Federal Reserve Bank of New York
Staff Reports

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Staff Report no. 280
March 2007

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JEL classification: F32, F36, F41

Abstract

The sharp increase in both gross and net international capital flows over the past two decades has prompted renewed interest in their determinants. Most existing theories of international capital flows are based on one-asset models, which have implications only for net capital flows, not for gross flows. Moreover, because there is no portfolio choice, these models allow no role for capital flows as a result of assets' changing expected returns and risk characteristics. In this paper, we develop a method for solving dynamic stochastic general equilibrium open-economy models with portfolio choice. After showing why standard first- and second-order solution methods no longer work in the presence of portfolio choice, we extend these methods, giving special treatment to the optimality conditions for portfolio choice. We apply our solution method to a particular two-country, two-good, two-asset model and show that it leads to a much richer understanding of both gross and net capital flows. The approach identifies the time-varying portfolio shares that result from assets' time-varying expected returns and risk characteristics as a potential key source of international capital flows.

Key words: international capital flows, portfolio allocation, home bias

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1 Introduction

The last two decades have witnessed a remarkable growth of both gross and net international capital flows and external positions. The near-tripling of gross positions among industrialized countries has also given rise to large valuation effects as asset price and exchange rate changes interact with much bigger external assets and liabilities.[1] These developments have lead to a renewed interest in understanding the driving forces behind capital flows and their macroeconomic implications. Most of what we know about capital flows is within settings where only one risk-free bond is traded. These models only have implications for net capital flows, not gross flows. Capital flows are not driven by differences in expected returns or risk characteristics of assets since there is only one risk-free asset and therefore no portfolio choice. Finally, since these are generally one-period bonds there is no role for valuation effects. At the other extreme are models where financial markets are complete. But capital flows do not really matter in these models and are rarely ever computed as the real allocation is independent of the exact structure of asset markets.[2]

A broad consensus has therefore recently developed of the need for general equilibrium models of portfolio choice in which financial markets are not restricted to be complete.[3] Such models feature a limited number of assets, such as stocks and bonds, with both gross and net capital flows. Portfolio choice is then key and leads to capital flows associated with changes in expected returns and risk

[1] Lane and Milesi-Ferretti (2005) offer a detailed review of these developments.

[2] The magnitude of capital flows in complete markets models depends on the particular structure through which the market completeness is implemented. In a setup where a full set of Arrow Debreu securities covering all possible future contingencies is traded in an initial period, subsequent capital flows will be always be zero. In other asset market structures with complete markets capital flows will generally be non-zero (e.g. Kollman (2006)), but Obstfeld and Rogoff (1996) argue that then they are "...merely an accounting device for tracking the international distribution of new equity claims foreigners must buy to maintain the efficient global pooling of national output risks."

[3] Typical of current views, Gourinchas (2006) writes "Looking ahead, the next obvious step is to build general equilibrium models of international portfolio allocation with incomplete markets. I see this as a major task that will close a much needed gap in the literature...". Also emphasizing the need for incomplete market models, Obstfeld (2004) writes: "at the moment we have no integrative general-equilibrium monetary model of international portfolio choice, although we need one."

characteristics of assets. One would expect that such models are widely adopted in open economy macroeconomics, but they are not, largely due to the difficulty of solving models of portfolio choice in a fully dynamic stochastic general equilibrium (DSGE) setting.

The goal of this paper is twofold. First, we develop a tractable method for solving DSGE open-economy models with portfolio choice that can be implemented both when asset markets are complete and incomplete.[4] Second, the method is applied to a particular two-country, two-good, two-asset model to both illustrate the solution technique and to show that it can lead to a much richer understanding of both *gross* and *net* capital flows and positions, and corresponding adjustments of goods and asset prices. The approach highlights a potential key source of international capital flows, associated with changes over time in portfolio allocation.[5] We show that capital flows can be broken down into a component associated with portfolio growth through savings and a component associated with the optimal reallocation of portfolios across various assets due to changing expected returns and risk-characteristics of assets. The model also allows us to study the impact of both expected and unexpected valuation effects that have received significant attention in recent years, e.g. Gourinchas and Rey (2006), Lane and Milesi-Ferretti (2005) and Tille (2005).

Standard solution methods for DSGE models separately analyze model equations at different orders (zero-order, first-order, and so on). The zero-order component of a variable is its level when the variance of innovations in the model goes to zero. The first-order component of a stochastic variable is proportional to model innovations, while the second-order component depends on the product of model innovations (product of first-order variables). The standard solution method computes the zero-order component of the variables from the zero-order component of the model equations, then the first-order component of the variables from the first-order component of the model equations (after linearization), and so on.

Unfortunately the standard method cannot be applied to a model with portfolio choice. For example, the zero-order component of portfolio shares cannot

[4] The method is in fact broader than just open-economy applications and can be broadly applied to both partial and general equilibrium models of portfolio choice.

[5] Even in complete market models authors generally only solve the steady portfolio allocation rather than its time variation, e.g. Engel and Matsumoto (2005), Heathcote and Perri (2005) and Kollman (2006).

be computed from the zero-order component of model equations because portfolio choice is not well-defined in a deterministic environment. Portfolio allocation, including its zero-order component, depends on the variance and covariance of asset returns. These second-order moments only show up in the second-order component of the optimality conditions for portfolio choice. Therefore solving the zero-order component of portfolio allocation is based on the second-order component of the optimality conditions for portfolio choice. Analogously, the first-order component of portfolio allocation is based on the third-order component of the optimality conditions for portfolio choice. This captures the impact on portfolio choice over time due to the time-variation in second moments of returns and time variation in expected return differences.[6] While the third-order component of model equations is generally considered to be very small and best ignored, we show that this is misleading as it is key to obtaining the first-order solution of portfolio shares and therefore capital flows.

We show that the technical challenge is associated with the *difference* between portfolio shares of Home and Foreign investors (i.e. the share of one asset in the Home investor's portfolio minus the share of that asset in the Foreign investor's portfolio). In contrast, the first-order component of *average* portfolio shares can be solved from the first-order component of asset market clearing conditions.

Overall the method can be summarized as follows. The zero-order component of portfolio share differences is solved jointly with the first-order component of other model variables. This uses the second-order component of the optimality conditions for portfolio choice and the first order component of other model equations. The second-order component of the optimality conditions for portfolio choice leads to a solution of the zero-order component of portfolio share differences as a function of various second moments. These second moments in turn depend on the first-order solution of other model variables. The latter can be solved from the first-order component of other model equations, conditional on the zero-order component of portfolio share differences. Overall this therefore leads to a fixed point problem in the zero-order component in portfolio share differences. Taking this one step further, the first-order component of portfolio share differences is solved

[6] The latter are third-order. For example, in a standard two-asset model the portfolio shares depend on the expected excess return divided by the variance of the excess return. Since the latter is second-order, the first-order component of portfolio allocation depends on the third-order component of the expected excess return.

jointly with the second-order component of other model variables. This uses the third-order component of the optimality conditions for portfolio choice and the second-order component of other model equations, which in combination lead to a fixed point problem for the first-order component of portfolio share differences.

Solving for the first-order component of portfolio share differences is technically challenging as it is based on the second and third-order components of model equations. However, we show that this is only needed to solve *gross* capital flows and *gross* external assets and liabilities, and to conduct welfare analysis. It is not needed to solve for the first-order component of *net* capital flows and the *net* external asset position.

The remainder of the paper is organized as follows. In section 2 we connect the paper to related literature. Section 3 describes the solution method in general terms. Section 4 describes a particular model, to which the solution method is applied in section 5. Focusing on a particular parameterization, section 6 discusses the implications of the model for gross and net capital flows and positions, as well as asset prices and the real exchange rate, external adjustment issues, and welfare. Section 7 concludes.

2 Related Literature

Most closely related to this paper is the work by Devereux and Sutherland (2006a,b,c). Devereux and Sutherland (2006c) have independently and simultaneously developed a solution method for DSGE models with portfolio choice that is essentially the same as ours.[7] They focus on the solution of the first-order component of portfolio allocation, building on Devereux and Sutherland (2006a) that discusses the solution of the zero-order component of portfolio allocation. While the solution method is exactly the same as in our paper, the emphasis is different. Devereux and Sutherland emphasize the most efficient way to obtain a solution to the fixed point problem for portfolio allocation that we described in the introduction, and show that there is an analytical solution to this problem in a broad class of models. Our focus is different in two ways. First, we characterize at a general level why

[7] Judd and Guu (2000) develop a solution method for portfolio choice in a partial equilibrium, static, context that is nonetheless related as well. While they adopt a different method, combining bifurcation and implicit function theorems, it delivers a solution for portfolio allocations that is the same as ours at least for the zero-order component.

standard solution methods for DSGE models break down with portfolio choice, and present an iterative solution method to solve for portfolio choice that applies to any order of approximation. Second, we illustrate the implications of this method for the dynamics of gross and net international capital flows in a simple model. Since our ultimate goal is to have a better understanding of capital flows we focus on the intuitive driving forces behind the optimal portfolio allocation. Such intuition is best obtained from the implicit solution for portfolio choice that follows from the optimality conditions for portfolio choice, before combining them with the other model equations. This delivers an expression for portfolio allocation as a function of the expected excess return, various intuitive second moments and time-variation in these second moments.[8]

Also closely related is the work by Evans and Hnatkovska (2005,2006a,b) and Hnatkovska (2006). Evans and Hnatkovska (2006a) develop a solution method for DSGE models with portfolio choice that combines a variety of discrete time approaches (perturbation and projection methods) with various continuous time approximations (of portfolio return and second-order dynamics of the state variables). Evans and Hnatkovska (2005) and Hnatkovska (2006) apply the solution method to discuss implications for issues such as the volatility of asset prices and capital flows and the magnitude of portfolio home bias. Evans and Hnatkovska (2006b) use the method to discuss the welfare implications of financial integration. While these are the first papers to tackle the difficult problem of portfolio choice in typical DSGE models, the method employed is an unusual hybrid that departs significantly from standard first and second-order solution methods commonly used to solve DSGE models. The solution described in this paper stays much closer to these existing methods, modifying them in a way that accommodates portfolio choice.

There is also a related literature in finance that solves stochastic equilibrium models with portfolio choice. Examples are Brennan and Cao (1997) and Albuquerque, Bauer and Schneider (2006). These are rich models in that there is a wide range of assets, there are gross capital flows among many countries, agents have both public and private information that differs across countries, and the models are framed in a full general equilibrium setting. Nonetheless these models are far

[8] This is an implicit solution since second moments and their time-variation themselves depend on the portfolio allocation. We first solve the fixed point problem and then compute the various intuitive drivers in the implicit solution.

removed from standard DSGE models used in macroeconomics. The main missing link from these models is that they are largely static. While there are multiple trading rounds, assets have only one terminal payoff. The solution methods also strongly rely on constant absolute risk-aversion preferences, as is standard in noisy rational expectations models.

Finally, closely related as well are an instructive set of papers by Kraay and Ventura (2000,2003). While they consider partial equilibrium models, they do allow for portfolio reallocation across assets, which yields important insights. There is trade in a riskfree international bond, with a fixed return, and domestic and foreign capital. The expected return on capital can change due to diminishing returns to capital. In that case there is a reallocation across assets that affects net capital flows. This is distinguished from capital flows associated with changes in savings for a given portfolio allocation of savings (holding expected returns given).

3 A general description of the solution method

3.1 Overview

This section describes the key features of our approach. We start by presenting the breakdown of the model equations and variables into components of different orders. We then discuss how the allocation of investors' portfolios enter the model. We review the standard solution method and explain why it no longer works in a model with portfolio choice. The section ends by describing how the method is adapted to encompass portfolio choice and discusses the solution algorithm.

3.2 The various orders of approximation

DSGE models generally lead to a set of equations of the form:

$$E_t f(x_t, x_{t+1}) = 0 \tag{1}$$

where x_t contains a vector of both control and state variables at time t. A subset of the state variables, denoted y_t, usually follows an exogenous process:

$$y_{t+1} = \Omega y_t + \epsilon_{t+1}$$

where ϵ_{t+1} are the model innovations. Each variable has components that are zero-order, first-order, and higher order:

$$x_t = x(0) + x_t(1) + x_t(2) + ... \tag{2}$$

$x(0)$ is the zero-order component of x_t. It is defined as the level to which x_t converges when the variance of model innovation approaches zero. $x_t(O)$ is the order O component, for $O > 0$. Normalizing the standard deviation of all model innovations to σ, the order of a variable is defined as follows:

Definition 1 *The component of a variable is of order O if:*

$$\lim_{\sigma \to 0} \frac{x_t(O)}{\sigma^O}$$

is either a well-defined stochastic variable whose variance is not zero or infinity or a non-zero constant whose value is not zero or infinity.

Components of order O are proportional to σ^O. Stochastic variables that are proportional to model innovations are first-order. An example is the dynamics of goods prices in response to a shock: $p_{t+1}(1) = p_1 \epsilon_{t+1}$. Stochastic variables that depend on the product of model innovations are second-order, such as $p_{t+1}(2) = p_2 (\epsilon_{t+1})^2$. Other examples of second-order variables are σ^2 or $\sigma \epsilon_{t+1}$. Examples of third-order variables are the product of three model innovations, or the product of σ^2 and a model innovation.

Model equations have to hold at all orders.[9] They are derived by writing (1) as an infinite order Taylor expansion around the allocation $x_t = x_{t+1} = x(0)$ and substituting the order decomposition (2). Let f_1 and f_2 denote the first-order derivatives of f with respect to respectively x_t and x_{t+1}, both evaluated at $x(0)$. Second-order derivatives f_{11}, f_{22} and f_{12} are defined analogously. Writing $\hat{x}_t = x_t - x(0)$, and limiting ourselves for illustrative purposes to a second-order expansion, we have:

$$f(x_t, x_{t+1}) = f(x(0), x(0)) + f_1 \hat{x}_t + f_2 \hat{x}_{t+1} + \frac{1}{2} \hat{x}_t' f_{11} \hat{x}_t + \frac{1}{2} \hat{x}_{t+1}' f_{22} \hat{x}_{t+1} + \hat{x}_t' f_{12} \hat{x}_{t+1} + ...$$

[9] Formally, this can be seen as follows. Write f as shorthand for $f(x_t, x_{t+1})$ and let $f(O)$ be the order O component of f. We have $f(O) = \sigma^O \lim_{\sigma \to 0} (f - \sum_{o=0}^{O-1} f(o))/\sigma^O$. Adding expectations, and applying this equation recursively starting at $O = 0$, using $E(f) = 0$, we have $E(f(O)) = 0$ for all O.

Substituting $\hat{x}_t = x_t(1) + x_t(2) + ...$ in this relation and taking expectations, we write the zero-order component of (1) as

$$f\left(x\left(0\right), x\left(0\right)\right) = 0 \tag{3}$$

Similarly, the first-order component is

$$f_1 x_t\left(1\right) + f_2 E_t x_{t+1}\left(1\right) = 0 \tag{4}$$

which consists only of linear terms. The second-order component is

$$\begin{aligned} & f_1 x_t(2) + f_2 E_t x_{t+1}(2) + \frac{1}{2} x_t'\left(1\right) f_{11} x_t\left(1\right) + \\ & + \frac{1}{2} E_t x_{t+1}'\left(1\right) f_{22} x_{t+1}\left(1\right) + E_t x_t'\left(1\right) f_{12} x_{t+1}\left(1\right) = 0 \end{aligned} \tag{5}$$

Notice that the second-order component includes linear terms. Therefore, while first-order components are linear, linear terms are not necessarily made only of first-order components.

3.3 Introducing portfolio choice

Before describing the solution method, it is useful to specify how portfolio shares enter the model.

Assumption 1 *The only two ways that portfolio shares enter model equations are (i) through the return on the overall portfolio and (ii) through asset demand.*

This assumption holds in almost any general equilibrium model with portfolio choice. For concreteness, assume that there are two countries, Home and Foreign, and N assets with asset i providing a gross stochastic return $R_{i,t+1}$ from t to $t+1$, with the return expressed in units of a numeraire currency. Consider an investor in the Home country. In period t she invests a share $k_{i,t}^H$ of her wealth in asset i, with the shares summing up to 1. Treating asset 1 as a base asset, portfolio shares clearly affect the overall portfolio return:

$$R_{t+1}^{p,H} = \sum_{i=1}^{N} k_{i,t}^H R_{i,t+1} = R_{1,t+1} + \sum_{i=2}^{N} k_{i,t}^H E R_{i,t+1}$$

where $ER_{i,t+1} = R_{i,t+1} - R_{1,t+1}$ is the excess return on asset i.

In addition portfolio shares affect the model through asset demand. Consider the asset market clearing condition for asset i:

$$Q_{i,t}K_{i,t} = k_{i,t}^{H}W_t + k_{i,t}^{F}W_t^{*} \tag{6}$$

The left hand side of (6) is the value of the asset supply, which is the product of the asset price $Q_{i,t}$ and the quantity of the asset available for trading, $K_{i,t}$. The right hand side of (6) is the asset demand from both Home and Foreign investors. The Home investor invests a share $k_{i,t}^{H}$ of her wealth W_t in asset i, and the Foreign investor invests a share $k_{i,t}^{F}$ of her wealth W_t^{*} in the asset.

Rather than conducting the analysis in terms of the portfolio shares of each country, it is useful to do so in terms of *average* portfolio shares and *differences* in portfolio shares. These are respectively

$$k_{i,t}^{A} = 0.5\left(k_{i,t}^{H} + k_{i,t}^{F}\right) \qquad ; \qquad k_{i,t}^{D} = k_{i,t}^{H} - k_{i,t}^{F} \tag{7}$$

If asset i is equity in Home firms then $k_{i,t}^{D} > 0$ corresponds to the well-known *home bias* in portfolios. The shares in each portfolio are simple combinations of the elements of (7): $k_{i,t}^{H} = 0.5k_{i,t}^{D} + k_{i,t}^{A}$ and $k_{i,t}^{F} = -0.5k_{i,t}^{D} + k_{i,t}^{A}$. We similarly define average wealth and its cross-country difference as $W_t^{A} = 0.5\left(W_t + W_t^{*}\right)$ and $W_t^{D} = W_t - W_t^{*}$. Although this is not key to the argument, we assume that the zero-order components of wealth of the two countries are the same, equal to $W(0)$.[10]

Optimal portfolio choice implies

$$E_t m^{s}(x_t, x_{t+1}) ER_{i,t+1} = 0 \qquad i = 2, .., N \qquad s = H, F \tag{8}$$

where $m^{s}(x_t, x_{t+1})$ is the asset pricing kernel for country s. Investors choose their portfolio to equalize the expected return on each asset, discounted by the pricing kernel. Note that portfolio shares do not directly enter (8). They only enter indirectly by affecting the overall portfolio return, which affects next period's wealth and therefore the asset pricing kernels. An immediate implication of (8) is that the zero-order components of excess returns are zero: $ER_i(0) = 0$. Furthermore, the first-order component of (8) implies that the first-order component of expected excess returns is zero: $E_t ER_{i,t+1}(1) = 0$.[11]

[10] Otherwise average portfolio shares need to be defined as a weighted average, using the zero-order components of wealth shares as weights.

[11] Without loss of generality, the zero-order component of the asset pricing kernels are normalized at 1.

3.4 The limitation of the standard solution method

The standard method for solving DSGE models solves the order O component of model variables from the order O component of model equations. Specifically, the zero-order component of variables is obtained from (3) and is also known as the deterministic steady state. The method is sequential, as the zero-order solution is required to compute the first-order solution: the terms f_1 and f_2 in (4) are evaluated at $x(0)$. In turn, the zero- and first-order solution is required to solve for the second-order solution: the terms f_1, f_2, f_{11}, f_{22} and f_{12} in (5) are evaluated at $x(0)$, while $x_t(1)$ and $x_{t+1}(1)$ use the first-order solution. This solution method only works if the following two conditions are satisfied:

Condition 1 *The order O components of all model variables affect the order O component of at least one model equation.*

Condition 2 *The order O components of all model equations depend on the order O component of at least one model variable.*

Unfortunately neither of these conditions holds in the presence of portfolio choice. First, Condition 1 is not satisfied because the order O components of the $N-1$ portfolio share differences $k_{i,t}^D$ do not affect the order O component of model equations for any $O \geq 0$. This can be seen from the order O components of the Home portfolio return and total asset demand (the right-hand side of (6)):

$$R_{t+1}^{p,H}(O) = R_{1,t+1}(O) + \sum_{i=2}^{N}\sum_{o=0}^{O}\left(0.5k_{i,t}^D(o) + k_{i,t}^A(o)\right) ER_{i,t+1}(O-o) \tag{9}$$

$$\sum_{o=0}^{O}\left(0.5k_{i,t}^D(o)W_t^D(O-o) + 2k_{i,t}^A(o)W_t^A(O-o)\right) \tag{10}$$

$k_{i,t}^A(O)$ enters (10) and can therefore be identified from the order O component of the asset market clearing equations. By contrast, $k_{i,t}^D(O)$, does not enter either (9) or (10), and we therefore cannot compute it from the order O components of model equations. Specifically, $k_{i,t}^D(O)$ appears in (9) and (10) multiplied with respectively $ER_{i,t+1}(0)$ and $W_t^D(0)$, which are both zero. While the order O component of $k_{i,t}^D$ does not affect the order O component of model equations, the lower order components of $k_{i,t}^D$ do affect the order O component of model equations (they affect both (9) and (10)).

Condition 2 is not satisfied either because there are $N-1$ equations whose order O components do not depend on the order O components of variables. This can be seen by considering the order O component of the optimality conditions for the Home and Foreign investors' portfolio choice (8), and taking the difference between the two conditions. We refer to this expression as the portfolio Euler equation differential. The zero and first-order components of the difference are zero. For $O \geq 2$ the difference is

$$E_t \sum_{o=1}^{O} \left[m_{t+1}^H(o) - m_{t+1}^F(o) \right] ER_{i,t+1}(O-o) = 0 \qquad i = 2, .., N \tag{11}$$

(11) does not depend on the order O component of variables because $ER_{i,t+1}(0) = 0$.[12]

While the order O components of the portfolio Euler equation differentials do not depend on the order O components of model variables, they do depend on the order $O-1$ components of model variables, as reflected in both $m_{t+1}^H(O-1) - m_{t+1}^F(O-1)$ and $ER_{i,t+1}(O-1)$. Therefore the order O components of portfolio Euler equation differentials can be written as a function of components of order $O-1$ and less of model variables other than portfolio share differences. The latter only impact the asset pricing kernels indirectly through the portfolio return, which affects next period's wealth.

3.5 Solution algorithm

Assume that the model contains a total of Z equations and variables. In developing the solution method, we start from the fact that Conditions 1 and 2 are satisfied for $\tilde{Z} = Z - (N-1)$ equations and variables. This includes all model variables other than the vector k_t^D of $N-1$ portfolio share differences and all model equations other than the $N-1$ portfolio Euler equation differentials (11).[13] From now on

[12](11) is derived under the assumption that the return on asset i is the same for Home and Foreign investors, in terms of the numeraire. The model presented in Section 3 relaxes this assumption by introducing a trading friction, which appears as an additional term in (11). But the presence of this additional term does not affect our point that the order O component of (11) does not depend on the order O component of model variables.

[13]For model equations and variables that do not involve portfolio choice we simply assume that Conditions 1 and 2 hold as those are standard even in DSGE models without portfolio choice. It is easily verified that Condition 2 holds for the average of portfolio Euler equations. We have also seen that it holds for the portfolio return and asset market clearing equations. Finally, we have seen that Condition 1 holds for average portfolio shares k_{it}^A.

we will simply refer to these as the "other" model variables and "other" model equations. The solution algorithm is then summarized as follows.

Solution Algorithm *In sequence $O = 1, 2, ..$ solve the order $O - 1$ component of k_t^D jointly with the order O components of all "other" model variables, using (i) the order $O + 1$ components of the portfolio Euler equation differentials and (ii) the order O components of all "other" model equations.*

Consider the case of $O = 1$. We know from (9)-(10) that the first-order component of model equations is only affected by the zero-order component of k_t^D, namely $k^D(0)$. Using the first-order component of the $\tilde{Z}$ "other" model equations, we can then solve the first-order component of the $\tilde{Z}$ "other" variables as a function of the unknown $k^D(0)$. To solve for $k^D(0)$, we then use the second-order component of the portfolio Euler equation differentials. These depend on the first-order components of the "other" model variables, which have been solved as a function of $k^D(0)$. The use of second-order components of portfolio Euler equations to solve for $k^D(0)$ is consistent with the intuition discussed in the introduction as $k^D(0)$ depends on second moments that show up in the second-order components of the portfolio Euler equations.

We proceed similarly for $O = 2$. We solve jointly for the first-order component of k_t^D and the second-order component of the $\tilde{Z}$ "other" model variables. In this case we use the second-order components of the "other" model equations together with the third-order component of the portfolio Euler equation differentials. This is where we stop in the paper as we are only interested in the first-order components of gross and net capital flows. But in principle one can keep following this algorithm for higher orders.

Solving for the first-order component of k_t^D requires second and third-order components of model equations and is therefore substantially more complicated than solving the first-order component of "other" model variables. However, the first-order solution of k_t^D is only needed to compute the first-order component of *gross* capital flows and *gross* external positions. We can gain substantial insights on the solution of the model, while avoiding technical complications, by focusing on the *net* asset positions and *net* capital flows, which depend only on the zero-order solution of k_t^D and the first-order solution of the other model variables. Intuitively, net capital flows reflect the extent to which investors worldwide accumulate assets of one country relative to another, which is driven by the first-order component of k_t^A (one of the "other" variables). They do not depend on whether investors in particular countries do so to different extents, which is captured by the first-order component of k_t^D.

Algebraically this can be seen as follows. If the first J assets are claims on the Home country, the net value of Home external assets minus liabilities is

$$W_t \sum_{i=J+1}^{N} k_{i,t}^H - W_t^* \sum_{i=1}^{J} k_{i,t}^F = W_t - W_t \sum_{i=1}^{J} k_{i,t}^H - W_t^* \sum_{i=1}^{J} k_{i,t}^F$$

The first-order component of the net external asset position is proportional to:

$$W_t(1) - 2W_t^A(1) \sum_{i=1}^{J} k_{i,t}^A(0) - 2W(0) \sum_{i=1}^{J} k_{i,t}^A(1) - \frac{1}{2} W_t^D(1) \sum_{i=1}^{J} k_{i,t}^D(0)$$

It clearly depends on the zero and first-order components of the average portfolio shares, but only on the zero-order component of the difference in portfolio shares. Net capital flows are simply equal to the change in the net external asset position, after controlling for valuation changes associated with asset prices, and can also be solved without needing the first-order component of the difference in portfolio shares.

4 A two-country, two-good, two-asset model

This section describes a symmetric two-country, two-good, two-asset model to which the solution technique will be applied. In order to both simplify the model and focus on portfolio choice, we abstract from other decisions by agents (e.g. consumption and investment decisions) in order to focus squarely on what has been the key obstacle so far in solving models with incomplete financial markets.

4.1 Two goods: production and consumption

The two countries, Home and Foreign, each produce a different good that is available for consumption in both countries. Production uses a constant returns to scale technology combining labor and capital:

$$Y_{i,t} = A_{i,t} K_{i,t}^{1-\theta} N_{i,t}^{\theta} \qquad i = H, F$$

where H and F denote the Home and Foreign country respectively. Y_i is the output of the country i good, A_i is an exogenous stochastic productivity term, K_i is the capital input and N_i the labor input. A share θ of output is paid to labor, with the remaining going to capital. The capital stocks and labor inputs are fixed and normalized to unity. Outputs therefore simply reflect the levels of productivity, which follow an exogenous auto-regressive process:

$$Y_{i,t} = A_{i,t} \qquad ; \qquad a_{i,t+1} = \rho a_{i,t} + \epsilon_{i,t+1} \tag{12}$$

where lower case letters denote logs and $\rho \in (0,1)$. The productivity innovations are iid, with a $N(0,\sigma^2)$ distribution and uncorrelated across countries.

Consumers in both countries purchase both Home and Foreign goods, with a preference towards towards domestic goods. The CES consumption indices are given in the first column of the table below, where C is the overall consumption of the Home consumer, C_H denotes her consumption of Home goods and C_F denotes her consumption of Foreign goods. The corresponding variables for the Foreign consumer are denoted by an asterisk. λ is the elasticity of substitution between Home and Foreign goods, and α captures the relative preference towards domestic goods, with $\alpha > 0.5$ corresponding to home bias in consumption. The second column shows the corresponding consumer price indexes in both countries, with the Home good taken as the numeraire and P_F representing the relative price of the Foreign good:

Consumption indices	Price indices
$C_t = \left[(\alpha)^{\frac{1}{\lambda}} (C_{H,t})^{\frac{\lambda-1}{\lambda}} + (1-\alpha)^{\frac{1}{\lambda}} (C_{F,t})^{\frac{\lambda-1}{\lambda}}\right]^{\frac{\lambda}{\lambda-1}}$	$P_t = \left[\alpha + (1-\alpha) [P_{F,t}]^{1-\lambda}\right]^{\frac{1}{1-\lambda}}$
$C_t^* = \left[(1-\alpha)^{\frac{1}{\lambda}} \left(C_{H,t}^*\right)^{\frac{\lambda-1}{\lambda}} + (\alpha)^{\frac{1}{\lambda}} \left(C_{F,t}^*\right)^{\frac{\lambda-1}{\lambda}}\right]^{\frac{\lambda}{\lambda-1}}$	$P_t^* = \left[(1-\alpha) + \alpha [P_{F,t}]^{1-\lambda}\right]^{\frac{1}{1-\lambda}}$

The allocation of consumption across goods is computed along the usual lines, reflecting the relative price of Foreign goods and the elasticity of substitution λ. The presence of home bias in consumption implies that the Home and Foreign consumer price indexes do not move in step, so movements in the relative price of the Foreign good lead to movements in the real exchange rate P_t^*/P_t. The model therefore has implications for real exchange rate adjustments that can be expected when faced with external imbalances, as in Obstfeld and Rogoff (2005) and Engel and Rogers (2006).

4.2 Two assets: rates of return

Two assets are traded, claims on the Home capital stock K_H and claims on the Foreign capital stock K_F. We refer to these as Home and Foreign equity. The price at time t of a unit of Home equity is denoted by $Q_{H,t}$, measured in terms of the numeraire Home good. The holder of this claim gets a dividend in period $t+1$, which is a share $1-\theta$ of output (12), and can sell the claim for a price $Q_{H,t+1}$. The overall return on a Home equity, in terms of Home goods, is then:

$$R_{H,t+1} = 1 + \left(Q_{H,t+1} - Q_{H,t}\right)/Q_{H,t} + (1-\theta)A_{H,t+1}/Q_{H,t} \tag{13}$$

Similarly, the price at time t of a unit of Foreign equity is denoted by $Q_{F,t}$, expressed in terms of the numeraire Home good. The return on Foreign equity is:

$$R_{F,t+1} = 1 + \left(Q_{F,t+1} - Q_{F,t}\right)/Q_{F,t} + (1-\theta)P_{F,t+1}A_{F,t+1}/Q_{F,t} \tag{14}$$

(13)-(14) show that the returns consist of a capital gain or loss due to movements in equity prices and a dividend yield.

While agents can invest in equity abroad, this entails a cost. Specifically, the agent receives only the returns in (13)-(14) times an iceberg cost $e^{-\tau}$. It is a simple way to capture the hurdles of investing outside the domestic country, reflecting the cost of gathering information on an unfamiliar market for instance.[14] This cost is second-order (τ is proportional to σ^2) to ensure a well-behaved portfolio allocation. This friction also ensures that financial markets are incomplete.[15]

In period t a Home agent invests a fraction $k_{H,t}^H$ of her wealth in Home equity, and a fraction $1-k_{H,t}^H$ in Foreign equity. The overall real return on her portfolio, expressed in terms of the Home consumption basket, is then:

$$R_{t+1}^{p,H} = \left[k_{H,t}^H R_{H,t+1} + (1-k_{H,t}^H)e^{-\tau}R_{F,t+1}\right] P_t/P_{t+1} \tag{15}$$

Similarly, a Foreign agent invests a fraction $k_{H,t}^F$ of her wealth in Home equity, and a fraction $1-k_{H,t}^F$ in Foreign equity, leading to an overall real return in terms of

[14] It is by now quite common to introduce such exogenous financial frictions. Other examples, with more detailed motivating discussions, are Martin and Rey (2004), Coeurdacier (2005) and Coeurdacier and Guibaud (2005).

[15] Even in the absence of this financial friction financial markets are incomplete when $\lambda\gamma \neq 1$, where γ is the rate of relative risk-aversion discussed below. See the discussion in Obstfeld and Rogoff (2000), page 364. Their model is one with trade costs, but that is observationally equivalent to home bias in preferences.

the Foreign consumption basket of:

$$R_{t+1}^{p,F} = \left[k_{H,t}^F e^{-\tau} R_{H,t+1} + (1 - k_{H,t}^F) R_{F,t+1}\right] P_t^* / P_{t+1}^* \tag{16}$$

4.3 Stationarity and wealth accumulation

It is well-known that when financial markets are incomplete even transitory shocks can have a permanent effect on the distribution of wealth, leading to a non-stationary distribution of wealth. This is unappealing as a country will eventually own the entire world, so that the long run wealth distribution is not determined. In addition, approximation methods around an allocation cannot be used as the economy can move far away from it.

In order to induce stationarity we assume that agents die with constant probability ψ and new agents are born at the same rate. We do so by adopting the framework of Caballero, Fahri and Gourinchas (2006). Agents only consume in the last period of life, during which they liquidate all their assets. Since the probability of death is the same for all agents, total consumption is then simply equal to aggregate wealth times the probability of death.

We assume that newborn agents work only in the first period of their life and therefore face no risk on any future labor income. After that the wealth of a particular Home investor j accumulates according to

$$W_{t+1}^j = W_t^j R_{t+1}^{p,H} \tag{17}$$

The portfolio return will be the same for all Home investors as they all choose the same portfolio.

Aggregate wealth accumulation is different for three reasons. First, only a fraction $1 - \psi$ of wealth is reinvested since the rest is consumed by agents who will die. Second, labor income of the newborns raises aggregate wealth. Third, we assume that the cost of equity investment abroad does not represent lost resources, but instead is a fee paid to a broker, which we take to be the newborn agents for simplicity. These fees therefore do not affect aggregate wealth. Let W_t and W_t^* be real aggregate wealth of the Home and Foreign countries, measured in terms of their respective consumption baskets. They then accumulate according to

$$W_{t+1} = (1 - \psi)\left[k_{H,t}^H R_{H,t+1} + (1 - k_{H,t}^H) R_{F,t+1}\right] \frac{P_t}{P_{t+1}} W_t + \frac{\theta A_{H,t+1}}{P_{t+1}} \tag{18}$$

$$W_{t+1}^* = (1 - \psi)\left[k_{H,t}^F R_{H,t+1} + (1 - k_{H,t}^F) R_{F,t+1}\right] \frac{P_t^*}{P_{t+1}^*} W_t^* + \frac{\theta P_{F,t+1} A_{F,t+1}}{P_{t+1}^*} \tag{19}$$

4.4 Markets clearing

There are goods and asset market clearing conditions for both countries. Consumption by the Home and Foreign dying agents has to equal the output of Home and Foreign goods. Using (12) and the allocation of consumption between Home and Foreign, goods market clearing conditions are

$$A_{H,t} = \alpha (P_t)^{\lambda} \psi W_t + (1-\alpha) (P_t^*)^{\lambda} \psi W_t^* \tag{20}$$

$$A_{F,t} = (1-\alpha) (P_{F,t})^{-\lambda} (P_t)^{\lambda} \psi W_t + \alpha (P_{F,t})^{-\lambda} (P_t^*)^{\lambda} \psi W_t^* \tag{21}$$

Turning to asset markets, the total values of Home and Foreign equity supply are equal to $Q_{H,t}$ and $Q_{F,t}$ since the capital stocks are normalized to 1. The amounts invested by Home and Foreign agents at the end of period t, measured in Home goods, are $(1-\psi) W_t P_t$ and $(1-\psi) W_t^* P_t^*$ respectively. The market clearing conditions for Home and Foreign asset markets are then

$$Q_{H,t} = (1-\psi) \left[k_{H,t}^H W_t P_t + k_{H,t}^F W_t^* P_t^* \right] \tag{22}$$

$$Q_{F,t} = (1-\psi) \left[(1-k_{H,t}^H) W_t P_t + (1-k_{H,t}^F) W_t^* P_t^* \right] \tag{23}$$

4.5 Portfolio allocation

The only decision faced by agents is the allocation of their investment between Home and Foreign equity. A Home agent j who dies in period $t+1$ consumes her entire wealth and gets utility

$$U_{t+1}^j = \left(W_{t+1}^j\right)^{1-\gamma} / (1-\gamma)$$

From the point of view of period t, the agent faces a probability ψ of dying the next period. We denote the value of wealth in period t by $V(W_t^j)$. The Bellman equation is

$$V(W_t^j) = \beta(1-\psi) E_t V(W_{t+1}^j) + \beta\psi E_t \left(W_{t+1}^j\right)^{1-\gamma} / (1-\gamma) \tag{24}$$

where β is the discount rate.

We conjecture the following form for the value of wealth:

$$V(W_t^j) = e^{v+f_H(S_t)} \left(W_t^j\right)^{1-\gamma} (1-\gamma) \tag{25}$$

where S_t is the state space discussed below. The function $f_H(S_t)$ captures time variation in expected portfolio returns. For given wealth utility is higher ($f_H(S_t)$

is lower) the larger are expected future portfolio returns. These expected returns will vary with the state. In the steady state $S = 0$ and we normalize $f_H(0) = 0$. The constant term v can have components of zero, first and higher order, written as $v = v(0) + v(1) + ...$, with $v(i)$ proportional to σ^i. For Foreign investors the function $f_H(S_t)$ is replaced by $f_F(S_t)$.

Agent j of the Home country chooses the portfolio allocation to maximize the right hand side of (24), subject to (17) and (15). The first-order conditions for Home and Foreign investors are:

$$E_t\Lambda_t\left(R_{H,t+1} - e^{-\tau}R_{F,t+1}\right) = 0 \quad ; \quad E_t\Lambda_t^*\left(e^{-\tau}R_{H,t+1} - R_{F,t+1}\right) = 0 \tag{26}$$

where

$$\begin{aligned}\Lambda_t &= \left((1-\psi)e^{v+f_H(S_{t+1})} + \psi\right)\left(R^{p,H}_{t+1}\right)^{-\gamma} P_t/P_{t+1} \\ \Lambda_t^* &= \left((1-\psi)e^{v+f_F(S_{t+1})} + \psi\right)\left(R^{p,F}_{t+1}\right)^{-\gamma} P_t^*/P_{t+1}^*\end{aligned}$$

are the asset pricing kernels of the Home and Foreign investors respectively. The optimality condition for portfolio choice therefore sets the expected product of the asset pricing kernel and the excess return equal to zero.

Using (25), the Bellman equation for a representative investor in counry i is

$$e^{v+f_i(S_t)} = \beta E_t\left((1-\psi)e^{v+f_i(S_{t+1})} + \psi\right)\left(R^{p,i}_{t+1}\right)^{1-\gamma} \qquad i = H, F \tag{27}$$

which gives an implicit solution to the function $f_i(S_t)$.

5 Solution of the model

We now apply the general solution method discussed in section 3 to the specific model of section 4. After substitution of the expressions for asset and portfolio returns, the model can be summarized by 12 equations: the two processes for technology (12), the two wealth accumulation equations (18)-(19), the two goods market equilibrium equations (20)-(21), the two asset market clearing conditions (22)-(23), the two Euler equations for portfolio choice (26) and the two Bellman equations (27). The Foreign goods market equilibrium condition (21) can be dropped due to Walras' law, which gives a total of 11 equations.

Dropping country subscripts due to symmetry, the zero-order components of the equations imply that $W(0) = 1/\psi$, $R(0) = (1-\psi\theta)/(1-\psi)$, $Q(0) = (1-\psi)/\psi$,

$A(0) = P_F(0) = 1$ and $v(0) = ln(\psi) - ln(R(0)^{\gamma-1}/\beta - 1 + \psi)$. For portfolio allocation we need to make a distinction between average portfolio shares and the difference across countries. Define $k_t^A = 0.5(k_{H,t}^H + k_{H,t}^F)$ as the average share invested in the Home country. From the asset market clearing conditions $k^A(0) = 0.5$. Define $k_t^D = k_{H,t}^H - k_{H,t}^F$ as the difference in portfolio shares invested in the Home country. A positive number reflects positive portfolio home bias. Its zero order component, $k^D(0)$, can only be computed from the second-order component of portfolio Euler equations. We take first and higher order log-expansions around the zero-order solution of all variables. Appendix A lists all model equations with variables in logarithmic form. Logs are denoted with lower case letters.

We now follow the solution method described in section 3. We keep the description of the solution method as non-technical as possible, focusing on the methodology rather than the details. Appendices B and C provide an abbreviated version of technical details associated with the first and second-order components of Bellman equations and the third-order components of Euler equations for portfolio choice, with a full description of all the algebra left to a Technical Appendix that is available on request.

5.1 The easy part

We start with the first-order solution of all variables other than the portfolio share difference, conditional on $k^D(0)$. For technology, wealth and portfolio shares we use the differences and averages of the variables across countries rather than the country-specific variables themselves. For example, $a_t^D = a_{H,t} - a_{F,t}$ and $a_t^A = 0.5(a_{H,t} + a_{F,t})$. The vector of state variables is

$$S_t = \begin{pmatrix} a_t^D & w_t^D & a_t^A \end{pmatrix}' \tag{28}$$

The state consist of the average and difference in technology variables, as well as the difference in wealth levels that matters when asset markets are incomplete.[16].

First consider the 9 equations of the model other than the Bellman equations. After linearization we obtain the first-order components of these equations. There is one redundancy since the first-order component of the portfolio Euler equations for Home and Foreign investors both imply that $E_t er_{t+1}(1) = 0$, where $er_{t+1} =$

[16]The average wealth level is not a separate state variable and the first-order components of w_t^A and a_t^A are identical.

$r_{H,t+1} - r_{F,t+1}$ is the excess return between Home and Foreign equity. This leaves us with 8 equations. Taking expectations of all equations, they take the form $E_t f(x_t, x_{t+1}) = 0$, where x_t consists of the 3 state variables in (28) plus the 5 control variables $cv_t = (w_t^A, p_{F,t}, k_t^A, q_{H,t}, q_{F,t})'$.

Using the standard first-order solution technique applied to the first-order components of the log-linearized equations, we solve for the first-order component of control variables as a function of state variables and for the dynamic process of the first-order component of state variables:

$$cv_t(1) = BS_t(1) \qquad ; \qquad S_{t+1}(1) = N_1 S_t(1) + N_2 \epsilon_{t+1} \tag{29}$$

where B, N_1 and N_2 are matrices and $\epsilon_{t+1} = (\epsilon_{H,t+1}, \epsilon_{F,t+1})'$.

The first-order component of k_t^A, the average fraction invested in Home assets, is solved using only the first-order component of the asset market clearing conditions. A higher average portfolio share implies a higher demand for Home equity, which raises the relative price of Home equity and lowers its expected return relative to Foreign equity. Imposing that the first-order components of expected returns must be equal then identifies the equilibrium average portfolio share.

$k^D(0)$ affects the first-order solution in two ways. First, it affects the responsiveness of $k_t^A(1)$ to the state variables (through the difference in the two asset market clearing conditions), but does not affect the responsiveness of the other control variables to the state variables.[17] Second, it affects the sensitivity of the second state variable to model innovations as $k^D(0)$ multiplies excess return innovations in the wealth accumulation equations.

The final two equations are the Bellman equations (27). Let the first-order component of $f_H(S_t)$ be $H_{1,H} S_t(1)$, where $H_{1,H}$ is the first-order derivative of f_H with respect to S_t at $S(0) = (0,0,0)'$. Appendix B shows that $H_{1,H}$ can be computed from the first-order component of the Home Bellman equation, which also gives $v(1) = 0$. For the Foreign country the first-order component of $f_F(S_t)$ is $H_{1,F} S_t(1)$, with $H_{1,F}$ solved analogously from the first-order component of the Foreign Bellman equation.

[17] $k^D(0)$ does not affect the other control variables since when adding the expectation operator to the wealth accumulation equations (which is needed to solve for the control variables), portfolio shares are multiplied by the expected excess return. Both the zero and first-order components of the expected excess return are zero.

5.2 A bit more difficult

The first-order solution (29) is conditional on the unknown $k^D(0)$, which is solved from the difference across countries of the second-order component of the portfolio Euler equations. Abstracting from the algebraic details, we get

$$\begin{aligned} k^D(0) &= 2\frac{\tau}{\gamma var(er_{t+1}(1))} + \frac{\gamma-1}{\gamma}\frac{cov(p_{t+1}(1)-p^*_{t+1}(1), er_{t+1}(1))}{var(er_{t+1}(1))} \\ &+ \frac{(1-\psi')cov(f_{Ht+1}(1)-f_{Ft+1}(1), er_{t+1}(1))}{\gamma var(er_{t+1}(1))} \end{aligned} \tag{30}$$

The first-order component of the excess return between Home and Foreign equity is $er_{t+1}(1) = r_\epsilon \epsilon_{t+1}$ for a 1 by 3 vector r_ϵ that follows from the first-order solution (29). $f_{Ht+1}(1) = H_{1,H}S_{t+1}(1)$ is the first-order component of the function $f_H(S_{t+1})$, and $\psi' = 1-\beta(1-\psi)R(0)^{1-\gamma}$.

A positive value of (30) implies home bias, while a negative value implies foreign bias. (30) shows that there are three sources of portfolio bias. The first reflects the cost of investing abroad, τ, with a higher cost making investing in domestic equity more attractive. The second reflects the co-movements of the real exchange rate and excess return. Assuming $\gamma > 1$, it is attractive for Home investors to invest in the Home equity if the excess return on Home equity is high in states where the Home price index is relatively high.

The final source reflects a hedge against changes in future expected portfolio returns, which are captured by the functions $f_H(S_{t+1})$ and $f_F(S_{t+1})$ in the value function of Home and Foreign investors next period. An increase in these functions imply a drop in welfare because of low expected future returns. It is attractive for Home investors to invest in Home equity when the excess return on Home equity is high in states where expected future portfolio returns are low ($f_H(S_{t+1})$ high). This source is positive when there is consumption home bias ($\alpha > 0.5$). Consider for example a positive shock to Home productivity relative to Foreign productivity in period $t+1$. This will lower the expected real portfolio return of Home investors in subsequent periods, relative to Foreign investors. The reason is that the relative price of Foreign goods rises at time $t+1$ and is then expected to fall, leading to an expected fall of the Foreign price index relative to the Home price index (i.e. a lower real portfolio return for Home investors). At the same time the return on Home equity increases at time $t+1$, relative to Foreign equity. Home equity is then a better hedge against changes in expected real portfolio returns for Home than for Foreign investors.

Notice that each of the three components of $k^D(0)$ in (30) is a ratio of second-order variables. Both the numerator and the denominator of these terms are proportional to σ^2, and the ratio is therefore zero-order. This illustrates why the second-order components of portfolio Euler equations are necessary to compute the zero-order component of portfolio shares.

With the exception of τ, all the second-order components in the three ratios are based on variances and covariances of first-order components of model variables. These are based on the first-order solution (29), which is in turn conditional on $k^D(0)$.[18] This leads to a fixed point problem. We solve $k^D(0)$ as a fixed point of the function that maps $k^D(0)$ into itself: $k^D(0)$ maps into the first-order solution (29), which maps into $k^D(0)$ in (30). The solution described so far implements the solution algorithm in section 3 for $O = 1$.

It is worthwhile pointing out that the difficulty in solving DSGE models with portfolio choice lies in assumptions that distinguish Home investors from Foreign investors. Otherwise $k_t^D = 0$ and we can solve the model in exactly the same way as for DSGE models without portfolio choice. In the model described here there are two differences between Home and Foreign investors. First, the financial friction impacts returns asymmetrically for Home and Foreign investors. Second, the home bias in preferences leads to different consumer price indices that they hedge against when choosing their portfolio. When we set $\tau = 0$ and $\alpha = 0.5$ these differences disappear.

5.3 The hard part

The final step is only necessary to compute gross external holdings or gross capital flows, which requires the first-order component of the portfolio share difference k_t^D. In order to solve for the first-order component of k_t^D we need to implement the solution algorithm in section 3 for the case $O = 2$. It proceeds along the same line as the solution described above, but now one order higher for all equations and variables. We combine the third-order component of the difference in portfolio Euler

[18] The solution for $k^D(0)$ in (30) depends on the first-order components $er_{t+1}(1)$, $p_{t+1}(1) - p^*_{t+1}(1)$ and $f_{H,t+1}(1) - f_{F,t+1}(1)$. These all depend on $S_{t+1}(1)$ in a way that is independent of $k^D(0)$. But $S_{t+1}(1)$ depends on ϵ_{t+1} in a way that does depend on $k^D(0)$ as the portfolio allocation affects the impact of shocks on wealth accumulation.

equations with the second-order components of all 10 "other" model equations, and solve for the first-order component of k_t^D and the second-order component of all "other" variables.

We start by solving the second-order component of the "other" variables conditional on a first-order solution for the portfolio share difference: $k_t^D(1) = k_s S_t(1)$, with k_s a 1 by 3 vector. The second-order components of the "other" variables are obtained after substituting the first-order solutions of all variables into the second-order components of the 10 "other" model equations. Since such second-order solutions are by now quite standard, we omit a full description of the algebra for brevity.[19] The solution for control variables, for example p_{Ft}, takes the form

$$p_{F,t}(2) = p_s S_t(2) + S_t(1)' p_{ss} S_t(1) + k_p \sigma^2 \tag{31}$$

where p_s is a vector, p_{ss} a matrix and k_p a scalar. The second-order solution for state space accumulation takes the form

$$S_{t+1}(2) = N_1 S_t(2) + \begin{bmatrix} S_t(1)' N_{3,1} S_t(1) + \epsilon_{t+1}' N_{4,1} \epsilon_{t+1} + S_t(1)' N_{5,1} \epsilon_{t+1} \\ S_t(1)' N_{3,2} S_t(1) + \epsilon_{t+1}' N_{4,2} \epsilon_{t+1} + S_t(1)' N_{5,2} \epsilon_{t+1} \\ S_t(1)' N_{3,3} S_t(1) + \epsilon_{t+1}' N_{4,3} \epsilon_{t+1} + S_t(1)' N_{5,3} \epsilon_{t+1} \end{bmatrix} + N_6 \sigma^2 \tag{32}$$

where $N_{3,i}$, $N_{4,i}$ and $N_{5,i}$ are matrices and N_6 is a vector. Finally, Appendix B shows that the second-order component of the Bellman equations yield the second-order derivative of the functions $f_H(S_t)$ and $f_F(S_t)$ at $S = S(0)$.

In parallel to the first-order solution, $k_t^D(1)$ affects the second-order solution in two ways. First, of the control variables it only affects $k_t^A(2)$, through the difference in the two asset market clearing equations.[20] Second, it affects the dynamic process of the second-order component of the second state variable as $k_t^D(1)$ multiplies the excess return innovation $er_{t+1}(1) = r_\epsilon \epsilon_{t+1}$ in the wealth accumulation equations.

The second-order components of time $t+1$ variables depend, among other terms, on the product of elements of $S_t(1)$ and ϵ_{t+1}. For example, using (29), (31) and (32), the sum of these terms in the solution for $p_{F,t+1}$ is

$$S_t(1)' \left(\sum_{i=1}^{3} p_{s,i} N_{5,i} + N_1' (p_{ss} + p_{ss}') N_2 \right) \epsilon_{t+1}$$

[19] For descriptions of second-order solutions see Kim et.al. (2003), Schmitt-Grohe and Uribe (2004) and Lombardo and Sutherland (2005).

[20] In the *expected* second-order component of the wealth accumulation equations $k_t^D(1)$ is multiplied by the first-order component of the expected excess return, which is zero.

where $p_{s,i}$ is element i of the vector p_s. This captures time-variation in the impact of shocks on the relative price next period. The impact varies with the current state $S_t(1)$.

This naturally leads to time-variation in conditional second moments, which shows up in the third-order component of variances and covariances. In order to see that, consider two variables x and y for which the expected first-order components are zero. The third-order components of the variance and covariance are then $\hat{var}(x) = 2Ex(1)x(2)$ and $\hat{cov}(x,y) = Ex(1)y(2) + Ex(2)y(1)$.[21] These third-order components take the form $\sigma^2 S_t(1)$. For example $er_{t+1}(1) = r_\epsilon \epsilon_{t+1}$, while in line with the discussion above $er_{t+1}(2)$ involves a term of the form $S_t(1)' A \epsilon_{t+1}$ (with A a matrix of constants), as well as squared terms in model innovations and $S_t(1)$. Therefore $Eer_{t+1}(1)er_{t+1}(2) = r_\epsilon A' \sigma^2 S_t(1)$. This third-order component therefore varies with the state space.

In order to solve $k_t^D(1)$, we combine the second-order solution described above with the third-order component of the difference in portfolio Euler equations across countries. The latter is derived in Appendix C. The resulting first-order solution for k_t^D is

$$k_t^D(1) = -k^D(0)\frac{\hat{var}(er_{t+1})}{var(er_{t+1}(1))} + \frac{\gamma - 1}{\gamma}\frac{\hat{cov}(p_{t+1} - p_{t+1}^*, er_{t+1})}{var(er_{t+1}(1))} + \frac{\hat{cov}\left(f_{Ht+1} - f_{Ft+1}, er_{t+1}\right) + 0.5\psi' E_t\left[(f_{Ht+1}(1))^2 - (f_{Ft+1}(1))^2)\right] er_{t+1}(1)}{\gamma var(er_{t+1}(1))/(1-\psi')} \tag{33}$$

In this expression the variance in the denominator of each ratio is second-order, while the terms in the numerator are all third-order, so that the ratios are all first-order. The three ratios capture respectively time variation in the variance of the excess return, in the covariance between the real exchange rate and the excess return, and in the hedge against changes in expected portfolio returns. These same elements without their time variation are present in the zero order component (30) of portfolio shares.

An increase in the variance of the excess return by itself reduces home bias. For instance, (30) shows that home bias is affected by the financial friction τ, relative to the variance of the excess return. An increase in the variance then reduces the relevance of the financial friction for the portfolio decision, which translates into a

[21] For example, $var(x) = E(x^2) - (Ex)^2$. Substituting $x = x(0) + x(1) + x(2) + ...$ and using $Ex(1) = 0$, the third- order component of $var(x)$ is $2Ex(1)x(2)$.

smaller home bias. This is captured by the first ratio in (33). An increase in the covariance between the real exchange rate and the excess return leads to increased home bias as it implies that for both Home and Foreign investors their domestic asset has a relatively high payoff when the domestic price index is high. This is captured by the second ratio in (33). Similarly, an increase in the covariance between the hedging term $f_H(S_{t+1}) - f_F(S_{t+1})$ and the excess return leads to increased home bias as it implies that for both Home and Foreign investors their domestic asset has a relatively high payoff when their utility is low due to low future expected portfolio returns. This is captured by the last ratio in (33).

(33) implies that $k_t^D(1)$ is of the form $k_s S_t(1)$. We solve for the vector k_s by solving the fixed point of a function that maps k_s into itself. For a given vector k_s we can solve the second-order components of the "other" model variables. Together with the first-order components of the "other" model variables it allows us to solve the time varying moments $\hat{var}$ and $\hat{cov}$ in (33). This in turn yields a new vector k_s. Solving the fixed point problem yields the first-order solution of k_t^D.

The solution for $k_t^D(1)$ is based on the *difference* across countries in the third-order component of the portfolio Euler equations. So far we have not used the *average* of the portfolio Euler equations across the two countries. One can show that the average of the second-order components of portfolio Euler equations implies that $E_t er_{t+1}(2) = 0$, so that the zero, first and second-order components of the expected excess return are all zero.[22] Taking the average of the third-order component of the portfolio Euler equations we get (see Appendix C):

$$\begin{aligned} k_t^A(1) = {} & \frac{E_t er_{t+1}(3)}{\gamma var(er_{t+1}(1))} + \frac{\gamma-1}{\gamma}\frac{\hat{cov}(p_{t+1}+p_{t+1}^*, er_{t+1})}{2var(er_{t+1}(1))} + \\ & -0.5\frac{\gamma-1}{\gamma}\frac{\hat{var}(r_{H,t+1}) - \hat{var}(r_{F,t+1})}{var(er_{t+1}(1))} + \\ & \frac{\hat{cov}\left(f_{Ht+1}+f_{Ft+1}, er_{t+1}\right) + 0.5\psi' E_t\left[(f_{Ht+1}(1))^2 + (f_{Ft+1}(1))^2)\right] er_{t+1}(1)}{2\gamma var(er_{t+1}(1))/(1-\psi')} \end{aligned} \tag{34}$$

The first-order component of the average portfolio share depends on time-varying second moments, just as the first-order component of the difference in portfolio shares. But it also depends on the third-order component of the expected excess return. This term did not show up in the difference of portfolios since Home

[22] The result that the expected second-order component is zero depends critically on the symmetry of the model. More generally it would be a non-zero constant term that is proportional to σ^2.

and Foreign agents respond in the same way to changes in expected returns. Note however that $k_t^A(1)$ is solved in the first step of the solution method, based asset market clearing conditions. The expected excess return adjust to ensure that agents are willing to hold the portfolio that clears asset markets. Therefore (34) can be used to solve for the third-order component of the expected excess return given the first-order solution $k_t^A(1)$ and the first and second-order components of $r_{H,t+1}$, $r_{F,t+1}$, $p_t + p_t^*$, $f_{H,t+1}$ and $f_{F,t+1}$ that are needed to compute the time-varying second moments.

6 A numerical illustration

6.1 Parametrization

The implications of our simple model can be illustrated through a numerical example. The parameterization we adopt is for illustrative purposes only, not to match the data of any particular country. Various extensions of the model will need to be introduced before it can be seriously confronted to the data.

We assume a labor share of output, θ, of 0.7. Productivity shocks are assumed to be highly persistent, with $\rho = 0.99$, and productivity innovations have a standard deviation of $\sigma = 5\%$. Turning to consumers' preferences, we assume home bias in preferences by setting $\alpha = 0.8$. The elasticity of substitution between Home and Foreign goods is set at $\lambda = 2$. The rate of relative risk-aversion, γ, is set at 10 and $\beta = 1$. Agents face a probability of death of $\psi = 0.05$, leading to a consumption-wealth ratio of 5%. The transaction cost on investing abroad, τ, is set at 0.419%. These parameters generate a sizable home bias in equity holdings, with the zero-order component of the fraction invested in domestic equity equal to 0.8.[23] We illustrate the dynamic response to a one standard deviation increase in Home productivity through nine charts.

[23] This implies that agents invest 30% more in the domestic country than under perfect diversification. Of this, there is a bias of +67% invested in the domestic country due to the financial friction τ, a negative bias of -40% due to a negative correlation between the real exchange rate and excess return (this is a foreign bias) and a positive home bias of +3% due to the hedge against changes in expected portfolio returns.

6.2 Real exchange rate and equity prices

Chart 1 illustrates the dynamic response of the relative price of the Foreign good. The persistent increase in Home productivity boosts the supply of the Home good, leading to an immediate 2.6% increase in the relative price of the Foreign good (a Home real depreciation). This is followed by a gradual drop in the relative price of the Foreign good (Home real appreciation) as the shock dissipates.

Chart 2 shows the dynamic response of equity prices, depicting the Home equity price in units of the Home good and the Foreign equity price in units of the Foreign good. The persistent Home productivity shock immediately raises the Home equity price by 4.7%. The Foreign equity price rises by a small 0.3% because the higher productivity boosts wealth, some of which is invested in Foreign equity. While the increase in Foreign equity prices is larger when expressed in Home goods (2.9%), Home equity prices still increase by more on impact. Following the initial jump, equity prices gradually drop back to their steady state, which implies a larger expected drop in the Home equity price than in the Foreign equity price.

6.3 Financial positions

Chart 3 shows the dynamic response of gross external assets and liabilities of the Home country, as well as its net external asset position. All are shown as a fraction of the initial GDP. Gross positions change both as a result of valuation effects and capital flows. It is therefore useful to view Chart 3 jointly with Chart 4, which shows net external assets along with the cumulative net capital outflows. The initial response of both gross assets and liabilities is almost entirely due to unexpected valuation effects. Chart 4 shows that initial net capital outflows are small in comparison. Gross liabilities rise due to the increase in the Home equity price. Gross assets rise both as a result of the rise in the Foreign equity price (in units of the Foreign good) and the large immediate real depreciation of the Home currency. Overall the net external position becomes negative at -6.2% of GDP.

After the initial shock gross liabilities drop much faster than gross assets and soon the Home country becomes a net creditor. Chart 4 shows that this is driven to a large extent by cumulative net capital outflows. On top of that the Home country also receives fully expected valuation gains that increase its net external position, reflecting the gradual fall in Home equity prices in Chart 2. This is illustrated by the decreasing gap between cumulative capital outflows and the net external position in Chart 4.

6.4 Capital flows

Chart 5 shows the dynamic response of both gross and net capital flows as a fraction of initial GDP. Positive gross capital outflows capture purchases of Foreign equity by Home investors, while positive gross capital inflows capture purchases of Home equity by Foreign investors. Net capital outflows measure the difference between gross outflows and inflows. Initially both capital inflows and outflows go down, while subsequently they almost perfectly mirror each other. The theory can therefore account both for periods of positive co-movements between inflows and outflows and periods of negative co-movements.

A first step towards understanding the drivers of capital flows is to break them down into *portfolio growth* and *portfolio reallocation* components, a breakdown also emphasized by Kraay and Ventura (2000,2003).[24] Without any changes in portfolio shares, an increase in national savings leads to capital outflows equal to the rise in national savings times the portfolio share of Foreign assets. This portfolio growth represents the first source of capital flows. The second source, portfolio reallocation, is associated with an active reallocation of wealth across assets. While it is related to a change in portfolio shares, it is important to realize that changes in portfolio shares do not necessarily translate into capital flows. In particular, changing asset prices affect portfolio shares without any asset trade, a dimension that we refer to as the passive portfolio. Capital flows associated with portfolio reallocation reflect a change in portfolio shares away from this passive portfolio.

Charts 6 and 7 document the breakdown of gross capital outflows and inflows into the portfolio reallocation and portfolio growth components. The shock leads to a rise in Home savings and an offsetting drop in Foreign savings. The portfolio growth effect then leads to positive capital outflows and negative capital inflows. While this channel is not negligible under our parameterization, Charts 6 and 7 show that the portfolio reallocation effect dominates the overall dynamics of gross capital flows. At the time of the shock, there is a retrenchment in that both Home and Foreign investors reallocate their portfolios towards domestic assets, leading

[24] A decomposition of capital flows along this line is derived in the Technical Appendix based on standard balance of payments accounting.

to negative values for both capital inflows and outflows. In subsequent periods, both Home and Foreign investors reallocate their portfolio towards Foreign equity, which translates into positive capital outflows and negative capital inflows.

The portfolio reallocation is further illustrated in Chart 8. It shows the portfolio share invested in Home equity by both Home and Foreign investors, as well as the passive portfolio share. Without any asset trade, the increase in Home equity prices automatically boosts the value of investors' holdings of Home equity, thereby raising the passive share of Home equity in all portfolios. Chart 8 shows that there is a gap between the optimal Home and Foreign portfolio shares in the immediate response to the shock. The Home portfolio share is higher than the passive portfolio share, so that Home investors actively reallocate their portfolio towards Home assets. In contrast, the Foreign portfolio share is lower than the passive portfolio share, so that Foreign investors actively reallocate their portfolio towards Foreign assets. This retrenchment towards domestic assets implies negative capital inflows and outflows. After the initial shock the portfolio shares of both Home and Foreign investors drop much faster than the passive portfolio share. This means that both Home and Foreign investors actively reallocate their portfolio towards Foreign assets, leading to positive capital outflows and negative capital inflows.

Portfolio reallocation is a result of both changes in the expected excess return and time-varying second moments. In the immediate response to the shock, changes in second moments in (33) have the biggest impact on gross capital flows. The volatility of the excess return increases, leading to a reduction in home bias $k_t^D(1)$. But this is more than offset by an increase in the covariance between the excess return and the real exchange rate and between the excess return and the hedging component $f_{Ht+1} - f_{Ft+1}$, leading to increased home bias and therefore a shift to domestic assets by both countries.

The subsequent reallocation towards Foreign assets by investors from both countries is mainly driven by changes in the expected excess return, which is shown in Chart 9. Not all changes in expected excess return lead to capital flows though, and it is useful to break down the change in the expected excess return into three components, as illustrated in Chart 9. Each of these components is associated with a different source of change in demand or supply of assets. For each source, asset demand and supply are brought to equilibrium through changes in the expected excess return. The first component is associated with the increase in the relative supply of Home equity following the increase in the relative price of Home equity. A rise in demand for Home equity is needed to clear asset markets, which is achieved through a higher expected excess return. Since this change in expected returns induces agents to hold the passive portfolio, it does not give rise to capital flows.

Second, changes in the second moments affect the *average* portfolio share, as shown in (34). In our example, this translates into a substantial shift of investors towards Foreign assets. By contrast, the first-order component of relative equity supply is not affected by changes in second moments.[25] The clearing of equity markets requires that demand be brought back in line with supply. This is achieved through a rise in the expected excess return on Home equity that undoes the shift towards Foreign equity. This aspect does not lead to any capital flows either. The first two aspects therefore illustrate the need to be careful when linking capital flows with changes in expected returns. Most of the changes in the expected excess return are not related to capital flows at all.

Third, the rise in Home savings leads to an increase in demand for Home equity due to the portfolio home bias. The expected excess return on Home equity then needs to fall to clear equity markets. This leads to a portfolio reallocation towards Foreign equity by investors from both countries, so that capital outflows are positive and capital inflows are negative.[26] This last component is therefore the only one that is associated with capital flows. Notice that it moves in opposite direction from the overall expected excess return, which rises after the shock, again illustrating the pitfalls in empirically linking capital flows to changes in expected returns.

6.5 Channels of external adjustment

Our setup allows us to explore the channels through which the net external position of the Home country adjusts after the initial jump. Standard balance of payments

[25] The relative equity supply depends exclusively on relative equity prices, which depends on the first-order component of the expected present value of differences in dividend payments and is solved from the first-order component of model equations.

[26] While we have abstracted from investment in the model, it plays a similar role. For example, an increase in Home investment, holding everything else constant, raises the Home equity supply and would lead to a rise in the expected excess return on Home equity, leading to net capital inflows.

accounting implies that

$$-nfa_t(1) = \sum_{s=1}^{\infty} \frac{E_t tb_{t+s}(1)}{R(0)^s} + GA(0) \sum_{s=1}^{\infty} \frac{E_t(r_{F,t+s}(1) - r_{H,t+s}(1))}{R(0)^{s-1}} \tag{35}$$

where nfa is the net foreign asset position, tb is the trade balance and $GA(0)$ is the zero-order component of gross assets. (35) shows that a net external debt can be financed by either expected future trade surpluses or by more favorable expected future returns on external assets (Foreign equity) than external liabilities (Home equity).

As expected future excess returns are zero to the first-order, the net external debt is simply equal to the present value of expected trade surpluses. The model can therefore not account for empirical findings by Gourinchas and Rey (2006) that net external debt is to some extent financed by differences in expected returns. Our finding that first-order expected excess returns are zero is a standard arbitrage condition found in virtually any asset pricing model, and can only be relaxed by introducing elements that break the arbitrage across various assets.[27]

While the expected excess return is zero to a first-order, it is nonetheless of interest to look at its components. Differences in expected returns are associated with different expected dividend yields, different expected Home and Foreign equity price changes and expected real exchange rate changes. Chart 10 breaks down the components of net external adjustment. In the immediate response to the shock the net external debt of the Home country reaches 6.2% of GDP, which is financed entirely through expected future trade surpluses. As Home productivity is persistently higher, the expected dividend yield is larger for Home than Foreign equity. In present value terms this adds 2.1% to the external debt. After the shock, the expected appreciation of the Home real exchange rate leads to a capital loss on Home investors' holdings of Foreign equity, adding 5.1% to the external debt in present value terms. Finally, the expected fall in Home equity prices, relative to Foreign equity prices, translates into an expected capital loss for Foreign investors on their holdings of Home equity which reduces the external debt of the Home country by 7.2% in present value terms.

[27] One example is Bacchetta and van Wincoop (2006), who introduce a portfolio decision making cost (or asset management cost), leading to infrequent portfolio decisions.

6.6 Welfare

The method can also be used to conduct welfare analysis, which requires the second-order solution of the model. With portfolio choice this means combining the third-order component of portfolio Euler equations with the second-order component of other model equations, the same step that is needed to compute gross capital flows. As an illustrative exercise we compute the impact of the financial friction τ on the welfare of a representative investor, varying the friction from 0 to 0.5%. Welfare is measured by the value function (25). We assess how the financial friction τ affects the welfare for a given wealth W_t in a situation where the state variables are equal to their zero-order component, so $f_H(S) = 0$. Welfare then depends only on the constant v, which is affected by the second-order component of the financial friction that we compute from the second step of the solution algorithm.

The welfare loss is reported in Chart 11, expressed in terms of the percentage drop in wealth that leads to an equal welfare loss. The welfare loss rises to about 1.2% when $\tau = 0.4\%$ as in the benchmark parameterization. In addition, the loss is concave in τ. When τ gets close to 0.5%, the portfolio approaches full home bias with investors holding only domestic equity. With little exposure to foreign equity, investors are little affected by further changes in the financial friction.

7 Conclusion

We have developed a method for solving DSGE open-economy models of portfolio choice with the aim of better understanding the nature of international capital flows. The method has the advantage that it closely connects to existing first and second-order solution methods of DSGE models, while giving special treatment to optimality conditions for portfolio choice. It highlights the need to go to higher orders of these optimality conditions to solve for zero and first-order components of the portfolio allocation and therefore capital flows.

The method also has the advantage that it can be broadly applied. The simple two-country, two-asset, two-good example discussed in the paper illustrates what we can learn from such models. The next natural step is to extend this framework by introducing consumption and investment decisions. Other natural extensions are to introduce monetary elements through price rigidities, fiscal policy and additional assets. A potentially rewarding strategy may also be to introduce information asymmetries as in the noisy rational expectations literature in finance. All these extension will put us in a better position to confront the model to data on gross and net capital flows, analyze policy questions related to capital flows, and make meaningful predictions related to the external adjustment process faced by countries with large external imbalances like the United States.

Appendix

A Equations of the model

As discussed at the beginning of section 4, the model can be summarized by 11 equations. Writing variables other than portfolio shares in logarithmic form these equations are

$$a_{H,t+1} = \rho a_{H,t} + \epsilon_{H,t+1} \tag{36}$$

$$a_{F,t+1} = \rho a_{F,t} + \epsilon_{F,t+1} \tag{37}$$

$$e^{w_{t+1}+p_{t+1}} = (1-\psi)\left[k^H_{H,t}e^{r_{H,t+1}} + (1-k^H_{H,t})e^{r_{F,t+1}}\right]e^{w_t+p_t} + \theta e^{a_{H,t+1}} \tag{38}$$

$$e^{w^*_{t+1}+p^*_{t+1}} = (1-\psi)\left[k^F_{H,t}e^{r_{H,t+1}} + (1-k^F_{H,t})e^{r_{F,t+1}}\right]e^{w^*_t+p^*_t} + \theta e^{p_{Ft+1}+a_{F,t+1}} \tag{39}$$

$$e^{a_{H,t}} = \alpha\psi e^{w_t+\lambda p_t} + (1-\alpha)\,\psi e^{w^*_t+\lambda p^*_t} \tag{40}$$

$$e^{q_{H,t}} = (1-\psi)\left[k^H_{H,t}e^{w_t+p_t} + k^F_{H,t}e^{w^*_t+p^*_t}\right] \tag{41}$$

$$e^{q_{F,t}} = (1-\psi)\left[k^H_{F,t}e^{w_t+p_t} + k^F_{F,t}e^{w^*_t+p^*_t}\right] \tag{42}$$

$$E_t\left((1-\psi)e^{v+f_H(S_{t+1})}+\psi\right)e^{-\gamma r^{p,H}_{t+1}}\left(e^{r_{H,t+1}} - e^{r_{F,t+1}-\tau}\right)e^{p_t-p_{t+1}} = 0 \tag{43}$$

$$E_t\left((1-\psi)e^{v+f_F(S_{t+1})}+\psi\right)E^{-\gamma r^{p,F}_{t+1}}\left(e^{r_{H,t+1}-\tau} - e^{r_{F,t+1}}\right)e^{p^*_t-p^*_{t+1}} = 0 \tag{44}$$

$$e^{v+f_H(S_t)} = \beta E_t\left((1-\psi)e^{v+f_H(S_{t+1})}+\psi\right)e^{(1-\gamma)r^{p,H}_{t+1}} \tag{45}$$

$$e^{v+f_F(S_t)} = \beta E_t\left((1-\psi)e^{v+f_F(S_{t+1})}+\psi\right)e^{(1-\gamma)r^{p,F}_{t+1}} \tag{46}$$

(36) and (37) are the autoregressive processes for productivity. (38)-(39) are the wealth dynamics in the Home and Foreign countries. (40) is the Home goods market clearing condition (we can omit the Foreign goods market clearing condition due to Walras's law). (41)-(42) are the market clearing conditions for Home and Foreign equities. (43)-(44) are the optimal portfolio conditions for Home and Foreign investors. Finally, (45)-(46) are the Bellman equations for Home and Foreign investors.

These equations depend on consumer price indices, asset and portfolio returns, which in logarithmic form can be written as

$$e^{(1-\lambda)p_t} = \alpha + (1-\alpha)\, e^{(1-\lambda)p_{Ft}} \tag{47}$$

$$e^{(1-\lambda)p_t^*} = (1-\alpha) + \alpha e^{(1-\lambda)p_{Ft}} \tag{48}$$

$$e^{r_{H,t+1}} = e^{q_{H,t+1}-q_{H,t}} + (1-\theta)e^{a_{H,t+1}-q_{H,t}} \tag{49}$$

$$e^{r_{F,t+1}} = e^{q_{F,t+1}-q_{F,t}} + (1-\theta)e^{p_{Ft+1}+a_{F,t+1}-q_{F,t}} \tag{50}$$

$$e^{r_{t+1}^{p,H}} = \left[k_{H,t}^{H} e^{r_{H,t+1}} + (1-k_{H,t}^{H})e^{r_{F,t+1}-\tau}\right] e^{p_t - p_{t+1}} \tag{51}$$

$$e^{r_{t+1}^{p,F}} = \left[k_{H,t}^{F} e^{r_{H,t+1}-\tau} + (1-k_{H,t}^{F})e^{r_{F,t+1}}\right] e^{p_t^* - p_{t+1}^*} \tag{52}$$

(47)-(48) define the consumer prices indexes. (49)-(50) define the rates of return on Home and Foreign equity. Finally, (51)-(52) define the rates of return on the portfolios of Home and Foreign investors.

B Expansions of the Bellman equation

The elements of the Bellmann equation for the Home investor (45) are solved by taking a second-order expansion around $S = 0$. The resulting expression contains both first- and second-order components. The first-order components are:

$$v\,(1) + H_{1,H} S_t\,(1) = (1-\psi')\left[v\,(1) + H_{1,H} E_t S_{t+1}\,(1)\right] + E_t(1-\gamma) r_{t+1}^{p,H}\,(1) \tag{53}$$

where $v\,(1)$ is the first-order component of v and $H_{1,H}$ is a 1x3 vector with the first derivative of $f_H(S)$, evaluated at $S = 0$. ψ' is a transformation of the probability of death ψ: $\psi' = 1 - \beta(1-\psi)R(0)^{1-\gamma}$. (53) is solved by $v\,(1) = 0$ and:

$$H_{1,H} = (1-\gamma) r_s \left(I_3 - (1-\psi')N_1\right)^{-1}$$

where r_s is a 1x3 matrix taken from the first-order solution of the portfolio return for the Home investor from (29): $r_{t+1}^{p,H}\,(1) = r_s S_{t+1}\,(1)$, I_3 is a 3x3 identity matrix and N_1 is the 3x3 matrix from (29).

The second-order components of (45) are:

$$\begin{aligned} & H_{1,H} S_t\,(2) + \frac{1}{2}\left[\left[H_{1,H} S_t\,(1)\right]^2 + 2v\,(2) + S_t\,(1)' H_{2,H} S_t\,(1)\right] = \\ & (1-\psi') H_{1,H} E_t S_{t+1}\,(2) + (1-\gamma) E_t r_{t+1}^{p,H}\,(2) + \frac{\psi'}{2} E_t \left[(1-\gamma) r_{t+1}^{p,H}\,(1)\right]^2 + \\ & \frac{1-\psi'}{2} E_t \left[\left[H_{1,H} S_{t+1}\,(1) + (1-\gamma) r_{t+1}^{p,H}\,(1)\right]^2 + 2v\,(2) + S_{t+1}\,(1)' H_{2,H} S_{t+1}\,(1)\right] \end{aligned} \tag{54}$$

where $v\,(2)$ is the second-order component of v and $H_{2,H}$ is a 3x3 matrix with the second derivative of $f_H(S)$, evaluated at $S = 0$.

(54) entails cross-products of the first-order components of the state variables, $S_{t+1}\,(1)$, and the portfolio return, $r^{p,H}_{t+1}\,(1)$. These terms are taken from the first-order solution (29). (54) also includes the second-order components of the state variables, $S_{t+1}\,(2)$, which are taken from (32), as well as the second-order component of the expected the portfolio return, $E_t r^{p,H}_{t+1}\,(2)$, which takes a form similar to (31):

$$E_t r^{p,H}_{t+1}\,(2) = r_s S_t\,(2) + S_t\,(1)'\, r_{ss} S_t\,(1) + \hat{r}\sigma^2 \tag{55}$$

where r_{ss} is a 3x3 matrix and $\hat{r}$ is a scalar.

We use (54), along with the solution for $S_{t+1}\,(1)$, $S_{t+1}\,(2)$, $r^{p,H}_{t+1}\,(1)$ and $E_t r^{p,H}_{t+1}\,(2)$ to solve for $H_{2,H}$.[28] The 9x1 vector $H^{vec}_{2,H}$ is the "vectorized" form of the 3x3 matrix $H_{2,H}$. Specifically, the first three elements of $H^{vec}_{2,H}$ are the first row of $H_{2,H}$, the next three elements are the second row of $H_{2,H}$ and the last three elements are the third row of $H_{2,H}$. $H^{vec}_{2,H}$ is solved from (54) as:

$$H^{vec}_{2,H} = (I_9 - (1-\psi')\hat{N})^{-1} H^{vec}_3$$

where I_9 is a 9x9 identity matrix. $\hat{N}$ is a 9x9 matrix that consists of cross-products of various elements of the N_1 matrix from (29). The 9x1 vector H^{vec}_3 is the "vectorized" form of a 3x3 matrix H_3. The matrix H_3 includes cross-products of the matrices $H_{1,H}$ and r_s, as well as the matrix r_{ss} in the second-order component of the expected portfolio return (55), specifically:

$$\begin{aligned}
H_3 &= -H'_{1,H}H_{1,H} + 2F_1 + 2(1-\gamma)r_{ss} + (1-\psi')N'_1 H'_{1,H} H_{1,H} N_1 \\
&\quad +2(1-\psi')(1-\gamma)N'_1 H'_{1,H} r_s + (1-\gamma)^2 r'_s r_s \\
F_1 &= (1-\psi')\sum_{v=1}^{3} H_{1,H}(v) N_{3,v}
\end{aligned}$$

where $H_{1,H}(v)$ is the v'th element of the 1x3 vector $H_{1,H}$ and the 3x3 matrices $N_{3,v}$ are the same as in (32).

The corresponding matrices for the Foreign investor, $H_{1,F}$ and $H_{2,F}$, are computed analogously.

[28] We also solve for $v\,(2)$, but this element does not affect portfolio choice.

C First-order difference in portfolio shares

The solution of the first-order component of the portfolio share difference k_t^D relies on the third-order components of the optimal portfolio conditions (43)-(44). The expansion of the condition for the Home investor (43) leads to:

$$\begin{aligned}
&E_t er_{t+1}(3) + E_t er_{t+1}(1)\, r_{t+1}^A(2) + E_t er_{t+1}(2)\, r_{t+1}^A(1) \\
&+E_t er_{t+1}(1)\left[(1-\psi')f_{Ht+1}(2) - \gamma r_{t+1}^{p,H}(2) + p_t(2) - p_{t+1}(2)\right] \\
&+E_t er_{t+1}(2)\left[(1-\psi')f_{Ht+1}(1) - \gamma r_{t+1}^{p,H}(1) + p_t(1) - p_{t+1}(1)\right] \\
&+\tau E_t\left[r_{t+1}^A(1) + (1-\psi')f_{Ht+1}(1) - \gamma r_{t+1}^{p,H}(1) + p_t(1) - p_{t+1}(1)\right] + O_3 = 0
\end{aligned} \tag{56}$$

where $er_{t+1}(i) = r_{H,t+1}(i) - r_{F,t+1}(i)$, $r_{t+1}^A(i) = 0.5\left[r_{H,t+1}(i) + r_{F,t+1}(i)\right]$, and τ is second-order. The first term in (56) is the third-order component of the expected excess return. The next two terms are the third-order components of the cross-product between excess returns and the average return, and consists of products of first- and second-order terms. Similarly, the fourth and fifth terms are the third-order components of the cross-product between excess returns and the pricing kernel. The sixth term reflects the friction in investing abroad, τ. The last term in (56) consists of cubic-products of first-order elements:

$$\begin{aligned}
O_3 \;=\;& \frac{1}{6}E_t\left[\left(r_{H,t+1}(1)\right)^3 - \left(r_{F,t+1}(1)\right)^3\right] \\
&+E_t\left[(1-\psi')f_{Ht+1}(1) - \gamma r_{t+1}^{p,H}(1) + p_t(1) - p_{t+1}(1)\right] r_{t+1}^A(1)\, er_{t+1}(1) \\
&+\frac{1}{2}(1-\psi')E_t\left[f_{Ht+1}(1) - \gamma r_{t+1}^{p,H}(1) + p_t(1) - p_{t+1}(1)\right]^2 er_{t+1}(1) \\
&+\frac{1}{2}\psi' E_t\left[-\gamma r_{t+1}^{p,H}(1) + p_t(1) - p_{t+1}(1)\right]^2 er_{t+1}(1)
\end{aligned}$$

The various components of O_3 are solved using the first-order solution (29). We can show that the resulting expression is:

$$O_3 = 2r_{DE}B_H A_H \sigma^2 S_t + \frac{\psi'(1-\psi')}{2}E_t\left[f_{Ht+1}(1)\right]^2 er_{t+1}(1)$$

where A_H is a 1x3 vector and r_{DE} and B_H are scalars. r_{DE} reflects the sensitivity of the first-order excess return to innovations:

$$er_{t+1}(1) = r_{DE}\epsilon_{t+1}^D$$

where $\epsilon^D_{t+1} = \epsilon_{H,t+1} - \epsilon_{F,t+1}$. A_H and B_H reflect the first-order solution of a combination of the average rate of return $r^A_{t+1}(1)$ and the pricing kernel:

$$r^A_{t+1} + (1-\psi')f_{Ht+1}(1) - \gamma r^{p,H}_{t+1}(1) + p_t(1) - p_{t+1}(1) = A_H S_t + B_H \epsilon^D_{t+1} + C_H \epsilon^A_{t+1}$$

where C_H is a scalar and $\epsilon^A_{t+1} = 0.5\left(\epsilon_{H,t+1} + \epsilon_{F,t+1}\right)$.

We undertake similar steps using the condition for the Foreign investor (44). Taking the difference between (56) and its equivalent for the Foreign investor, we write:

$$\begin{aligned}
&\frac{\psi'(1-\psi')}{2} E_t\left[[f_{Ht+1}(1)]^2 - [f_{Ft+1}(1)]^2)\right] er_{t+1}(1) \\
&+E_t er_{t+1}(1)\left[\begin{array}{c} (1-\psi')\left[f_{Ht+1}(2) - f_{Ft+1}(2)\right] - \gamma\left(r^{p,H}_{t+1}(2) - r^{p,F}_{t+1}(2)\right) \\ +\left(p_t(2) - p^*_t(2)\right) - \left(p_{t+1}(2) - p^*_{t+1}(2)\right) \end{array}\right] \\
&+E_t er_{t+1}(2)\left[\begin{array}{c} (1-\psi')\left[f_{Ht+1}(1) - f_{Ft+1}(1)\right] - \gamma\left(r^{p,H}_{t+1}(1) - r^{p,F}_{t+1}(1)\right) \\ +\left(p_t(1) - p^*_t(1)\right) - \left(p_{t+1}(1) - p^*_{t+1}(1)\right) \end{array}\right] = 0
\end{aligned} \tag{57}$$

The first-order component of the difference in portfolio shares, $k^D_t(1)$, enters (57) through the second-order components of the portfolio returns. Taking the second-order components of (51)-(52) leads to:

$$\begin{aligned}
&r^{p,H}_{t+1}(2) - r^{p,F}_{t+1}(2) = k^D(0) er_{t+1}(2) + \left(p_t(2) - p^*_t(2)\right) \\
&- \left(p_{t+1}(2) - p^*_{t+1}(2)\right) + k^D_t(1)\, er_{t+1}(1)
\end{aligned}$$

Similarly, taking the first-order component of (51)-(52) leads to:

$$r^{p,H}_{t+1}(1) - r^{p,F}_{t+1}(1) = k^D(0) er_{t+1}(1) + \left(p_t(1) - p^*_t(1)\right) - \left(p_{t+1}(1) - p^*_{t+1}(1)\right)$$

Using this result, (57) becomes:

$$\begin{aligned}
&\frac{\psi'(1-\psi')}{2} E_t\left[[f_{Ht+1}(1)]^2 - [f_{Ft+1}(1)]^2)\right] er_{t+1}(1) \\
&+(1-\psi')\hat{cov}\left(f_{Ht+1} - f_{Ft+1}, er_{t+1}\right) - \gamma k^D(0)\hat{var}(er_{t+1}) \\
&+(\gamma - 1)\,\hat{cov}(p_{t+1} - p^*_{t+1}, er_{t+1}) - \gamma k^D_t(1)\, var(er_{t+1}) = 0
\end{aligned} \tag{58}$$

where $\hat{cov}\left(x_{t+1}, y_{t+1}\right) = E_t x_{t+1}(1)\, y_{t+1}(2) + E_t x_{t+1}(2)\, y_{t+1}(1)$ and $\hat{var}\left(x_{t+1}\right) = \hat{cov}\left(x_{t+1}, x_{t+1}\right)$ and $var(er_{t+1}) = E_t\left[er_{t+1}(1)\right]^2$. (33) follows simply from (58).

The elements of (58) are computed by using the first-order solution (29), the second-order dynamics of the state variables, (32), and the second-order solution for the control variables, which are of the form of (31). For instance, the excess returns are:

$$er_{t+1}(1) = r_\epsilon' \epsilon_{t+1} \qquad er_{t+1}(2) = S_t(1)' M \epsilon_{t+1}$$

where r_ϵ' is a 1x2 vector, $\epsilon_{t+1} = [\epsilon_{H,t+1}, \epsilon_{F,t+1}]'$ and M is a 3x2 matrix. Using these expression, we write:

$$\hat{var}(er_{t+1}) = 2E_t er_{t+1}(1)\, er_{t+1}(2) = 2\sigma^2 r_\epsilon' M' S_t(1) \tag{59}$$

(59) shows that the third-order components of the variances and covariances in (58) reflect the second-order variance of the innovations, σ^2, along with the first-order state variables, $S_t(1)$. Solving for all the third-order components of the variances and covariances in (58) along similar lines we compute the first-order difference in portfolio shares as a function of the first-order components of state variables:

$$k_t^D(1) = k_s S_t(1)$$

where k_s is a 1x3 vector.

Taking the average of (56) and its equivalent for the Foreign investor, we write:

$$\begin{aligned} 0 &= E_t er_{t+1}(3) + \frac{1}{4}\psi'(1-\psi')E_t\left[f_{H,t+1}(1)^2 + f_{F,t+1}(1)^2\right] er_{t+1}(1) \\ &\quad + \hat{cov}_t \left[\begin{array}{c} er_{t+1}, r_{t+1}^A + (1-\psi')\frac{1}{2}\left[f_{Ht+1} + f_{Ft+1}\right] \\ -\gamma\frac{1}{2}\left(r_{t+1}^{p,H} + r_{t+1}^{p,F}\right) + \frac{1}{2}(p_t - p_t^*) - \frac{1}{2}\left(p_{t+1} + p_{t+1}^*\right) \end{array} \right] \end{aligned}$$

Using the first- and second-order components of (51)-(52) this becomes:

$$\begin{aligned} 0 &= E_t er_{t+1}(3) + \frac{1}{4}\psi'(1-\psi')E_t\left[f_{H,t+1}(1)^2 + f_{F,t+1}(1)^2\right] er_{t+1}(1) \\ &\quad + \frac{\gamma-1}{2}\hat{cov}_t\left(p_{t+1} + p_{t+1}^*, er_{t+1},\right) \\ &\quad - \frac{\gamma-1}{2}E_t\left[2r_{H,t+1}(1)r_{H,t+1}(2) - 2r_{F,t+1}(1)r_{F,t+1}(2)\right] \\ &\quad + \frac{1}{2}(1-\psi')\hat{cov}_t\left(f_{Ht+1} + f_{Ft+1}, er_{t+1}\right) - \gamma k_t^A(1)E_t\left(er_{t+1}(1)\right)^2 \end{aligned}$$

where we used the fact that $E_t\left(er_{t+1}(1)\right)^3 = 0$. (34) follows from a simple re-arrangement of terms.

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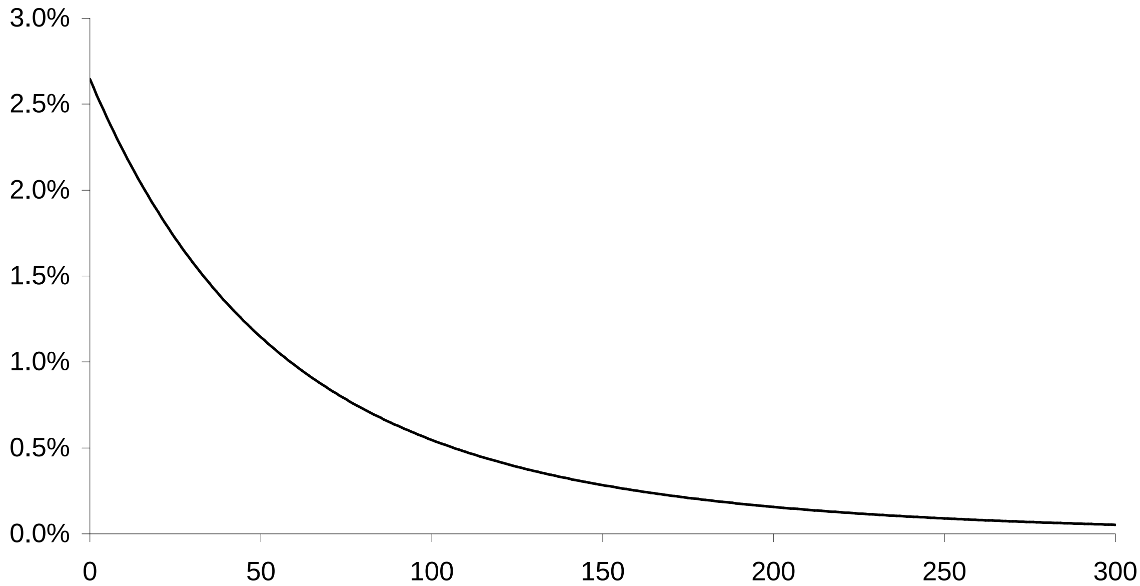
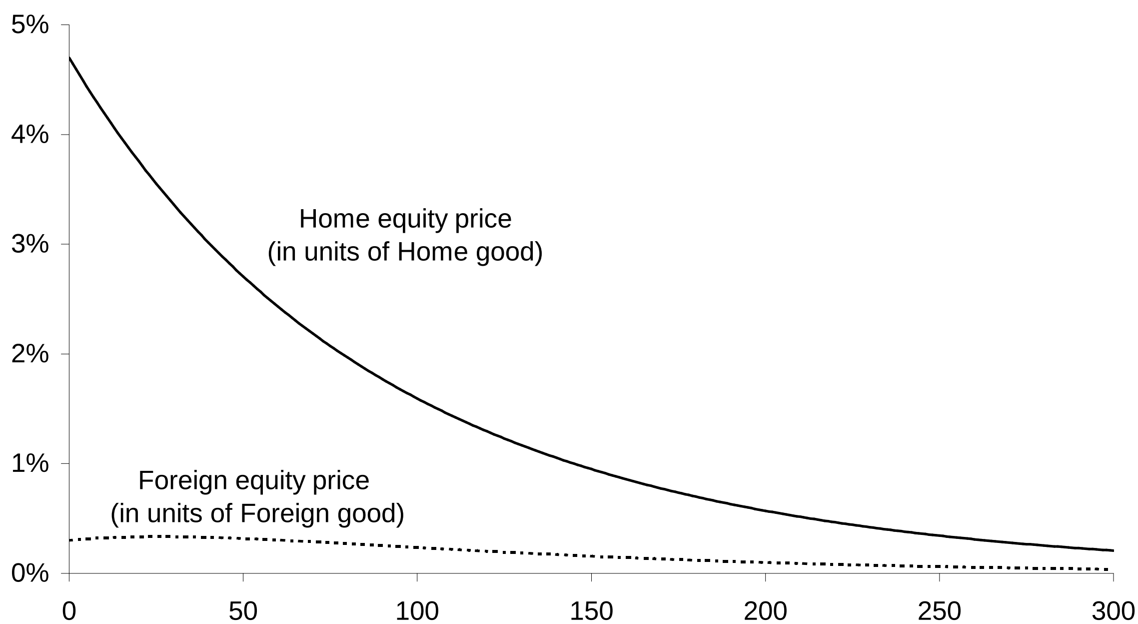
Chart 1: Relative price of the Foreign good*



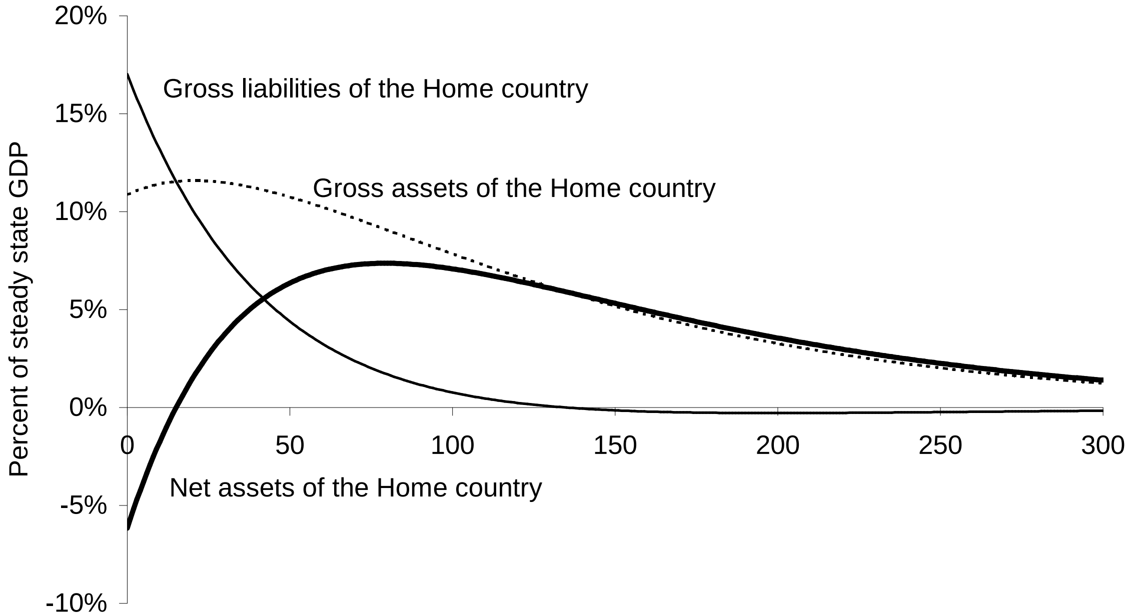
* Impulse response of the relative price of the Foreign good to a 5% increase in Home productivity. An increase in the relative price of the Foreign good corresponds to a real depreciation for the Home country.

Chart 2: Equity prices*



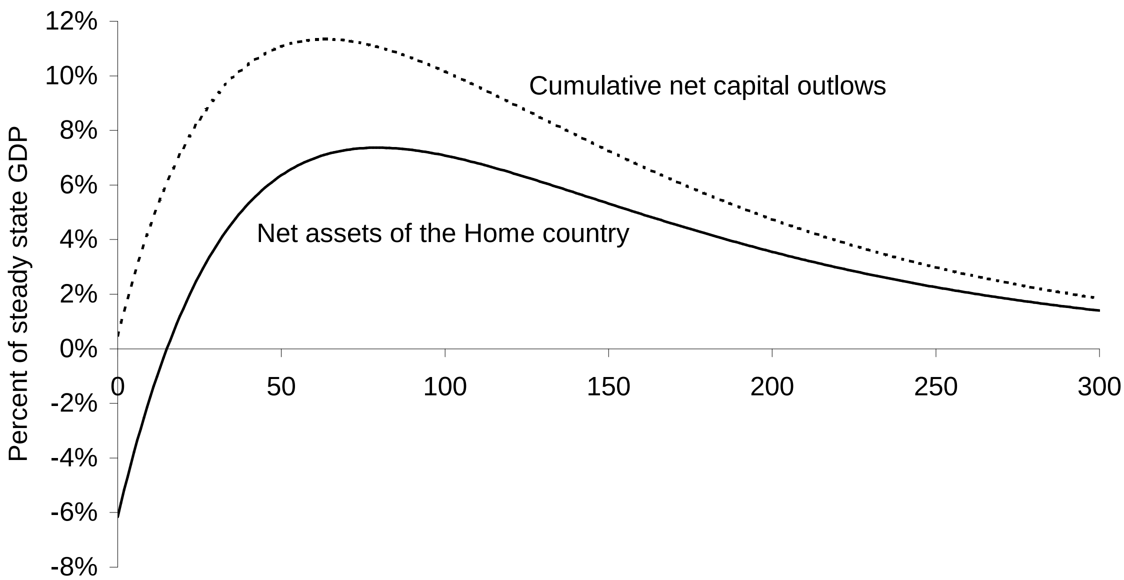
* Impulse response after a 5% increase in Home productivity. In terms of the notation in the text the lines represent q_H and q_F-p_F.

Chart 3: International assets and liabilities*



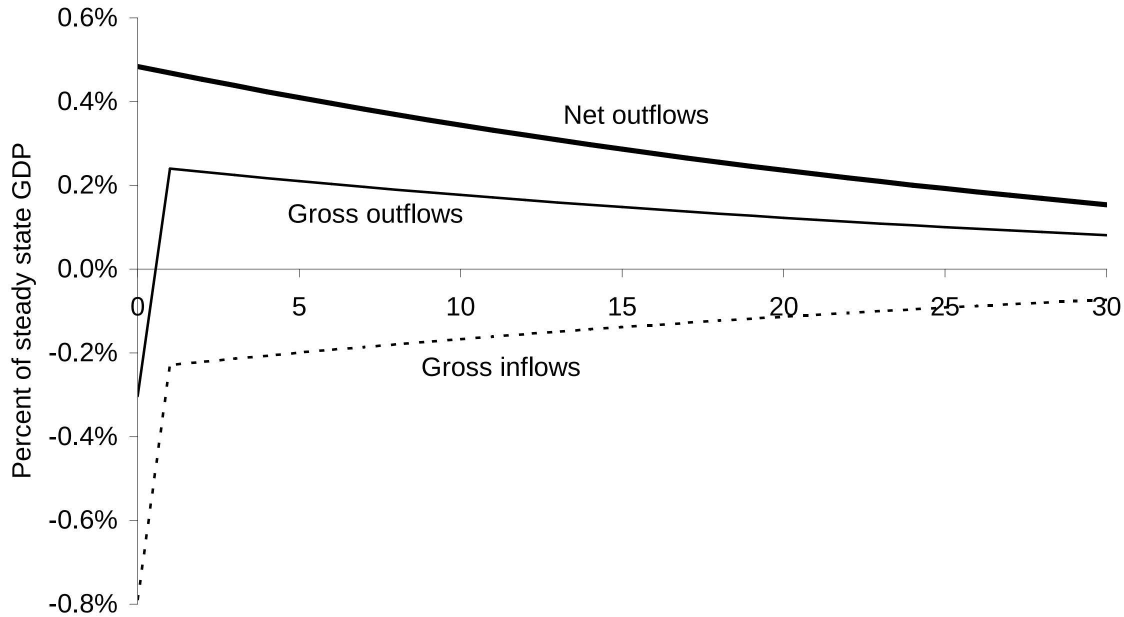
*Impulse response after a 5% increase in Home productivity.The gross assets of the Home country are the gross liabilities of the Foreign country.

Chart 4: Net assets and cumulative net capital outflows*



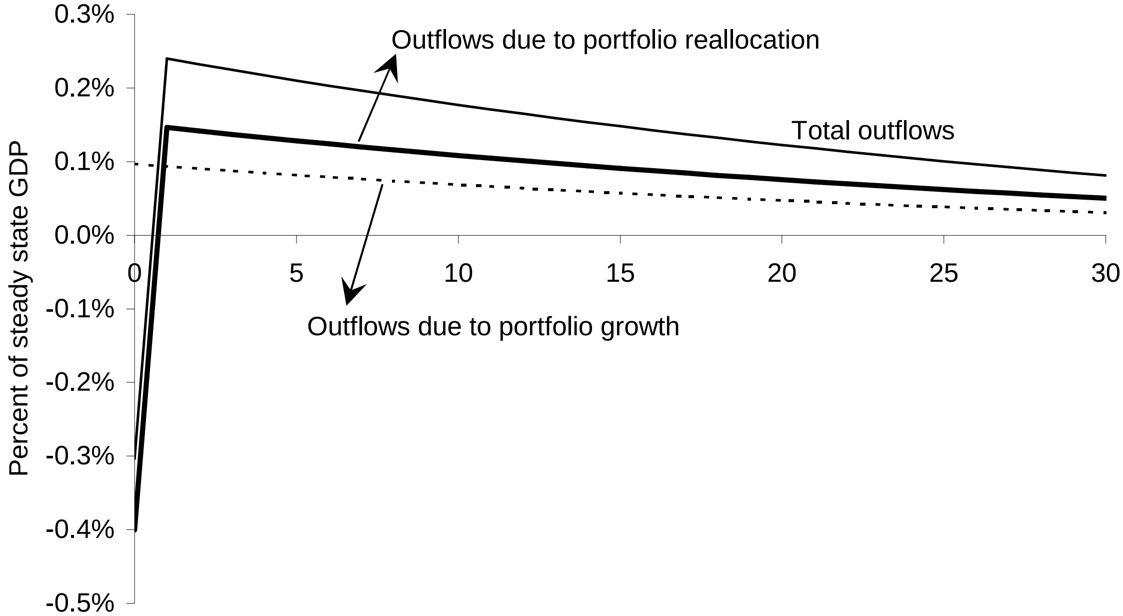
* Impulse response after a 5% increase in Home productivity. The 'cumulative net capital outflows' line at period t denotes the sum of net capital outflows from Home to Foreign between period zero and period t.

Chart 5: Gross and net capital flows*



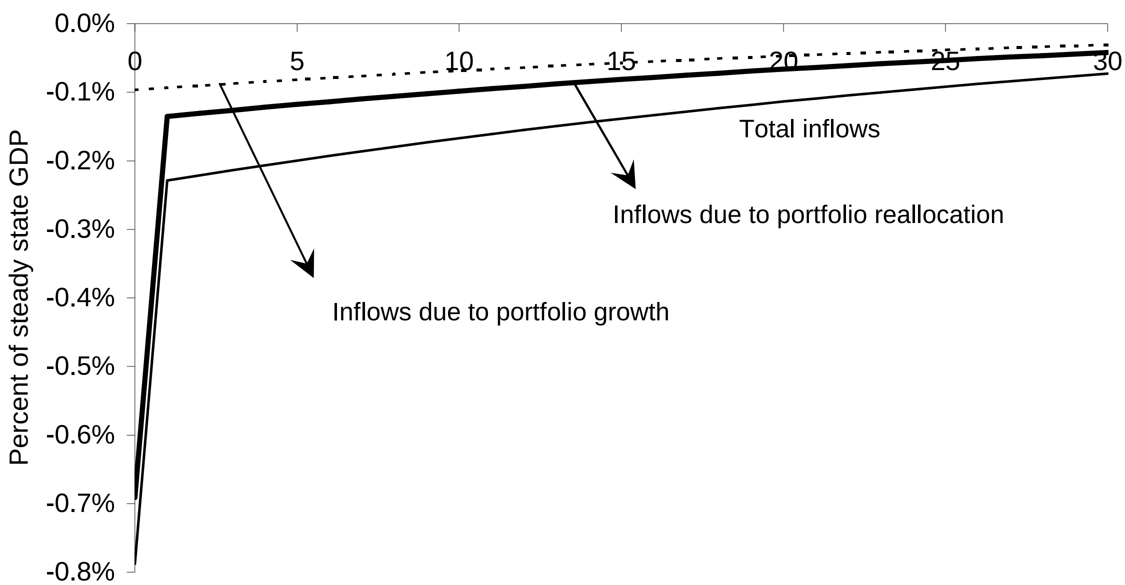
* Impulse response after a 5% increase in Home productivity. Positive values for gross outflows indicate a purchase of Foreign equity by Home investors. Positive values for gross intflows indicate a purchase of Home equity by Foreign investors.

Chart 6: Breakdown of gross capital outflows*



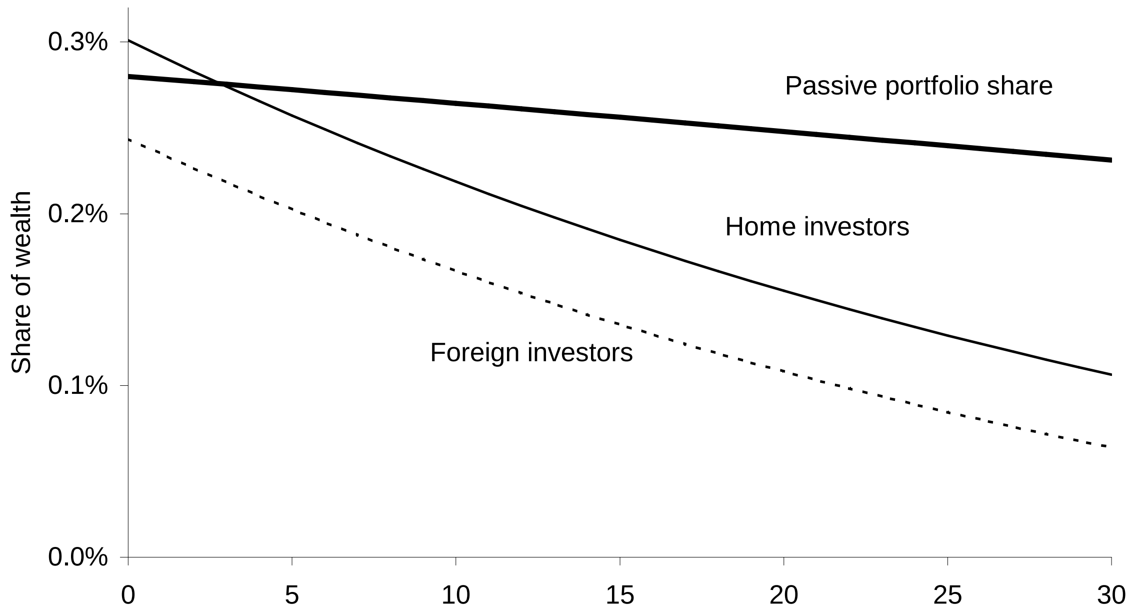
* Portfolio reallocation indicates capital outflows due to active reallocation towards Foreign equity. Portfolio growth indicates capital outflows due to increased saving, allocated across assets at steady state portfolio shares.

Chart 7: Breakdown of gross capital inflows*



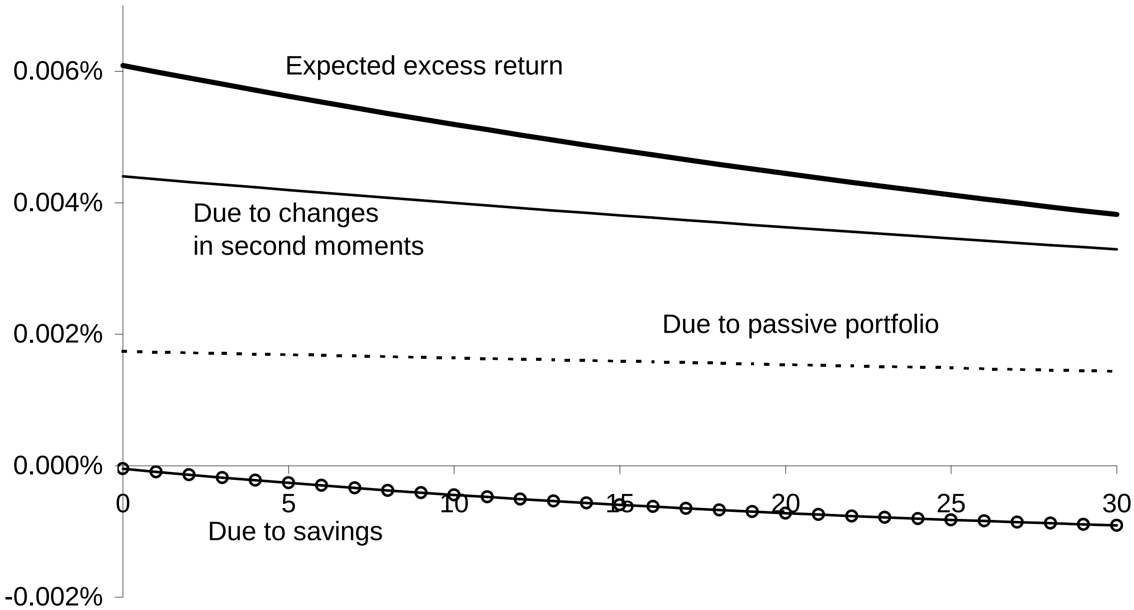
* Portfolio reallocation indicates inflows due to reallocation towards Home equity. Portfolio growth indicates inflows due to increased saving, allocated at steady state portfolio shares. Both are negative as Foreign saving drops and the portfolio is reallocated to Foreign equity.

Chart 8: Share of Home equity in portfolio*



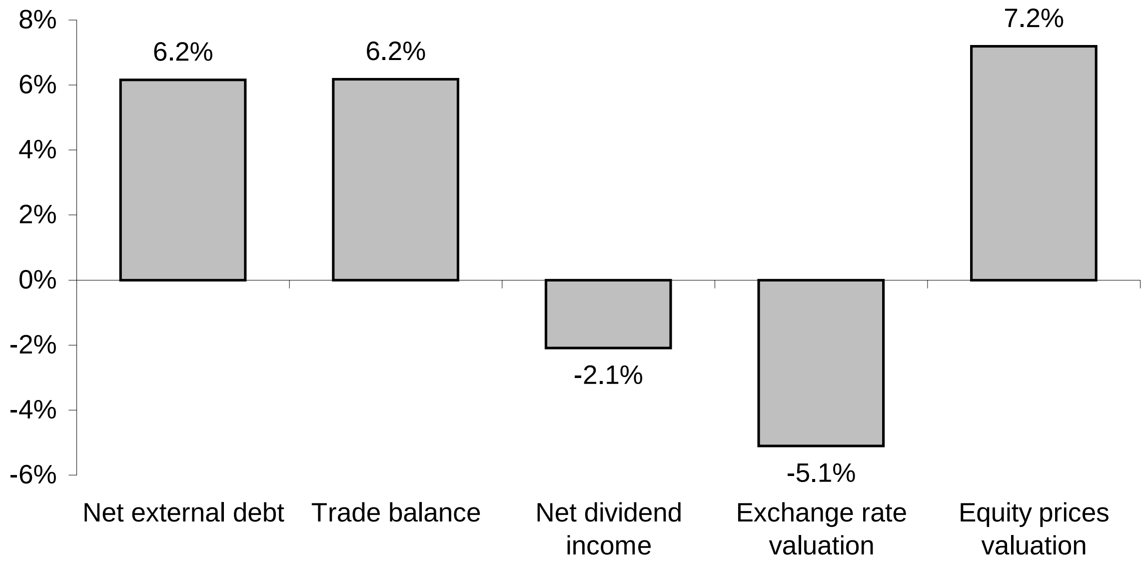
* Impulse response after a 5% increase in Home productivity. The chart shows the change in the share invested in Home equity. The passive portfolio share reflects the direct impact of movements in equity prices (the change in the portfolio share without equity trade).

Chart 9: Expected excess return



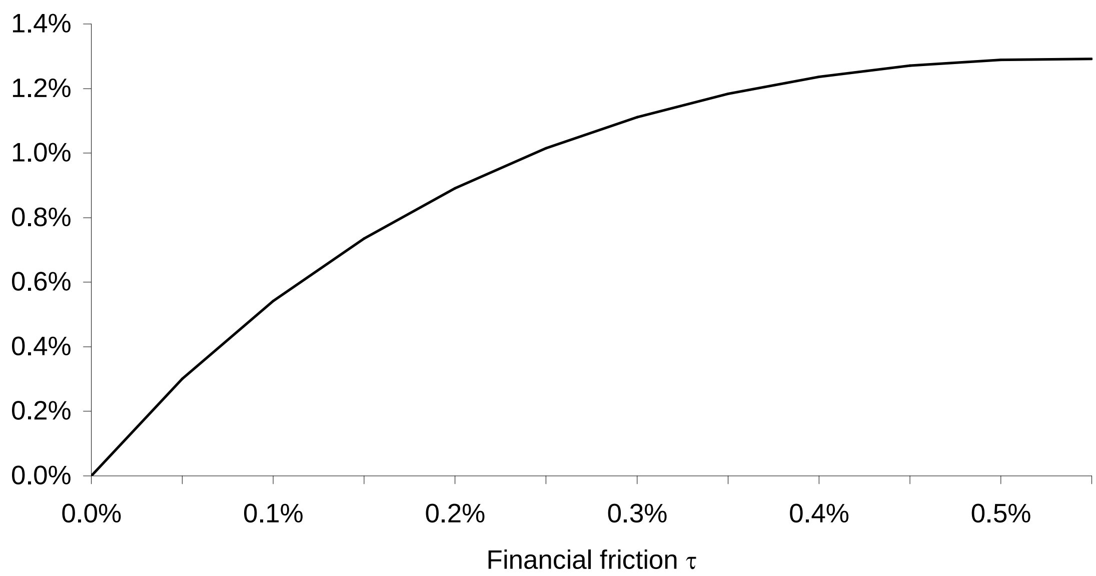
* Impulse response after a 5% increase in Home productivity. The chart shows the third-order expected excess return on Home equity, relative to Foreign equity. The component due to passive portfolio shows the expected excess return needed to induce investors to hold the passive portfolio. The component due to changes in second moments is the expected excess return needed to undo the demand shift toward Foreign equity driven by changing second moments. The component due to savings shows the reduction in the expected excess return needed to offset the demand shift driven by the higher savings of Home investors.

Chart 10: External adjustment channel: net present values*



* Changes after 5% increase in Home productivity. All values are measured at the end of the period when the shock occurs, after any initial jump in response to the shock. The 'net external debt' column indicates the value of the Home net external debt as a fraction of GDP. The 'trade balance' column is the net present value of expected future trade surpluses of the Home country. The 'net dividend income' column is the present value of expected net dividend income of the Home country (negative value=expected positive net dividend payments to Foreign country). The 'exchange rate valuation' column is the present value of expected future valuation gains due to a real depreciation of Home currency (negative value=valuation losses due to expected real appreciation). The 'equity prices valuation' column is the net present value of expected future valuation gains due to equity price changes.

Chart 11: Welfare loss from financial friction



* The chart shows the welfare loss from the financial friction τ, measured in terms of the percentage loss in wealth that leads to an identical drop in utility