concept of anchored expectations is easily formalized" (speech at the NBER Monetary Economics Workshop, July 2007).

[60] Under this learning model, agents form expectations through recursive least squares with decreasing gain, and their perceived laws of motion usually take the minimum state variable (MSV) form.

[61] Despite some differences in the assumptions regarding the learning process across the two papers, the main result seems quite robust across different specifications of the Taylor rule (backward, forward, and inertial). Kurozumi (2013b) shows, however, that the result that the shrinkage of the E-stability region with trend inflation may not hold for low levels of the Frish elasticity of labor supply and high levels of trend inflation.

[62] See Ellison and Pearlman (2011) for a formal analysis of the link betwen the determinacy and the E-stability region.

process; as a result, inflation would be below target, and the central bank would reduce the nominal interest rate. Given the specification of the learning process, agents will detect a structural change in the persistence of inflation, which will lead to higher inflation expectations that feed back into higher inflation rates. Eventually agents come to believe that inflation follows a random walk, and inflation could then largely overshoot the target before returning to its new long-run value. Imperfect information on the inflation target can thus generate instability in the inflation rate and in inflation expectations, which temporarily follow non-stationary dynamics. To prevent that, monetary policy should be perfectly credible and transparent, informing agents about the size and the timing of the change in the target.

A related work by Cogley et al. (2010) studies optimal disinflation policy with Taylor-type rules under learning. They analyze the options of a central bank that inherits high inflation. Under full information, the optimal disinflation policy in that case implies more aggressive response coefficients; whereas under learning, optimal disinflation occurs more gradually and implies a larger sacrifice ratio. Under learning, the optimal disinflation is gradual because the equilibrium law of motion in that case is potentially explosive. However, the authors find that the crucial source of potential explosiveness is the imperfect information about the policy feedback parameters rather than about the long-run inflation target. The authors also show that the form of the Taylor rule – whether backward or contemporaneous – matters in generating potentially unstable dynamics.

3.3 Trend inflation, macroeconomic dynamics, and monetary policy trade-offs

Trend inflation alters the transmission mechanism of monetary policy and the dynamics of the macroeconomy in response to shocks. These alterations arise because trend inflation affects the parameters of the log-linearized model; in particular, as we have shown, it flattens the Phillips curve. To analyze these effects, we specify the following autoregressive processes for the three shocks of our baseline model:

(i) the technology shock:

$$\hat{A}_t = \rho_A \hat{A}_{t-1} + u_{At}; \tag{50}$$

(ii) the labor supply shock in (24) (which is equivalent to a cost push shocks in the marginal cost equation (12)):

$$\varsigma_t = \rho_\varsigma \varsigma_{t-1} + u_{\varsigma t}; \tag{51}$$

and (iii) the monetary policy shock in the Taylor rule (39):

$$v_t = \rho_v v_{t-1} + u_{vt}. \tag{52}$$

The innovations u_{At}, $u_{\varsigma t}$, and $u_{\nu t}$ are assumed to be *i.i.d.* standard normal processes.

To obtain analytical expressions for the responses of output and inflation to the shocks, we use the following simplifying assumptions: log preferences in consumption ($\sigma = 1$), indivisible labor ($\varphi = 0$), and no persistence in the shocks ($\rho_i = 0$ for $i = A, \varsigma, v$).[63] With these assumptions, the log-linearized *GNK* model can be described by the following equations:

[63] Note that $\sigma = 1$ and $\varphi = 0$ imply $\lambda(\bar{\pi}) = \kappa(\bar{\pi})$.

$$\hat{\pi}_t = \lambda(\bar{\pi})\left(\hat{Y}_t - \hat{A}_t + \varsigma_t\right) + b_1(\bar{\pi}) E_t\hat{\pi}_{t+1} - b_2(\bar{\pi}) E_t\hat{\psi}_{t+1}, \tag{53}$$

$$\hat{\psi}_t \equiv (1-\theta\beta\bar{\pi}^{\varepsilon})\left(\hat{Y}_t - \hat{A}_t + \varsigma_t\right) + (\theta\beta\bar{\pi}^{\varepsilon}) E_t\left(\hat{\psi}_{t+1} + \varepsilon\hat{\pi}_{t+1}\right), \tag{54}$$

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - (\hat{\imath}_t - E_t\hat{\pi}_{t+1}), \tag{55}$$

$$\hat{\imath}_t = \phi_\pi\hat{\pi}_t + \phi_y\hat{Y}_t + v_t. \tag{56}$$

The model is completely forward-looking, as in the case of zero steady-state inflation, because the assumption of indivisible labor makes price dispersion irrelevant for the dynamics of output and inflation (but not for the dynamics of hours), as is evident from (43) and (44). Moreover, given that the shocks are i.i.d., there are no transitional dynamics, and the economy returns to the steady state in the period after the shock. The impact of the different shocks is derived as:[64]

$$\hat{\pi}_t = \frac{\lambda(\bar{\pi})}{1+\lambda(\bar{\pi})\frac{\phi_\pi}{1+\phi_y}}\left(\varsigma_t - \hat{A}_t - \frac{1}{1+\phi_y}v_t\right), \tag{57}$$

$$\hat{y}_t = \frac{1}{\frac{1+\phi_y}{\phi_\pi} + \lambda(\bar{\pi})}\left(\lambda(\bar{\pi})\left(\hat{A}_t - \varsigma_t\right) - \frac{1}{\phi_\pi}v_t\right). \tag{58}$$

As in the standard *NK* model, a positive technology (labor supply) shock reduces (increases) inflation and increases (lowers) output, while a positive monetary policy shock contracts both output and inflation.[65] However, an increase in $\bar{\pi}$ reduces the impact effect of a technology (labor supply) shock on output and inflation because it flattens the Phillips curve by reducing $\lambda(\bar{\pi})$.[66] With higher trend inflation the impact of a monetary shock on output is larger, and the effect on inflation is smaller. Thus, the effect of trend inflation on the volatility of aggregate variables depends on the type of shock.

If we remove the assumption of $\varphi = 0$, which was used to obtain the analytical derivations above, the dynamic response to shocks is affected also by the dynamics of price dispersion. As we discussed, when trend inflation increases, deviations of price dispersion from the steady state are more persistent. This feeds back into marginal costs - and hence into the *NKPC* - so that trend inflation tends to increase the persistence of macroeconomic variables, particularly of inflation. This in turn also affects the unconditional variance of most economic variables. We display these effects in the following figures.

Technology Shock. Figure 13 shows the impulse response functions of output, inflation, and price dispersion to a positive 1 percent technology shock for three values

[64] Under our simplifying assumptions, the labor supply shock is equivalent to a negative technology shock. We will thus comment only on the effects of technology shocks, with the opposite effects being valid for labor supply shocks.

[65] Moreover, the higher the $\phi_\pi(\phi_y)$, the lower (higher) the response of inflation to shocks and the higher (lower) the response of output.

[66] Very often in the literature a cost-push shock is simply added to the *NKPC* equation. If we had added a shock to (53), then the output and inflation impact response to such a shock would have increased with trend inflation. This explain the result in Ascari and Ropele (2007) that the policy frontier, i.e., the locus of feasible combinations of output and inflation variances attainable by monetary policy, shifts outward with trend inflation (see proposition 3 therein).

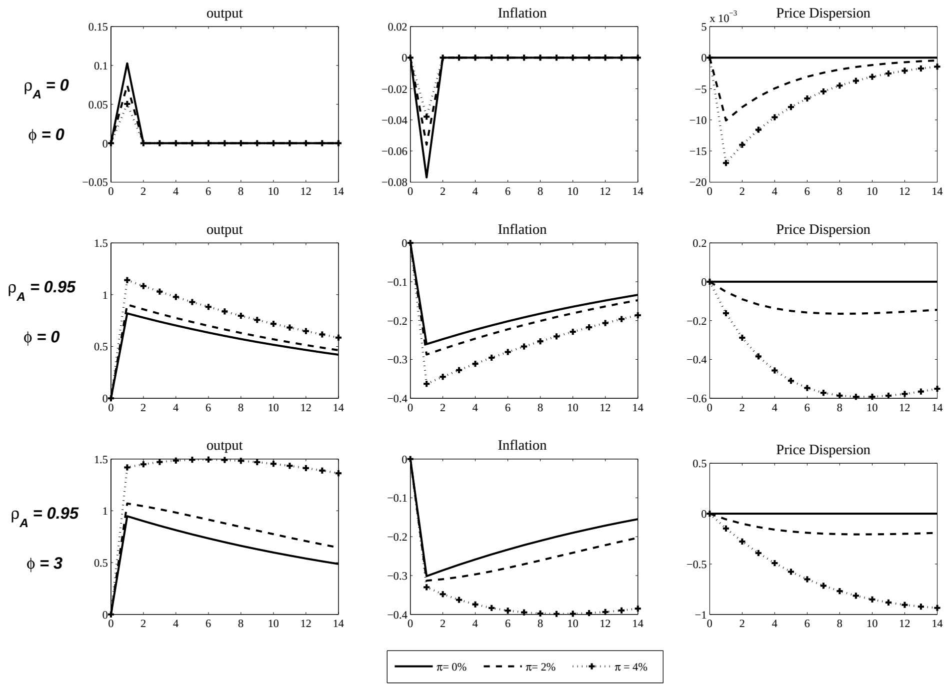


Figure 13: Impulse response functions to a 1% positive technological shock

of trend inflation: 0, 2, and 4 percent. Each row of the figure corresponds to a different parametrization of the shock persistence, ρ_A, and the Frish elasticity, φ.[67] The first row assumes the same parameters as in our analytical investigation; the second line assumes $\rho_A = 0.95$ and the third sets $\rho_A = 0.95$ and $\varphi = 3$. The first line displays the analytical result we discussed above: trend inflation dampens the impact of a technology shock on output and inflation and amplifies that on price dispersion. Output and inflation return immediately to the steady state after the impact because they do not inherit any persistence from the shock (since $\rho_A = 0$) or from price dispersion (because s does not enter the GNKPC when $\varphi = 0$). Price dispersion does not move much because the change in inflation is small and short-lived.[68] However, when the technology shock displays high persistence, as is often assumed in quantitative studies, trend inflation *amplifies* the impact of a technology shock on output and inflation because price-setting firms are in this case more forward-looking. The price dispersion response is also quite sizable and persistent in this case, as expected from (45). Finally, when $\varphi > 0$ (bottom row of Figure 13), price dispersion increases the persistence of output and inflation because in this case there is mutual feedback between inflation and price dispersion, whose strength is governed by the parameter φ.

Monetary Policy Shock. Figure 14 shows the impulse responses of output, in-

[67] Calibration of the Taylor rule parameters is standard: $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$.

[68] As previously explained, price dispersion does not move in the case of zero steady-state inflation because, in that case, it does not matter to a first- order.

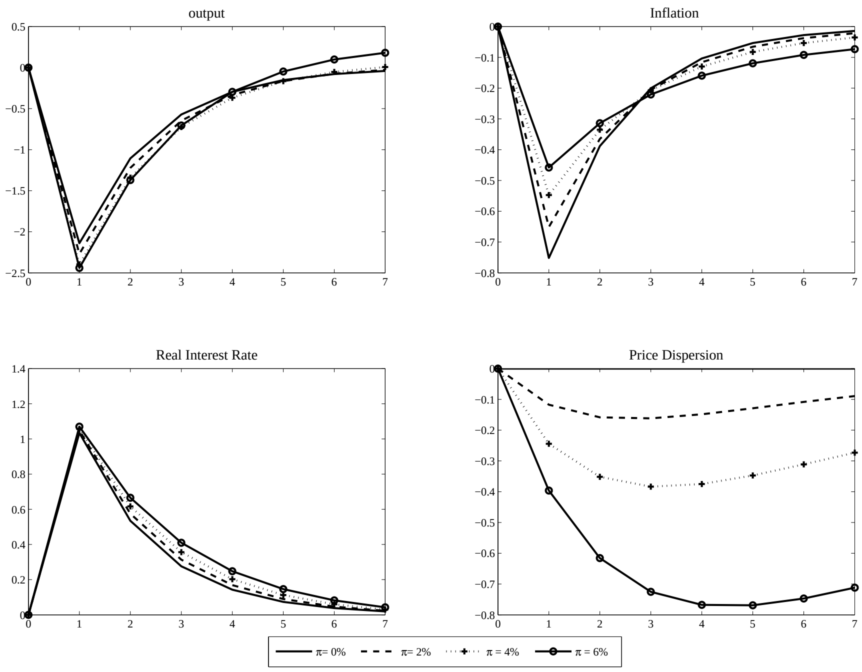


Figure 14: Impulse response functions to a 1% positive monetary policy shock

flation, and price dispersion to a 1 percent shock to the Taylor rule.[69] At higher rates of trend inflation, price resetting firms react less to the current drop in output, so that inflation reacts less, and the interest rate increase induces a larger reaction of output according to the Euler equation. Trend inflation matters for these impulse responses through the GNKPC slope $\lambda\left(\bar{\pi}\right)$: by reducing this slope, higher trend inflation reduces the impact of monetary policy shocks on inflation.[70] Note also that, through price dispersion, higher trend inflation increases inflation persistence.

The main message from these simulations is that trend inflation tends to increase the persistence of macroeconomic variables. Its effect on volatility, however, depends on the type of shocks, the variable, and the calibration. In particular, when supply shocks are very persistent, as is commonly calibrated in the literature, trend inflation strongly increases the volatility of macroeconomic variables. For example, in our benchmark model, an increase in the inflation target from 0 to 4 percent more than doubles the variance of output (from 1.3 to 2.85) and that of inflation (from 0.41 to 0.85), assuming

[69] Here we set $\phi_\pi = 2$ (rather than 1.5, with the other parameters as in the benchmark calibration of footnote 36) to allow determinacy for the 6% trend inflation case. The change in parameters has no effect on the shape of the impulse responses. For realism, we also add inertia to the Taylor rule, which is now specified as:

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + (1-\rho_i)\left(\phi_\pi \hat{\pi}_t + \phi_Y \hat{Y}_t\right) + v_t, \tag{59}$$

with $\rho_i = 0.8$.

[70] Ascari and Ropele (2007) show that this result is valid also for shocks to the rate of growth of the money supply when monetary policy is described by an exogenous autoregressive process for money growth.

only technology shocks with persistence and variance equal to 0.95 and 0.45, respectively (as in Smets and Wouters, 2007).

These results are in line with those of Amano et al. (2007), one of the first papers in the literature to analyze the macroeconomic effects of trend inflation. Amano et al. (2007) also uncover the effects of trend inflation on the stochastic means of variables by simulating the model dynamics using a second-order approximation. They show that the spread between the variables' deterministic steady states and their stochastic means increases with trend inflation. With zero trend inflation, the stochastic mean of inflation is slightly higher than the deterministic trend inflation rate, while the stochastic means of output, consumption, and employment are slightly lower than their steady-state counterparts. With positive trend inflation, these spreads increase dramatically. The effect is particularly large for the stochastic mean of inflation which, in the case of a 4 percent inflation target, is very large – 7.8 percent. The mechanism again works through price dispersion, whose effects are nonlinear: persistent increases in the deviations of price dispersion from the steady state have a greater impact on the model's endogenous variables than do decreases, leading to a spread between the deterministic steady state and the stochastic mean. This result has an important policy implication: the higher the inflation target, the more often will inflation exceed its target (unless the monetary authority is willing to counteract these effects by forcing output, on average, to fall short of its deterministic steady-state value).

In sum, trend inflation affects the dynamics of the model in a significant way. By changing the trade-off faced by monetary policy, it also affects the optimal monetary policy response to shocks, to which we turn next.

3.4 Trend inflation and optimal stabilization policy

To evaluate the effects of trend inflation on stabilization policy we revisit, in the context of our GNK model, the well-known Clarida et al. (2000) analysis of optimal stabilization policy under discretion and commitment. Ascari and Ropele (2007) conduct such an exercise in a model like our baseline, adding a cost-push shock to the GNKPC (53*)* as in Clarida et al. (2000) and assuming an ad hoc quadratic loss function of the form:

$$\mathcal{W} = \frac{1}{2} E_t \sum_{j=0}^{\infty} \beta^j \left(\hat{\pi}_{t+j}^2 + \chi \hat{Y}_{t+j}^2 \right), \tag{60}$$

where χ represents the relative weight between output and inflation stabilization around the target.

Optimal policy under discretion has the same prescription in our GNK model that Clarida et al. (2000) obtained for the standard model approximated around a zero inflation steady state: optimal policy has to "lean against the wind", distributing the effects of the cost-push shock between output and inflation according to:[71]

$$\hat{Y}_t = -\frac{\lambda(\bar{\pi})}{\chi} \hat{\pi}_t. \tag{61}$$

However, since the slope of the Phillips curve $\lambda(\bar{\pi})$ is decreasing in $\bar{\pi}$, inflation is less sensitive to a change in output engineered by monetary policy. As a consequence,

[71] Ascari and Ropele (2009) assume $\sigma = 0$ to derive condition (61).

the optimal policy response to a cost-push shock depends on trend inflation: as trend inflation rises, the shock is passed more into inflation and less into output because higher trend inflation lowers the gain in terms of reduced inflation for each unit of output loss. This result carries two implications. First, the optimal policy is at first more aggressive as $\bar{\pi}$ increases but becomes less aggressive for further increases in $\bar{\pi}$. The reason for this non-monotone response is that the interest rate is a weaker policy instrument in the presence of trend inflation: at some point the output costs of controlling inflation become too high, so the optimal response is increasingly cautious. Second, the variability of inflation relative to output increases with trend inflation.

Turning to optimal policy under commitment, a positive inflation target strengthens the policy features of the optimal commitment in Clarida et al. (2000), such as a lower impact on endogenous variables and greater persistence in impulse responses. Inflation dynamics is in fact more forward-looking with positive trend inflation, so that, according to the model, there are greater incentives for the monetary authority to use forward guidance to manage expectations. The nominal interest rate also reacts less aggressively under commitment because monetary policy becomes less effective as trend inflation increases. Hence, in this case the relative volatility of inflation and output also increases. Finally, the gains from commitment are initially increasing with positive trend inflation because trend inflation increases the importance of managing future expectations; after a certain trend inflation rate, however, such gains diminish because higher trend inflation reduces the effectiveness of monetary policy. At some point, inflation becomes so costly to control that policymakers find it optimal, both under commitment and discretion, to keep the output gap almost constant rather than trying to stabilize inflation; hence, commitment does not produce significant gains.

In the standard optimal policy literature, the NKPC is usually expressed in terms of an output gap, $\tilde{Y}_t$, defined as the log-deviation of current output from the output that would arise in a flexible-price environment, namely the natural level of output Y_t^n (Woodford, 2003; and Galí, 2008): $\tilde{Y}_t = \log Y_t - \log Y_t^n$. $\tilde{Y}_t$ is a measure of the nominal distortion implied by sticky prices. In our benchmark model under zero steady state inflation, the output gap can be written as $\tilde{Y}_t = (\log Y_t - \log \bar{Y}) - (\log Y_t^n - \log \bar{Y}) = \hat{Y}_t - \frac{\varphi+1}{\sigma+\varphi}\hat{A}_t + \frac{1}{\sigma+\varphi}\varsigma_t$, where $\bar{Y}$ is the flexible-price steady-state output. Thus, the Phillips curve (43) can be written in the familiar form:

$$\hat{\pi}_t = \lambda \tilde{Y}_t + \beta E_t \hat{\pi}_{t+1}. \tag{62}$$

An optimal policy that closes the output gap also stabilizes inflation: this is what Blanchard and Galí (2007) called the "divine coincidence" in the NK model.

To write an expression analogous to (62) under trend inflation, we introduce the following decomposition:

$$\begin{aligned} \tilde{Y}_t &= (\log Y_t - \log \bar{Y}(\bar{\pi})) - (\log Y_t^n - \log \bar{Y}) + (\log \bar{Y}(\bar{\pi}) - \log \bar{Y}) \\ &= \hat{Y}_t - \frac{\varphi+1}{\sigma+\varphi}\hat{A}_t + \frac{1}{\sigma+\varphi}\varsigma_t + \tilde{Y} \end{aligned} \tag{63}$$

where we defined $\bar{Y}(\bar{\pi})$ as the steady-state output for a given trend inflation. The extra term, $\tilde{Y}$, arises from the fact that the model is log-linearized around a positive $\bar{\pi}$. The term $\tilde{Y} = \log \bar{Y}(\bar{\pi}) - \log \bar{Y}$ is the deviation of the level of output associated with steady-state inflation $\bar{\pi}$ from the level of long-run output under flexible prices (or when steady-state inflation is zero). As such, $\tilde{Y}$ represents a *long-run output gap* (recall Figure 9). Using (63), we can write the GNKPC (43) and (44) as:

$$\begin{aligned}\hat{\pi}_t &= \lambda(\bar{\pi})\left(\tilde{Y}_t - \tilde{Y}\right) + \kappa(\bar{\pi})\,\varphi\hat{s}_t + b_1(\bar{\pi})\,E_t\hat{\pi}_{t+1} \\ &+ b_2(\bar{\pi})\left(\left(\tilde{Y}_t - \tilde{Y} + \frac{\varphi+1}{\sigma+\varphi}\hat{A}_t - \frac{1}{\sigma+\varphi}\varsigma_t\right)(1-\sigma) - E_t\hat{\psi}_{t+1}\right),\end{aligned} \tag{64}$$

$$\hat{\psi}_t \equiv (1-\theta\beta\bar{\pi}^{\varepsilon})\left[\varphi\hat{s}_t + (1+\varphi)\left(\tilde{Y}_t - \tilde{Y} + \frac{1-\sigma}{\sigma+\varphi}\hat{A}_t\right) - \frac{1-\sigma}{\varphi+\sigma}\varsigma_t\right] + \theta\beta\bar{\pi}^{\varepsilon}E_t\left(\hat{\psi}_{t+1} + \varepsilon\hat{\pi}_{t+1}\right). \tag{65}$$

It is now evident that closing the output gap is not sufficient to stabilize inflation, so divine coincidence does not hold when trend inflation is positive.[72]

Using this output gap definition, Coibion et al. (2012) and Lago-Alves (2012) derive a second-order approximation to the utility function of the representative agent under trend inflation, following the approach in Woodford (2003).[73]

Woodford (2003) shows that in the zero inflation steady-state approximation the steady-state inefficiency Φ_y is simply due to the monopolistic distortion: $\Phi_y = 1 - \frac{1}{\overline{\mu}} = \frac{1}{\varepsilon}$. The second-order approximation of the utility function around the efficient level of output (Rotemberg and Woodford, 1999) is then derived under the assumption that the inefficiency Φ_y is "small enough".[74] When trend inflation is positive, however, Lago-Alves (2012) shows that the monopolistic distortion is potentially larger (see (37) and (30)), since the steady state markup is now :

$$\overline{\mu} = \frac{\varepsilon}{\varepsilon-1}\frac{1-\beta\theta\bar{\pi}^{\varepsilon-1}}{1-\beta\theta\bar{\pi}^{\varepsilon}}\left(\frac{1-\theta\bar{\pi}^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{\varepsilon-1}}. \tag{66}$$

Lago-Alves (2012) also illustrates two results from the analysis of the welfare function. First, the non-linear welfare function decreases rapidly and becomes increasingly concave as the inflation rate rises. Hence the microfounded loss function derived under the zero inflation steady-state assumption can be a poor approximation to the true welfare function when average inflation is positive. At 2 percent inflation, for example, the curvature of the true loss function is two and a half times as large as the curvature at the steady state with zero inflation; at 4 percent inflation it is 10 times as large. An approximation around positive trend inflation would be more precise. The second result pertains to the relative weight on output in the welfare function, which is now:[75]

$$\chi(\bar{\pi}) = \frac{1-\theta\bar{\pi}^{\varepsilon-1}}{1-\theta\bar{\pi}^{\varepsilon}}\frac{\lambda(\bar{\pi})}{\varepsilon} \tag{67}$$

rather than $\chi = \frac{\lambda}{\varepsilon}$, as in the case of $\bar{\pi} = 1$. The weight $\chi(\bar{\pi})$ is rapidly decreasing with trend inflation: with $\bar{\pi}$ at 3 percent, for example, $\chi(\bar{\pi})$ is about 75 percent of

[72] In other words, the shocks in our benchmark GNK model do generate policy trade-offs under positive trend inflation. See Lago-Alves (2012).

[73] This provides an extension of the Ascari and Ropele (2007) analysis to the case of a microfounded loss function. This extension is important because the parameters of the microfounded loss function provide another channel for the effects of trend inflation.

[74] Alternatively, one may assume another policy tool (e.g., a subsidy) to take care of this inefficiency.

[75] See Proposition 2 in Lago-Alves (2012) and the subsequent discussion therein.

the corresponding value under zero steady state inflation. Thus, optimal policy has a lower weight on output gap volatility (or a higher weight on inflation volatility) when the target level for inflation is higher. This is very intuitive. Inflation fluctuations are costlier when trend inflation is higher because they have a larger and more prolonged impact on price dispersion; hence the weight on inflation stabilization should be higher.

Finally, Lago-Alves shows that the features of optimal policy under a positive inflation target, which we discussed for the case of an ad hoc loss function, also hold when the optimal weight χ depends on trend inflation. The welfare effects of trend inflation are obviously critical to the issue of the optimal inflation target, which we discuss in the next section.

Our discussion of stabilization policy has assumed away an important issue: model uncertainty. We have illustrated how policy prescriptions change under positive trend inflation. But how is the policymaker's choice affected by uncertainty regarding the true model of the economy, and especially for our discussion, uncertainty about inflation persistence? As we discussed in Section 2, an empirical specification of the Phillips curve that accounts for drifts in trend inflation tends to attribute the larger part of inflation persistence to the persistence of its underlying trend. Under this interpretation, inflation persistence is not an intrinsic feature of the economy but might instead result from the behavior of monetary policy itself; hence the New Keynesian Phillips Curve should have a purely forward-looking specification. Which specification one chooses affects the optimal policy prescriptions because those are sensitive to the degree of intrinsic inflation persistence.

Sbordone (2007) illustrates this point for the case of optimal response to cost-push shocks. She considers a small New Keynesian model similar to the model developed here, but she allows for two specifications of the Phillips curve. In one specification the curve has a backward-looking component, as in Galí and Gertler (1999), while in the other it is purely forward-looking, and the inflation persistence is picked up by the time-varying inflation trend (the specification is similar to that of Cogley and Sbordone, 2008). She then evaluates the consequences for stabilization policy of assuming a wrong degree of intrinsic inflation persistence in the model. Her simulations suggest that the cost of implementing a stabilization policy that overestimates the degree of intrinsic inflation persistence is generally higher than the cost of responding to shocks without regard to possible structural persistence. The results in Sbordone (2007) sharpen as the inflation stabilization weight increases in the policymaker's loss function; this is the case in her model when the loss function is welfare-based.

Uncertainty about the form of the Phillips curve is one of many types of model uncertainty that policymakers must confront when designing optimal monetary policy. In a Bayesian approach for dealing with uncertainty, policymakers form a prior over models and choose policy to minimize expected loss given that prior (see for example Soderstrom, 2002). Svensson and Williams (2007, 2008) have pushed the frontier by adapting and extending Markov jump-linear-quadratic control algorithms for the study of monetary policy. They approximate policymakers' uncertainty using different discrete modes in a Markov chain and take mode-dependent linear-quadratic approximations of the underlying model.[76] In contrast to the Bayesian approach, robust control seeks to

[76] Applying this methodology to the study of the effect of uncertainty about the degree to which the New Keynesian Phillips Curve is forward-looking, Svensson and WIlliams (2008) show that the optimal

minimize the worst case loss over the set of possible models. This approach presumes that decisionmakers are either unable or unwilling to form a prior over the forms of model misspecification and instead just bound the set of possibilities rather than fully specifying each alternative (see Hansen and Sargent, 2007).

The literature on optimal policy under uncertainty addresses many kinds of uncertainty and is now quite vaste; further discussion goes beyond the scope of this paper.

3.5 The optimal trend inflation rate

In our baseline model, as in New Keynesian models more generally, trend inflation is under the control of the central bank so long as its policy is credible. This assumption is not unrealistic. The experience of inflation-targeting central banks, for example, is evidence that, with appropriate monetary policy, trend inflation can be brought to a desired level. Similarly, in Europe, the reduction and convergence of inflation rates in EMU countries leading up to the 1999 monetary union is a real-world example of controllability of trend inflation.[77] Even in the case of liquidity traps, monetary policy tools have been deployed to maintain reasonable control of trend inflation. Controllability of trend inflation doesn't imply that one should not observe time variations in trend inflation of the kind we analyzed earlier. Such variations may arise from adjustments in the policymaker's target, from the public's imperfect information about the target, or from persistent shocks. On the other hand, controllability raises the question of what long-run rate of inflation a policymaker should choose.

We have shown so far that incorporating a positive long-run rate of inflation in the benchmark NK model tends to unanchor inflation expectations, enhance macroeconomic volatility, and lower welfare. These results are consistent with the vast literature on the optimal long-run rate of inflation, recently covered in an extensive survey by Schmitt-Grohè and Uribe (2011). In most New Keynesian models the optimal long-run rate of inflation is indeed zero (Goodfriend and King, 1998; Yun, 2005; Benigno and Woodford, 2005; Khan et al., 2003; and Amano et al., 2009), a conclusion shown to be quite robust in cashless models with staggered price setting.[78] Moreover, Schmitt-Grohè and Uribe (2011) showed that the Ramsey optimal policy calls for price stability (i.e., zero inflation) at all times in such a model. For plausible calibrations, the inflation rate is always very close to zero, even in the presence of uncertainty. This result is perhaps not surprising, given the emphasis we have put on the cost of price dispersion. Nor it is surprising that the optimality of zero inflation disappears only in the extreme and unrealistic case of full indexation. When prices are fully indexed, there is no price dispersion in the steady state, so the optimal steady-state rate in this case is equal to the initial one (Schmitt-Grohè and Uribe, 2011).

policy is more aggressive under uncertainty in responding to inflation movements than is optimal policy in the absence of uncertainty.

[77] Among other criteria, the Maastricht treaty established that, in order to join the European monetary union, a country was to have inflation no more than 1.5 percentage points above that of the 3 lowest inflation rates among the EMU members.

[78] If there is a demand for money, the optimal inflation rate can be negative, since it will counteract the opportunity cost of holding money attributable to a positive nominal interest rate and the cost of price dispersion attributable to a positive inflation rate. Since we consider only a cashless framework, we do not address the money demand case, which is comprehensively treated in Schmitt-Grohé and Uribe (2011).

The pivotal role of price dispersion in this class of models is even more evident by considering how the optimal policy is affected by a positive inherited level of price dispersion. In this case the optimality of zero inflation at all times is no longer valid. Although the optimal policy does converge to the steady state characterized by zero inflation, the transitional dynamics call for a quicker reduction of the inherited level of price dispersion than under a zero inflation policy. This result is easy to show in our framework, following Yun (2005). The Ramsey problem is to maximize the utility function of the representative consumer under just the resource constraint and with the evolution of price dispersion as described by (34).[79] The optimal policy should satisfy the following two conditions (see the online appendix):

$$\pi_t = \frac{s_t}{s_{t-1}} \tag{68}$$

and

$$s_t = s_{t-1} \left[\theta + (1 - \theta) \, s_{t-1}^{\varepsilon - 1} \right]^{\frac{1}{1-\varepsilon}} . \tag{69}$$

From these, it immediately follows that: (i) the optimal policy implies zero inflation in the steady state ($\bar{\pi} = 1$); (ii) if there is no inherited price dispersion ($s_{t-1} = 1$), then a policy setting $\pi_t = 1$ for all t is optimal; (iii) however, if there is inherited persistence ($s_{t-1} > 1$), then optimal policy would prescribe a gradual reduction in price dispersion to satisfy (69) toward a steady state with no price dispersion.[80] In the latter case, then, the inflation rate is negative ($\pi_t < 1$) along the adjustment path to exactly engineer a reduction in price dispersion faster than the reduction implied by a zero inflation policy. Furthermore, along the optimal path, as long as there is price dispersion, the optimal reset price is below the average price level:

$$p_{i,t}^* = \frac{1}{s_t}. \tag{70}$$

To sum up, the GNK model would predict an optimal inflation rate either equal to zero (assuming no money demand distortions) or negative during the transition whenever the initial level of price dispersion is non-zero, since the goal of policy is to minimize the distortions due to price dispersion. These results show once again how the costs of inflation arise naturally from the key feature of price dispersion in an NK model with trend inflation.

3.6 Trend inflation, alternative models of price setting and generalization of the basic *NK* model

In this section we discuss some generalizations of the framework we have used throughout this paper. We consider alternative price-setting models as well as other features within the Calvo price-setting model.[81]

[79] Using (30) to substitute for $p_{i,t}^*$ in (34).

[80] Since $\varepsilon > 1$, if $s_{t-1} > 1$, then (69) implies $s_t < s_{t-1}$.

[81] We do not discuss models with growth. However, Amano et al. (2009), Vaona and Snower (2008); and Vaona (2012) show that growth generally strengthens the effects of trend inflation in an environment with nominal rigidities.

3.6.1 Microeconomic features

Firm-specific inputs. When factors of production are firm specific, the effects of trend inflation described in the previous sections are amplified. In our baseline model, we assumed a common-factor market, which implies strategic substitutability in pricing decisions; in contrast, specific-factor markets (either in capital or in labor) generate strategic complementarity in price setting. Bakhshi et al. (2007) show that firm-specific factors increase the long-run negative effects of trend inflation and the elasticity of the slope of the Phillips curve with respect to trend inflation. The presence of firm-specific factors also strengthens the effects of trend inflation on the determinacy and E-learnability regions we analyzed in Section 3.2 (Hornstein and Wolman, 2005 and Kurozumi and Van Zandweghe, 2012).

The effect of differences in microeconomic features across models is often muted when the models are approximated around a steady state with zero inflation. This is what Levin et al. (2008) call *macroeconomic equivalence* and *microeconomic dissonance*: when two different models are log-linearized around the flexible-price equilibrium, their different microfoundations may become irrelevant. They show this phenomenon by considering two types of strategic complementarity (or real rigidity): one generated by a kinked demand curve, the other by the presence of firm-specific inputs. Up to a first-order approximation, these two models yield the same standard NKPC, which is flatter than the standard case because of the real rigidities. This is, however, an effect of their approximation: if they had taken the approximation around a generic trend inflation level (or even better, worked with the non-linear model) the model differences would have been preserved.[82]

In Levin et al. (2008), the differening microfoundations of the two models carry implications only for welfare, since welfare is approximated to second order (around the efficient allocation). Price dispersion is not very costly in the kinked demand model, where relative-demand reactions (and thus equilibrium allocations of resources) have a low price sensitivity, while it is very costly in the model with firm-specific factors. This in turn leads to very different policy prescriptions, since the two models imply different optimal weights on output and inflation volatility in the welfare function.

Finally, contrary to firm-specific factors, a kinked demand curve implies a lower long-run inflation elasticity of output, which weakens the effects of trend inflation on the determinacy and E-learnability regions (see Kurozumi and Van Zandweghe, 2013).

Discounting. We saw above that another microeconomic feature that interacts with trend inflation is discounting. Graham and Snower (2008, 2013 demonstrate that hyperbolic discounting could generate a positive relationship between inflation and output in steady state, so that the optimal rate of inflation could be positive in this case. Hyperbolic discounting strengthens the discounting effect described above, which eventually dominates the relative price and price dispersion effect.[83]

Indexation. In the NK models literature, non-optimized prices are often assumed to be somewhat indexed to steady-state inflation (Yun, 1996), or to past inflation

[82] Taking second-order approximations around the zero inflation steady state will generally reveal the difference in microfoundations, but this would matter only at second order.

[83] Graham and Snower use a model with overlapping contracts à la Taylor. We conjecture that this result will be harder to get in a Calvo setup, where the price dispersion effect is stronger.

rates (Christiano et al., 2005), or to both (Smets and Wouters, 2007). The purpose of backward-looking indexation is mainly to capture persistence in inflation data, as discussed in Section 2. Indexation counteracts the effects of trend inflation (Ascari, 2004): the theoretical results discussed in previous sections are mitigated when assuming partial indexation of prices that have not been reset, and they are actually muted if indexation is full. In fact, under full indexation, the steady state of the model is exactly that of flexible prices; there is no price dispersion, and all the firms charge the same price. The assumption of indexation, however, does not seem justified on either theoretical or empirical grounds. Theoretically, indexation is not typically derived from the optimal behavior of price setters, and empirically, analyses based on price microdata show that prices do not change all the time, as automatic indexation would imply. Furthermore, microdata on price changes do not exhibit a spike at the previous period or trend inflation level, as indexation to previous-period inflation or to trend inflation would imply. Finally, as we discussed in Section 2.2.3, indexation doesn't appear to be needed to fit the NKPC, once trend inflation is taken into account.

Capital. Little work has been conducted on the effects of trend inflation in models with capital. Capital accumulation is typically included in medium-scale NK models, together with other features that allow one to better capture specific aspects of the data. Some features are introduced to generate more inertia in the model and are likely to have an impact on, for example, the determinacy region (see Sveen and Weinke, 2013). Gerko and Sossounov (2011) investigate the introduction of capital accumulation in the GNKPC model of Ascari and Ropele (2009) and confirm that positive trend inflation substantially shrinks the determinacy region. In contrast, Arias (2013) and Ascari et al. (2012) find that real frictions that smooth aggregate demand make the determinacy region less sensitive to variations in trend inflation. This suggests that the results on determinacy are sensitive to the particular features of the models and the specification of the policy rule. More work is needed on the interaction of other features of medium-scale models with trend inflation and how it affects the properties of the baseline model that we have discussed; and more work is needed to assess the quantitative implications of trend inflation in empirically realistic models.

3.6.2 Other time-dependent pricing models

The New Keynesian literature often employs two other time-dependent pricing models: the Taylor (1979, 1980) overlapping contracts model and the Rotemberg (1982) cost-of-adjustment model.

The effects of trend inflation we discussed for the Calvo model still hold qualitatively for the Taylor model, but they are quantitatively far less important because the Taylor model has only limited price dispersion. Two features in the Taylor model limit its price dispersion: (i) the price resetting firms in each period are those that reset their prices in the more distant past; (ii) price resetting occurs a fixed number of times (typically four, so there are only four prices in each period). Ascari (2004) shows that the steady state and the dynamics of the Taylor model are much less sensitive to changes in trend inflation than those of the Calvo model.[84] In line with Ascari and Ropele's (2009) results, Kiley

[84] See also Ascari (2000) and Graham and Snower (2004), which analyze the effects of trend inflation in a wage-staggering model à la Taylor.

(2007) shows that, in a Taylor model with an interest rate rule, trend inflation makes the determinacy of the rational expectation equilibrium more difficult to achieve, and it increases inflation volatility following cost-push shocks. Finally, Amano et al. (2007) present simulations both for the Taylor model and for a truncated Calvo model[85] and show that in both models the effects of trend inflation on the dynamics and properties of the model are dampened. These results reinforce the general message of the present paper that trend inflation matters because it increases price dispersion: the degree to which trend inflation increases price dispersion, though, depends on the price-staggering model considered.

The same intuition reveals why the Rotemberg price-setting model implies qualitatively different results from the Calvo model: in the Rotemberg model, which uses a quadratic cost of price adjustment, price dispersion is always zero because all firms set the same price. The two models are indeed an example of *macroeconomic equivalence* and *microeconomic dissonance* in the NK literature: to a first-order approximation around a zero inflation steady state, they have the same reduced-form dynamics (Rotemberg, 1987; Roberts, 1995) and equivalent welfare implications (Nisticò, 2007). As a consequence, these two models are treated in the literature as equivalent from a macroeconomic perspective, despite their differing microfoundations and welfare implications. Under positive trend inflation, the two models differ because there is no price dispersion in the Rotemberg model, hence none of the effects of relative prices on price-setting that we described in Section 3.1.1 are present.

The discounting effect remains, however. In changing their prices, firms weight the cost of today's adjustment more heavily than the cost of tomorrow's adjustment. It follows that in the Rotemberg model the steady-state average markup decreases, and the steady-state output increases, with trend inflation.[86] The Calvo model implies a wedge between aggregate output and aggregate employment through price dispersion (see (33)); but in the Rotemberg model the cost of nominal rigidities is the quadratic adjustment cost that enters the resource constraint, thus creating a wedge between consumption and output. Ascari and Rossi (2009) show in particular that in the Rotemberg model, unlike in Calvo's, (i) the long-run NKPC is positively sloped; (ii) the log-linear GNK model is qualitatively very different, implying different dynamics; and (iii) when monetary policy follows a Taylor rule, positive trend inflation enlarges the determinacy region and dampens the dynamic response of output and inflation to a persistent technology shock. Using U.S. data, Ascari et al. (2011) compared the empirical fit of the log-linearized Calvo and Rotemberg models under trend inflation; the results support the view that trend inflation is an empirically relevant feature and point in favor of the Calvo model.

3.6.3 Endogenous frequency of price adjustment

Finally, throughout our analysis, we treated the frequency of price adjustment (the Calvo parameter) as constant and independent of trend inflation. However, with higher inflation, price resetting may occur more often. In the standard NK model, higher price rigidity implies higher price dispersion for any given level of trend inflation. As a

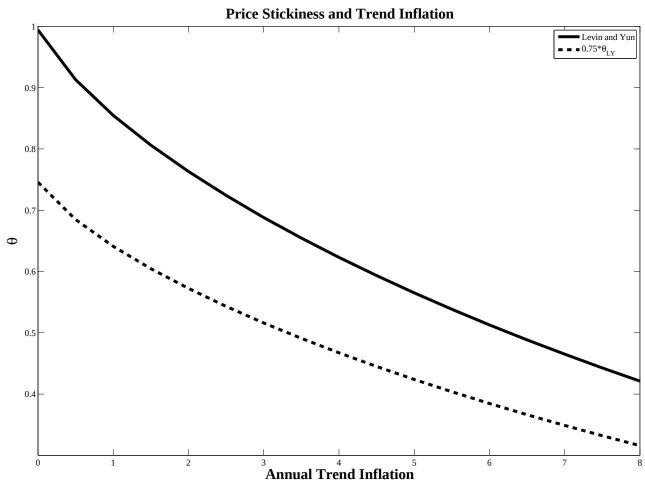
[85] This is a hybrid version of the Calvo model, which borrows from the Taylor model the idea that prices are fixed for a limited period, after which they are always readjusted.

[86] However, consumption and welfare decrease with trend inflation, as in the Calvo model, because as trend inflation rises, firms have to pay a larger adjustment cost.

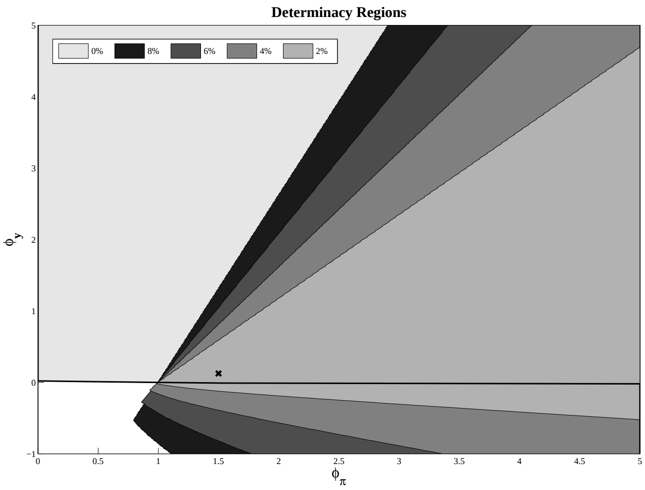
consequence, more flexible prices (a lower Calvo parameter) would dampen the effects of trend inflation.

Bakhshi et al. (2007) obtain the GNKPC in the state-dependent model of Dotsey et al. (1999). In this model the coefficients of the GNKPC (which in this case exhibits more inflation inertia) are still a function of steady-state inflation; but they are also a function of the steady-state distribution of price vintages and the number of price vintages, which evolve endogenously. When trend inflation rises, firms adjust their prices more frequently; thus, the number of price vintages declines while the number of firms in recent vintages increases, and so price dispersion decreases. In this framework (under high policy inertia), monetary policy shocks have a larger impact on inflation when trend inflation is higher; and the persistence of inflation is lower. The opposite occurs for output. These effects are, however, quantitatively small (see Bakhshi et al., 2007). Moreover, Dotsey et al. (1999) show that the relationship between inflation and the impact of money on output could be nonmonotonic. That is, with high inflation, firms know that they will be adjusting prices in the near future. Hence, they may not adjust their prices in response to a shock, which leads to a greater impact of monetary shocks on output.

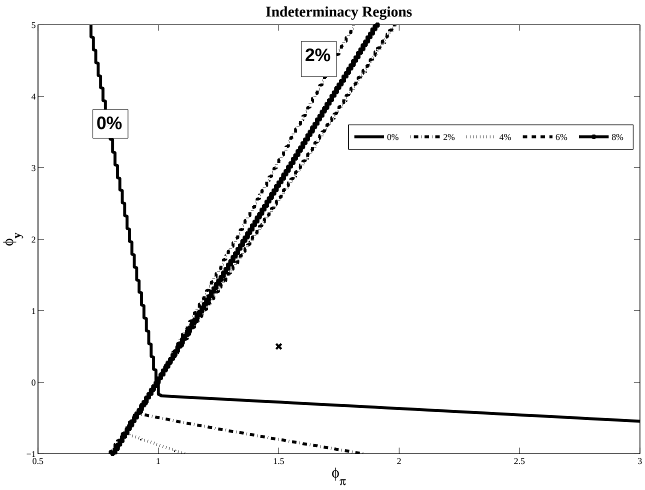
Levin and Yun (2007) analyze the steady-state properties of a model in which firms are subject to a cost of price adjustment but are allowed to choose the frequency of price changes once and for all, given a constant average steady-state inflation. The equilibrium average frequency of price adjustment is then endogenous and increases with steady-state inflation. The solid line in Figure 15a shows the steady-state relationship, given our benchmark calibration, between trend inflation and the probability of price adjustment, θ, implied by the Levin and Yun (2007) model. In this model, the steady-state relationship between output and inflation is different from that shown in Figure 9. For very high rates of inflation, firms adjust prices every period so that the equilibrium is the same as the flexible price one and the model exhibits superneutrality. For low and intermediate rates of trend inflation, the relationship is first positive and then negative as in Figure 9, but the effects are far less dramatic. Since, under endogenous price stickiness, the long-run Phillips curve is different, it follows that the term $\left.\frac{\partial \hat{Y}}{\partial \hat{\pi}}\right|_{LR}$ will be different, and this may change some of the results surveyed above. Kurozumi (2013) follows this intuition and employs the steady-state relationship defined in Levin and Yun (2007) to study how much endogenous price stickiness affects the results in Ascari and Ropele (2007, 2009). For our standard calibration, Figure 15b shows that with high trend inflation (the cases of $\overline{\pi} = 4$, 6, and 8 percent are shown in the figure), the determinacy region increases, contrary to what we showed in Figure 11. However, when trend inflation rises from 0 to 2 percent, the determinacy region shrinks, as we showed in Figure 11; and, for moderate values of trend inflation, it is still true that the slope of the generalized Taylor principle (49) switches sign.


Price Stickiness and Trend Inflation
Levin and Yun
0.75*θ_LY
θ
Annual Trend Inflation

(a)


Determinacy Regions
0%
8%
6%
4%
2%
ϕ_y
ϕ_π

(b)


Indeterminacy Regions
2%
0%
0%
2%
4%
6%
8%
ϕ_y
ϕ_π

(c)

The Levin and Yun (2007) results have the unpalatable implication that, at zero inflation, firms would choose to fix their prices once for all, so $\theta = 1$; in this case, the model is determinate as long as $\phi_\pi, \phi_y > 0$. If we assume instead that $\theta = 0.75$ at zero inflation (a much more common calibration), and if we maintain the elasticity of θ with respect to trend inflation that is implied by the Levin and Yun (2007) model (i.e., θ is scaled down by 0.75), we would obtain the dashed line in Figure 15a, which would deliver Figure 15c. In that figure we recognize the standard Taylor principle at zero inflation. For a low rate of inflation, the effects described by Figure 11 are still present because the effect of the increase in trend inflation dominates the effect of the change in the probability of readjustment. However, the relative strength of these two effects changes as trend inflation increases. Considering just the positive orthant, the determinacy regions at 4 and 6 percent rates of trend inflation basically overlap. Then, for further increases in trend inflation, the determinacy region expands because the effect of the change in θ dominates that of the increase in trend inflation. The general message is that, at low inflation, the non-linearity of the model allows the effect of trend inflation to prevail. For high rate of trend inflation, the opposite is true.

The sensitivity of the frequency of price adjustment to the average inflation rate is ultimately an empirical question. But because our interest lies in the effects of low to moderate rates of trend inflation, we believe it is reasonable for us to treat the Calvo parameter as constant. The lines in Figure 15a assume a price stickiness parameter that is fairly elastic with respect to changes in trend inflation, a condition that does not seem consistent with either macro or micro estimates. Many of the macroeconomic studies that estimated various versions of the NK model in the pre- and post-Volcker periods report no significant changes in the Calvo parameter.[87] Microeconomic data also show no evidence of sizable changes in the frequency of price adjustment. Alvarez et al. (2011), for example, using a uniquely rich dataset from Argentina, conclude that the steady-state frequency of price changes is unresponsive to inflation for low inflation rates (below 10 percent).[88] In their comprehensive survey, they show that their results are similar to those obtained for other countries. But a similar result holds for price dispersion. From the perspective of a price-staggering framework useful for macroeconomic models, this may suggest that, rather than the fixed probability of changing prices, a more problematic feature of the Calvo model could be the existence of very long tails; that is, very old, and then relatively very low, prices lead to a large cost of price dispersion that is very sensitive to the degree of trend inflation.

4 Trend inflation and the zero lower bound (ZLB)

This section brings our surveyed results to bear on the debate about whether to raise the inflation target.

The debate involves two somewhat separate questions. On the one hand, does the presence of a zero lower bound alter traditional considerations regarding the optimal long-run inflation rate (see Section 3.5)? On the other hand, should alternative policies

[87] E.g., see Fernández-Villaverde and Rubio-Ramirez (2008) and Cogley, Primiceri, and Sargent (2010). In the working paper version of their 2008 AER article, Cogley and Sbordone (2005) show that the estimate of the Calvo probability on U.S. data appears quite stable in the post-World War II period.

[88] See also Gagnon (2009).

be preferred even if a higher inflation target can in principle mitigate the zero bound constraint? We discuss these questions in turn.

4.1 Is low long-run inflation optimal in a ZLB environment?

The first question is whether the central bank should adopt a higher inflation target in normal times to reduce the probability of hitting the ZLB. In the course of our survey, we discussed the effects of long-run inflation on economic stability and overall welfare. The general lesson we drew from our model is that higher trend inflation tends to decrease average output, unanchor inflation expectations, increase the volatility of the economy, worsen policy trade-offs, and reduce welfare. Hence, a higher inflation target would seem to be a bad policy prescription.

How can these results be made compatible with the fact that central banks typically target a positive level of inflation, most often around 2 percent per year? The literature has long pointed out some reasons why policymakers may want to target a higher rate of inflation. Because of measurement errors, inflation indexes likely overstate true inflation by failing to completely adjust for quality improvements. Also, nominal wages are arguably downwardly rigid; therefore, a moderate inflation can facilitate a reduction in real wages in response to negative shocks. Schmitt-Grohè and Uribe (2011) discuss both these reasons for a positive target rate of inflation – other reasons they discuss are uncertainty about parameter values and alternative price-staggering models, both time- and state-dependent. But none of these modifications, they argue, generates more than a very low optimal inflation rate, typically much lower than 2 percent.

The most prominent argument against targeting a low rate of inflation is the risk of hitting the ZLB constraint. As argued by Summers (1991), stabilizing the economy may occasionally require real negative interest rates, and nominal rates cannot fall below zero; therefore a very low average inflation rate may limit the ability of central banks to conduct effective stabilization policy. Summers (1991) and Fischer (1996) suggested that an inflation target in the 1 to 3 percent range would accommodate the need for a negative real rate of interest when a zero lower bound is binding. Krugman (1998) made a similar argument and suggested indeed that, in its then-current economic situation, Japan needed an inflation target of 4 percent for several years to generate the needed negative real rates and curb deflation.

Raising the inflation target, however, implies increasing the distortions associated with a positive rate of inflation that we discussed at length, and the distortions would remain when the natural real rate ceases to be negative. Evaluating whether a higher inflation target is appropriate would then depend upon the assessment of how likely it is for the ZLB constraint to be hit. This in turn depends on the size of the exogenous shocks that affect the economy and on the transmission mechanism of these shocks; hence, the evaluation must be conducted in the context of a theoretical model.

Schmitt-Grohè and Uribe (2011) analyze the issue in an estimated medium-scale macroeconomic model with both nominal and real frictions. Rather than directly imposing the ZLB, they evaluate its relevance by assessing how often the optimal interest rate violates the zero bound. For a reasonable calibration of the model's exogenous shocks, they find that the optimal inflation rate remains mildly negative.[89] They also

[89] The shocks driving the model are a permanent, neutral labor-augmenting technology shock; a per-

find that under optimal policy, the standard deviation of the interest rate is only 0.9 percentage points at an annual rate. With a mean Ramsey optimal nominal interest rate estimated at 4.4 percent, that implies that "for the nominal interest rate to violate the zero bound, it must fall by more than 4 standard deviations below its target level" (p. 703). They conclude that the probability that the Ramsey optimal nominal rate will hit the ZLB constraint is practically zero. One important assumption underlying these results is the value of the subjective discount rate, which is set at 3 percent per year. Combined with an average growth rate of per capita output of 1.8 percent, that implies a real rate of interest in the deterministic steady state of 4.8 percent. Reducing the discount rate increases the likelihood of hitting the ZLB constraint. However, the authors note that a discount rate of 1 percent per year will still require the nominal rate to fall by almost three standard deviations below its mean for the ZLB to bind. The conclusion that the ZLB constraint would rarely bind under an optimal monetary policy regime implies that considerations of the constraint do not essentially alter the optimal rate of inflation.[90]

Coibion et al. (2012) compute the optimal inflation rate in a setting very similar to our baseline model, in which steady-state inflation is positive and monetary policy follows a Taylor rule subject to the zero bound constraint.[91] They find that for plausible calibrations and with costly but infrequent instances of the ZLB constraint (defined as eight-quarter episodes), optimal inflation is above 0 but typically lower than 2 percent. The intuition for this result is straightforward: "the unconditional cost of the ZLB is small even though each individual ZLB event is quite costly" (p. 1373). The unconditional cost is low because, in their model, eight-quarter ZLB events are rare, occurring about once every 20 years. So while it is true that raising the inflation target could significantly reduce the costs associated with a ZLB situation, the expected long-run gain of such a policy is rather small.[92] Conversely, while the cost of higher trend inflation is much more modest, it must be paid every period. That is why the optimal inflation rate remains below 2 percent even in the presence of a ZLB concern.

The frequency and cost of ZLB episodes depend on the structure of the model and the calibration of its parameters, particularly those driving the distribution of the shocks. Results obtained by Coibion et al. (2012), for example, are sensitive to the calibration of the risk premium shock because an increase in the persistence and volatility of that shock has a large effect on the frequency and duration of being at the ZLB and hence raises the benefit of a higher steady-state inflation. In their baseline calibration, they choose a combination of parameter values that closely matches the historical incidence of the ZLB in the U.S.; they show, however, that despite "parameter values that double or even triple the frequency of hitting the ZLB at the historical average rate of inflation

manent investment-specific technology shock; and a temporary shock to government spending. The model also features money demand distortions in the form of a transaction technology.

[90] This is in line with the conclusion of Adam and Billi (2006) and Billi (2011), who explicitly impose the zero bound constraint on the interest rate, although they use a model without trend inflation.

[91] They follow the approach of Bodenstein et al. (2009) by solving for the duration of the zero bound endogenously. The duration of the zero bound episodes as well as their frequency affect the volatility of inflation and output, which are in turn important determinants of the welfare cost of inflation.

[92] They calculate that a ZLB situation lasting for eight quarters at 2 percent trend inflation is equivalent to a 6.2 percent permanent reduction in consumption; but the unconditional permanent reduction in consumption is only 0.08 percent because the probability of this ZLB event is very low.

for the U.S., the optimal inflation rate rises only to about 3 percent. This suggests that the evidence for an inflation target in the neighbourhood of 2 percent is robust to a wide range of plausible calibrations of hitting the ZLB" (p. 1394).

4.2 Incidence of the ZLB constraint in more complex models

The implications we reviewed so far are normative and were obtained in the context of a single framework. The likelihood and the consequences of hitting the ZLB, however, have also been examined in alternative classes of structural and statistical models and through a variety of policy rules; this literature is more supportive of the need to set a higher inflation target in normal times.

Reifschneider and Williams (2000) provide one of the first estimates of the effects of the ZLB for macroeconomic stability under different rates of trend inflation. They use the FRB/US model, in which monetary policy follows a standard Taylor rule. Simulating the model under historical disturbances (whose variance is estimated from the equation residuals over for the 1966-95 period), they study how the steady-state distribution of output, inflation, and interest rate vary as policymakers change the target rate of inflation. In their simulations, the zero bound gives rise to a trade-off between the average rate of inflation and the variability of output but doesn't generate any significant trade-off between the average rate of inflation and inflation variability. They estimate that if the inflation target in the standard Taylor rule were set at 2 percent, the federal funds rate would be near zero only 5 percent of the time, and the typical ZLB episode would last about four quarters. Raising target inflation to 4 percent would reduce the incidence of the zero bound to less than 1 percent of the time and reduce the length of the episode to two quarters. Overall, their results suggest that "macroeconomic stability would likely deteriorate somewhat if the target rate of inflation were to fall below 1 or 2 percent" (p. 956).[93] The authors also discuss how the negative impact of the ZLB in a low-inflation environment can be mitigated by simple modifications of the Taylor rule. Specifically, Taylor rule that respond to the cumulative deviations of inflation and output from their target levels (which is the form of efficient simple rules in FRB/US) would lead to a small efficiency loss from the ZLB.[94] In other words, policies that are efficient in the absence of the zero bound implicitly incorporate the response to past constraints that provide the best set of policy trade-offs between output and inflation variability.[95]

Williams (2009) revisits the constraint imposed by the ZLB, again using the FRB/US model. His objective is to estimate how much the ZLB has limited monetary policy during the recent recession and to evaluate the optimal inflation rate in light of this experience. He comes to two main conclusions. First, the ZLB has been a significant

[93] The simulations were conducted under the assumptions that fiscal policy is characterized by the historically average degree of activism and the equilibrium real rate of interest is around 2-1/2 percent. Of course, for the present paper, too, the results depend on the features of the model. In his comments on Reifschneider and Williams (2000), Sims (2000) is skeptical about their conclusion because the FRB/US model lacks a careful treatment of real balances and of fiscal policy, which may either mitigate or aggravate the effects of the ZLB constraint.

[94] Efficient simple rules are those delivering the best combinations of standard deviations of inflation and the output gap, known as the policy frontier (see Williams, 2009).

[95] Price-level targeting rules are a particular case of the modified Taylor rules considered; hence they can overcome the effect of the zero bound, as shown in Wolman (1998). We discuss this point later.

factor in slowing the recovery both in the U.S. and other economies. The ZLB constraint, he computes, has imposed a cost in terms of lost output of about $1.8 trillion. Second, if current conditions imply a permanently lower equilibrium real rate, then a 2 percent steady-state inflation may indeed be an inadequate buffer against the ZLB, and it may be prudent to aim at a moderately higher rate.[96] The Williams (2009) analysis is expanded by Chung et al. (2012) to assess whether estimates of the frequency of the ZLB in the literature are too benign. They use an array of statistical and structural models to investigate what we can learn from recent events about the expected frequency, duration, and magnitude of ZLB episodes, and how severe the zero bound constraint has been in the current crisis. The authors consider both structural models - FRB/US, EDO (a medium-size DSGE model developed at the Federal Reserve Board), and the Smets and Wouters (2007) model - and statistical models that impose less tight constraints and allow for the possibility of structural changes.[97] For each model they conduct stochastic simulation exercises.

From the standpoint of 2007, the authors compare the actual course of events with what each model would have predicted prior to the crisis (the authors base the forecast distribution on historic shocks, starting from 1960). They observe that the drop in output and the federal funds rate as well as the increase in unemployment were big surprises, while the mild disinflation was not a surprise. Furthermore, the structural models deliver only a very low probability of hitting the ZLB; by contrast, the statistical models give it a probability of 2 to 9 percent. In the structural models, the probability of hitting the ZLB and the probability of being stuck there for four or more quarters in general increase when the estimation sample is extended to 2010 (which gives three more years of information about the shocks) and when the simulations account for the uncertainty about model parameters and latent variables. These probabilities decrease when using only the observations for the Great Moderation period.

The broad message of Chung et al. (2012) is that uncertainty about model parameters significantly increases the probability of hitting the zero bound; that parametrizations based on the Great Moderation period tend to understate the incidence and severity of the ZLB; and that the propagation mechanism of typical DSGE models is unable to generate sustained periods during which policy is stuck at the ZLB.

The question, however, is whether pinning down the probability of hitting the zero bound can resolve the issue of whether the long-run target for inflation should be raised, or whether one should instead consider alternative policies that could help respond to the kind of negative shocks that brought about the Great Recession. We turn to this issue next.

[96] This conclusion is based on a calibration of the equilibrium real rate at 1 percent, and a distribution of shocks estimated from data covering the pre-Great Moderation period only. Both assumptions are arguably quite extreme (see Woodford, 2009a).

[97] The statistical models considered are a time-varying parameter VAR; the Laubach and Williams (2003) model, which allows a unit root in output growth and the equilibrium interest rate; and a univariate model with GARCH error processes.

4.3 History-dependent policy as an alternative to a higher inflation target

Interpreting the conclusion of simulations performed under a simple Taylor rule for monetary policy requires some caution. As noted by Reifschneider and Williams (2000) and demonstrated by Eggertsson and Woodford (2003), such a rule is a poor form of policy when the zero bound binds because it is a commitment to a purely forward-looking policy. That is, the simple Taylor rule takes into account at each point in time only the evolution of the economy from that point on; as soon as the constraint is no longer binding, policy is again conducted as it would have been if the constraint had never hit. As we discussed above, Reifschneider and Williams (2000) note indeed that the economy would perform better under a modified Taylor rule that responds "to the *cumulative* deviations of output and inflation from their respective target levels" (p. 961, emphasis in the original). Eggertsson and Woodford (2003) address this issue and show how rules that incorporate a form of history dependence, as those indeed do, can approximate optimal policy in the presence of the zero bound constraint. They recast the Krugman (1998) analysis in a simple dynamic general equilibrium model in which the policymaker's objective function trades off inflation and output stabilization, and characterize optimal policy subject to the zero bound constraint. They find that the zero bound constrains stabilization policy in a low inflation environment when a real shock lowers the natural rate of interest. They show, however, that it is possible to mitigate the effects of the zero bound constraint by creating "the right kind of expectations regarding how monetary policy will be used after the constraint is no longer binding, and the central bank again has room to maneuver" (p. 143).

In the sort of dynamic models that we discussed, the effectiveness of monetary policy rests on its ability to affect private sector expectations about the future path of the short-term rates because these determine equilibrium long-term rates, which in turn affect most spending decisions. Hence "a commitment to create subsequent inflation involves the commitment to *keep interest rates low* for some time in the future" (Eggertsson and Woodford, 2003, p.144, emphasis in the original) and this commitment can help stimulate aggregate demand today even if current nominal rates cannot be lowered any further.

The commitment to a history-dependent policy is key to generating expectations of higher inflation and to exiting the zero lower bound. This commitment to future policy can achieve what the advocates of a higher inflation target want to achieve, namely generating expectations of higher inflation in the future (when the constraint would no longer bind) in order to obtain the desired decline in the current real rate. Rather than permanently raise the inflation target, policymakers can generate these expectations by being explicit about how they will conduct policy *after* the zero bound constraint is no longer binding.

Eggertsson and Woodford (2003) also show that the optimal commitment takes the form of price-level targeting: people should expect inflation to be only temporarily high following periods in which the price level has fallen below its target path; the price-level target implements a sort of history-dependent inflation target.[98]

[98] A similar argument holds if, instead of targeting the price level, monetary policy uses the exchange rate as an instrument (Svensson, 2003; McCallum, 2000).

The "forward guidance" adopted by the Federal Reserve in the recent crisis is consistent with the history-dependence feature of the Eggertsson and Woodford (2003) optimal policy. Indeed, their recommendation of compensating for the current zero bound constraint by providing more accommodation in the future is echoed in the FOMC's post-meeting statement language adopted since September 2012: "... the Committee expects that a highly accommodative stance of monetary policy will remain appropriate for a considerable time after the economic recovery strengthens."[99] It has also found expression in a proposal advanced by the president of the Federal Reserve Bank of Chicago (Evans, 2012). He argued for the adoption of a state-contingent price-level targeting regime and illustrated the way it would work in the specific case of a desired growth rate of prices of 2 percent per year (the FOMC's long-run inflation objective; see fn. 1). He argued that when the annual rate of inflation is below 2 percent (where it has been on average since the recession started), the price level will have to grow at a faster rate afterward in order to close the gap between the actual and the desired price paths. The expected path of inflation would therefore have to be above 2 percent for some time, even though 2 percent would continue to be the long-run objective.[100]

A similar recommendation has been made by Coibion et al. (2012), the contribution we discussed above. They derive optimal policy rules under commitment and simulate the model under baseline parameters. They find that when the central bank can commit to the Ramsey policy, the cost of the ZLB can be negligible. The reason is that, in the event of a large shock, the credible commitment to keep the interest rate low for an extended period reduces the impact of the shock and helps in exiting the ZLB sooner. The impact of a negative shock is therefore smaller and the ZLB events are shorter despite the fact that the optimal inflation rate is very close to zero and the zero bound binds more frequently. Commitment is the key for this result: the optimal policy under discretion would entail much higher costs in a ZLB situation and a higher inflation rate – about 2.7 percent. Evaluating price-level targeting rules, Coibion et al. (2012) find that such policies stabilize expectations, substantially reduce the volatility of output and inflation, and increase welfare. Under those rules, the zero bound binds less frequently even at a low steady-state inflation.[101]

Rules that target nominal GDP also contain the history-dependence feature of price-level targeting rules. Nominal GDP targeting has therefore been advocated by some in the academic and business communities as another promising alternative to an increase in the long-run inflation objective for addressing the ZLB constraint (e.g., Woodford, 2012; Hatzius and Stehn, 2011). The discussion of such alternatives is beyond the scope of this paper.

[99]FOMC post-meeting statement, September 13, 2012 (http://www.federalreserve.gov/newsevents /press/monetary/20120913a.htm.) From October 2012 on, that sentence of the statement was amended to include a reference to the purchase program: ".. the Committee expects that a highly accommodative stance of monetary policy will remain appropriate for a considerable time after the purchase program ends and the economic recovery strengthens." This language has further evolved in the FOMC statement of March 19, 2014.

[100]Price-level targeting has been examined by researchers at the Bank of Canada for some time, e.g. Côté (2007), Ambler (2009), and Crawford et al., (2009).

[101]Analyzing Japan's "lost decade," Leigh (2010) shows that simply increasing the inflation target would have had only limited and short-lived effects on output, while a price-level targeting rule would have proved to be a much better stabilization tool.

5 Summary

The literature lacks a comprehensive discussion of trend inflation and the macroeconomic effects of conducting monetary policy with a higher inflation target. We help fill this gap by reviewing key empirical, theoretical, and policy issues arising from an assumption of positive trend inflation in a workhorse macroeconomic model for monetary policy.

The theoretical underpinnings of time variation in trend inflation and its relation to the policymakers' inflation target remain an active area of research. We show that accounting for the evolution of trend inflation in empirical models of inflation dynamics allows a better definition of the properties of the inflation gap, the variable that matters to policymakers. In our estimate, trend inflation—which we measure with the GDP deflator—peaked in the '70s and then started a slow decline before stabilizing in the mid-'90s at around a 2 percent annual rate.

We also show that the persistence of the inflation gap is lower when measured as the deviation of inflation from trend than as the deviation from a constant mean. As the deviation from trend, the persistence of the inflation gap has declined significantly since the mid-'80s, an important consideration in monetary policy decisions.

Many economists have recently proposed trend inflation rates higher than current central bank targets to reduce the incidence of the zero lower bound constraint on monetary policy. We evaluate the implications of this proposal through a more general version of the standard log-linearized New Keynesian model, which we obtain as an approximation around positive steady-state inflation. In this model, even a moderate rate of trend inflation has important consequences for the evaluation of monetary policy. It tends to destabilize inflation expectations and thus requires a more aggressive policy response to inflation deviations from target. Moreover, because positive trend inflation increases price dispersion, it flattens the Phillips curve; this in turn reduces the impact of technology shocks on output and inflation, amplifies the impact of monetary shocks, and increases the persistence of the effects of shocks on the macroeconomic variables. Although our model employs the Calvo price-setting mechanism, the results continue to hold under variations in this and other microeconomic features. Extensions of our baseline model to environments with richer specifications are the subject of ongoing research.

Our results imply that raising the long-run rate of inflation would carry significant costs. However, these must be weighed against the costs of hitting the zero bound as well as against the uncertainties surrounding the efficacy of alternative monetary policy tools in such an environment.

The debate on this matter hinges significantly on the frequency and severity of zero bound episodes, whose likelihood is difficult to evaluate. Research centered on the historical record may underestimate that likelihood because it doesn't take full account of the current situation, in which the zero bound has been binding for more than 20 quarters. We argue, however, that the literature provides effective responses to the zero lower bound problem that do not require a permanent increase in the long-run inflation objective. Specifically, the literature shows that the commitment to a history-dependent policy can raise inflation expectations, thus reducing the current real interest rate and providing economic stimulus. Although this result is the same as that sought by advocates of a higher inflation target, it would be realized via only a temporary increase in expected inflation. Hence, it would be less likely to carry the negative implications of a higher trend rate of inflation.

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