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Abstract

In standard Walrasian macro-finance models, pecuniary externalities due to fire sales lead to excessive borrowing and insufficient liquidity holdings. We investigate whether imperfect competition (Cournot) improves welfare through internalizing the externality and find that this is far from guaranteed. Cournot competition can overcorrect the inefficiently high borrowing in a standard model of levered real investment. In contrast, Cournot competition can exacerbate the inefficiently low liquidity in a standard model of financial portfolio choice. Implications for welfare and regulation are therefore sector-specific, depending both on the nature of the shocks and the competitiveness of the industry.

Key words: fire sales, pecuniary externalities, overinvestment, liquidity, financial regulation, macroprudential regulation

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1 Introduction

The macro-finance literature has taken great interest in fire-sale externalities. Canonical models show that such pecuniary externalities lead to overinvestment in risky capital (e.g. Lorenzoni, 2008) and overinvestment in illiquid assets (e.g. Allen and Gale, 2004) because perfectly competitive agents do not internalize how their ex-ante choices affect fire-sale prices after adverse shocks. In reality, however, agents may not be perfectly competitive. Industry concentration has increased substantially over recent decades, both in the real economy and in the financial sector (see e.g. Gutiérrez and Philippon, 2018, and Corbae and Levine, 2018, respectively). These increased concentrations raise the possibility that firms do internalize price impacts in asset markets.

In the standard macro-finance models, pecuniary externalities would be mitigated if agents internalized their effects on prices: agents would invest in less capital (i.e. borrow less) or invest in fewer illiquid assets (i.e. hold more liquidity), so that asset prices would be higher when bad aggregate shocks occur. We show that this mitigating effect of imperfect competition is not robust to simple modifications of the standard macro-finance models. Instead of being mitigated, the inefficiencies can be overcorrected or even exacerbated, depending on whether fire sales occur due to productivity or liquidity shocks and whether the shocks are purely aggregate or have an idiosyncratic component.

Our analysis covers two standard macro-finance models of fire sales — a model of firms with risky production funded with debt, in the spirit of Lorenzoni (2008), and a model of banks that invest in illiquid projects to issue liquid deposits, in the spirit of Allen and Gale (2004). What distinguishes the two settings is the force that causes fire sales. In the first setting, fire sales occur when leveraged agents' investments experience bad productivity shocks, forcing them to sell part of their illiquid assets to second-best users in order to repay debts. In the second setting, fire sales occur when liquidity shocks force liquidity-transforming institutions to sell all of their illiquid assets to cash-strapped buyers. In both of these settings, a Social Planner would choose less investment in illiquid assets, leading to higher asset prices (less severe fire sales).

To these standard setups, we introduce the following crucial modifications: (i) "Cournot behavior" of agents, i.e. internalizing the marginal impact an agent's ex-ante balance sheet decisions have on ex-post asset prices, and (ii) a combination of both aggregate and idiosyncratic risk. When fire sales occur because some agents receive bad shocks, then other agents receive good shocks and are therefore in a favorable position to buy fire-sale assets. Agents strategically consider how their ex-ante choices affect ex-post prices, both when they receive bad shocks and contribute to fire sales, and when they receive good shocks and benefit from fire sales.

Our settings nest the standard macro-finance variants of these models, and we confirm that, in the standard setting, Cournot mitigates the externalities. However, the strategic considerations of potential buyers and sellers have important consequences when the nature of idiosyncratic and aggregate risk differ from the standard formulations. The Social Planner's first-order condition considers how initial decisions will affect fire-sale prices *in the aggregate*, and then weights the combined marginal utilities for buyers and sellers by the aggregate marginal price impact. Cournot agents instead consider separately how their initial decisions affect prices *when they are a buyer* and *when they are a seller*. In contrast to the Social Planner perspective, the price impact is different when buying than when selling, and therefore Cournot agents weight differently the respective marginal utilities in their first-order conditions.

Cournot competition can therefore overcorrect or exacerbate the inefficiency, depending on the relative magnitude of buyer and seller price impacts. As a result, to study the effect of industry concentration within macro-finance models of fire sales, there is no alternative but to go into specifics and understand the subtleties within different classes of models. Accordingly, we make small, relevant modifications that leave the direction of the externality intact while the "direction" of strategic Cournot behavior may differ. We first present a unified model with both productivity shocks and liquidity shocks that clearly identifies under which conditions market power overcorrects, mitigates, or exacerbates the inefficiency. The unified model nests general versions of two of the most important models of fire sales in the macro-finance literature, and thus provides the appropriate representative setting to study the effects of industry concentration on fire sales. We then turn to the canonical models to provide the necessary structure to verify the empirical plausibility of our results in each setting.

The general intuition for our results is as follows. A higher price benefits agents in the state of the world where they are selling assets but hurts agents in the state where they are buying. How much an increase in ex-ante investment affects expected utility through these price effects depends on how much the higher investment impacts the price in either state. Importantly, buyers always affect the price in the same way — they use available cash flow to buy fire-sale assets — but how sellers affect the price depends on whether they are partially liquidating their asset holdings (to raise a fixed value of funds) or completely liquidating their asset holdings. When partially liquidating, the supply of assets sold depends on the price, but when fully liquidating the supply of assets is inelastic. As a result, the price impact of selling is very different depending on whether the equilibrium liquidation regime features partial or full liquidation.

When shocks force partial liquidation of assets, as is typically the case when agents face productivity shocks, the relative price impacts of buying and selling, and therefore the level of investment under Cournot equilibrium, are mainly determined by the degree of idiosyncratic risk, i.e. the difference between a good and a bad productivity shock. Higher idiosyncratic risk increases the price impact as a seller and reduces it as a buyer, and therefore reduces the incentive to invest in illiquid assets. With no idiosyncratic risk, Cournot agents partially mitigate the inefficiently high investment of the Walrasian equilibrium, but sufficiently high idiosyncratic risk pushes down Cournot agents' investment below the level chosen by the Social Planner, thus overcorrecting the inefficiency.

In contrast, when shocks force "early consumers" to fully liquidate their assets, as is the case when agents face liquidity shocks, sellers' supply of assets is inelastic and their price impact is proportional to the level of the equilibrium fire-sale price. Importantly, the fire-sale price is determined by the aggregate level of liquidity in the market, which is primarily determined by the likelihood of a fire-sale state. The level of investment in the Cournot equilibrium is therefore mainly determined by the degree of aggregate risk. A lower likelihood of the fire-sale state, and therefore a lower fire-sale price, reduces the price impact as a seller, when agents like higher prices, and increases the incentive for investment. With a high likelihood of the fire-sale state, Cournot agents partially mitigate the inefficiently high investment of the Walrasian equilibrium, but a sufficiently low likelihood of a fire sale pushes up Cournot agents' investment above the Walrasian level, thus exacerbating the inefficiency.

These contrasting mechanisms are clearly seen within the standard settings of the two canonical models. While Cournot competition can mitigate the inefficiency arising from fire sales, in the canonical model with productivity shocks Cournot can instead *overcorrect* the inefficiency, and in the canonical model with liquidity shocks Cournot can also *exacerbate* the inefficiency.

First, standard models of fire sales due to productivity shocks and borrowing constraints typically consider "pure aggregate risk" so that all agents receive a bad shock at the same time (e.g. Lorenzoni, 2008). Bad shocks force firms to sell some of their capital to repay debts, pushing down asset prices and requiring even more sales in order to raise funds. If firms borrowed less initially, then fire sales would be smaller, and less capital would be reallocated to inefficient users. Hence, the standard model features *overinvestment* in capital in the Walrasian equilibrium. To this standard setup we introduce idiosyncratic productivity risk in the bad state, so that some firms have good productivity and can buy up capital at cheap prices. With Cournot competition, firms know that when they receive bad shocks they will sell capital, and so they strategically would like to hold less capital to minimize the price impact. Firms also know that when they receive good shocks they will buy capital, and they would like to purchase capital at lower prices, which they would do by having fewer funds available to buy capital — which occurs by holding less capital. So whether a buyer or a seller, firms strategically would like to have invested in less capital *in either case*. As a result, the Cournot equilibrium can feature *under-investment* relative to the constrained efficient level chosen by the Social Planner because shocks to capital determine the funds available to repay debts or buy new capital. We discipline our model by matching some key empirical moments, and show that it is empirically plausible for Cournot competition to overcorrect the pecuniary externality, so that market power leads to *under-investment* in capital.

Second, standard models of fire sales due to liquidity transformation typically consider idiosyncratic liquidity shocks that cannot be adequately insured as a result of incomplete markets (e.g. Allen and Gale, 2004). Investors receiving liquidity shocks are forced to sell all of their illiquid assets, and thus their consumption is a function of the asset price. If investors held fewer illiquid assets, the interim asset price would be higher, providing better insurance to investors selling their assets because of liquidity shocks. Hence, the standard model features *over-investment* in illiquid assets in Walrasian equilibrium. In our model, investors know that holding fewer illiquid assets will push up the asset price, which is good when they are sellers but bad when they are buyers. When the price is sufficiently low, investors have a greater strategic incentive to push down the price (to buy at cheap prices when they are buyers). As a result, fire sales are more extreme, and Cournot competition can lead to even *lower* asset prices. We find that it is empirically plausible for Cournot competition to exacerbate the pecuniary externality rather than mitigate it, so that market power leads to *over-over-investment* in illiquid assets.

Related literature. The literature on generic inefficiency arising from pecuniary externalities dates to Geanakoplos and Polemarchakis (1986) and Greenwald and Stiglitz (1986), which provide justifications for policy interventions when private agents do not internalize their effects on prices. Dávila (2015) and Dávila and Korinek (2018) provide recent analysis of pecuniary externalities in macro-finance models with borrowing constraints, showing that terms of trade and collateral externalities are distinct, as are the issues of efficiency and amplifications. Stein (2013) is an example of policy thinking based on academic insights.

Closely related to the literature on pecuniary externalities are the papers on fire sales and limits to arbitrage: Shleifer and Vishny (1992), Gromb and Vayanos (2002), and Shleifer and Vishny (2011). All of these papers on pecuniary externalities share the feature that inefficiencies arise because price-taking agents do not internalize how their portfolio decisions affect prices, affecting risk sharing and borrowing capacities.

Our paper relates to the literature on over-investment, which includes Caballero and Krishnamurthy (2001), He and Kondor (2016), and Lorenzoni (2008). Other recent work has considered the possibility of under-investment due to market power. In particular, Gutiérrez and Philippon (2017, 2018) document that investment is low based on Tobin's Q and that the shortfall is related to industry concentration. In theoretical work, Kurlat (2019) shows that the canonical over-investment result can also be reversed if the microfoundation for fire sales is based on adverse selection, as opposed to slow moving capital or other constraints on potential buyers.

The literature on liquidity provision includes Diamond and Dybvig (1983), Bhattacharya and Gale (1987), Jacklin (1987), and Allen and Gale (2004). Recently, Farhi et al. (2009) and Geanakoplos and Walsh (2018) study inefficient liquidity provision with private trades in financial markets. These papers study how incomplete markets lead to under-provision of liquidity (typically, though, different specifications of shocks can lead to over-provision).

A few macro-finance papers consider the implications of agents internalizing their effect on prices. Corsetti, Dasgupta, Morris, and Shin (2004) consider how the presence of a large trader affects the likelihood of currency crises, as small traders take into account strategically the behavior of the large trader (small traders are more aggressive). Korinek (2016) considers how international policy cooperation depends on whether countries internalize the impact of their policies on exchange rates. Dávila and Walther (2019) consider the leverage decisions of large and small banks when banks internalize how their leverage and size affect bailout probabilities (small banks use more leverage in the presence of large banks). In a complementary paper to ours, Neuhann and Sockin (2020) consider Cournot agents in a model with investment and complete Arrow–Debreu markets and study how market power leads to distortions in risk sharing and investment decisions. In contrast, we consider models with *incomplete* markets where pecuniary externalities have welfare effects, and study the impact of market power on the (in)efficiency of equilibrium allocations. Babus and Hachem (2019, 2020) show that differences in financial market structure, through the relative market power of buyers and sellers, have rich effects on endogenous security design and ultimately welfare.

Diamond and Rajan (2011) study how anticipating potential future fire sales can affect asset markets today, reducing buyers' willingness to pay and, in turn, sellers' willingness to sell. Gale and Yorulmazer (2013) argue that costly bankruptcy and incomplete markets cause inefficient liquidity hoarding. Malherbe (2014) argues that liquidity provision can exacerbate adverse selection. Perotti and Suarez (2002) highlight the incentive to be the "last bank standing." Kuong (2016) shows that the pecuniary externality leads to interactions between firms' borrowing and risk-taking decisions. Finally, Morrison and Walther (2020) consider how market discipline and systemic risk interact in a competitive setting with both aggregate and idiosyncratic risk where banks may be in a position to buy or sell assets.

2 Cournot in a unified macro-finance model

We first present a unified model that nests the canonical fire-sale models of Lorenzoni (2008) and Allen and Gale (2004). Agents invest in an asset that is productive in the long term but illiquid in the short term and face shocks that can require them to sell the illiquid asset early. Within this unified framework, the key distinction between the two canonical models is whether the equilibrium regime features fire sales that are caused by partial liquidations — akin to the entrepreneurs of Lorenzoni (2008) — or complete liquidations — the early consumers of Allen and Gale (2004). In this section we analyze optimizing behavior in the unified model within each endogenous regime in terms of endogenous variables. In Sections 3 and 4 we show that the two standard macro-finance settings provide the general equilibrium structure determining the equilibrium fire-sale regime allowing us to fully characterize how internalizing price impacts affects equilibrium investment.

2.1 Model setup

There are three periods, $t = 0, 1, 2$, and an even number of agents, $2N$ (to ensure that they can be split into to equal-sized groups). Agents have utility over consumption at $t = 1, 2$ given by $u(c_1) + \beta u(c_2)$ with u risk averse and $\beta \leq 1$. At $t = 0$, agents start with a unit endowment that they can invest in either a liquid or an illiquid asset. Denote the fraction invested in the liquid asset by ℓ and the fraction invested in the illiquid asset by k, with $\ell + k = 1$. The liquid asset has a gross return of 1 per period (storage). The illiquid asset has a deterministic gross return $R > 1$ per unit at $t = 2$ and can be traded at $t = 1$ at an endogenous price p. In addition to the agents' endogenous illiquid asset demand and supply, there are outside investors with a downward sloping demand $D(p)$ with demand elasticity $\xi_p > 1$.[1]

Agents are subject to two types of shocks at $t = 1$. First, each agent receives an idiosyncratic liquidity shock θ_1, which is independent of the portfolio decision (ℓ, k). This shock is

[1] This constraint on the elasticity is needed in the regime featuring partial liquidations but can be ignored in the regime with complete liquidations.

meant to capture an early need for funds, i.e. debt repayments or depositors who want to consume. We model the liquidity shock as an exogenous required level for consumption c_1. We suppose that the liquidity shock θ_1 is sufficiently large to yield a corner solution for intertemporal substitution between $t = 1$ and $t = 2$ and may be so large that an agent is forced to sell all illiquid assets (see Appendix A for details). Second, each agent receives an idiosyncratic cash flow shock δ_1 per unit of illiquid assets. This cash flow is meant to capture a net flow resulting from a stochastic dividend from a (possibly leveraged) capital investment. The cash flow shock can therefore be negative and add to the agent's need to sell illiquid assets.

In sum, there are two reasons why agents may need to sell from their illiquid asset holdings k at $t = 1$ to raise cash in addition to their liquid asset holdings ℓ: (i) a sufficiently high liquidity shock θ_1 or (ii) a sufficiently low (negative) cash flow δ_1. Note that the effect of the liquidity shock θ_1 does not depend on the portfolio decision (ℓ, k), though the ability to address the shock does, while the effect of the cash flow shock δ_1 does depend on the portfolio decision since the cash flow is per unit of k. The combination of the two shocks therefore generates $t = 1$ liquidity needs that are affine in the $t = 0$ portfolio decision, i.e. $\theta_1 + \delta_1 k$, which allows us to capture both the debt repayments of agents with net worth in the model of Lorenzoni (2008) as well as the early consumption needs of agents in the model of Allen and Gale (2004).

There are two aggregate states: a good state occurring with probability α and a bad state occurring with probability $1 - \alpha$. Fire sales will occur in the bad state only, but the likelihood of the bad state affects whether Cournot behavior can overcorrect or exacerbate the inefficiency. In the good state, there is no risk — the the liquidity shock is $\theta_1 = \overline{\theta}$ and the cash flow is $\delta_1 = 0$ for all agents. In the bad aggregate state, there is idiosyncratic risk. First, liquidity shocks in the bad aggregate state can be high or low, denoted by θ_H and θ_L. Second, the illiquid asset pays either a high or low cash flow, $\delta_H = \delta + \epsilon$ or $\delta_L = \delta - \epsilon$, where δ is the average (possibly zero), and ϵ captures the amount of idiosyncratic risk. The case of a negative average cash flow, $\delta < 0$, captures the interpretation of a negative aggregate productivity shock when capital is used for production. As a parameter restriction, we suppose that $\delta_H < 1$ to generate equilibria with fire sales. For tractability, we suppose shocks are perfectly correlated for an agent: an agent either receives two favorable shocks or two unfavorable shocks. Thus, a "lucky" agent receives a low liquidity shock and a high cash flow shock, while an "unlucky" agent receives a high liquidity shock a low cash flow shock. We suppose that half of the $2N$ agents, randomly selected, end up lucky and the other half unlucky, such that each agent is lucky with probability $1/2$ and the aggregate shares of lucky and unlucky agents are deterministic.

2.2 Trade at $t = 1$ and resulting consumption

We first study trade between lucky and unlucky agents (and outside investors) at $t = 1$ and how the resulting consumption depends on the asset price at $t = 1$ and the portfolio choice at $t = 0$. While we assume that Cournot agents behave strategically at $t = 0$, we assume them to be price takers at $t = 1$. This is to clearly contrast the Cournot decision at $t = 0$ with the Social Planner decision at $t = 0$ by keeping other periods unchanged. However, we show in Appendix C that allowing for strategic behavior at $t = 1$ would not affect our results.

In the good aggregate state without risk there is no trade in the illiquid asset, and consumption is simply $\bar{c}_1 = \bar{\theta}$ and $\bar{c}_2 = \ell - \bar{\theta} + Rk$, with the intertemporal corner solution guaranteed by a sufficiently large $\bar{\theta}$.[2] In the bad aggregate state, we have to distinguish between lucky and unlucky agents. Since lucky agents will have high consumption, we denote lucky agents by H and unlucky agents by L (even though the liquidity shocks they receive are reversed).

Lucky agents. Lucky agents have a high cash flow shock δ_H and a low liquidity shock θ_L. We assume that the cash flow shock is sufficiently large so that a lucky agent has spare cash to buy assets at $t = 1$, i.e. $\ell + \delta_H k > \theta_L$. Lucky agents' demand for illiquid assets is therefore

$$d_H = \frac{\ell + \delta_H k - \theta_L}{p},$$

and their consumption is $c_{1H} = \theta_L$ and $c_{2H} = R\,(k + d_H)$.

Unlucky agents. Unlucky agents have a low cash flow shock δ_L and a high liquidity shock θ_H. The size of the liquidity shock, θ_H, relative to the total cash value of the portfolio, $\ell + \delta_L k + pk$, determines if an unlucky agent fully or only partially liquidates the portfolio at $t = 1$.

If the liquidity shock θ_H is smaller than the cash value of the portfolio, the unlucky agent sells only part of the portfolio, and consumes the desired θ_H at $t = 1$ and the payoff of the remaining assets at $t = 2$. In this case, unlucky agents' supply of illiquid assets is given by

$$s_L^{\text{part}} = \frac{\theta_H - (\ell + \delta_L k)}{p},$$

and their consumption is $c_{1L}^{\text{part}} = \theta_H$ and $c_{2L}^{\text{part}} = R\,(k - s_L)$. If, instead, the liquidity shock

[2] Appendix A provides the conditions on the liquidity shock θ_1 that guarantee $c_1 = \theta_1$, i.e. a corner solution for intertemporal substitution between $t = 1$ and $t = 2$.

θ_H is larger than the cash value of the portfolio, the unlucky agent sells the full portfolio, $s_L^{\text{full}} = k$, consumes the entire proceeds at $t = 1$, $c_{1L}^{\text{full}} = \ell + \delta_L k + pk$, and nothing at $t = 2$, $c_{2L}^{\text{full}} = 0$.

There is a crucial difference between the supply of assets in two regimes, s_L^{part} and s_L^{full}. Under partial liquidation, the quantity of assets sold is a decreasing function of the price p. Under full liquidation, the quantity of assets sold is not a function of p.

2.3 Portfolio choice at $t = 0$

We now study portfolio choice at $t = 0$ and consider the Walrasian equilibrium, where agents take the asset price at $t = 1$ as given, as well as the Cournot equilibrium, where agents perceive the impact of their portfolio choice on the asset price. Comparing the perspective of Cournot agents to that of the Social Planner, we show the potential for market power to overcorrect or exacerbate the constrained inefficiency of the canonical models.

When considering portfolio choice at $t = 0$, we can ignore the utility terms where consumption is at a corner solution irrespective of the equilibrium liquidation regime and write the agents' objective function in general form as

$$\underbrace{\alpha\beta u(\bar{c}_2)}_{\text{good state}} + \underbrace{\frac{1-\alpha}{2}\left(u(c_{1L}) + \beta u(c_{2L})\right)}_{\text{bad state, unlucky}} + \underbrace{\frac{1-\alpha}{2}\beta u(c_{2H})}_{\text{bad state, lucky}}.$$

The consumption terms are as derived in Section 2.2 above, with only the unlucky agents' consumption depending on the equilibrium regime, i.e. partial liquidation or full liquidation:

$$\bar{c}_2 = \ell - \bar{\theta} + Rk, \quad c_{2H} = R\left(k + \frac{\ell + \delta_H k - \theta_L}{p}\right),$$
$$\text{and} \quad c_{1L}^{\text{part}} = \theta_H, \quad c_{2L}^{\text{part}} = R\left(k - \frac{\theta_H - \ell - \delta_L k}{p}\right),$$
$$\text{or} \quad c_{1L}^{\text{full}} = \ell + \delta_L k + pk, \quad c_{2L}^{\text{full}} = 0.$$

2.3.1 Walrasian optimization

Price-taking agents consider the marginal effects of investment k on utility in each state, so the Walrasian first-order condition with respect to investment k is given by:

$$\underbrace{\alpha\beta u'(\bar{c}_2)\frac{\partial \bar{c}_2}{\partial k}}_{\text{good state: benefit}} + \frac{1-\alpha}{2}\left(\underbrace{u(c_{1L})\frac{\partial c_{1L}}{\partial k} + \beta u'(c_{2L})\frac{\partial c_{2L}}{\partial k}}_{\text{unlucky: cost}} + \underbrace{\beta u'(c_{2H})\frac{\partial c_{2H}}{\partial k}}_{\text{lucky: cost or benefit}}\right) = 0 \qquad (1)$$

In the good state, more investment benefits the agent due to the illiquid asset's return in excess of the unit return on storage, $\partial\bar{c}_2/\partial k > 0$. In the bad state, more investment is costly when the agent is unlucky and has to sell assets at a price below the return on storage; this cost is irrespective of equilibrium regime as $\partial c_{1L}^{\text{full}}/\partial k < 0$ and $\partial c_{2L}^{\text{part}}/\partial k < 0$. For lucky agents, additional investment has the opportunity cost of less "dry powder" in the form of cash to buy fire-sold assets at $t = 1$ but the benefit of the higher return at $t = 2$; more investment is therefore costly ($\partial c_{2H}/\partial k < 0$) when the asset price is sufficiently low but beneficial otherwise.

Whatever the equilibrium liquidation regime, Walrasian agents' investment in the illiquid asset trades off the benefit of higher consumption in the good state against the cost of lower consumption in the unlucky state, and possibly also missed opportunities in the lucky state.

2.3.2 Social Planner optimization

To keep the objectives of the Social Planner as close to the objectives of the agents as possible, we suppose that the Social Planner maximizes the ex-ante welfare of the agents, ignoring the utility of outside investors.[3] Compared to the Walrasian optimization, a Social Planner explicitly accounts for the effect of aggregate investment on fire-sale prices in the bad state. The Social Planner considers the effect of investment on the price, dp/dk, and considers how changing the price affects consumption, and thus utility, for agents in each state.

Thus, the Social Planner's first-order condition will incorporate an additional term rel-

[3] As discussed in Dávila and Korinek (2018), this assumption is without loss of generality if the Planner can also engage in initial transfers to make Pareto improvements. Caring about outside investors will encourage the Social Planner to decrease the asset price (since outside investors buy at $t = 1$), which pushes against the main objective of the Social Planner, which is to minimize fire sales (i.e. wanting a higher price). Given our focus on the fire-sale externality, ignoring or minimizing the role of outside investors is the natural way to proceed.

ative to the Walrasian first-order condition (1), reflecting the price impact:

$$\frac{1-\alpha}{2}\left(u'(c_{1L})\frac{\partial c_{1L}}{\partial p}+\beta u'(c_{2L})\frac{\partial c_{2L}}{\partial p}+\beta u'(c_{2H})\frac{\partial c_{2H}}{\partial p}\right)\frac{dp}{dk} \tag{2}$$

A higher price is beneficial for unlucky agents who sell as $\partial c_{1L}^{\text{full}}/\partial p > 0$ and $\partial c_{2L}^{\text{part}}/\partial p > 0$ but costly for lucky agents who buy, $\partial c_{2H}/\partial p < 0$. As in the Walrasian case, the extra term in the FOC from price effect depends on the equilibrium liquidation regime, which we explicitly consider below. Nonetheless, in both equilibrium regimes, the additional Social Planner term is negative when evaluated at the Walrasian equilibrium allocation (as shown below), and therefore the Social Planner chooses lower asset holdings compared to the Walrasian equilibrium. This is why we say that asset markets feature fire sales: the price is inefficiently low in the Walrasian equilibrium.

2.3.3 Cournot optimization

Like the Social Planner, Cournot agents internalize the price impact of their initial portfolio choice. However, while the Social Planner considers the aggregate consequences of initial investment on price at $t = 1$ as it affects all agents, dp/dk, Cournot agents consider separately their price impacts when they turn out lucky and buy or when they turn out unlucky and sell. When lucky, the agent's initial investment affects the price through the demand for assets, denoted by dp/dk_H, and when unlucky, through the supply of assets, denoted by dp/dk_L. We study these price impacts, and how they very across equilibrium liquidation regimes, in the next section. Hence, the additional term in the Cournot FOC term can generally be written as

$$\frac{1-\alpha}{2}\left(\left(u'(c_{1L})\frac{\partial c_{1L}}{\partial p}+\beta u'(c_{2L})\frac{\partial c_{2L}}{\partial p}\right)\times\frac{dp}{dk_L}+\beta u'(c_{2H})\frac{\partial c_{2H}}{\partial p}\times\frac{dp}{dk_H}\right). \tag{3}$$

Compared to the Social Planner term (2), in which the price impact dp/dk factors out, the Cournot price impacts dp/k_L and dp/dk_H act as weights on the individual marginal utility terms.

Since the price impacts weight the benefit of a higher price to a seller and the cost of a higher price to a buyer, the net effect on the Cournot term (3) is ambiguous. First, recall that the Social Planner term (2) is unambiguously negative in equilibrium. In contrast, when evaluated at the allocation in the Walrasian equilibrium, the Cournot term could be positive, implying that Cournot agents would choose a higher investment compared to the Walrasian equilibrium (exacerbating the overinvestment externality), or the Cournot term could be negative, implying that Cournot agents choose lower investment compared to the Walrasian equilibrium (mitigating the externality).

Second, the Cournot term could be negative like the Social Planner term, but the magnitude could be quite different. When evaluated at the Social Planner allocation, the Cournot term could be greater than the Social Planner term so that Cournot agents under-correct the externality or less than the Social Planner term so that Cournot agents overcorrect the externality. It may therefore seem that the effect of Cournot optimization on equilibrium is ambiguous. This is not the case. A careful analysis of the price impacts in each regime allows us to make precise predictions regarding the sign of the Cournot term and how it compares to the Social Planner term. Specifically, the *seller* price impact is very different in each regime. Whether Cournot agents amplify, mitigate, or overcorrect the externality depends in systematic ways on whether liquidation is partial or full.

2.4 Price impacts and equilibrium allocations

We now explicitly solve for the price impacts that appear in the first-order conditions of the Social Planner and of Cournot agents and show how Cournot behavior can overcorrect or exacerbate pecuniary externalities, depending on the equilibrium regime of partial or full liquidation. To simplify the exposition, in this section we fix the total number of agents at two so there is always one lucky and one unlucky agent in the bad state (i.e. $N = 1$). In later sections we explicitly consider variations in the number of agents, N. Given the demand and supply of assets from Section 2.2, market clearing is given by

$$d_H(p) + D(p) = s_L(p)\,. \tag{4}$$

2.4.1 Partial liquidation

With partial liquidation by unlucky agents, both demand and supply depend on price. Substituting the expressions for d_H and s_L^{part} into equation (4) we can rewrite the market clearing condition as[4]

$$2\ell + 2\delta k + pD(p) = \theta_H + \theta_L. \tag{5}$$

First, we can solve for the effect of aggregate asset holdings on the equilibrium price by

[4]Note that for an equilibrium with outside investors purchasing assets, we need δ sufficiently small and/or θ_i sufficiently large as well as $pD(p)$ decreasing in p, i.e. price elasticity of outside demand exceeding 1: $\xi_p = -D'(p) \times p/D(p) > 1$.

implicitly differentiating the market clearing equation (5), taking into account $\ell = 1 - k$:

$$\frac{dp}{dk} = -\frac{2\,(1-\delta)}{D(p)\,\left(\xi_p - 1\right)}.$$

Given the assumption about the outside demand elasticity, $\xi_p > 1$, and that $\delta < 1$, the price is decreasing in agents' aggregate holdings of the illiquid asset.

Second, we can solve for the price impacts separately of buyers and sellers. Splitting up the portfolio holdings for each agent the market clearing condition (5) is

$$\ell_H + (\delta + \epsilon)\, k_H + \ell_L + (\delta - \epsilon)\, k_L + pD(p) = \theta_H + \theta_L. \tag{6}$$

Implicitly differentiating gives the price impact of lucky buyers and unlucky sellers as

$$\frac{dp}{dk_H} = -\frac{1-\delta-\epsilon}{D(p)\,\left(\xi_p - 1\right)} \quad \text{and} \quad \frac{dp}{dk_L} = -\frac{1-\delta+\epsilon}{D(p)\,\left(\xi_p - 1\right)},$$

where both price impacts are negative given the assumption about the outside demand elasticity and $\delta + \epsilon < 1$. Without idiosyncratic risk ($\epsilon = 0$), the price impacts are identical and equal to half the aggregate price impact (which sums the impact of both agents). With idiosyncratic risk, the difference in price impacts is systematically important. The impact on the price p of an agent increasing illiquid asset holdings k (and decreasing liquid asset holdings ℓ) depends on the difference between the coefficients on the agent's k and ℓ in the market clearing condition (6). Taking into account $\ell = 1 - k$, the total effect of a change in k is $-(1 - \delta - \epsilon)$ for buyers and $-(1 - \delta + \epsilon)$ for sellers. More idiosyncratic risk ϵ therefore decreases the effect of a portfolio shift for buyers and hence their price impact dp/dk_H (in absolute value) and increases the effect and price impact for sellers (again in absolute value).

With partial liquidation, the extra term (2) in the Social Planner first-order condition due to the price effect dp/dk becomes

$$\frac{1-\alpha}{2}\left(u'(c_{2L})\, s_L^{\text{part}} - u'(c_{2H})\, d_H\right)\frac{R}{p}\frac{dp}{dk}.$$

Substituting in the price effect, this term can be written as

$$-\,2\,(1-\delta)\left(u'(c_{2L})\, s_L^{\text{part}} - u'(c_{2H})\, d_H\right) X, \tag{7}$$

with $X = \frac{1-\alpha}{2}\frac{R}{p}\frac{1}{D(p)\left(\xi_p - 1\right)} > 0$. Since $c_{2L} < c_{2H}$ and $s_L^{\text{part}} \geq d_H$, the Social Planner term is unambiguously negative at the Walrasian allocation, and so the Social Planner chooses lower illiquid asset holdings, $k^{\mathrm{SP}} < k^{\mathrm{WE}}$, the standard result. The extra term (3) in the Cournot first-order condition with partial liquidation becomes

$$\frac{1-\alpha}{2}\left(u'(c_{2L})\,s_L^{\text{part}}\frac{dp}{dk_L} - u'(c_{2H})\,d_H\frac{dp}{dk_H}\right)\frac{R}{p},$$

which, when substituting in the price effects with partial liquidation, can be written as[5]

$$\underbrace{-\,(1-\delta)\left(u'(c_L)\,s_L^{\text{part}} - u'(c_H)\,d_H\right)X}_{1/2\text{ SP term}} \underbrace{-\,\epsilon\left(u'(c_L)\,s_L^{\text{part}} + u'(c_H)\,d_H\right)X}_{\text{Value of redistributive trades}}. \tag{8}$$

Compared to the Social Planner, the expression in (8) shows that we can separate the strategic behavior of Cournot agents into two forces. First, Cournot agents do not completely internalize their aggregate impact on the price, which is the first term that is half of the Social Planner term in (7). This force alone would lead Cournot agents to partially mitigate the externality. Second, Cournot agents value separately the price impact when a buyer (when they don't want to push up the price) and when a seller (when they don't want to push down the price). How important this second force is depends on the amount of idiosyncratic risk ϵ, which determines the difference between lucky and unlucky agents' shocks. The greater is ϵ, the more cash lucky agents have to buy assets (and hence the more they will increase the price when they buy) and the more unlucky agents need to sell assets (and hence the more they will decrease the price when the sell). Compared to the Social Planner term, higher idiosyncratic risk makes the Cournot term more negative, and if ϵ is sufficiently large then Cournot agents will even overcorrect the externality.[6]

To understand why Cournot generates these two effects, it is helpful to consider that there are two different types of trades occurring in equilibrium, each with different welfare implications. First, there are net sales to outside investors, which are inefficient, so the Social Planner wants to minimize these trades. But second, there are trades between lucky and unlucky agents. These trades are purely redistributive ex post and do not affect overall welfare because agents are symmetric ex ante. The Social Planner does not care about these redistributive trades per se, but individual agents do care. As a result, it is privately optimal for Cournot agents to consider how they will impact the price in these redistributive trades between lucky and unlucky agents — and this occurs only be-

[5] We thank an anonymous referee for suggesting this decomposition.

[6] Note that the demand elasticity ξ_p of outside investors shows up in the same way for the Planner and Cournot terms (through X), scaling the overall effect together.

cause agents consider the price impacts of buying and selling separately. When this effect is strong, there is overcorrection of the externality.[7]

Showing this result explicitly requires closing the model. In Section 3 we explicitly consider a variation of the Lorenzoni (2008) model to see precisely when the the previous intuition carries through in equilibrium.

2.4.2 Full liquidation

With full liquidation by unlucky agents, asset demand is unchanged but supply no longer depends on price ($s_L^{\text{full}} = k$). Substituting the expressions for d_H and s_L^{full} into equation (4) and rewriting market clearing in terms of cash supplied and demanded yields

$$\ell + \delta_H k + pD(p) = pk + \theta_L. \tag{9}$$

First, we can implicitly differentiate the market clearing equation (9) for the effect of aggregate asset holdings on the equilibrium price:

$$\frac{dp}{dk} = -\frac{1 - \delta_H + p}{D(p)\left(\xi_p - 1\right) + k}.$$

Given the assumption about the outside demand elasticity and $\delta_H < 1$, the price is decreasing in agents' aggregate holdings of the illiquid asset.

Second, we again solve for the price impacts separately of buyers and sellers. Splitting up the portfolio holdings for each agent, the market clearing condition (9) is

$$\ell_H + \delta_H k_H + pD(p) = pk_L + \theta_L.$$

Implicitly differentiating gives the price impact of lucky buyers and unlucky sellers as

$$\frac{dp}{dk_H} = -\frac{1 - \delta_H}{D(p)\left(\xi_p - 1\right) + k} \quad \text{and} \quad \frac{dp}{dk_L} = -\frac{p}{D(p)\left(\xi_p - 1\right) + k},$$

where both price impacts are negative given the assumption about the outside demand elasticity. In contrast to the partial liquidation case, due to the fact that under full liq-

[7] If we supposed that the Social Planner also cared about the welfare of outside investors, then this would further strengthen our result. Caring about outside investors would decrease the magnitude of the Social Planner term, but the Cournot term would be unaffected. In the language of this paragraph, the Planner would put less weight on decreasing inefficient trades between agents and outside investors, but Cournot agents would continue to put the same weight. Thus, Cournot agents would be even more likely to "overcorrect" the externality in this case.

uidation the unlucky agents' supply is fully inelastic, their price impact dp/dk_L is now proportional to the price. As a result, the price impact of a seller can be much lower than that of a buyer if p is low.

With full liquidation, the extra term (2) in the Social Planner first-order condition due to the price effect dp/dk becomes

$$\frac{1-\alpha}{2}\left(u'(c_{1L})\,k-\beta u'(c_{2H})\,\frac{R}{p}\,d_H\right)\frac{dp}{dk}.$$

Substituting in the price effect, this can be written as

$$-\left(1-\delta_H+p\right)\left(u'(c_{1L})\,k-\beta u'(c_{2H})\,\frac{R}{p}\,d_H\right)Z,$$

with $Z=\frac{1-\alpha}{2}\frac{1}{D(p)\left(\xi_p-1\right)+k}>0$. With $k\geq d_H$ (net sales to outside investors) and supposing the standard condition that $u'(c_{1L})>\beta u'(c_{2H})\,\frac{R}{p}$ (e.g. Diamond and Dybvig, 1983), the Social Planner term is negative, and so the Social Planner chooses lower illiquid asset holdings, $k^{\text{SP}}<k^{\text{WE}}$, the standard result.

The extra term (3) in the Cournot first-order condition with full liquidation becomes

$$\frac{1-\alpha}{2}\left(u'(c_{1L})\,s_L^{\text{full}}\frac{dp}{dk_L}-\beta u'(c_{2H})\,\frac{R}{p}\,d_H\frac{dp}{dk_H}\right).$$

Substituting in the price effects with full liquidation, this can be written as

$$-\left(pu'(c_{1L})\,k-(1-\delta_H)\beta u'(c_{2H})\,\frac{R}{p}\,d_H\right)Z.$$

Thus, Cournot agents choose higher illiquid asset holdings than the Walrasian level, $k^{\text{CN}}>k^{\text{WE}}$, if and only if, at the Walrasian allocation we have

$$u'(c_{1L})\,kp<\beta u'(c_{2H})\,\frac{R}{p}d_H\left(1-\delta_H\right).$$

While this condition is given in terms of endogenous objects, notably investment and the asset price, we see that high illiquid asset holdings in Cournot are more likely the lower the equilibrium price p is. Note that all else equal, the weight on the marginal utility as a seller goes to zero as the price p decreases since the price impact goes to zero; similarly, the weight on the marginal utility as a buyer explodes as the price decreases. This suggests that the Cournot term can be *positive* at the Walrasian allocation, i.e. Cournot agents would prefer to hold even more illiquid assets than in the Walrasian equilibrium, and thus internalizing price impact would exacerbate the externality.

In contrast to the partial liquidation case, with full liquidation redistributive trades between lucky and unlucky agents are at the very heart of addressing the externality (we can even remove outside investors entirely, $D(p) = 0$, and the model goes through). Because markets are incomplete, agents have no way to insure against receiving a bad liquidity shock; the only thing unlucky agents can do is sell their entire asset holdings to lucky agents. The asset price p thus plays a critical role in "providing insurance": a higher price transfers resources from lucky agents to unlucky agents, which is precisely what insurance would do. The Social Planner therefore considers how aggregate investment affects the ability of these redistributive trades to provide insurance between agents, similar to Geanakoplos and Polemarchakis (1986). The utility benefit from these redistributive trades is given by $u'(c_{1L})\, k - \beta u'(c_{2H})\, \frac{R}{p}\, d_H$, which is multiplied overall by the aggregate price impact dp/dk. For the Social Planner, lower investment in the illiquid asset (and thus a higher price p) is unambiguously good for providing insurance.

Cournot agents consider how they privately fare when they are unlucky or when they are lucky. Because of the difference between the price impact when selling and when buying, the Cournot term puts a weight of p on the marginal utility when unlucky and a weight of $1 - \delta_H$ on the marginal utility when buying — but the Social Planner makes no such distinction. When the asset price is very low, the marginal impact of selling additional illiquid assets is very low, and thus it is not privately optimal for agents to worry about pushing down the price when selling. However, the marginal impact of buying additional assets is comparatively much higher, and so it is privately optimal to worry about pushing up the price when buying. When the price is low, Cournot agents therefore care ex-ante more about pushing up the price compared to pushing it down, which is why they hold more illiquid assets and less liquidity. Hence, the marginal private value of illiquid investment can be positive for Cournot agents, whereas it is strictly negative for the Planner.

Showing this result explicitly requires closing the model to solve for the price as a function of investment to also consider the joint behavior of consumption/marginal utilities and the price. In Section 4 we explicitly consider a variation of the Allen and Gale (2004) model to see precisely when the asset price is low in equilibrium.

3 Cournot in a productivity shock model

The setting for our model with productivity shocks is similar to Lorenzoni (2008). In this model, the key choice is the ex-ante scale of debt-funded investment in productive but risky capital. Since unlucky agents sell capital to repay debts but continue operating, this setting corresponds to the partial liquidation case of the model in Section 2. Ex post, more investment is preferred if hit by a good productivity shock, while less investment, with less debt to repay, is preferred if hit by a bad productivity shock. The canonical result in this type of model is that a pecuniary externality leads to inefficiently high borrowing in the Walrasian equilibrium — the "inefficient credit booms" of Lorenzoni (2008). We show that internalizing the pecuniary externality through Cournot behavior can *overcorrect* the standard inefficiency by leading to underinvestment even compared to the Social Planner.

3.1 Model setup

We modify some of the notation from Section 2 to match the notation common in the literature. There are three periods, $t = 0, 1, 2$. The $2N$ agents are now referred to as firms and the outside investors are $2N$ households.[8] Households are risk neutral with deep pockets and do not discount consumption. Firms consume at $t = 2$ and have risk-averse utility $u(c)$ with $\lim_{c\to 0} u'(c) = \infty$. Capital can be irreversibly produced from consumption goods at unit cost and is perfectly durable. Firms have access to a linear production technology using capital in each period. Capital k_i held by firm i at $t = 0$ produces $A_i k_i$ consumption goods at $t = 1$, where A_i is uncertain. Period 2 functions as a continuation value, so we assume that every unit of capital held at $t = 1$ produces one unit of consumption at $t = 2$.[9] Firms are each endowed with $n > 0$ units of capital at $t = 0$ and can borrow at a rate of $r \geq 1$; for simplicity, we assume that borrowing is risk free.[10] We assume that $\mathbb{E}[A_i] > r$ so firms will leverage to invest. Denoting borrowing by $b_i \geq 0$, firm i's balance sheet at $t = 0$ satisfies $k_i = n + b_i$.

Households have access to an inferior production technology that yields $F(k)$ consumption goods at $t+1$ for capital holdings k at t, with $F(k) = a\log(1+k)$. This technology implies that households buy capital to produce if the capital price is below a. To ensure that households only buy capital following a fire sale at $t = 1$, we suppose that

[8] We do not need the number of firms to equal the number of households. We only need the number of households to be proportional to the number of firms to ensure that the economy properly scales as N varies.

[9] Modeling firms as risk-averse with linear production is a tractable way to generate a motive for insurance. We could also model firms as risk-neutral with curvature in their production technology (see Holmström and Tirole, 1998).

[10] We could endogenize r as an outside option available to impatient lenders (see Appendix B).

$a < 1$.[11]

As in the unified model, there are two aggregate states in the economy at $t = 1$. In the good state, all firms have productivity $\overline{A} > r$ and are therefore able to repay their debt. In the bad state, half of the $2N$ firms, randomly selected, are unlucky and have low productivity A_L, and the other half are lucky and have high productivity A_H with $A_H > A_L$ and low average productivity:

$$\underline{A} = \tfrac{1}{2}A_L + \tfrac{1}{2}A_H < r$$

We mainly consider the case $A_H > r > A_L$ but also discuss the case $r > A_H > A_L$ below. We assume that firms cannot borrow more in the bad state at $t = 1$, so an unlucky firm i has a cash shortfall $A_L k_i - rb_i < 0$, forcing it to sell capital. A lucky firm j has a cash surplus $A_H k_j - rb_j > 0$, allowing it to buy capital. The low average productivity $\underline{A}$ ensures that households, in addition to lucky firms, will buy capital in the bad state.

This setting corresponds to the partial liquidation regime of the unified model in Section 2 with the shocks mapped as $\delta_i = A_i - r$ and $\theta_i = -rn$ and with households as the outside investors. Allowing firms to hold liquid assets in addition to capital is equivalent to having firms simply hold less debt. Accordingly, we will consider firms' investment and implied borrowing decisions and discuss over-borrowing or over-investment, though the reader should understand that over-borrowing is equivalent to under-provision of liquidity (i.e. holding too few liquid assets).

In the bad aggregate state, capital trades at an endogenous price p. A firm i with low productivity sells part of its capital to repay debts and supplies

$$s_{iL} = \frac{rb_i - A_L k_i}{p} = \frac{(r - A_L)\,k_i - rn}{p}$$

units of capital, while a firm j with high productivity uses its cash surplus to buy capital and demands

$$d_{jH} = \frac{A_H k_j - rb_j}{p} = \frac{rn + (A_H - r)\,k_j}{p}$$

units. Households are perfectly competitive and their demand for capital is $D(p) = a/p - 1$.[12] Thus, market clearing for capital, $\sum s_{iL} = \sum d_{jH} + \sum D(p)$ with N low types, N high

[11] In this case, letting ℓ denote investments in liquid assets (e.g., cash), the budget constraint would be $p_0 k + \ell = n + b$. It is easy to verify that consumption in each state, as well as quantities of assets sold/purchased, are just a function of $b - \ell$, and so ignoring liquidity holdings is equivalent to folding liquidity holdings into the debt in our baseline analysis.

[12] Households' demand is the solution to $\max_D \{a \log(1 + D) - pD\}$ with first-order condition

types and $2N$ households, implies that the price of capital in the bad state is

$$p = a + rn + \sum_{j\in H} \frac{(A_H - r)\, k_j}{2N} - \sum_{i\in L} \frac{(r - A_L)\, k_i}{2N}. \tag{10}$$

In the good state, all firms have the same productivity and consumption. In the bad state, a low-productivity firm sells capital, resulting in low consumption while a high-productivity firm buys capital, resulting in high consumption. Accordingly, the expected utility of firm i at $t = 0$ is given by

$$\alpha u(\bar{c}_i) + \frac{1-\alpha}{2}\, u(c_{iL}) + \frac{1-\alpha}{2}\, u(c_{iH})\,. \tag{11}$$

with

$$\bar{c}_i = rn + \left(\overline{A} + 1 - r\right) k_i, \quad c_{iL} = \frac{rn}{p} - \frac{r - A_L - p}{p}\, k_i, \quad c_{jH} = \frac{rn}{p} + \frac{A_H - r + p}{p}\, k_j.$$

3.2 Walrasian equilibrium

In the Walrasian equilibrium, all firms act as price takers with respect to the $t = 1$ price of capital p when choosing their level of borrowing at $t = 0$ to maximize their expected utility from (11). Taking p as exogenous, the first-order condition of a firm in the Walrasian equilibrium is

$$\underbrace{\alpha\left(\overline{A} + 1 - r\right) u'(\bar{c})}_{\text{good state: benefit}} + \frac{1-\alpha}{2}\left(\underbrace{-\frac{r - A_L - p}{p}\, u'(c_L)}_{\text{low productivity: cost}} + \underbrace{\frac{A_H - r + p}{p}\, u'(c_H)}_{\text{high productivity: benefit}} \right) = 0. \tag{12}$$

The first term is the benefit of more capital in the good state, where everyone receives a high productivity shock and holding more capital yields a net return $\overline{A} + 1 - r > 0$. The second term is the cost or benefit of more capital in the bad state, depending on whether the firm receives a a low or a high productivity shock. Holding more capital hurts a firm in the bad state if it has low productivity since it forces more costly sales of capital, yielding a net return $-(r - A_L - p)/p$ but benefits a firm with high productivity since it allows for more profitable purchases of fire-sold capital, yielding a net return $(A_H - r + p)/p$.

$a\,(1 + D)^{-1} = p$.

3.3 Social Planner

The Social Planner maximizes firm utility while being constrained to a choice of investment (and implied borrowing) at $t = 0$, just like the firms themselves. To enable a direct comparison to the firms' first-order condition (12), we do not explicitly consider household welfare in the Social Planner's problem.[13] We replicate the standard (and intuitive) result that the Walrasian equilibrium invests in too much capital.

The Social Planner chooses a single level of capital for all firms to maximize their expected utility from (11) but takes into account the effect on the equilibrium price of capital (10) which, setting $k_i = k_j = k$ for all i and j, simplifies to

$$p = a + rn - \left(r - \frac{1}{2}\left(A_L + A_H\right)\right)k = a + rn - \left(r - \underline{A}\right)k.$$

Compared to the Walrasian equilibrium first-order condition (12), the Social Planner's first-order condition contains an extra term that considers the impact of capital holdings on the price:

$$\frac{1-\alpha}{2}\left(u'(c_L)\frac{\partial c_L}{\partial p} + u'(c_H)\frac{\partial c_H}{\partial p}\right)\frac{dp}{dk}. \tag{13}$$

A higher level of capital decreases the fire-sale price, $dp/dk = -\left(r - \underline{A}\right) < 0$, which is bad for low types who sell capital and have consumption increasing in p (for sufficiently small n), $\partial c_L/\partial p = ((r - A_L)k - rn)/p^2 > 0$, but good for high types who buy capital and have consumption decreasing in p, $\partial c_H/\partial p = -((A_H - r)k + rn)/p^2 < 0$. Thus, the Social Planner trades off the loss to low types against the gain to high types (marginal-utility weighted). We can simplify the term in parentheses in (13), capturing the trade-off, as

$$u'(c_L)\frac{\partial c_L}{\partial p} + u'(c_H)\frac{\partial c_H}{\partial p} = \frac{1}{p}\left(u'(c_L)\,s_L - u'(c_H)\,d_H\right).$$

By assumption, capital sales by low types exceed capital purchases by high types, i.e. $s_L > d_H$, and marginal utility of high types is less than that of low types. Hence, $s_L u'(c_L) > d_H u'(c_H)$ and the additional Social Planner term in (13) is negative so that the Social Planner chooses a lower level of capital.

Proposition 1 (Standard inefficiency of Walrasian equilibrium)**.** *The pecuniary externality leads to inefficiently high investment in the Walrasian equilibrium, $k^{\text{WE}} > k^{\text{SP}}$.*

This is the standard result as shown by Lorenzoni (2008); the Social Planner holds less

[13] Including household welfare would only strengthen our result of Cournot agents overcorrecting the externality since it would reduce the Social Planner's incentive to mitigate fire sales that benefit households.

capital, which reduces fire sales, increasing the asset price in the bad state and increasing production since less capital is sold to low-productivity households.

3.4 Cournot equilibrium

In the Cournot equilibrium, firms take into account the effect of their own investment choice at $t = 0$ on the equilibrium price at $t = 1$, i.e. they maximize their expected utility from (11) subject to (10). A Cournot firm's first-order condition therefore also contains a price-effect term but, in contrast to the Social Planner, the Cournot firm considers separately the price effect it has as a high or low type:

$$\frac{1-\alpha}{2}\left(u'(c_L)\,\frac{\partial c_L}{\partial p}\,\frac{dp}{dk_L} + u'(c_H)\,\frac{\partial c_H}{\partial p}\,\frac{dp}{dk_H}\right) \tag{14}$$

Recall how the equilibrium price of capital (10) depends on individual firms' level of capital:

$$\begin{aligned} p &= a + rn + \sum_{i\in H}\frac{(A_H - r)\,k_i}{2N} - \sum_{j\in L}\frac{(r - A_L)\,k_j}{2N}, \\ &= a + rn - (r - \underline{A})\,k \quad \text{for} \quad k_i = k_j = k. \end{aligned}$$

While the relationship between the capital price and the aggregate level of capital is negative, the effect on the capital price as a low or high type differs. A low type firm has a negative effect on the price since its cash shortfall, which forces sales, increases with its initial investment; a high type firm has a positive effect on the price since its cash surplus, which is used for purchases, also increases with its initial investment:

$$\frac{dp}{dk_L} = -\frac{r - A_L}{2N} < 0 \quad \text{and} \quad \frac{dp}{dk_H} = \frac{A_H - r}{2N} > 0.$$

Combining the effects on consumption, $\partial c_L/\partial p > 0$ and $\partial c_H/\partial p < 0$, with the price impacts, we therefore have

$$\frac{\partial c_L}{\partial p}\,\frac{dp}{dk_L} < 0 \quad \text{and} \quad \frac{\partial c_H}{\partial p}\,\frac{dp}{dk_H} < 0. \tag{15}$$

That is, both as a seller *and* as a buyer, the extra term in a Cournot firm's first-order condition is negative, biasing *downward* investment at $t = 0$.

Comparison to the Walrasian allocation. The Walrasian first-order condition (12) and the Cournot first-order condition differ only in the price-effect term (14). From (15), we know that the extra term is negative, so the Cournot equilibrium will always have less capital than the Walrasian equilibrium. In this productivity shock model, internalizing the price impact therefore does correct the pecuniary externality, $k^{\mathrm{CE}} < k^{\mathrm{WE}}$. The question is how much.

Comparison to Social Planner allocation. Cournot firms will hold *even less* capital than the Social Planner if, at the Social Planner allocation, the extra term (14) in the Cournot first-order condition is smaller than the extra term (13) in the Social Planner first-order condition. Substituting in for the derivatives in (13) and (14), and simplifying, we obtain a simple condition for when the Cournot term is smaller than the Social Planner term.

Proposition 2 (Overcorrection in Cournot equilibrium)**.** *The pecuniary externality leads to inefficiently low investment in the Cournot equilibrium, $k^{\mathrm{CE}} < k^{\mathrm{SP}}$, if and only if*

$$\frac{2N\left(r-\underline{A}\right)-r+A_L}{2N\left(r-\underline{A}\right)-r+A_H} < \frac{u'(c_H)\, d_H}{u'(c_L)\, s_L}. \tag{16}$$

While the right-hand side of condition (16) is positive, the left-hand side is negative for small N and large $A_H - A_L$, holding r and $\underline{A}$ constant with $r - \underline{A}$ not too large. This yields the following comparative statics

Corollary 1. *Cournot behavior is more likely to overcorrect the pecuniary externality in the productivity shock model if (i) the degree of idiosyncratic productivity risk is larger (high $A_H - A_L$) and (ii) the number of Cournot agents is smaller (low N).*

Figure 1 illustrates the potential for Cournot to not only mitigate the inefficiently high investment of the Walrasian equilibrium but to over-correct it. The figure compares the levels of investment in capital in the Walrasian and Cournot equilibria to the efficient level.[14] As the degree of productivity risk increases, the efficient level of investment declines and is always lower than the level of investment in the Walrasian equilibrium. The Cournot equilibrium corrects this inefficiency as long as productivity risk is sufficiently low. Once idiosyncratic risk is sufficiently high, the Cournot equilibrium over-corrects the

[14] For graphical clarity we use $N = 1$ (two Cournot firms) as a baseline and also show the case with $N = 3$ (six Cournot firms), but choosing higher N would not qualitatively change the results so long as N is not too large (see the discussion of empirical plausibility below). The figure varies the degree of idiosyncratic risk by varying $A_H - A_L$ on the horizontal axis while keeping average productivity $\underline{A}$ constant.

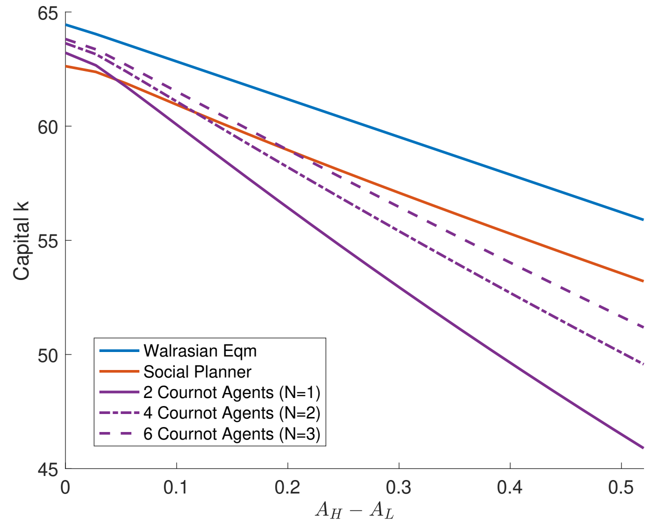

Figure 1: Comparison of investment of Walrasian and Cournot equilibrium to the efficient level. The figure shows the levels of investment k in the Walrasian equilibrium, the Cournot equilibrium, and the Social Planner allocation for different levels of idiosyncratic productivity risk, $A_H - A_L$. With constant relative risk aversion 1, $\alpha = 0.85$, $a = 0.93$, $n = 1$, $N \in \{1, 2, 3\}$, $\mathbb{E}[A] = 1.05$, $r = 1.02$, and $\underline{A} = 0.99$.

over-investment of the Walrasian equilibrium, leading to inefficiently *low* investment. Naturally, the region of overcorrection is larger the smaller the number of Cournot agents is.

Of course, for sufficiently low idiosyncratic risk $A_H - A_L$, condition (16) for under-investment compared to the Social Planner reverses and the Cournot equilibrium leads to investment higher than efficient but lower than in the Walrasian equilibrium. In particular, this is what happens with Cournot in the standard model of Lorenzoni (2008), which our model nests in the case of no idiosyncratic risk ($A_H = A_L < r$).

In sum, while Cournot does mitigate the pecuniary externality as in the standard formulation of the model, for sufficiently high idiosyncratic risk, Cournot will overcorrect, leading to under-investment relative to the Social Planner.

3.5 Empirical plausibility and welfare

The model in this section is simple and the interesting results depend on parameters. Whether Cournot overcorrects the inefficiency mainly depends on the degree of idiosyncratic risk. In this section, we argue that the parameter values necessary for the surprising Cournot effects are not implausible. Given the concavity of agents' utility, welfare decreases as the level of investment k moves away from the efficient level. How much welfare in the Cournot and Walrasian equilibria suffers relative to the Planner allocation depends

Table 1: Leverage Model Parameters.

α	a	n	r	$\mathbb{E}[A]$	$\underline{A}$	$A_H - A_L$
0.85	0.93	1	1.02	1.05	0.99	0.418

on how much the level of capital differs from the efficient level, and the utility cost of that deviation (i.e. risk aversion). In particular, the welfare cost of Cournot behavior depends on the level of idiosyncratic risk and the utility benefit associated with internalizing the price impacts.

First, whether Cournot overcorrects the inefficiency empirically mainly depends on the degree of idiosyncratic risk. We argue that the parameter values necessary for the surprising Cournot effects are not implausible. We let various moments from data determine likely values for parameters within this stylized model and find that, in reality, internalizing price impact likely overcorrects the externality. Second, given the calibrations of our simple model, we can also compare welfare across the allocations of Cournot, Walras and the Social Planner. The welfare losses can be meaningful, but whether welfare losses are worse under Cournot or Walrasian behavior depends critically on the level of curvature in firms' objective function. However, given the very stylized nature of the models, the quantitative welfare effects should be taken with a grain of salt.

The most important variable that determines the region we are in is the level of idiosyncratic risk facing firms in the bad aggregate state. There is substantial evidence that productivity dispersion is counter-cyclical (see Kehrig, 2015). Bloom et al. (2018) find that the standard deviation of micro-productivity shocks in recessions is 20.9%, which implies $A_H - A_L = 0.418$ in our model.[15] Apart from the level of risk aversion, our results are not sensitive to the remaining variables, which we set in relatively standard ways. We set $\alpha = 0.85$, which corresponds to the frequency of expansions post-WWII. We set the real rate to 2% and the expected return on capital to 5% so that capital earns 3% excess return in expectation. We let $a = 0.93$, so that the second-best user of capital has a 7% productivity loss, and we set $\underline{A} = 0.99$, corresponding to an aggregate shock 5% below average. Table 1 contains the parameters we use.

Figure 2 plots capital holdings relative to the efficient level and consumption equivalent losses for several values of risk aversion σ and varying the market size N. Regardless of the level of risk aversion, the Cournot equilibrium with $N = 1$ has capital investment that is about 10% *below* the efficient level. In other words, the level of idiosyncratic risk is

[15] Bloom et al. (2018) find that the unconditional standard deviation of micro-productivity shocks is 5.1% and that it is 4.1 times higher in recessions.

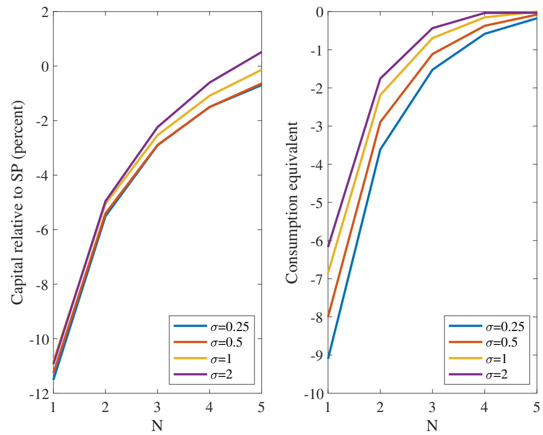

Figure 2: Quantitative effects in the productivity shock model. The figure shows the effect of market size N on the capital investment of the Cournot equilibrium relative to the Social Planner allocation (left panel), and on the welfare of the Cournot equilibrium relative to the Social Planner allocation in terms of consumption equivalent (right panel) under the calibration in Table 1 for different levels of risk aversion.

high enough that Cournot overcorrects the externality. Importantly, the fire-sale discount is on the order of 80%, implying substantial efficiency losses from capital being allocated to second-best users (households). A critical element of our model is that some first-best users (firms) are positioned to buy capital cheaply during downturns, which may seem at odds with the intuition in Shleifer and Vishny (1992), where fire sales occur because first-best users must sell to second-best (inefficient) users of capital. Our results show that, indeed, the primary force driving fire sales is the reallocation to households, which pushes down the price of capital substantially. Opportunistic buying by lucky firms is important for our mechanism without violating the intuition of Shleifer and Vishny (1992).

It is thus empirically plausible that Cournot can overcorrect the pecuniary externality, leading to inefficiently *low* levels of real investment. The levels of idiosyncratic risk present in the data are well above the level of idiosyncratic risk required for Cournot to overcorrect in our model. Nonetheless, there are several caveats that could push against our results. First, a high level of competition (high N) would bring the level of capital closer to the Walrasian level, thus weakening the overcorrection. Second, in the model debt is the only vehicle available for firms to borrow, implying that firms retain all their idiosyncratic risk. If in reality firms can shed some of this productivity risk, then bad shocks need not lead to forced sales (and good shocks need not lead to higher levels of cash).

The welfare implications of the overcorrection depend on the level of risk aversion for firms. Welfare decreases as the level of capital moves *away* from the efficient level, whether due to an overcorrection or due to an under-correction of the externality. If risk aversion is relatively high ($\sigma > 1$), then the overcorrection from Cournot is preferred over the under-correction from the Walrasian equilibrium. With risk aversion $\sigma = 2$ (a plausible estimate for risk aversion for households), welfare losses are 52% of consumption in the Walrasian equilibrium and 6% in the Cournot equilibrium. With $\sigma = 1$, the consumption equivalent losses are 20% and 6.8% respectively. In these cases, the overcorrection from Cournot is not so severe, and thus welfare is higher with a low level of competition. The policy implications in this case would be to allow industry concentration but to provide incentives for investment. One could easily argue that firms should be modeled as less risk averse than the typical household. For low levels of risk aversion, the welfare results change substantially. With $\sigma = 0.5$, the welfare loss in the Walrasian equilibrium is 2.8% while the welfare loss from Cournot is 8%; with $\sigma = 0.25$, the welfare losses are 0.16% and 9.1%, respectively. In this case, the Cournot overcorrection is very costly in terms of welfare losses and the policy implications are quite different because industry concentration is quite bad for welfare.

4 Cournot in a liquidity shock model

We now consider a standard model of fire sales due to liquidity shocks, in a setting based on Diamond and Dybvig (1983) with interim trade à la Allen and Gale (2004), potentially at fire-sale prices. In this model, the key choice is an ex-ante portfolio allocation between a liquid low-return asset and an illiquid high-return asset. Since unlucky agents (early consumers) sell all assets to meet liquidity needs, this setting corresponds to the full liquidation case of the model in Section 2. Ex post, the liquid asset is preferred if hit by a liquidity shock, while the illiquid asset is preferred otherwise.

The canonical result in this type of model is that a pecuniary externality leads to inefficiently low liquidity holdings in the Walrasian equilibrium (Allen and Gale, 2004). We show that internalizing the pecuniary externality through Cournot behavior can *exacerbate* the standard inefficiency, leading to even lower liquidity holdings than in the Walrasian equilibrium.

4.1 Model setup

We again modify some of the notation from Section 2 to match the notation common in the literature. There are three periods $t = 0, 1, 2$, and $2N$ agents, referred to as banks, that start with one unit of endowment at $t = 0$ and have two investment opportunities: (i) liquid assets, which, for each unit invested at $t = 0$, deliver 1 at $t = 1$ or $t = 2$; and (ii) illiquid assets, which, for each unit invested at $t = 0$, deliver $R > 1$ at $t = 2$ but nothing before. Denote by ℓ_i the fraction of bank i's funds invested in liquid assets (hence, $1 - \ell_i$ is invested in illiquid assets).

In the spirit of Diamond and Dybvig (1983), banks can be subject to liquidity shocks, in which case they only value early consumption at $t = 1$; otherwise they discount utility from consumption at $t = 2$ by $\beta \leq 1$:

$$U(c_1, c_2) = \begin{cases} u(c_1) & \text{with liquidity shock,} \\ u(c_1) + \beta u(c_2) & \text{without liquidity shock.} \end{cases}$$

We will suppose throughout our analysis that banks' utility u has relative risk aversion of at least 1. Together with $\beta R > 1$, our assumptions on preferences imply a standard demand for liquid claims and therefore a role for banks in providing liquidity insurance. We could instead assume that banks may have a high or low fraction of depositors withdraw, rather than a binary zero-one type of shock. This would have quantitative implications for our results without affecting the qualitative results. See Appendix B for a micro-foundation of such banks pooling resources from many households with correlated liquidity needs.

As before, there are two aggregate states at $t = 1$. In the good state, no liquidity shocks occur and no bank is forced to liquidate early. In the bad state, half the banks, randomly selected, receive liquidity shocks. These banks sell their illiquid assets to the other half that did not receive liquidity shocks at an endogenous price p. This setting corresponds to the full liquidation regime of the unified model in Section 2 with the shocks mapped as $\delta_i = 0$, $\theta_L = 0$ and $\theta_H = \infty$ and no outside investors.

A bank with a liquidity shock supplies $s_{iL} = 1 - \ell_i$ while a bank without a liquidity shock demands $d_{jH} = \ell_j / p$. Market clearing at $t = 1$ therefore corresponds to cash-in-the-market pricing, where the price is such that the total value of assets being sold equals the total cash available to buy assets (Allen and Gale, 1994):

$$\sum_{i \in L} (1 - \ell_i) \times p = \sum_{j \in H} \ell_j \tag{17}$$

Note that the market clearing condition (17) in the liquidity shock model differs from the market clearing condition (10) in the the productivity shock model of Section 3, which has additional demand from households. We show in Appendix D that adding such outside buyers to the liquidity shock model does not materially affect our results, as is also the case in the unified model of Section 2.[16]

In the good state, no-one has a liquidity shock and all banks consume at $t = 2$. In the bad state, a bank receiving a liquidity shock sells its illiquid assets and and consumes at $t = 1$. A bank that does not receive a liquidity shock uses its liquid assets to buy illiquid assets and consumes at $t = 2$. Accordingly, the expected utility of banks is given by

$$\alpha\beta u(\bar{c}_i) + (1-\alpha)\left(\tfrac{1}{2}u(c_{iL}) + \tfrac{1}{2}\beta u(c_{iH})\right), \tag{18}$$

with

$$\bar{c}_i = \ell_i + (1-\ell_i)\,R, \quad c_{iL} = \ell_i + (1-\ell_i)\,p, \quad c_{iH} = \ell_i \frac{R}{p} + (1-\ell_i)\,R.$$

It is clear that $p \leq R$ in equilibrium since high types would not be willing to pay more than R for illiquid assets. We suppose that the only buyers of illiquid assets are other banks, and liquidity shocks therefore lead to an asset price strictly below R in the bad aggregate state.[17]

4.2 Walrasian equilibrium

In the Walrasian equilibrium, all banks act as price takers with respect to the $t = 1$ price of illiquid assets when choosing their portfolio at $t = 0$ to maximize their expected utility from 18. Taking p as exogenous, a bank's first order condition in the Walrasian equilibrium is

$$\begin{aligned} \overbrace{\alpha\beta\,(R-1)\,u'(\bar{c})}^{\text{good state: cost}} &= \overbrace{\frac{1-\alpha}{2}\left((1-p)\,u'(c_L) + \beta\left(\frac{1}{p}-1\right)Ru'(c_H)\right)}^{\text{bad state: benefit with or without liquidity shock}}, \\ &= \frac{1-\alpha}{2}\,(1-p)\left(u'(c_L) + \beta\frac{R}{p}u'(c_H)\right). \end{aligned} \tag{19}$$

The left-hand side is the cost of holding extra liquidity in the good aggregate state, where no one receives a liquidity shock and holding more illiquid assets instead of liquid assets

[16] In the productivity shock model, L types have a cash shortfall forcing asset sales, i.e. supply a fixed dollar amount, and H types have a cash surplus to buy assets, i.e. demand a fixed dollar amount; without a decreasing residual demand from outside buyers (households), asset market equilibrium would not be well defined. In contrast, in the liquidity shock model, L types sell all their assets, i.e. supply a fixed asset amount. Together with H types demanding a fixed dollar amount, asset market equilibrium is well defined even without outside buyers.

[17] The model easily generalizes to additional buyers of illiquid assets as long as they are second-best users or have limited resources as is standard in the fire-sale literature (Shleifer and Vishny, 2011), which was the case in Section 2.

yields a net return $R - 1 > 0$. The right-hand side is the benefit of extra liquidity in the bad state; since the left-hand side is positive, it must be that the equilibrium price satisfies $p < 1$. Holding extra liquidity in the bad state then is good both as a seller of assets, since it requires fewer sales at net cost $1 - p > 0$, and as a buyer of assets since it allows more asset purchases with net return $\frac{1}{p} - 1 > 0$. Note the contrast to the productivity shock model in Section 3 where, in the Walrasian equilibrium, holding more capital benefits a buyer since it allows more purchases but hurts a seller since it requires more sales.

If there is no aggregate risk ($\alpha = 0$) so that only the bad state can occur, then the first-order condition 19 implies $p = 1$ in equilibrium. If $p < 1$, then assets are traded below cost so no-one wants to invest in them; sellers (state L) would rather hold liquidity and buyers (state H) would rather buy assets cheaply; vice versa for $p > 1$. Equilibrium in the case of no aggregate risk is pinned down by the no-arbitrage condition, $p = 1$, which leads to $c_L = 1$ and $c_H = R$. The resulting wedge in marginal utilities, $u'(c_L) > \beta R u'(c_H)$, represents the standard insufficient liquidity risk sharing of Diamond and Dybvig (1983). If there is aggregate risk ($\alpha > 0$), then the wedge in marginal utilities is $u'(c_L) > \beta \frac{R}{p} u'(c_H)$, which maintains the insufficient risk sharing.[18]

4.3 Social Planner

The Social Planner maximizes bank utility while being constrained to a choice of liquidity holdings at $t = 0$, just like the banks themselves. We replicate the standard (and intuitive) result that the Walrasian equilibrium provides inefficiently low liquidity.

The Social Planner chooses a single level of liquidity holding for all banks to maximize their expected utility from (18) but takes into account the effect on the equilibrium asset price (17), which, setting $\ell_i = \ell_j = \ell$ for all i and j, simplifies to $p = \ell/(1-\ell)$. Compared to the Walrasian first-order condition (19), the Social Planner's first-order condition has an additional term on the right-hand side, which considers how liquidity holdings will affect the asset price:

$$\frac{1-\alpha}{2}\left(u'(c_L) - \beta \frac{R}{p} u'(c_H)\right)(1-\ell)\frac{dp}{d\ell} \tag{20}$$

The Social Planner considers that more liquidity increases the price by $dp/d\ell = 1/(1-\ell)^2$, which benefits sellers who gain $u'(c_L)$, and hurts buyers who lose $\beta \frac{R}{p} u'(c_H)$. Since $u'(c_L) > \beta \frac{R}{p} u'(c_H)$, the Social Planner chooses higher liquidity than the Walrasian equilibrium.

[18] Making use of the proof in Diamond and Dybvig (1983), we have $2\ell u'(2\ell) > 2(1-\ell)\beta R u'(2(1-\ell)R)$. Since $c_L = 2\ell$, $c_H = 2(1-\ell)R$ and $p = \ell/(1-\ell)$ in equilibrium, this implies $u'(c_L) > \beta \frac{R}{p} u'(c_H)$.

Proposition 3 (Standard inefficiency of Walrasian equilibrium)**.** *The pecuniary externality leads to inefficiently low liquidity holdings in the Walrasian equilibrium, $\ell^{\mathrm{WE}} < \ell^{\mathrm{SP}}$.*

The intuition for the standard constrained inefficiency of the Walrasian equilibrium is that the market incompleteness prevents full insurance agains the liquidity risk. The constrained Social Planner, by changing the price, can perform a reallocation that is outside the asset span and thereby increase welfare (Geanakoplos and Polemarchakis, 1986).

In the case without aggregate risk ($\alpha = 0$), the Social Planner's first order condition yields the standard optimal risk sharing condition, $u'(c_L) = \beta R u'(c_H)$, of Diamond and Dybvig (1983). Without aggregate risk, our setup with trading at $t = 1$ essentially corresponds to the Jacklin (1987) model, and our result that liquidity under-provision can be corrected by increasing the asset price is found also in Farhi, Golosov, and Tsyvinski (2009) and Geanakoplos and Walsh (2018).

4.4 Cournot equilibrium

In the Cournot equilibrium, banks take into account the effect of their own liquidity choice at $t = 0$ on the equilibrium price at $t = 1$, i.e. they maximize their expected utility from (18) subject to (17), also resulting in an additional term in the first order condition:

$$\frac{1-\alpha}{2}\left(u'(c_L)\frac{dp}{d\ell_L} - \beta\frac{R}{p}u'(c_H)\frac{dp}{d\ell_H}\right)(1-\ell_i) \tag{21}$$

Similar to the productivity shock model, a Cournot bank distinguishes between the price it faces as a seller and the price it faces as a buyer as well as the effect extra liquidity holdings have in the two states. However, in contrast to the productivity shock model where the $t = 0$ choice has *opposite effects* on the two prices (more capital increases the buyer price and decreases the seller price), here the $t = 0$ choice has *the same effect* on the two prices (more liquidity increases the buyer price and the seller price). Specifically, from the equilibrium condition (17), the asset price is

$$p = \frac{\sum_{i\in H}\ell_i}{\sum_{j\in L}\left(1-\ell_j\right)}.$$

As a seller, the bank affects the denominator while as a buyer it affects the numerator; with $2N$ banks and taking as given other banks' (symmetric) equilibrium choice ℓ, we have

$$\frac{dp}{d\ell_L} = \frac{1}{N}\frac{\ell}{(1-\ell)^2} > 0 \quad \text{and} \quad \frac{dp}{d\ell_H} = \frac{1}{N}\frac{1}{1-\ell} > 0. \tag{22}$$

Compared to the Social Planner's price impact, $dp/d\ell = 1/(1-\ell)^2$, the Cournot bank's price impacts are uniformly lower, biasing downward the Cournot liquidity choice. We now consider how the allocation in the Cournot equilibrium compares to the constrained efficient allocation chosen by the Social Planner and the allocation in the Walrasian equilibrium.

Comparison to Social Planner allocation. Cournot leads to inefficiently low liquidity if, at the Social Planner allocation, the price-effect term of the Cournot first-order condition (21) is less than the price-effect term of the Social Planner first-order condition (20). Substituting in for the price effects, the condition becomes

$$\frac{1}{N}\left(u'(c_L)\,\ell - \beta\frac{R}{p}u'(c_H)\,(1-\ell)\right) < u'(c_L) - \beta\frac{R}{p}u'(c_H)\,. \tag{23}$$

In the natural case where the good aggregate state lowers the efficient level, i.e. for $p < 1$ at the Social Planner allocation, we have $u'(c_L) > \beta\frac{R}{p}u'(c_H)$ and $\ell < 1/2$ so condition (23) is satisfied and, as expected, Cournot liquidity is inefficiently low, $\ell^{\text{CE}} < \ell^{\text{SP}}$.[19]

Comparison to Walrasian allocation. To assess whether internalizing the price impact attenuates or exacerbates the inefficiently low liquidity holdings of the Walrasian equilibrium, the key comparison is between the allocations in the Cournot equilibrium and in the Walrasian equilibrium. Cournot yields less liquidity than the Walrasian equilibrium and therefore exacerbates the inefficiency if, at the Walrasian allocation, the price-effect term of the Cournot first-order condition (21) is negative.

Proposition 4 (Exacerbation in Cournot equilibrium)**.** *The pecuniary externality leads to even lower liquidity in the Cournot equilibrium than in the Walrasian equilibrium, $\ell^{\text{CE}} < \ell^{\text{WE}}$, if and only if*

$$u'(c_L)\,\frac{dp}{d\ell_L} - \beta\frac{R}{p}u'(c_H)\,\frac{dp}{d\ell_H} < 0. \tag{24}$$

We know that, at the Walrasian allocation, the marginal benefit of additional liquidity exceeds the marginal cost, $u'(c_L) > \beta\frac{R}{p}u'(c_H)$. But in the Cournot first-order condition (21), the price impacts act as weights on the benefit and the cost. Condition (24) is therefore satisfied if the seller price impact $dp_L/d\ell_i$ is sufficiently low relative to the buyer price impact $dp_H/d\ell_i$. From (22) we have that the seller price impact relative to the buyer

[19] The Social Planner will find it optimal to implement $p < 1$ as long as the good state is sufficiently likely and/or the illiquid asset sufficiently productive (high α and/or R).

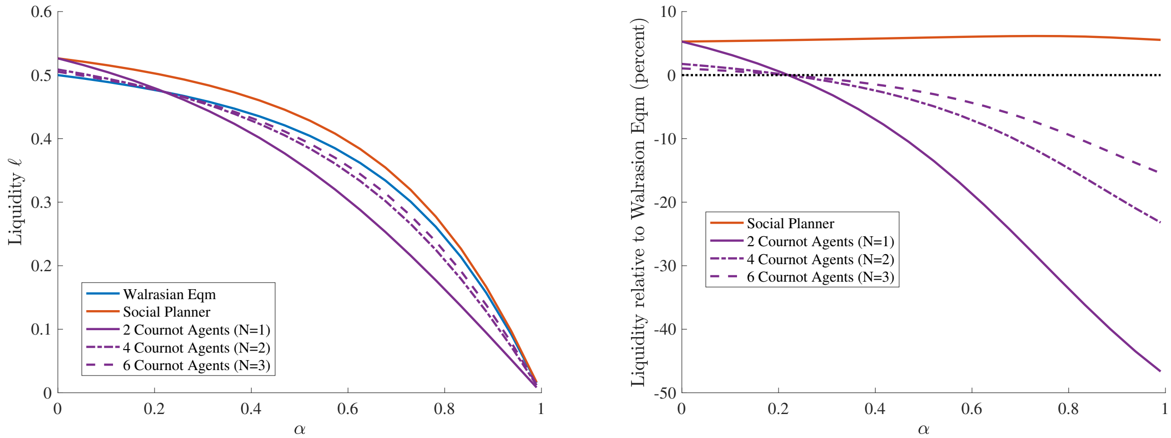

Figure 3: Comparison of liquidity provision of Walrasian and Cournot equilibrium to the efficient level. The left panel shows the equilibrium levels of liquidity ℓ in the Walrasian equilibrium, the Cournot equilibrium and the Social Planner allocation for different values of the probability of the good state, α. The right panel shows the ratios of the Social Planner and Cournot liquidities to the Walrasian liquidity. With log utility, $\beta = 0.9$, $R = 1.5$, and $N \in \{1, 2, 3\}$.

price impact depends on the level of the asset price p:

$$\frac{dp_L/d\ell_i}{dp_H/d\ell_i} = p.$$

This implies that if liquidity ℓ (and thus price p) is sufficiently low in the Walrasian equilibrium, then Cournot yields even less liquidity than the Walrasian equilibrium, exacerbating the inefficiency. For log utility, the "sufficiently low" condition simplifies to the fairly weak condition $p < \beta$, and more generally this case of sufficiently low p arises, e.g. if the bad state is not too likely.

Corollary 2. *Cournot behavior is more likely to exacerbate the pecuniary externality in the liquidity shock model if (i) the likelihood of the bad aggregate state is smaller (high α) and (ii) the number of Cournot agents is smaller (low N).*

Figure 3 illustrates the potential for Cournot to exacerbate the inefficiently low liquidity holdings of the Walrasian equilibrium. The figure compares the levels of liquidity provision in the Walrasian and Cournot equilibria to the efficient level.[20] As the good state without liquidity shocks becomes more likely (α increases), the efficient level of liquidity

[20]For graphical clarity we use $N = 1$ (2 banks) as a baseline and include $N = 3$ (6 banks), but choosing higher N would not qualitatively change the results so long as N is not too large (e.g. the results are similar with 10 banks with our preferred calibrations; see the discussion of empirical plausibility below).

declines but is always higher than the one provided by the Walrasian equilibrium. The Cournot equilibrium corrects this inefficiency only if the good state is sufficiently unlikely (liquidity risk is sufficiently high). Once the good state is sufficiently likely and liquidity risk therefore sufficiently low, the Cournot equilibrium exacerbates the under-provision of liquidity in the Walrasian equilibrium. The right panel shows that, for high α, the Cournot level of liquidity is substantially below the Walrasian level. Naturally, the region of exacerbation is larger the smaller the number of Cournot agents is.

For intuition, note that in the limit $\alpha \to 1$, liquidity has little ex-ante value and endogenously $\ell \to 0$, resulting in $p \to 0$. The seller price impact $dp_L/d\ell_i$, weighting the benefit of liquidity in condition (24), goes to zero while the buyer price impact $dp_H/d\ell_i$, weighting the cost of liquidity, does not. The Cournot equilibrium then holds very little liquidity because more liquidity would have a negligible price benefit when agents receive liquidity shocks and sell assets but a non-zero cost when agents do not receive liquidity shocks and instead buy assets.[21]

To understand this difference in the limit behavior of buyer and seller price impact, consider the equilibrium condition (17) determining the price p. Additional liquidity of buyers enters directly in the form of more cash while additional liquidity of sellers enters indirectly in the form of more assets with a factor p. The marginal effect of cash on the equilibrium condition is therefore always 1 but the marginal effect of additional assets is low if the price p is low.

In sum, with aggregate risk, Cournot can provide even less liquidity than the Walrasian equilibrium, in violation of the hypothesis that internalizing the pecuniary externality should lead to an allocation closer to the Social Planner's.

4.5 Empirical plausibility and welfare

Internalizing price impact in the liquidity shock model can either mitigate or exacerbate the pecuniary externality, depending on parameter values. Given the concavity of agents' utility, welfare decreases as the level of liquidity ℓ moves away from the efficient level. How much welfare in the Cournot and Walrasian equilibria suffers relative to the Planner allocation depends on how much the level of liquidity differs from the efficient level and the utility cost of that deviation (i.e. risk aversion). In particular, the welfare cost of Cournot behavior depends on the marginal price impacts (driven primarily by the equilibrium fire-sale price p) and the utility benefit associated with internalizing the price impacts.

[21] This requires that $\frac{dp}{d\ell_L} u'(c_L)$ goes to zero as long as the marginal utility does not increase too quickly, which holds as long as if risk aversion is not too high or marginal utilities are bounded.

Table 2: Liquidity Model Parameters.

Calibration	1	2	3
β	0.96	0.92	0.99
βR	1.03	1.03	1.03
α	0.98	0.97	0.91
σ	1.01	1.05	1.00

First, we find that, in reality, internalizing price impact likely exacerbates the externality. Empirically, whether Cournot exacerbates the inefficiency mainly depends on the severity of the fire sale in the bad state. We argue that the parameter values necessary for the surprising Cournot effects are not implausible. Internalizing price impact in the liquidity shock model can either mitigate or exacerbate the pecuniary externality, depending on parameter values. We let various moments from data determine likely values for parameters within this stylized model and find that, in reality, internalizing price impact likely exacerbates the externality. Second, given the calibrations of our simple model, we can also compare welfare across the allocations of Cournot, Walras and the Social Planner. The welfare consequences from Cournot behavior are orders of magnitude larger than the negligible welfare losses from Walrasian behavior. However, given the very stylized nature of the models, the quantitative welfare effects should be taken with a grain of salt.

Before considering the full exercise, consider the following back-of-the-envelope exercise. The risk aversion of banks is probably low; in the model, the lowest we can set risk aversion to is 1 (log utility). The impatience parameter β determines how much banks discount illiquid relative to liquid claims. Estimates of liquidity premia are typically on the order of basis points (20bps in Gertler and Kiyotaki, 2015) and so β should be close to 1. From our analytical results, Cournot will exacerbate the externality with log utility whenever the Walrasian fire-sale price is below β. Thus, if fire sales are a meaningful discount of fair value (more than 10% seems very conservative) and fair value is not too much greater than 1, then internalizing price impact will exacerbate the externality.

We consider two strategies to calibrate our parameters: target liquidity holdings to be 13% of banks assets[22] or target the fire sale to a 35% discount relative fair value. Table 2 contains the parameters used for each calibration, which we discuss in detail below. Figure 4 plots Cournot liquidity holdings relative to the Walrasian level and consumption equivalent losses for each calibration varying the market size N.

We show two parameterizations to hit 13% liquidity. In the first, we let $\beta = 0.96$, which

[22] This corresponds to the ratio of bank liquid reserves to bank assets in the U.S. (IMF International Financial Statistics).

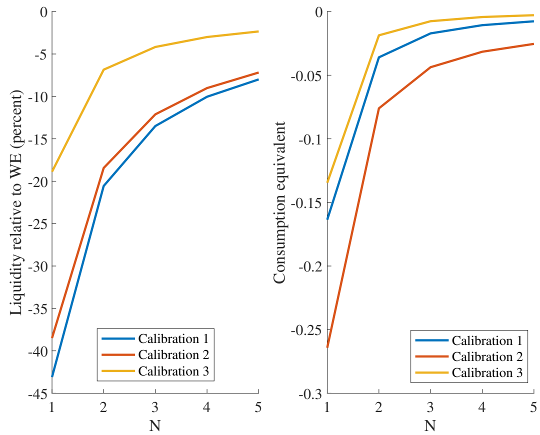

Figure 4: Quantitative effects in the liquidity shock model. The figure shows the effect of market size N on the liquidity provision of the Cournot equilibrium relative to the Walrasian equilibrium (left panel), and on the welfare of the Cournot equilibrium relative to the Social Planner allocation in terms of consumption equivalent (right panel) under the calibrations in Table 2.

corresponds to a standard annual discount rate; we suppose that $\beta R = 1.03$, so that illiquid assets earn about 3% excess returns; we suppose that $\alpha = 0.98$, so that financial crises occur 2% of the time (see Gertler and Kiyotaki, 2015, for similar estimates). With relative risk aversion $\sigma = 1.01$, the model then delivers 13% liquidity holdings in the Walrasian equilibrium. At this calibration, the efficient level of liquidity is 3.6% higher than the Walrasian level, but a Cournot equilibrium with $N = 1$ holds 43% *less* liquidity than the Walrasian equilibrium (i.e. banks hold 7.4% liquid assets), exacerbating the externality. In terms of welfare losses, the Walrasian equilibrium has welfare that is negligibly below the efficient level, while welfare in the Cournot equilibrium corresponds to a 0.16% loss in terms of consumption equivalent compared to the efficient outcome.

While this parameterization is entirely plausible, the probability of crises is somewhat low. Instead we now let $\alpha = 0.97$ (following Gertler and Kiyotaki, 2015), which on its own would significantly increase liquidity holdings in equilibrium. To hit our liquidity target, we set $\beta = 0.92$ and get $\sigma = 1.05$. For this calibration, the efficient level of liquidity is 9.8% higher than the Walrasian level, but a Cournot equilibrium with $N = 1$ holds 38.5% *less* liquidity than the Walrasian equilibrium, exacerbating the inefficiency. In terms of welfare, the loss in the Walrasian equilibrium is 0.008% in terms of consumption equivalent, while the loss in the Cournot equilibrium is 0.26%. In either case, the parameters are well within the range of parameters for which Cournot competition exacerbates the externality.

In the model, the level of liquidity directly determines the fire sale price in the bad state. Liquidity holdings of 13% imply a fire sale price of $p = 0.15$. It is fair to wonder if the right variable to target is liquidity holdings and not the level of fire sales directly, since the externality is after all determined by the fire sale in the asset price. We now target $p = 0.65 \times R$, which corresponds to a 35% discount over fair value for financial assets. One could reach this number, e.g. by considering the history of prices for ABX during the financial crisis and comparing trough levels to what prices ultimately returned to. It is more difficult to get the model to provide liquidity holdings high enough so that the fire sale price is this high. To do so, we set $\beta = 0.99$ and $\alpha = 0.91$, implying a very high likelihood of financial crises (higher than we believe to be empirically plausible). Maintaining $\beta R = 1.03$, the model requires $\sigma = 1$ in order to hit the target for fire sales. At this calibration, the efficient level of liquidity is 0.57% higher than the Walrasian level, but a Cournot equilibrium with $N = 1$ holds 19.3% *less* liquidity than the Walrasian equilibrium, exacerbating the inefficiency. In terms of welfare losses, the Walrasian equilibrium is negligibly below the efficient level, while welfare in the Cournot equilibrium corresponds to a 0.13% loss in terms of consumption equivalent.

In sum, all three calibrations are well within the range where Cournot exacerbates the externality, and the results for liquidity provision and welfare are meaningful. We do not take these results quantitatively seriously, but they do provide strong evidence at least for the direction of how the externality is affected (exacerbated, not mitigated). Furthermore, since we find that Cournot exacerbates the externality, the question "how much" depends on the competitiveness of the industry (i.e. on N). Less competition will lead to greater under-provision of liquidity. Thus, industry concentration is strictly bad for the fire sale externality, and policy should respond by providing greater incentives to hold liquid assets (disincentives to hold illiquid assets).

5 Conclusion

In light of increasing concentration in both real and financial markets, we have considered the effects of market power in standard macro-finance models of fire sales where pecuniary externalities lead to constrained inefficiency. We show that market power can not only mitigate but *overcorrect* the inefficiently high borrowing in a canonical model of leverage choice with productivity shocks, representative of firms with real investment. In contrast, we show that internalizing the price impact can *exacerbate* the inefficiently low liquidity holdings in a canonical model of portfolio allocation with liquidity shocks, representative of financial intermediaries engaged in liquidity transformation.

In terms of policy implications, our results highlight that intervention has to be tailored to both the type of activity and the competitiveness of a given sector. We interpret the liquidity risks faced by the real sector as generally corresponding to partial liquidation, whereas we interpret the liquidity risks faced by the financial sector as corresponding to full liquidation. Accordingly, the policy implications for the real and financial sectors are completely different. For the real sector, where market power overcorrects the tendency toward inefficient credit booms, our results imply a need for stronger investment stimulus as the sector becomes more concentrated, e.g. through an expansion of the favorable tax treatment of debt financing. For the financial sector, where market power exacerbates the tendency to hold insufficient liquidity, our results imply a need for stronger liquidity regulation as the sector becomes more concentrated, e.g. through a tightening of the Basel III Liquidity Coverage Ratio. Finally, our results speak to antitrust policy, highlighting additional welfare-relevant effects of market power. For example, in the debate whether concentration enhances financial stability (Bordo et al., 2015), our results show how important stringent liquidity regulation is for achieving stability benefits from concentration.

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Appendices for online publication

A Conditions for unified model

To capture both canonical frameworks in our unified model, we impose conditions on the liquidity shock θ_1 that guarantee $c_1 = \theta_1$, i.e. a corner solution for intertemporal substitution between $t = 1$ and $t = 2$. Here, we explicitly study the intertemporal optimization at $t = 1$ with extra consumption e_1 such that $c_1 = \theta_1 + e_1$ and derive necessary conditions for $e_1 = 0$.

In the good state, the liquidity shock is $\overline{\theta}$ and intertemporal optimization at $t = 1$ solves

$$\max_{e_1} \left\{ u\left(\overline{\theta} + e_1\right) + \beta u\left(\ell - \overline{\theta} - e_1 + Rk\right) \right\}.$$

For for $e_1 = 0$ we need

$$u'\left(\overline{\theta}\right) \leq \beta u'\left(\ell - \overline{\theta} + Rk\right)$$

which implicitly defines a unique lower bound on $\overline{\theta}$.

In the bad state, a lucky agent decides between extra consumption or extra asset purchases but, since $p \leq 1$, leaves no liquidity for $t = 2$ so the optimization is

$$\max_{e_1} \left\{ u(\theta_L + e_1) + \beta u\left(R\left(k + \frac{\ell + \delta_H k - \theta_L - e_1}{p} \right) \right) \right\}.$$

For for $e_1 = 0$ we need

$$u'(\theta_L) < \beta \frac{R}{p} u'\left(R\left(k + \frac{\ell + \delta_H k - \theta_L}{p} \right) \right)$$

which implicitly defines a unique lower bound on θ_L.

For an unlucky agent in the case of partial liquidation, the intertemporal optimization is

$$\max_{e_1} \left\{ u(\theta_H + e_1) + \beta u\left(R\left(k - \frac{\theta_H + e_1 - (\ell + \delta_L k)}{p} \right) \right) \right\}.$$

For for $e_1 = 0$ we need

$$u'(\theta_H) < \beta \frac{R}{p} u'\left(R\left(k - \frac{\theta_H - (\ell + \delta_L k)}{p} \right) \right)$$

which implicitly defines a unique lower bound on θ_H. However, note that the lower bound for θ_L is higher than that for θ_H so the assumption $\theta_H > \theta_L$ guarantees that θ_H is above its lower bound.

B A single model of banks and real investment

In the main text, we present two separate models, one with a choice of investment level funded with debt and one with a choice of portfolio allocation between liquid and illiquid assets. In this appendix, we show how the two models can be viewed as representing two parts of an economy with financial intermediation. The liquidity trade-off model then represents the main decision of banks in allocating households savings between liquid assets and illiquid loans to firms; the leverage trade-off model then represents the main decision of equity constrained firms borrowing from banks to invest in productive assets.

The economy consists of households, banks, and firms, active in a real sector with productivity shocks and a financial sector with liquidity shocks. Households have funds to invest but are inefficient at operating capital and face uncertain consumption needs, while firms are efficient at operating capital but have small initial endowments. Banks, which are mutually owned by households, sit squarely between the two agents, taking deposits from households and providing loans to firms. There are three periods, $t = 0, 1', 1'', 2$. Liquidity shocks determining banks' solvency occur at $t = 1'$, while productivity shocks determining firms' output occur at $t = 1''$. The shocks can lead to fire sales of financial assets and real assets in the financial and real sector, respectively. Period $t = 2$ functions as "the future" or a continuation value for production in the economy.

At $t = 0$ banks have access to liquid and illiquid investment technologies. With the liquid technology, one unit of capital invested at $t = 0$ produces one unit of consumption good ("output") either at $t = 1'$ or $t = 1''$. With the illiquid technology, one unit of capital invested at $t = 0$ produces $R > 1$ units of output at $t = 1''$ but nothing at $t = 1'$. Illiquid investments can be traded at $t = 1'$ at any endogenous price p.

In each period, firms have access to a linear production technology using capital. Production at $t = 1''$ is risky: capital k invested at $t = 0$ produces Ak units of output at $t = 1''$, where A is uncertain with $\mathbb{E}[A] > R$ (i.e. the interest rate r from the productivity shock model equals the project return R from the liquidity shock model). To simplify, production at $t = 2$ is risk-free, with every unit of period-1 capital producing one unit of output at $t = 2$. Firms have small endowments of capital at $t = 0$, denoted by n, and have utility function $v(c)$ over consumption in period 2.

In each period, households have access to a production technology that takes capital k and yields $F(k) = a \log(1 + k)$ units of consumption goods in the next period. We suppose that $a < 1$ so that households are never the efficient users of capital. Each period contains a new generation of households endowed with one unit of capital. Importantly, however, households born at $t = 0$ are subject to liquidity shocks à la Diamond and Dybvig (1983): they will either consume at $t = 1''$ (late types), or they will receive a liquidity shock and be forced to consume at $t = 1'$ (early types). Households receive utility $u(c)$ over early consumption and $\beta u(c)$ over late consumption, with $\beta \leq 1$ and $\beta R > 1$. To simplify the analysis, we suppose that households born at $t = 1''$ are not subject to liquidity shocks.

Banks pool resources from many households in order to offer deposit contracts that provide liquidity in the sense of Diamond and Dybvig (1983). Banks serve a restricted economic area as in Allen and Gale (2004), able to take deposits only from a set of households with correlated liquidity needs (i.e. banks cannot serve the entire population of households and completely diversify away liquidity shocks). To simplify, we assume that the households in each area have perfectly correlated liquidity needs. As a result, a bank whose consumers are early types will be forced to liquidate its assets in the interim period. Thus, we can say that the bank is itself subject to liquidity shocks.

At $t = 0$, firms can borrow from banks by issuing non-contingent debt due at $t = 1''$. Firms cannot default, and therefore bank loans to firms are identical to investments in the illiquid technology. Thus, firms can borrow at a gross interest rate R. Firms cannot borrow new funds at $t = 1''$ but must repay debt using proceeds from production or from selling capital at an endogenous price p.

The economy therefore features the following financial frictions. In the financial sector, banks and households cannot insure against liquidity risk, and the price of illiquid assets is determined by cash-in-the-market pricing (i.e. bank capital is slow moving and so demand for assets must come from banks who do not receive liquidity shocks). In the real sector, firms cannot insure against productivity shocks (i.e. they are restricted to borrow using non-contingent debt), and firms are subject to borrowing constraints at $t = 1''$ (they cannot borrow to repay/roll over debts).

We assume that, relative to the firm sector, household endowments are sufficiently large and liquidity demands sufficiently small so that in equilibrium banks' demands for illiquid investments exceed firms' demands for borrowing. As a result, the flow of funds in the economy in equilibrium can be described as follows. At $t = 0$, households deposit all capital with banks. Banks allocate a fraction ℓ of capital to liquid investments and a fraction $1 - \ell$ to illiquid investments, a portion of which are loans to firms at an interest rate R. Firms borrow b units from banks, allowing them to invest $k = n + b$ in capital for risky projects. (Our relative size assumption means that in equilibrium $b < 1 - \ell$.) At $t = 1''$, firms repay debts, perhaps by selling capital to households at price p in order to do so, and remaining capital is invested in projects to produce at $t = 2$.

Given these assumptions, we can solve for equilibrium in this economy by considering the financial and real sectors separately. First, we can consider the equilibrium provision of liquidity by banks at $t = 0$ and analyze how internalizing price impacts in the market for illiquid investments affects the price p and the level of liquidity ℓ. Second, we can consider the equilibrium borrowing decision of firms at $t = 0$ and analyze how internalizing price impacts in the market for capital affects the price p, the level of investment k, and borrowing b. Because all bank loans are risk-free, outcomes in the real sector (firm production and fire sales in capital) do not affect behavior or outcomes in the financial sector (provision of liquidity and fire sales in illiquid assets), and vice versa. As a result, we can also analyze pecuniary externalities in financial and real markets separately, and a Social Planner attempting to correct each externality can consider them separately without considering interactions between real and financial markets. The results therefore correspond to those in the main text.

C Strategic interim behavior

In the main analysis, we suppose that agents strategically choose portfolios at $t = 0$ (understanding that their portfolios will affect future prices), but in later periods agents act as price takers. In this section, we extend the previous analysis to allow agents to also act strategically when assets trade.

To incorporate strategic behavior in the interim period, we suppose that buyers choose a value of funds f with which they purchase assets, and sellers choose a quantity of assets s to sell. The price is determined given the funds supplied to purchase assets and the quantity of assets supplied, as in the canonical strategic market game of Shapley and Shubik (1977), which converges to Walras in the limit (Dubey and Geanakoplos, 2003).

C.1 Strategic interim behavior in the productivity shock model

Consider the productivity shock model and now suppose that firms act strategically in the market for capital at $t = 1$. We show that low-productivity firms still find it optimal to sell the least amount of capital necessary to repay their debt. Taking into account their effect on price means that the amount they sell is the solution to a fixed point condition, but their sales are still an increasing function of the capital they hold. We also show that, under mild conditions, high-productivity firms still find it optimal to use all their funds, just as in the non-strategic case. Allowing for strategic behavior at $t = 1$ therefore does not change the fact that additional investment in capital at $t = 0$ drives up the price paid as a buyer and down the price received as a seller. The potential for overcorrection of the externality is therefore unchanged.

Firms with low productivity shocks choose an amount s of capital to sell. Firms with high productivity shocks choose an amount f of funds to purchase capital. Market clearing with N low types, N high types and $2N$ households requires

$$\sum_{i \in L} s_i = \sum_{j \in H} \frac{f_j}{p} + 2N \left(\frac{a}{p} - 1 \right),$$

which implies a price of capital given by

$$p(s, f) = \frac{2Na + \sum_{j \in H} f_j}{2N + \sum_{i \in L} s_i}. \tag{25}$$

Sellers. Seller i chooses s_i, taking as given other sellers' choices s_{-i} and buyers choices f, to solve the problem

$$\begin{aligned}
&\max_{s_i} \{k_i - s_i + A_L k_i + p s_i - r b_i\} \\
&\text{s.t.} \quad p s_i \geq r b_i - A_L k_i \\
&\qquad\quad s_i \leq k_i \\
&\qquad\quad p = p(s_i, s_{-i}, f)
\end{aligned}$$

For the seller constraint $p s_i \geq r b_i - A_L k_i$ to be binding, i.e. for them not wanting to sell more than necessary to repay debt, we need the price elasticity with respect to s_i to satisfy:

$$-\frac{\partial p}{\partial s_i} \frac{s_i}{p} > 1 - \frac{1}{p} \tag{26}$$

For the case $p < 1$ that we are interested in, this condition is satisfied by a positive price elasticity, which we naturally have from the price function (25):

$$-\frac{\partial p}{\partial s_i} \frac{s_i}{p} = \frac{s_i}{2N + \sum_{j \in L} s}$$

A seller acting strategically at $t = 1$ therefore finds it optimal to sell the least amount of capital possible to repay their debt. Since they take into account their effect on the price, their optimal sales are given by a fixed point condition

$$p(s_i, s_{-i}, f)\, s_i = rb_i - A_L k_i.$$

Solving for s_i and substituting in $b_i = k_i - n$, we obtain

$$s_i = \frac{\left(2N + \sum_{j\in L\setminus i} s_j\right)\left((r - A_L)\, k_i - rn\right)}{2Na + \sum_{j\in H} f_j - \left((r - A_L)\, k_i - rn\right)},$$

which is increasing in k_i. Since the optimal level of sales with strategic behavior at $t = 1$ is increasing in the level of capital chosen at $t = 0$, the comparative statics underlying the results in the main text remain unchanged.

Buyers. Buyer i chooses f_i, taking as given other buyers' choices f_{-i} and sellers' choices s, to solve the problem

$$\begin{aligned}
\max_{f_i} &\left\{k_i + \frac{f_i}{p} + A_H k_i - f_i - rb_i\right\} \\
\text{s.t.} \quad & f_i \leq A_H k_i - rb_i \\
& p = p(s, f_i, f_{-i})
\end{aligned}$$

For the buyer constraint $f_i \leq A_H k_i - rb_i$ to be binding, i.e. for them to use all their funds, we the price elasticity with respect to f_i to satisfy:

$$\frac{\partial p}{\partial f_i}\frac{f_i}{p} < 1 - p \tag{27}$$

From the price function (25) we have an elasticity with respect to f_i given by

$$\frac{\partial p}{\partial f_i}\frac{f_i}{p} = \frac{f_i}{2Na + \sum_{j\in H} f_j}.$$

Substituting in this elasticity, using (25), and the equilibrium conditions $f_i = f$ and $s_i = s$ for all i, condition (27) becomes

$$p < \frac{2Na + (N-1)\, f}{2Na + Nf}. \tag{28}$$

Given that we are interested in the case $p < 1$, this is a weak condition that holds for sufficiently low p. A buyer acting strategically at $t = 1$ then finds it optimal to use all their

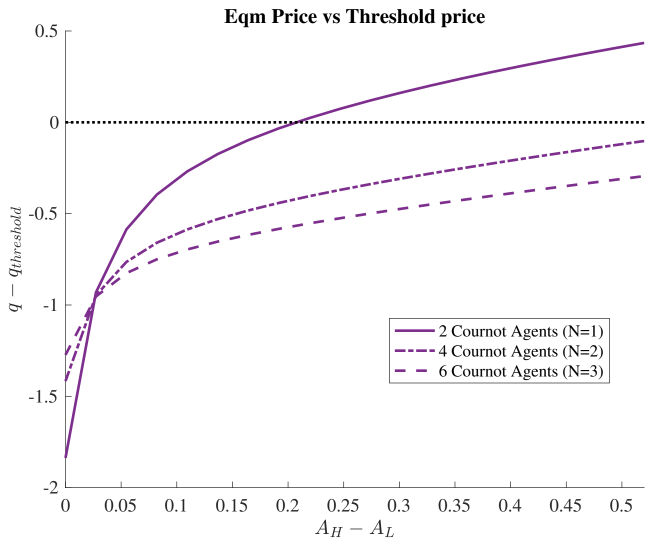

Figure 5: Condition for strategic interim behavior in productivity shock model. The figure shows the difference between the equilibrium price in the productivity shock model from the main text and the threshold from condition (28) such that non-strategic interim behavior is w.l.o.g. for different levels of idiosyncratic productivity risk, $A_H - A_L$. With relative risk aversion 1, $\alpha = 0.85$, $a = 0.93$, $n = 1$, $N \in \{1, 2, 3\}$, $\mathbb{E}[A] = 1.05$, $r = 1.02$, and $\underline{A} = 0.99$.

funds to buy capital, exactly as in the case without strategic interaction at $t = 1$.

Figure 5 illustrates that condition (28) is satisfied for almost all parameter combinations shown in Figure 1 in the main text. The figure shows the difference between the equilibrium price from the main text and the threshold from condition (28) which is negative if the condition is satisfied.

C.2 Strategic interim behavior in the liquidity shock model

We now consider the liquidity shock model and suppose that banks act strategically in the asset market at $t = 1$. We show that for $N > 1$, banks with liquidity shocks still find it optimal to sell all their assets. We also show that, under mild conditions, banks without liquidity shocks still find it optimal to use all their funds, just as in the non-strategic case. In sum, allowing for strategic behavior at $t = 1$ has no effect on the choices at $t = 1$ and therefore no effect on the optimization at $t = 0$. We derive the conditions in the more general setting with outside liquidity $N\phi \geq 0$ (see Appendix D).

Banks with liquidity shocks choose an amount s of assets to sell. Banks without liquidity shocks choose an amount f of funds to purchase assets. Market clearing implies an asset price given by

$$p(s,f) = \frac{N\phi + \sum_{j \in H} f_j}{\sum_{i \in L} s_i}. \tag{29}$$

Sellers. Seller i chooses s_i, taking as given other sellers' choices s_{-i} and buyers choices f, to solve the problem

$$\begin{aligned} &\max_{s_i} u(\ell_i + p s_i) \\ &\text{s.t.} \quad s_i \leq 1 - \ell_i \\ &\qquad\quad p = p\left(s_i, s_{-i}, f\right) \end{aligned}$$

For the seller constraint $s_i \leq 1 - \ell_i$ to be binding, i.e. for them to sell all their assets, we need the price elasticity with respect to s_i to satisfy:

$$-\frac{\partial p}{\partial s_i} \frac{s_i}{p} < 1$$

This is satisfied by the price function (29) for $N > 1$:

$$-\frac{\partial p}{\partial s_i} \frac{s_i}{p} = \frac{s_i}{\sum_{j \in L} s_j}$$

A seller acting strategically at $t = 1$ therefore finds it optimal to sell all their assets, exactly as in the case without strategic interaction at $t = 1$.

Buyers. Buyer i chooses f_i, taking as given other buyers' choices f_{-i} and sellers' choices s, to solve the problem

$$\begin{aligned} &\max_{f_i} u\left(\ell_i - f_i + R\frac{f_i}{p} + R\left(1 - \ell_i\right)\right) \\ &\text{s.t.} \quad f_i \leq \ell_i \\ &\qquad\quad p = p(s, f_i, f_{-i}) \end{aligned}$$

For the buyer constraint $f_i \leq \ell_i$ to be binding, i.e. for them to use all their funds, we need the price elasticity with respect to f_i to satisfy:

$$\frac{\partial p}{\partial f_i} \frac{f_i}{p} < 1 - \frac{p}{R} \tag{30}$$

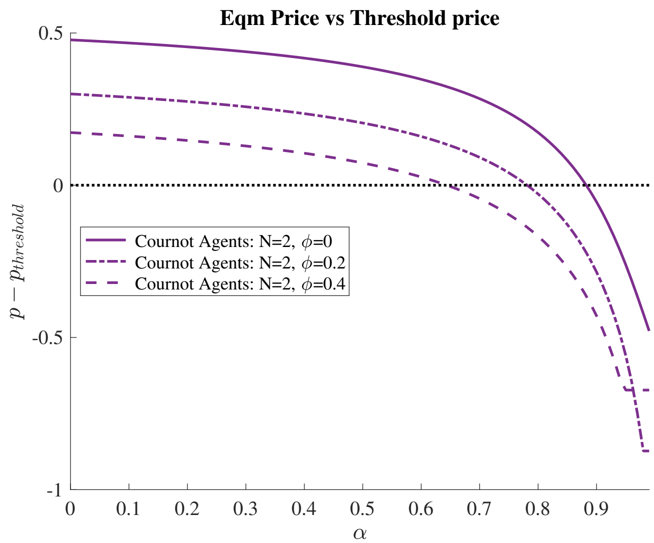

Figure 6: Condition for strategic interim behavior in liquidity shock model. The figure shows the difference between the equilibrium price in the liquidity shock model from the main text and the threshold from condition (31) such that non-strategic interim behavior is w.l.o.g. for different values of the probability of the good state, α. With log utility, $\beta = 0.96$, $R = 1.03/\beta$, and $N = 2$ (see Appendix **??**).

From the price function (29) we have an elasticity with respect to f_i given by

$$\frac{\partial p}{\partial f_i} \frac{f_i}{p} = \frac{f_i}{N\phi + \sum_{j \in H} f_j}$$

Substituting in this elasticity, using (29), and the equilibrium conditions $f_i = f$ and $s_i = s$ for all i, condition (30) becomes

$$p < \frac{N\phi + (N-1) f}{N\phi + Nf} R \tag{31}$$

Given that we are interested in the case $p < 1$ and have $R > 1$, this condition holds for sufficiently low p (i.e. high α) and/or large R. A buyer acting strategically at $t = 1$ then finds it optimal to use all their funds to buy assets, exactly as in the case without strategic interaction at $t = 1$. Figure 6 illustrates that condition (31) is satisfied for the relevant case of high α where Cournot leads to severe underprovision of liquidity (see Figure 3 in the main text). The figure shows the difference between the equilibrium price and the threshold from condition (28), which is negative if the condition is satisfied. Note that for more outside liquidity (higher ϕ), the region where strategic interim behavior is irrelevant increases considerably.

D Liquidity shock model with outside buyers

We now consider the case of outside buyers in the liquidity shock model of Section 4. Specifically, we assume that there are N outside buyers with $\phi \geq 0$ in cash to buy assets at $t = 1$. This collapses to the model in the main text for $\phi = 0$. With outside liquidity, the market clearing condition (17) becomes

$$\sum_{i \in \text{sell}} (1 - \ell_i) \times p = N\phi + \sum_{j \in \text{buy}} \ell_j.$$

The first-order condition (19) of Walrasian equilibrium remains unchanged and still implies $p < 1$ for $\alpha > 0$ and $p = 1$ for $\alpha = 0$.[23] However, due to the additional outside liquidity ϕ, the equilibrium inside liquidity ℓ will be lower. For example, in the case $\alpha = 0$, the equilibrium $p = 1$ implies that we can solve for ℓ in closed form and it is decreasing in ϕ:

$$\ell = \frac{1 - \phi}{2}$$

The Social Planner first-order condition (20) is affected by ϕ through the price effect

$$\frac{dp}{d\ell} = \frac{\phi + 1}{(1 - \ell)^2}.$$

Combined with the effect of ϕ on p, the additional outside liquidity ϕ will therefore also lower the efficient level of liquidity ℓ. Consider again the case $\alpha = 0$ where the Social Planner implements the standard risk sharing $u'(c_L) = \beta R u'(c_H)$ of Diamond and Dybvig (1983). Since outside liquidity substitutes for inside liquidity, the same risk sharing can be achieved with lower ℓ.

Finally, we turn to the Cournot first-order condition (21). Notably, the outside liquidity ϕ appears only in the price impact as perceived by an L type who sells assets, changing the price impacts in (22) as follows:

$$\frac{dp}{d\ell_L} = \frac{1}{N} \frac{\phi + \ell}{(1 - \ell)^2}$$

This price impact, which in the first-order condition weighs the benefit of holding extra liquidity, is increasing in ϕ. The presence of outside liquidity therefore biases downward the inside liquidity ℓ in the Cournot equilibrium as well. The Cournot equilibrium can

[23]Note that payoffs are in terms of the individual choice ℓ_i and the equilibrium price so their formulas are as in the main text.

still lead to lower liquidity than the Walrasian equilibrium if the seller price impact is sufficiently low relative to the buyer price impact. We still have that the ratio of the two satisfies

$$\frac{dp_L/d\ell_i}{dp_H/d\ell_i} = \frac{\phi + \ell}{1 - \ell} = p.$$

Since $\phi > 0$ bounds the price (and therefore the ratio of price impacts) away from zero, higher outside liquidity attenuates the underprovision of liquidity in the Cournot equilibrium.