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Abstract

Liquidity hoarding by banks and extreme volatility of the fed funds rate have been widely seen as severely disrupting the interbank market and the broader financial system during the 2007-08 financial crisis. Using data on intraday account balances held by banks at the Federal Reserve and Fedwire interbank transactions to estimate all overnight fed funds trades, we present empirical evidence on banks' precautionary hoarding of reserves, their reluctance to lend, and extreme fed funds rate volatility. We develop a model with credit and liquidity frictions in the interbank market consistent with the empirical results. Our theoretical results show that banks rationally hold excess reserves intraday and overnight as a precautionary measure against liquidity shocks. Moreover, the intraday fed funds rate can spike above the discount rate and crash to near zero. Apparent anomalies during the financial crisis may be seen as stark but natural outcomes of our model of the interbank market. The model also provides a unified explanation for several stylized facts and makes new predictions for the interbank market.

Key words: excess reserves, fed funds rate, hoarding, liquidity, limited participation, payments

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1 Introduction

"Cash-rich banks will hoard their money if they fear that the interbank market will cease to function, cutting them off from future supply." *Economist*, August 12, 2007

Throughout the 2007-08 financial crisis, banks have been widely accused of hoarding liquidity and being very reluctant to lend in the interbank market. Starting in August 2007, many banks realized that they had a dramatic increase in their liquidity risk because of the potential need to make intraday payments for ABCP liquidity lines. At the same time, many of these banks had new uncertainty about their ability to quickly borrow in the interbank market because of credit concerns about their sub-prime exposures. Additionally, after the Bear Stearns near-bankruptcy in March 2008 and the Lehman bankruptcy in September 2008, banks had increased uncertainty regarding their intraday payment liquidity shocks and credit constraints. Liquidity hoarding by banks and extreme fed funds rate volatility have been blamed for severely hampering the provision of credit and liquidity within the financial system and to the broader economy.

Our interest lies in studying the precautionary behavior of banks facing liquidity shocks and credit constraints, and how this affects the interbank market equilibrium. To achieve this, we develop a model with payment liquidity shocks, credit constraints and limited interbank market participation. Banks rationally hold large precautionary balances intraday and overnight, which may be described as "hoarding," and which leads to volatility in the interbank market rate. We show that extreme outcomes that occur during a crisis may be in part explained by our general model of interbank market frictions. The model also gives broad theoretical results about the effects of such interbank lending frictions during non-crisis times.

The model is based on the fed funds market in the U.S, in which banks lend reserves to each other in an uncollateralized interbank market. Banks hold reserve balances in accounts at the Federal Reserve. Excess reserves are non-interest bearing balances that banks hold beyond required reserves.[1] Banks withdraw from their Reserve accounts to pay for intraday liquidity shocks, which occur in the form of unexpected large-value payments

[1] The Federal Reserve started paying interest on reserves on October 9, 2008, which is after the sample periods for our empirical results. We discuss interest on reserves in the conclusion.

that must be made same-day. If banks are overdraft at the end of the day, they borrow from the discount window at a penalty rate.

In the model, constrained banks cannot borrow on the interbank market for credit reasons and do not participate in the market at day-end because of limited participation constraints. Constrained banks hold large reserve balances to self-insure against shocks. These constrained banks, which we label as "small," lend excess reserves to unconstrained banks, which we label as "large," during the day after the initial liquidity payment shock is realized. Such lending enables small banks to efficiently self-insure against liquidity shocks earlier in the day. This result is a novel intraday-liquidity based explanation for the stylized fact in the literature that small banks are on average large net lenders to large banks in the fed funds market. But small banks continue to hold some precautionary balances through the end of the day to self-insure against late-day shocks, implying that small banks are more reluctant to lend than large banks, controlling for available balances. Because of constrained banks' limited participation in the market, aggregate reserve balances can become trapped at the end of the day in the account of the small banks if the payments shocks flow to the small banks. The leads to "contagious hoarding," in which large unconstrained banks also need to hold precautionary balances.

The model may also explain the extreme volatility during the crisis of the fed funds rate, which traded on many days near zero percent and above the discount rate. In the model, if the hoarded reserves held by large banks are insufficient for late-day liquidity shocks, the fed funds rate spikes to the marginal cost of borrowing, which is (the shadow value of) the discount rate. Alternatively, if hoarded reserves are in excess of liquidity needs late in the day, large banks dump reserves in the market and drive the fed funds rate down to the marginal value of excess overnight reserves, which is zero (abstracting from reserve requirements during a maintenance period). In summary, the model shows that banks' limited participation constraints may explain our empirical findings of overnight excess reserves holding and fed funds rate volatility. Credit constraints may explain our observed findings of small banks' additional intraday precautionary reserve balances and net fed funds lending from small to large banks.

The assumptions and results of the model are generally consistent with our empirical findings. Using datasets that include minute-by-minute Federal Reserve account balances and all interbank Fedwire transactions, we estimate the complete set of overnight interbank fed funds trades using the methodology first developed by Furfine (1999, 2000). In particular, we show that small banks appear to have credit constraints that prevent them from actively borrowing in the market as large banks do. Small banks also appear to have limited participation in borrowing or lending in the fed funds market at the end of the day. In comparison with large banks, small banks hold larger intraday and overnight balances, are large positive net lenders, but show more reluctance to lend during the day when controlling for a bank's available balance.

The phenomena we examine of precautionary hoarding by banks also provides broader understanding of hoarding and the reduced maturity of lending seen during the crisis in the money markets more widely. Banks' holding of excess reserves is equivalent to the shortening of maturities in the limit—making a zero term maturity loan at a zero interest rate. Our paper shows that credit constraints and limited participation because of incomplete markets can explain hoarding and a shortening of maturities for lending as a more general result of liquidity problems for financial intermediaries more broadly. For banks in particular, liquidity shocks are understood in the literature as foundational. Liquidity shocks for banks in modern economies during contemporary times nearly always ultimately take the form of large-value payments withdrawals rather than, for example, currency withdrawals, which occurred historically during the Great Depression and in current times in emerging economies. Therefore, payments shocks are the fundamental source of shocks for studying liquidity problems for banks.

The literature on the fed funds market suggests a few different explanations for the pattern of small banks lending to large banks. Ho and Saunders (1985) develop a model in which small banks prefer taking deposits to borrowing on the fed funds market because of risk aversion. An alternative explanation for the reliance on deposits by small banks are the results of Rose and Kolari (1985) whose empirical results suggest that small regional banks have lower deposit-taking costs as a result of local monopoly power. Allen, Peristiani, and Saunders (1989) document that larger banks are net purchasers of fed funds, consistent with the hypothesis of small banks having greater adverse selection problems in the market, while the same pattern of net purchases does not exist in the repo market, a collateralized market that overcomes some of the adverse selection problems of the fed funds market.

Ashcraft and Bleakley (2005) document that privately-held banks appear to face financial constraints when borrowing in the federal funds market. Allen and Saunders (1986), give an explanation based on asymmetric information leading to adverse selection. Small banks' size and location outside of money centers makes information on their credit quality more difficult to discover. They further examine the roles of multi-period contracts and relationships to partially resolve those adverse selection problems in the fed funds market. We take the inability of small banks to borrow in the fed funds market as an assumption. This friction plays out through the banks' behavior in the fed funds market and in their choices of precautionary balance levels, which contrasts with Allen and Saunders (1986) who consider multi-period implicit contract remedies for the adverse selection problem.

A more recent literature examines the implementation of monetary policy based on incomplete markets or partial equilibrium models of payments shocks to bank reserves. Excess reserves are held because either no banks can trade after payments shocks occur, payments shocks are modeled as withdrawn from the banking system, or there are autonomous shocks to the supply of reserves held by banks that the Fed cannot fully offset. This literature includes Ennis and Weinberg (2007), Whitesell (2006a,b), Pérez-Quirós and Rodríguez-Mendizábal (2006) and Berentsen and Monnet (2007). In contrast, the fed funds market in reality is very active among large banks from 6-6:30pm after payment shocks end at 6pm. We provide a general equilibrium model of bank reserves and the fed funds market with a richer model of time-of-day payment shocks. Our model focuses on the heterogeneity of banks, by which only small banks have limited market participation end-of-day. The liquidity shocks in our model are a result of payments flowing between banks within a complete, closed system of banks. By modeling multiple trading rounds in the fed funds market, we can address the dichotomy between low and high volatility periods of trading within the day, as well as the evolution of banks' balances during the day, for which we also provide empirical evidence.

Section 2 gives empirical results. Section 3 presents and solves the model. The results of the model for precautionary reserves, bank lending and fed funds rate volatility are given in Section 4. Section 5 gives policy implications and conclusions.

2 Empirical Motivation

This section outlines some motivating facts for the model. The analysis sample refers to the approximately 700 banks that ever lend or borrow during September 2007 through August 2008. Figures for this section are in the Appendix. We measure size using percentiles of the cross-sectional distribution of average daily Fedwire send for the bank over this time period. While the smallest banks lend about one out of every five days, they rarely borrow (about 5 percent of business days). On the other hand, the largest decile of banks lends on about 8.5 out of every 10 days, and borrows on about 7.5 out of every 10. The key takeaway for our results below is that small banks do very little borrowing regardless of their available balance. However, small banks are net lenders, while large banks are net borrowers. But small banks lend only when they have large intraday balances relative to their typical intraday balance positions, and lend only earlier not later in the day. This suggests that small banks have credit and market participation constraints, and that small banks do not put themselves at risk of being overdraft or needing to borrow.

2.1 End of day problem for large banks

First, we focus on large banks, defined using the top quintile. Figures 1(a) and 1(b) illustrate the aggregate lending of large banks across different time intervals of the day and across percentiles of own institution balance. Panel (a) makes it clear that aggregate lending by large banks is concentrated in the last 90 minutes of the day, and followed by the interval from 3pm to 5pm. Moreover, the panel illustrates that for each of these two intervals, while large banks lend much more when balances are high, they are also willing to lend significant amounts even when balances are low, even at the end of the day. While some of this is driven by a small number of large institutions which are net lenders of funds every day, it also reflects the absence of financial constraints. Panel 7(b) illustrates a similar picture for large bank borrowing, with the most borrowing occurring during the last 90 minutes of the day, and followed closely by the 3pm to 5pm interval.

In contrast to the large banks, Figures 2(a) and 2(b) illustrate that small banks, defined using the bottom quintile of Fedwire sends, typically lend and borrow largely only for liquidity purposes. Most of this lending and borrowing occurs during the 3pm to 5pm period. Moreover, while there was a monotone relationship for large banks between balance and federal funds activity, the relationship for small banks is more non-linear: lending increases only when balances are in the highest two percentiles while borrowing increases sharply when balances are in the lowest percentile. These figures paint a picture of small institutions as being relatively constrained in borrowing, especially at the end of the day. Moreover, they suggest that small institutions do not lend meaningful amounts to large banks at the end of the day.

While the analysis here has focused on the two extreme quintiles, Figure 7(c) shows the fraction of lending and borrowing by banks for all size deciles. This figure shows that the asymmetry between lending and borrowing by banks holds for the smallest 80% of banks, but not for the largest 10% of banks. A more thorough analysis suggests that the market is segmented into two parts: one which includes roughly the top 100 institutions who lend and borrow throughout the day, for both funding and liquidity purposes, and one which includes the other 700 institutions who participate in the market only for liquidity purposes, and do so early in the day. This market structures suggests that at the end of the day, the federal funds market is largely a reallocation of reserves between large banks. This view is validated in Figure 3, which documents how the cross-sectional distribution of balances changes during the last 90 minutes of the business day. The figure focuses on the 100 accounts over September 2007 through August 2008. At the start of this window (17:00), note that a significant fraction of banks have negative balances. These typically large institutions make use of intraday credit throughout the day. This credit is provided by the Federal Reserve for a small fee (measured as 36 basis points at an annual rate, adjusted for the duration of the credit as a percentage of the day) to promote the timely sending of payments. As the end of the business day (18:30) nears, reserves are reallocated from institutions with positive balances to banks with negative balances, largely through federal funds loans.

Figure 4 illustrates that the last hour of the day is an increasingly volatile time for large banks. The graph plots the federal funds interest rate volatility measured by the time series standard deviation of the dollar-weighted average federal funds rate over the previous thirty minutes. The sample refers to loans between the top 100 banks over September 2007 through August 2008. It is clear from the figure that volatility starts to increase around 17:30 and has a significant spike at 18:20 when banks seem fairly certain of their end-of-day balances. Banks in need of reserves during this time are subject to a severe hold-up problem, as the penalty on an overnight overdraft is the effective federal funds rate plus 400 basis points, but the total cost could be much larger due to the presence of stigma.

The prospect of stigma is illustrated in Figures 5(a) and 5(b), which document instances in which banks are willing to pay a higher interest rate than the primary credit rate at the discount window. In Panel (a), the figure illustrates the fraction of days in each month where the intraday high is larger than the primary credit rate. In Panel (b), the figure illustrates a number of times when the stop-out rate for the Term Auction Facility was higher than the interest rate at which banks could have borrowed directly from the Federal Reserve (for the same term).

2.2 Financial frictions and small banks

Figures 6(a) illustrates the propensity of large banks to lend at different times of day and in different balance positions. For each bank, we measure the percentiles of the distribution of balance at a given minute of the day across all days of the sample period. The point of using bank-specific distributions is to take into account the fact that different banks have different standards for what is normal at a given time of day. While large banks are active lenders during the 3pm to 5pm window, they are also active lenders during the last 90 minutes of the day when faced with a favorable reserve position. The graph documents that more than 75 percent of the largest banks with the most favorable reserve position will lend during the last 90 minutes of the day. Moreover, note that 65 percent of the largest banks facing the most adverse reserve position are willing to lend during this late period. Together, these facts suggest that large banks are active lenders throughout the business day, but especially at the end of the business day, and especially when reserves are unusually high.

Figure 6(b) illustrates that large banks also borrow throughout the day, but do borrow the most when hit with an adverse reserve balance at the end of the day. For example, just under 65 percent of banks hit with the worst reserve position during he last 90 minutes borrow. This suggests that federal funds trading is a key component of the reserve management strategy of large banks throughout the day.

In contrast, Figure 7(a) focuses on the average propensity of the smallest banks to lend across different states of nature measured by the actual balance during different windows of the day. The figure documents that the smallest banks are most willing to lend in the 3pm to 5pm window, and that these institutes rarely lend during the last 90 minutes of the day. Moreover, the figure illustrates the natural phenomenon that banks are more likely to lend when faced when reserves are higher than normal. However, note that the willingness of these banks to lend is quite small, as only about 6 percent will lend during the 3pm to 5pm window when faced with the most favorable liquidity shock. The number is almost an order of magnitude smaller than the equivalent measure for the largest banks. These facts suggest that the smallest institutions do not participate in the federal funds market at the end of the day.

Figure 7(b) documents the average propensity of the smallest banks to borrow across percentiles of the balance distribution for different time windows. The smallest banks typically borrow during the 3pm to 5pm window when the reserve position is in one of the two most adverse deciles. However, small banks also borrow during the last 90 minutes of the day, but only when faced with the tail of the reserve balance distribution. Note that the mean probability of borrowing is quite low for small banks, and Figure 2(b) shows the amount of borrowing is very low, suggesting that reserve management is largely accomplished by holding large precautionary reserves and lending reserves, and not through borrowing.

In order to better illustrate this point, we illustrate the marginal in Figure 8(a) and cumulative distribution in Figure 8(b) of the 3pm balance for each of large and small banks, each scaled by the standard deviation of payment shocks over the next two hours. The figure clearly illustrates that small banks at 3pm hold more balances relative to the fundamental uncertainty about net payment flows, suggesting they are unable to use the federal funds market for borrowing.

In order to show that there is nothing special about the 3pm balance, we next focus in Figures 9(a) and 9(b) on the end-of-day balance ex federal funds lending and borrowing – which we denote the clean balance – scaled by payment uncertainty. As with the 3pm balance, small banks hold higher balances than large banks with the same amount of payment uncertainty. Consequently, the data suggest that the inability of small banks to use the federal funds market for liquidity purposes leads them to hoard reserves through the end of the day.

The inability of small banks to use borrowing in order to smooth payment shocks suggests that they would be net lenders in the federal funds market, both in absolute terms but also relative to the amount of payment uncertainty. Figure 10(a) and 10(b) illustrate the marginal and cumulative distribution of net lending, scaled by the standard deviation of net payment shocks over the next two hours. Since clean balances are a bank's day-end balance ex fed funds, the negative area under the large banks' curve in Figure 10(a) is the amount they borrow from small banks, and hence the small banks' net lending. The small banks' net lending is also reflected by the positive area under the small banks' curve in Figure 10(a). This positive area equals their net lending plus overnight excess reserves. These figures clearly indicate that a small institution has more net lending than a large institution with the same amount of payment uncertainty.

2.3 Precautionary hoarding and the recent financial crisis

Banks hoarded reserves with a large reluctance to lend during the 2007-08 financial crisis, and the federal funds rate in the interbank market traded at erratic extremes. Figure 11(a) shows banks' excess reserves, which are balances that banks hold at the Federal Reserve beyond required reserves through October 8, 2008. Excess reserves are calculated according to two-week reserve maintenance periods, and began receiving interest on October 9, 2008. Excess reserves were roughly one to two billions dollars through most of 2007 and the first half of 2008, but increased to roughly \$9 billion in August 2007 and spiked to over \$130 billion two weeks before October 8, 2008. The Federal Reserve determines the amount of reserves held by banks through open market operations to target the fed funds rate. The Federal Reserve's lending and other liquidity programs increased the size of its balance sheet, which was not fully sterilized. Figure 11(b) shows that the effective fed funds rate, which is the average lending rate between banks for reserve balances and represents the opportunity cost of holding excess reserves, was positive (even if below the target rate). The positive effective rate indicates that banks demanded holding excess reserves at a positive opportunity cost.

The fed funds rate also traded at erratic extremes. Figure 11(c) shows the 5th and

95th percentiles of the effective fed funds rate, calculated over five minute intervals, from the crisis period of August 9 through December 10, 2007, in comparison to the period of January 2 through August 8, 2007. The effective rate often deviated from the fed funds target rate, which is the policy rate chosen by the Federal Open Market Committee, during the last hour of trading from 5:30pm and 6:30pm by extreme amounts. The effective rate crashed more than 400 bps below target at the 5th percentile and spiked more than 100 bps above target at the 95th percentile.

This extreme funds rate volatility can be explained in part by the significant increase in the demand for reserves as term interbank lending migrated to an overnight maturity, as illustrated in Figure 12. As the Federal Reserve responded aggressively but imperfectly to large but uncertain increases in the demand for reserves, volatility increased significantly throughout the day, but most notably at the end of the day.

However, this simple explanation of the data is not complete, as Figure 13 documents that banks faced a significant increase in payment shocks.[2] The figure plots residuals from a regression of aggregate dollar volume of non-loan Fedwire sends on a time trend and day-of-month effects , which represent the unexpected level of payment activity.[3] As the model is estimated through July 2007, the mean daily unexpected payment volume is zero over that time period. However, the mean unexpected payment for August is over $200 billion. In response to this higher uncertainty about payments, banks responded by becoming more reluctant to lend excess reserves when reserves were high and by becoming more aggressive in bidding for borrowed reserves when balances were low. This is illustrated in Figure 14(a), which documents measures of reluctance to lend and desperation to borrow for every business day from January 2002 through March 2008.

Reluctance to lend is measured by the interest rate on which banks with unusually high reserves charge in order to lend relative to the effective federal funds rate for the previous 15 minutes. Desperation to borrow is measured by the interest rate at which banks with unusually low reserves receive to borrow relative to the effective federal funds rate for the previous 15 minutes. Unusually high or low levels of reserves are defined by the

[2]This increase in uncertainty about payments is in turn explained by draws on backup lines for asset-backed commercial paper (ABCP) as investors.

[3]The day-of-month effects include dummies for predictably high payment volume days, including the beginning and end of each month and quarter, principal and interest remittance dates for the GSEs, and the business day after a holiday. In addition, the regression employs dummy variables for each maintenance period day.

level of reserves being one standard deviation above or below the median level of reserves for that time of day for the institution over the quarter. Daily aggregates are constructed using the normal dollar value weights for federal funds activity by bank and time of day. These aggregates are regressed on the same day-of-month effects. The figure plots squared residuals from each regression. The levels of aggregate reluctance and desperation were at levels in August which had never been seen previously in the available data. While these levels quickly subsided, they have remained elevated and volatile relative to historical norms, and appear to have positive covariance with the measure of unexpected aggregate dollar payment volume illustrated in Figure 13.

Another explanation for the precautionary behavior banks might be heightened concerns about counterparty credit risk and participation constraints. In order to assess the importance of credit risk and participation constraints, we investigate how the empirical marginal probability distribution over the number of counterparties has changed over the crisis period. In particular, Figure 14(b) illustrates that from July 2007 to August 2008 or March 2008, there was little change in the number of banks a borrower funded itself. However, in each September 2008 and October 2008, there were significant decreases in the number of counterparties, most notably for institutions which only borrowed previously from one lender. Over this period the fraction of borrowers with zero counterparties increased from approximately 16 percent to 23 percent. Moreover, there appears to be a modest adverse impact even for institutions with 10 or more counterparties. Figure 14(c) tells a similar story for lenders, documenting significant increases in probability mass at zero counterparties during the most recent stress, offset by decreases in probability mass at one or two counterparties. The fraction of lenders with zero counterparties increased from about 22 percent in March 2008 to about 31 percent in October 2008. All of these results suggest that concerns about increased credit risk and participation constraints did not become extremely important until fall of 2008, suggesting that changes in reluctance starting in August 2007 were largely driven by changes in the volatility of payments.

3 Model

3.1 Environment

Banks are risk neutral and hold reserves for precautionary reasons in the face of random intraday shocks to avoid being overdrawn at the end of the day. There are L large banks called type 'l' and S small banks called type 's' and four periods $t \in \{1\text{pm}, 3\text{pm}, 6\text{pm}, 9\text{pm}\}$, abbreviated as $\{1, 3, 6, 9\}$. Banks receive payments shocks at $t \in \{3, 6\}$ that they must pay during the period. A bank can make any amount of payments intraday regardless of its reserve balance, which abstracts from any fees or caps for intraday credit from the Fed. But if a bank is overdrawn at the end of the day, it must borrow from the discount window at a penalty rate.

The time periods are stylized and broadly represent the actual intraday events of the fed funds market. Period $t = 1$ represents morning and early afternoon transactions, before banks realized many payments shocks and when the Fed conducts open market operations using collateralized repos. Period $t = 3$ represents late afternoon when many liquidity shocks are realized. Period $t = 6$ represents the end-of-day when large liquidity shocks still potentially occur but when there is little time until 6:30pm, when the fed funds market and Fedwire closes for the day. The fed funds market is dominated by rapid trading by large money center banks allocating available reserves among themselves. Collaterized repo lending is not possible during the late day interbank market because of the time and cost for securities collateral delivery. However, we assume that large banks do not need collateralization because they have no credit constraints, and we show that small banks efficiently overcome non-collateralized borrowing constraints through self-insurance with precautionary reserves.

Banks may have credit and end-of-day participation constraints for several reasons. Banks that are small may have greater risk aversion to the end-of-day rate volatility, relatively greater day-end participation fixed costs, or less credit information for lending to other banks. The standard market convention is for fed funds loans to be returned within 24 hours, which implies that banks that lend at day-end do not have access to their funds within the next day, which may impose relatively greater intraday burdens on small banks. Finally, European banks do not operate during Fedwire afternoon (Eastern

Standard Time) hours.

The model abstracts from reserve requirements. Many banks do not have binding reserve requirements because their vault cash is sufficient. Remaining reserve requirements imply that overnight reserves have a shadow value during the two-week maintenance period, and a more limited shadow value on the last day of the period. Up to 3% of reserves in excess of requirements may count forward to the following period's maintenance requirement. The model results are thus stylized and are mitigated by intra-maintenance period reserve smoothing and interperiod carryovers. During a crisis, increased demand for precautionary reserves met by the Fed may imply that banks are "locked-in," or have reserve requirements satisfied earlier in the maintenance period. This implies that the model's stark results for bank hoarding and rate spikes and crashes may be interpreted more literally, especially on day ten of the maintenance period. Also not considered are intraday overdraft fees of 36 bps per annum and caps, which may strengthen the effects of intraday precautionary reserves and rate volatility.

Positive values of the flow variables, payment shocks p_t^i and fed funds loans f_t^i, represent outflows from banks, while negative values represent inflows. Discount window loans w_6^i are always positive and represent inflows. The state variable m_t^i represents the reserve balances held by bank i entering period t.

Timeline The timeline is displayed in Figure 15.

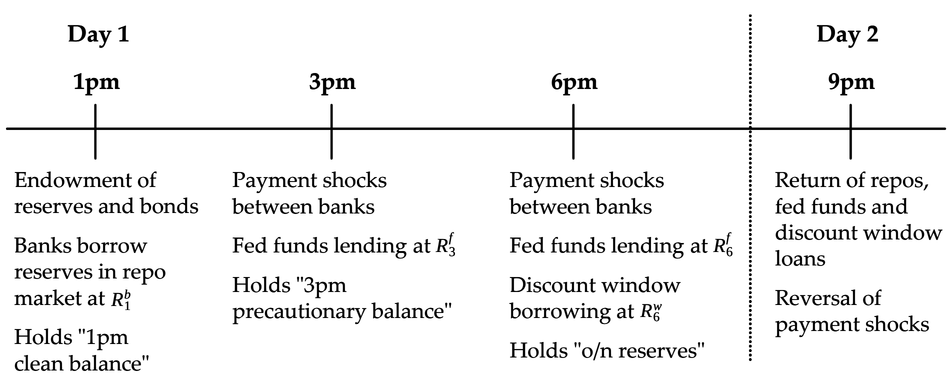

Figure 15: Timeline

1pm: Bank $i \in \{l, s\}$ holds $b_1^i \in \mathbb{R}$ bonds and $m_1^i \in \mathbb{R}$ Federal Reserve account balances at the start of the period. The Fed conducts open market operations (equivalent to a repo market) by buying and selling any amount of bonds to banks at a price of one and gross return that the Fed sets of $1 + R_1^b > 1$ at $t = 9$. The bank chooses $\Delta b_1^i \in \mathbb{R}$ bonds to buy.

3pm: Bank i holds $b_3^i = b_1^i + \Delta b_1^i$ and $m_3^i = m_1^i - \Delta b_1^i$.[4] Bank l has a payment shock of p_3^l to small banks and p_3^k to other large banks. Bank s has a payment shock of p_3^s to large banks. For simplicity, bank s has no payment shock to other small banks. (Bank l's shocks to other large banks at $t = 1$ and $t = 3$ below are not required for any results). Banks may then trade on the fed funds market, in which prices are taken as given. Bank s lends $f_3^s(R_3^s) \geq 0$ to large banks for a return due at $t = 9$ of R_3^s. Bank l borrows $-f_3^l(R_3^s) \geq 0$ from small banks and lends $f_3^k(R_3^k) \in \mathbb{R}$ to other large banks.

6pm: Bank l has a payment shock of p_6^l to small banks and p_6^k to other large banks. Bank s has a payment shock of p_6^s to large banks. Bank l lends $f_6^k(R_6^k) \in \mathbb{R}$ in the fed funds market to other large banks. Bank $i \in \{l, s\}$ must borrow $w_6^i \geq 0$ from the Fed discount window for a return due at $t = 9$ of $R_6^w \geq R_1^b$, such that it's balance at the end of the period is non-negative. R_6^w is interpreted as the actual discount rate plus the shadow cost of stigma and potential restriction on future ability to borrow at the discount window.

9pm: Period $t = 9$ can be considered as equivalent to occurring the next day before or at the beginning of the $t = 1$ period. Loans are returned and payment shocks are reversed, such that banks have no uncertainty outside of the 1pm through 9pm periods. Bank l has payment shocks of $-(p_3^l + p_6^l)$ to small banks and $-(p_3^k + p_6^k)$ to other large banks. Bank s has a payment shock of $p_9^s = -(p_3^s + p_6^s)$ to large banks. Bank l has a payment of $-(1+R_3^s)f_3^l - (1+R_3^k)f_3^k - (1+R_6^k)f_6^k$, and bank s has a payment of $-(1+R_3^s)f_3^s$, to repay fed funds. Bank i makes a payment of $(1+R_6^w)w_6^i$ to the Fed to repay its discount window loan, and the Fed redeems bonds to bank i for $(1 + R_1^b)b_3^i$ in reserve balances (equivalent to trading longer-dated bonds for balances).

Notation and distributions To summarize the notation, lowercase variables generally denote individual bank values. An 'l' or 's' superscript generally denotes a state variable for that bank type, a flow variable transaction from that bank type to the other bank type, or an interest rate R_t^i involving transactions of bank type. A 'k' superscript generally denotes a flow variable or interest rate for transactions among large banks. Subscripts

[4] We could equivalently assume bank s does not trade during $t = 1$, and rather that m_3^s is its steady-state level in a repeated game.

denote the period $t \in \{1, 3, 6, 9\}$.

For economy of notation, the superscript 'l', 's' or 'k' that indicates a bank or transaction type is also used as the index number for summations, where $l \in \{1, ..., L\}$, $k \in \{1, ..., K\}$ and $s \in \{1, ..., S\}$. For each lowercase variable, its uppercase P_t^i, F_t^i, M_t^i or W_6^i denotes the sum for type i at period t. For instance, $P_t^s = \sum_{s=1}^{S} p_t^s$ and $P_t^l = \sum_{l=1}^{L} p_t^l$ for $t \in \{3, 6\}$. Banks are competitive, so they take prices and aggregate quantities F_t^i and W_t^i as given. The aggregate payment shocks from small banks to large banks equals the aggregate payment shocks from large banks to small banks, implying $P_t^s = -P_t^l$. Aggregate payment shocks among large banks must aggregate to zero, implying $P_t^k = 0$ for $t \in \{3, 6\}$.

Payments shocks have zero mean, with a uniform distribution $p_t^i \sim U[-\overline{p}^i, \overline{p}^i]$, $i \in \{l, s\}$, and an unspecified distribution for p_t^k, for $t \in \{3, 6\}$. For simplicity, we assume that P_t^i has a uniform distribution, where $P_t^i \sim U[-\overline{P}, \overline{P}]$, for $i \in \{l, s\}$ and $t = \{3, 6\}$. $\overline{P} = \gamma^i \overline{p}^i$ for $i \in \{l, s\}$, where $\gamma^l \in (0, L)$ and $\gamma^s \in (0, S)$, which implies that shocks for type $i \in \{l, s\}$ are not perfectly positively or negatively correlated.[5] Bank i has combined liquid assets in the form of bonds and reserves greater that its potential payment shocks to other banks: $m_1^i + b_1^i \geq 2\overline{p}^i + \overline{p}^k \mathbf{1}_{i=l}$ for $i \in \{l, s\}$, where $\mathbf{1}_{[\cdot]}$ is the indicator function.

[5]It is natural to think of unexpected payments as having zero mean, because any expected payments would typically be funded by repos or fed funds traded in the morning fed funds market. The uniform distribution of P_t^i is assumed for simplification and should not qualitatively effect the results. Consider the correlation of p_t^i across all banks of a particular type $i \in \{l, s\}$ and period $t \in \{3, 6\}$. If the correlation is negative one, P_t^i has a degenerate uniform distribution of $U[0, 0]$ and corresponds to the limiting case of $\gamma^i = 0$. If the correlation is one, P_t^i has a uniform distribution of $U[-L\overline{p}^i, L\overline{p}^i]$ for $i = l$ and $U[-S\overline{p}^i, S\overline{p}^i]$ for $i = s$, which corresponds to the limiting case of γ^i equal to L and S, respectively. If the correlation is zero, the central limit theorem implies that as L and S go to infinity, the distributions of P_t^l and P_t^s, would approach normal given by $N(0, \frac{L(\overline{p}^l)^2}{3})$ and $N(0, \frac{S(\overline{p}^s)^2}{3})$, respectively. Instead, the variance of P_t^i with its assumed uniform distribution is $\frac{(\gamma^i \overline{p}^i)^2}{3}$. For $\gamma^l = L^{\frac{1}{2}}$ and $\gamma^s = S^{\frac{1}{2}}$, P_t^i has the same variance as it would under the central limit theorem. The difference is that a uniform distribution implies P_t^i has much "fatter tails," or extremely lower kurtosis, than P_t^i would have under a normal distribution. This can be interpreted as a positive correlation of p_t^i, with a particularly high correlation among tail values of p_t^i.

3.2 Bank Optimizations and Solutions

The bank $i \in \{l, s\}$ optimization problem to maximize profits is as follows:

$$\begin{aligned}
\max_{\boldsymbol{A}^i} \quad & E[\pi^i] && (1)\\
\text{s.t.} \quad & m_3^i \leq b_1^i + m_1^i && (2)\\
& -f_3^l \mathbf{1}_{i=l} + f_3^s \mathbf{1}_{i=s} \geq 0 && (3)\\
& w_6^i \geq 0 && (4)\\
& m_9^i \geq 0. && (5)
\end{aligned}$$

For bank l,

$$\begin{aligned}
m_6^l &= m_3^l - p_3^l - p_3^k - f_3^l - f_3^k && (6)\\
m_9^l &= m_6^l - p_6^l - p_6^k - f_6^k + w_6^l && (7)\\
\pi^l &= (1 + R_1^b) b_3^l + m_3^l - R_6^w w_6^l + R_6^k f_6^k + R_3^s f_3^l + R_3^k f_3^k - b_1^l - m_1^l\\
\boldsymbol{A}^l &= \{m_3^l, f_3^l, f_3^k, f_6^k, w_6^l\}. && (8)
\end{aligned}$$

For bank s,

$$\begin{aligned}
m_6^s &= m_3^s - p_3^s - f_3^s && (9)\\
m_9^s &= m_6^s - p_6^s + w_6^s\\
\pi^s &= (1 + R_1^b) b_3^s + m_3^s - R_6^w w_6^s + R_3^s f_3^s - b_1^s - m_1^s\\
\boldsymbol{A}^s &= \{m_3^s, f_3^s, w_6^s\}. && (10)
\end{aligned}$$

Constraint (2) gives the maximum reserve balances m_3^i that can be held at $t = 3$. We call m_3^i bank i's "clean balances," and is equal to the bank's daily starting reserve balances net of any fed funds or discount window loans, and before any payments shocks for the day. Constraint (3) gives the restriction that small banks cannot borrow from large banks. Constraint (4) restricts discount window loans to be non-negative, and constraint (5) requires that overnight reserve balances m_9^i are non-negative. The maximizers in (8) and (10) reflect that large banks participate in the fed funds market at $t = 6$ while small banks do not.

We examine equilibria that are symmetric among type $i \in \{l, s\}$, and for which constraint (3) does not bind. As equilibrium conditions, aggregate interbank lending among large banks nets to zero each period, implying $F_t^k = 0$ for $t \in \{3, 6\}$, and aggregate interbank lending between large and small banks satisfies $F_3^l(R_3^s) = -F_3^s(R_3^s)$.

We solve the model starting at $t = 6$. For a large bank, if payment shocks during $t = 6$ are larger than its balance entering the period, a large bank can borrow the difference from other large banks at a rate of zero if aggregate reserves of large banks are positive. If aggregate reserves of large banks are negative, the large bank must borrow from the discount window or from another large bank at $R_6^k = R_6^w$. In contrast, a small bank must always borrow at the discount window at R_6^w if its $t = 6$ payment shock is larger than its balance entering the period.

Lemma 1. *If large banks' aggregate balances at day-end $M_6^l - P_6^l < 0$, then $R_6^f = R_6^w$ and large banks' discount window borrowing is $W_6^l > 0$. If $M_6^l - P_6^l \geq 0$, then $R_6^f = 0$ and no large bank borrows from the discount window: $w_6^l = 0$ for all l. If and only if a small bank's individual balances at day-end $m_6^s - p_6^s < 0$, then its discount window borrowing $w_6^s > 0$.*

Proof. See Appendix. ■

At $t = 3$, banks choose interbank lending. Bank l chooses interbank lending $f_3^l(R_3^s)$ to small banks (in negative amounts) and $f_3^k(R_3^k)$ to large banks.

Lemma 2. *The large banks' aggregate demand for fed funds borrowing from small banks is*

$$-F_3^l(R_3^s) = -2\frac{R_3^s}{R_6^w}\overline{P} - M_3^l + P_3^l + \overline{P}, \tag{11}$$

and the fed funds rate at $t = 3$ is

$$R_3^k = R_3^s = E_3[R_6^k]. \tag{12}$$

Proof. See Appendix. ■

Arbitrage by large banks ensures result (12). The individual bank l first order conditions for f_3^l and f_3^k determine aggregate large bank borrowing F_3^l such that

$$R_3^s = R_6^w \frac{(\overline{P} + P_3^l + F_3^l - M_3^l)}{2\overline{P}}. \tag{13}$$

holds. The left-hand side of equation (13) is the return R_3^s on a marginal unit of fed funds borrowed by large banks in aggregate. This must equal the right-hand side of equation (13), which is the expected cost of large banks needing to borrow a marginal unit from the discount window. This expected cost is the discount rate R_6^w, multiplied by the probability that large banks have to borrow from the discount window, which is the last factor on the right-hand side of (13). For simplicity, we assume large banks trade at $t = 3$ to hold equal balances: $m_3^l = \frac{M_3^l}{L}$. Substituting for m_6^l from (6) into $m_6^l = \frac{M_6^l}{L}$, simplifying and solving for f_3^k,

$$f_3^k = -\frac{M_6^l}{L} + m_3^l - p_3^l - p_3^k - f_3^l. \tag{14}$$

Lemma 3. *A small bank's fed funds supply to lend to large banks is*

$$f_3^s(R_3^s) = 2\overline{p}^s \frac{R_3^s}{R_6^w} - p_3^s + m_3^s - \overline{p}^s. \tag{15}$$

Proof. See Appendix. ■

The first order condition for f_3^s implies

$$R_3^s = R_6^w \left[\frac{\overline{p}^s - (m_3^s - p_3^s - f_3^s)}{2\overline{p}^s} \right]. \tag{16}$$

Bank s chooses f_3^s to equate its return on a marginal unit of fed funds lending, R_3^s, with its expected cost of needing to borrow a marginal unit from the discount window. This expected cost is the discount rate R_6^w multiplied by the probability bank s has to borrow, which is the factor in brackets in (16).

The aggregate supply of interbank loans by small banks is

$$\begin{aligned} F_3^s(R_3^s) &= \sum_{s=1}^{S} f_3^s(R_3^s) \\ &= S[2\overline{p}^s \frac{R_3^s}{R_6^w} + m_3^s - \overline{p}^s] - \sum_{s=1}^{S} p_3^s, \end{aligned}$$

where $\sum_{s=1}^{S} m_3^s = Sm_3^s$ since banks of type $i \in \{l, s\}$ are ex-ante identical and choose the same m_3^i at $t = 1$. Solving for R_3^s gives

$$R_3^s = \frac{R_6^w(F_3^s + P_3^s - M_3^s + S\overline{p}^s)}{2S\overline{p}^s}.$$

Lemma 4. *The competitive market equilibrium for fed funds is*

$$\begin{aligned} F_3^s &= -P_3^s + \frac{\overline{P}M_3^s - S\overline{p}^s M_3^l}{S\overline{p}^s + \overline{P}} && (17) \\ R_3^s &= \tfrac{1}{2}R_6^w\{1 - \frac{M_3^s + M_3^l}{S\overline{p}^s + \overline{P}}\}. && (18) \end{aligned}$$

Proof. The equilibrium condition $F_3^s(R_3^s) = -F_3^l(R_3^s)$ determines F_3^s and R_3^s. ■

R_3^s does not depend on P_3^s. An early payment shock P_3^s shifts the aggregate small banks' supply curve and large banks' demand curve in equal amounts to the right, so the fed funds amount increases but the price is unchanged.

The amount borrowed from small banks is equal across large banks by assumption from above. By (15), bank lending across small banks is equal except for the p_3^s term. Thus, in equilibrium, $-f_3^l = \frac{F_3^s}{L}$ and $f_3^s = -p_3^s + \frac{F_3^s - P_3^s}{S}$, which gives

$$\begin{aligned} -f_3^l &= \frac{P_3^l}{L} + \frac{\overline{P}M_3^s - S\overline{p}^s M_3^l}{L\left(S\overline{p}^s + \overline{P}\right)} && (19) \\ f_3^s &= -p_3^s + \frac{\overline{P}M_3^s - S\overline{p}^s M_3^l}{S\left(S\overline{p}^s + \overline{P}\right)}. && (20) \end{aligned}$$

Proposition 1. *The deviation of the fed funds rate from target is greater at $t = 6$ than at $t = 3$. The deviation at $t = 6$ is based on payments shocks (and hence post-shock reserve* *balances) at $t = 6$:*

$$
\begin{aligned}
R_3^s &= R_1^b = E_3[R_6^k] && (21)\\
R_6^k &= \left\{ \begin{array}{ll} 0 & \text{if } P_6^l \le \overline{P}(1 - \frac{2R_1^b}{R_6^w}) \\ R_6^w & \text{if } P_6^l > \overline{P}(1 - \frac{2R_1^b}{R_6^w}) \end{array} \right. && (22)\\
R_6^k &= \left\{ \begin{array}{ll} 0 & \text{if } M_6^l - P_6^l \ge 0 \\ R_6^w & \text{if } M_6^l - P_6^l < 0. \end{array} \right. && (23)
\end{aligned}
$$

Proof. See Appendix. ■

The fed funds rate at $t = 3$ equals the rate targeted by Fed open market operations at $t = 1$. Small banks can efficiently fully self-insure against payments shocks at $t = 3$ since they hold precautionary balances and lend excess balances. Thus, payments shocks during this period do not effect the fed funds rate at $t = 3$ and there is no volatility. For large enough payments shocks to small banks at $t = 6$, reserves are trapped in small banks and the fed funds rate at $t = 6$ spikes to R_6^w. For payments shocks to large banks at $t = 6$, the fed funds rate crashes to 0. Since constrained banks have lending friction at day-end, this is the time when the fed funds rate volatility is greatest.

Solving for the aggregate clean balances by substituting R_1^b for R_3^s into (18) gives

$$M_3^s + M_3^l = (1 - \frac{2R_1^b}{R_6^w})(S\overline{p}^s + \overline{P}). \tag{24}$$

From the equilibrium solution for f_3^s in (20) and f_3^l in (19), if

$$\overline{P}M_3^s - S\overline{p}^s M_3^l > p_3^s S(S\overline{p}^s + \overline{P}) \text{ for all } s, \tag{25}$$

then $f_3^s > 0$ for all s, and $f_3^l < 0$ for all l, since $f_3^l = -\frac{S}{L}F_3^s$, so constraint (3) holds and does not bind.

The inequality (25) always holds if

$$\gamma^s M_3^s - SM_3^l > S\overline{p}^s(\gamma^s + S), \tag{26}$$

and implies that

$$F_3^s = \sum_{s=1}^{S} f_3^s > S\overline{p}^s - \overline{P} > 0. \tag{27}$$

This shows that when each bank s holds optimal balances so that its borrowing constraint is not binding, their precautionary reserves imply that there is always aggregate strictly positive lending to large banks. For solutions satisfying (24) and (26),

$$\begin{aligned} M_3^l &< \overline{P}(1 - \frac{2R_1^b}{R_6^w}) - S\overline{p}^s < 0 \\ M_3^s &> 2S\overline{p}^s(1 - \frac{R_1^b}{R_6^w}) > 0 \end{aligned}$$

which imply

$$m_3^l < \frac{\overline{P}}{L}(1 - \frac{2R_1^b}{R_6^w}) - \frac{S}{L}\overline{p}^s < 0 \tag{28}$$

$$m_3^s > 2\overline{p}^s(1 - \frac{R_1^b}{R_6^w}) > 0. \tag{29}$$

To satisfy constraint (2), $m_3^s < 2\overline{p}^s$, which implies $m_3^l \geq \frac{\overline{P}}{L}(1 - \frac{2R_1^b}{R_6^w}) - \frac{S}{L}\overline{p}^s(1 + \frac{2R_1^b}{R_6^w})$. Thus, to satisfy constraints (2) and (3),

$$\begin{aligned} m_3^l &\in \left(\frac{\overline{P}}{L}(1 - \frac{2R_1^b}{R_6^w}) - \frac{S}{L}\overline{p}^s(1 + \frac{2R_1^b}{R_6^w}), \frac{\overline{P}}{L}(1 - \frac{2R_1^b}{R_6^w}) - \frac{S}{L}\overline{p}^s\right) \\ m_3^s &\in \left(2\overline{p}^s(1 - \frac{R_1^b}{R_6^w}, 2\overline{p}^s\right), \end{aligned}$$

subject to (24).

4 Results for Precautionary Reserves and Bank Lending

Figure 16 summarizes the model's precautionary balances and bank lending results, which are explained in further detail in the Propositions in this section. The x-axis is a bank's balances scaled to the individual (large or small) bank's maximum payment shock size. The y-axis is a bank's lending as a percentage of available balances at $t = 3$. Period t precautionary balances are defined as $m_{t'}$, where t' is the period following t. These are the balances that a bank does not lend at period t in order to hold as a balance $m_{t'}$ entering period t' for shocks in period t'. For results in this section, we assume that aggregate reserve balances $M_3^l + M_3^s$, as determined in equation (24) by model parameters, are positive, which is the case in the U.S.

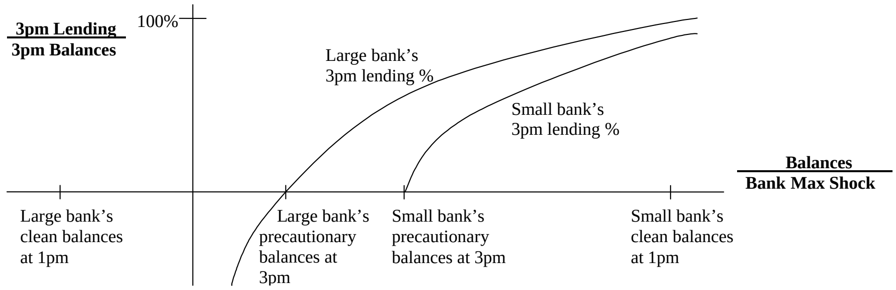

Figure 16: Precautionary reserve balances and bank lending percentages

As indicated in Figure 16, a small banks holds very large clean balances at $t = 1$ to self-insure against $t = 3$ and $t = 6$ payments shocks. These clean balances are large enough that the small bank's borrowing constraint at $t = 3$ never binds, so the small bank always lends balances to large banks at $t = 3$. A large bank holds negative clean balances. Small and large banks hold precautionary balances not lent at $t = 3$ for self-insurance against shocks at $t = 6$. Large banks borrow if necessary to acquire precautionary balances. The percentage of balances lent by small and large banks increases with balances above the precautionary balance level. For any positive scaled balance on the x-axis, a large bank lends a greater percentage than a small bank.

We first compare the percentage of available balances that large and small banks lend on the interbank market at $t = 3$. We show that for a given bank reserve balance, controlling for the size of the bank by scaling by the maximum $t = 6$ shock size, large banks lend a greater percentage of available reserve balances than small banks.

Proposition 2. *Small banks lend a smaller percentage of available reserve balances at $t = 3$ than large banks.*

Proof. See Appendix. ■

Proposition 3. *Small banks hold larger scaled precautionary balances at 3pm than large banks.*

Proof. The precautionary balances held are found by subtracting balances lent from balances available, and are equivalent to m_6^i balances held at the end of period $t = 3$. Banks target to hold the same amount of precautionary balances m_6^i across their type at the end of $t = 3$. The amount of precautionary balances that they do not lend out during $t = 3$ is m_6^i. Bank l holds (scaled) precautionary balances at $t = 3$ of

$$\begin{aligned} \frac{m_6^l}{\overline{p}^l + \overline{p}^k} &= \frac{\overline{P}}{L(\overline{p}^l + \overline{p}^k)}(1 - \frac{2R_1^b}{R_6^w}) \\ &< (1 - \frac{2R_1^b}{R_6^w}), \end{aligned} \tag{30}$$

compared to that of bank s, which holds

$$\frac{m_6^s}{\overline{p}^s} = (1 - \frac{2R_1^b}{R_6^w}). \tag{31}$$

■

Bank i holds fixed precautionary balances at $t = 3$ (and bank l will borrow if necessary to acquire them) to have available entering $t = 3$ regardless of the amount of reserve balances the bank has available to lend at $t = 3$. Hence, the percentage of balances that large or small banks lend increases with their available balances.

Taking the derivative of the left-hand side (right-hand side) of (50) with respect to the left-hand side (right-hand side) of (49) shows that the lending percentage of bank l (s) is a concave function of its scaled balances. The lending percentage increases for bank s and l with scaled balances, and the difference of lending percentage between bank s and l decreases with scaled balances.

Rewriting (30) and (31) as

$$R_6^w(\frac{\overline{P} - M_6^l}{2\overline{P}}) = R_3^s \tag{32a}$$

$$R_6^w(\frac{\overline{p}^s - m_6^s}{2\overline{p}^s}) = R_3^s, \tag{32b}$$

respectively, shows that these $t = 3$ precautionary balances equalize the expected marginal cost R_6^w of having to borrow from the discount window due to $t = 6$ shocks times the probability of discount window borrowing, with the marginal opportunity cost $R_3^s = R_1^b$

of holding excess precautionary balances at $t = 3$.

Bank s holds greater scaled precautionary balances because it cannot borrow at $t = 6$. Bank l can borrow from other large banks, so it only has to borrow at the discount window if the aggregate shock to large banks at $t = 6$ is greater than the aggregate balances held. This is why (32a) is written with the probability of overdraft of large banks in aggregate as a factor, whereas (32b) is written with the probability of overdraft of an individual small bank.

These precautionary balance and lending percentage results are derived assuming that large banks hold equal balances at the end of $t = 3$. However, large banks are indifferent to the relative balances held among themselves. The rate R_3^k at which they trade among themselves at $t = 3$ is equal to the expected rate they trade at $t = 6$. If there were a cost of trading, they would trade less at $t = 3$, which could possibly show that they lend a lower percentage of balances than small banks lend. However, if large banks were slightly risk averse, or if there were any trading frictions at $t = 6$, they would strictly prefer this amount of trading.

When $R_1^b = \frac{1}{2}R_6^w$, banks hold zero precautionary balances to give a one-half probability of borrowing at the discount window with a one-half probability of holding excess $t = 3$ precautionary balances. When $R_1^b < \frac{1}{2}R_6^w$, banks hold strictly positive precautionary balances since the cost of excess balances is less than the cost of the discount window.

Proposition 4. *Aggregate overnight reserve balances held by small and large banks decrease with the fed funds target rate and increase with the discount rate.*

Proof. From (32a) and (32b), M_6^l and m_6^s decrease with R_1^b and increase with R_6^w. ■

Proposition 5. *Large banks lending percentage of scaled balances increases with the $t = 6$ fed funds rate.*

Proof. The percentage of available balances that is lent by large banks at $t = 6$ is

$$\frac{f_6^k}{m_6^l - p_6^l - p_6^k} = \frac{m_6^l - p_6^l - p_6^k - \frac{1}{L}(M_6^l - P_6^l)}{m_6^l - p_6^l - p_6^k}.$$

For $W_6^l = 0$, this lending percentage is less than one since $M_6^l - P_6^l \geq 0$. Since there are excess balances, banks do not lend them all, and $R_6^k = 0$. As reserve balances increase for bank l, the percentage lent increases toward one.

For $W_6^l > 0$, $M_6^l - P_6^l < 0$, so the lending percentage is actually greater than one. This is because we assume large banks borrow equally from the discount window. Anticipating this, banks who need the least amount (or zero) borrowing at the discount window lend to others at the fed funds rate of $R_6^k = R_6^w$. An alternative assumption is that banks with $m_6^l - p_6^l - p_6^k \geq 0$ do not borrow from the discount window, and only banks with $m_6^l - p_6^l - p_6^k < 0$ do borrow from the discount window. This still implies that banks with available balances lend all of them at a rate of $R_6^k = R_6^w$. ■

The model also gives more general implications when there is any market friction that prevents a random positive epsilon amount of reserves from being tradable efficiently at the end of the day, such that the segment of the market that is trading at the end of the day is always in aggregate long or short of reserves. If this segment trades efficiently, then R_6^k is either zero or R_6^W. Greater end-of-day rate volatility implies greater market efficiency given that the full market does not trade. This also holds true if the random long or short for the market is due to "misses" by the Fed's open market operations desk that targets the supply of reserves in the market and if this "miss" information is only revealed throughout the day.

Proposition 6. *Discount window borrowing for small banks compared to that for large banks is less correlated among the bank type, occurs more frequently and is of larger average scaled amounts.*

Proof. The average (or expected) amount of discount window borrowing, scaled for size, for bank s is

$$
\begin{aligned}
E[\frac{w_6^s}{\overline{p}^s}] &= \left(\frac{p_3^s + f_3^s - m_3^s + \overline{p}^s}{2\overline{p}^s}\right)^2 \\
&= \left(\frac{R_1^b}{R_6^w}\right)^2,
\end{aligned}
$$

found by substituting for $E[w_6^s]$ from (43) and then for f_3^s from (47), whereas for bank l it is

$$
\begin{aligned}
E[\frac{w_6^l}{\overline{p}^l+\overline{p}^k}] &= E[\frac{(-M_6^l+P_6^l)^+}{L(\overline{p}^l+\overline{p}^k)}] \\
&= \frac{1}{L(\overline{p}^l+\overline{p}^k)}\int_{-\overline{P}}^{-M_6^l}(-M_6^l+P_6^l)\frac{1}{2\overline{P}}dP_6^l \\
&= \left(\frac{\gamma^l\overline{p}^l}{L(\overline{p}^l+\overline{p}^k)}\right)\left(\frac{R_1^b}{R_6^w}\right)^2 \\
&< \left(\frac{R_1^b}{R_6^w}\right)^2.
\end{aligned}
$$

■

The average amount of nonborrowed reserves held overnight, scaled for size, is equal to m_6^i, the precautionary reserves held at $t=3$, since banks' shocks (and large banks' fed funds lending) is zero on average at $t=6$. Thus, the scaled amount of nonborrowed reserves is also larger for small banks than large banks.

Proposition 7. *Small banks hold larger average scaled amounts of nonborrowed reserves overnight than do large banks.*

Proof. The scaled amount of nonborrowed reserves for bank s is

$$
\begin{aligned}
E[\frac{m_9^s-w_6^s}{\overline{p}^s}] &= \frac{m_6^s}{\overline{p}^s} \\
&= (1-\frac{2R_1^b}{R_6^w}), \qquad (33)
\end{aligned}
$$

whereas for bank l it is

$$
\begin{aligned}
E[\frac{m_9^l-w_6^l}{\overline{p}^l+\overline{p}^k}] &= \frac{m_6^l}{\overline{p}^l+\overline{p}^k} \\
&= \frac{\overline{P}}{L(\overline{p}^l+\overline{p}^k)}(1-\frac{2R_1^b}{R_6^w}) \qquad (34) \\
&< (1-\frac{2R_1^b}{R_6^w}).
\end{aligned}
$$

■

Note that while we include the shock size $\overline{p}^k$ for payments between large banks, all results hold for $\overline{p}^k = 0$. The term $\overline{p}^k$ shows that the results hold even more strongly as the amount of payments shocks among large banks increases.

The clean balances held by banks from (9) is

$$\begin{aligned} m_3^s &= m_6^s + p_3^s + f_3 \\ &> \overline{p}^s(1 - \frac{2R_1^b}{R_6^w}) + \overline{p}^s, \end{aligned}$$

where the second line is from (29) and (31). The first term of the second line is the $t = 3$ precautionary balances of bank s. The second term is the bank's pre- $t = 3$ precautionary balances to self-insure against p_3^s. Any excess $f_3^s = m_3^s - m_6^s - p_3^s$ is lent at $t = 3$. Thus, bank s always lends a strictly positive amount, even when it ends up borrowing at the discount window at day's end. The clean balances held by bank l is shown by (28) to be negative. In expectation, bank l rolls-over overnight fed funds borrowing every day to hold $t = 3$ precautionary balances during the day and positive balances overnight. Since bank s has to choose its lending before $t = 6$ shocks, it has to lend every day, whereas bank l can borrow on the aggregate market after $t = 6$ shocks, which explains why aggregate fed funds lending (27) from small to large banks is strictly positive

$$F_3^s = S\overline{p}^s - \overline{P} > 0.$$

The model offers a partial explanation for the large amount of interbank lending relative to bank reserves. The interbank market lends for an overnight term multiples of the amount of aggregate reserve balances held by banks. At first, this phenomenon may appear to imply that banks must lend the same funds multiple times among banks. However, this model offers a different explanation. In this model, large banks have negative clean balances, $M_3^l < 0$, and rely on borrowing from small banks to achieve non-negative overnight reserves. The amount of funds lent F_3^s may exceed the net supply of reserve balances $M_3^s + M_3^l$, even if there is no relending of reserves. The model also explains why fed funds lending that acts as a large source of financing from small to large banks is primarily of overnight term. Since the lending is a way for small banks to self-insure against daily shocks, the small banks require daily repayment for its potential liquidity needs.

The aggregate amount of clean balances equals the aggregate amount of nonborrowed reserves, and also equals the aggregate amount of $t = 3$ precautionary balances:

$$\begin{aligned} M_3^l + M_3^s &= (M_9^l - W_6^l) + (M_9^s - W_6^s) \\ &= M_6^l + M_6^s, \end{aligned}$$

found by substituting (34) and (33) into the right-hand side of (24). In aggregate, the only purpose for reserves is for precautionary reasons at $t = 3$, because the aggregate pre-$t = 3$ precautionary balances held by small banks that are not used for $t = 3$ shocks are lent to large banks. Anticipating this lending, large banks hold negative clean balances. The following proposition summarizes these results.

Proposition 8. *Small banks hold positive clean balances (balances net of fed funds and discount window loans) and large banks hold negative clean balances. Small banks lend positive amount of fed funds each night.*

Aggregate reserves can also be interpreted in the context of an interest rate corridor, with a deposit facility rate of zero and a lending facility rate of R_6^w. If $R_3^s = \frac{1}{2}R_6^w$, (24) shows aggregate reserves equal zero. The marginal opportunity cost of depositing excess reserves and borrowing needed reserves are equal since banks have a one-half probability of either occurring. As R_1^b decreases below the corridor midpoint, overnight shortages are costlier than overnight excesses, so aggregate reserves increase.

5 Policy Implications and Conclusion

In order to study bank hoarding of reserves, we document new stylized facts of the fed funds market and explain them in a simple model of trading frictions in the interbank market. We show that the concept of precautionary balances can help to explain the stylized facts that small banks hold relatively large amounts of excess reserves overnight, while lending net positive large funds to large banks overnight, despite lending a lower percentage of available balances during the day than large banks lend. We also show there is an increase in the volatility of the fed funds rate late in the day. Furthermore, we offer a new explanation for the phenomena of large amounts of fed funds lending that is multiples of aggregate bank reserves.

Small banks' credit constraints explain why they hold large intraday precautionary balances to self-insure against shocks and act as a structural net lender to large banks. Small banks' limited participation at the end of the day explains day-end fed funds rate volatility and overnight precautionary reserves held by both small and large banks.

The model shows that extreme spikes and crashes in the fed funds rate during the 2007-08 crisis are not surprising, especially for the last day of a maintenance period. The empirical evidence suggests that reserve requirements over a maintenance period helped to prevent extreme rate deviations during normal times but do not prevent these extreme rates during a crisis period. Future research can attempt to quantify the cost of banks' constraints to study what amount of the increase in excess reserves during the crisis that the model can quantitatively explain.

The model suggests that during the 2007-08 financial crisis, the supply of overnight fed funds increased as more banks became constrained and needed to self-insure. Based on anecdotal reports of reduced term lending, these banks likely substituted to overnight interbank lending away from term lending. However, the extreme volatility of the fed funds rate likely increased the demand for term rather than overnight borrowing. The Term Auction Facility (TAF) introduced by the Fed in December 2007 helped to meet the increased net demand for term borrowing by lending to banks for originally a 28 day term. Evidence from McAndrews et al. (2008) shows that the TAF had helped to reduce the term LIBOR spread.

The model allows for interpreting the current Fed regime as a corridor system of monetary policy implementation, with a lower bound of zero and an upper bound of the shadow cost of borrowing at the discount window. This may at first appear to suggest from a simplistic point of view that a narrow corridor paying positive interest on reserves near the fed funds target rate and a discount window lending rate at a small spread above the target would minimize spikes and crashes and provide a good outcome. Under Congressional authorization, the Fed began paying interest on reserves starting on October 9, 2008. However, the model shows that reduced interest rate volatility does not necessarily reduce bank hoarding of reserves and reluctance to lend. Furthermore, fed funds rates traded above the discount rate suggests that discount window stigma would hamper implementing a narrow corridor. Nonetheless, a system of paying interest on reserves near the target rate with a very large amount of reserves supplied to the banking system may reduce the impact of bank hoarding. An abundance of reserves would reduce the risk to banks caused by payment shocks and allow banks to be less dependent on interbank borrowing. This would allow outside of the model for banks to take on greater liquidity risk by holding more assets and extending more credit.

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Appendix

Proof of Lemma 1. For bank l,

$$\pi^l = (b_1^l + m_1^l - m_3^l)R_1^b - R_6^w w_6^l + R_6^k f_6^k + R_3^s f_3^l + R_3^k f_3^k.$$

Bank l chooses discount window borrowing w_6^l and interbank lending f_6^k. Constraints (4) and (5) imply that

$$w_6^l = \max\{0, -m_6^l + p_6^l + p_6^k + f_6^k\}, \tag{35}$$

which is greater than zero if the bank cannot borrow enough on the interbank market to ensure its overnight balance m_9^l is not overdrawn. For $m_9^l \neq w_6^l$, the first order condition for f_6^k implies

$$R_6^k = R_6^w \frac{dw_6^l}{df_6^k} = \{ \begin{array}{ll} 0 & \text{if } w_6^l = 0 \\ R_6^w & \text{if } w_6^l > 0. \end{array} \tag{36}$$

If $m_9^l = w_6^l$, then $w_6^l = 0$. If $m_9^l = w_6^l = 0$ for all l, then there is no trading in the interbank market and $R_6^k \in [0, R_6^w]$ is indeterminate. In order for the first order condition to hold for all large banks for which $m_9^l \neq w_6^l$, either they all borrow from the discount window or none do. This means that no large banks borrow at the discount window while others hold excess overnight balances. This allows for deriving the aggregate discount window borrowing $W_6^l = \sum_{l=1}^{L} w_6^l = \max\{0, -M_6^l + P_6^l\}$, where

$$M_6^l = M_3^l - P_3^l - F_3^l. \tag{37}$$

If $W_6^l = 0$, there is sufficient aggregate balances among large banks. No large banks borrow at the discount window, and those that need funds borrow from those with excess funds at $R_6^k = 0$. If $W_6^l > 0$, there is an aggregate shortage of balances among large banks, which requires borrowing at the discount window. The interbank lending rate equals the discount window rate, so it is arbitrary how large banks choose between w_6^l and f_6^k. For simplicity, we assume that all large banks borrow equally from the discount window according to

$$\begin{aligned} w_6^l &= \frac{1}{L} W_6^l \\ &= \max\{0, \frac{1}{L}(-M_6^l + P_6^l)\}, \end{aligned}$$

and trade in the interbank market to give themselves equal overnight balances. Banks are indifferent because if $R_6^k = 0$, then $w_6^l = 0$ and they borrow in the fed funds market at no cost. If $R_6^k = R_6^w$, then all large banks hold $m_9^l = 0$, and borrow at the same rate in the fed funds as at the discount window. This implies that for each large bank, $m_9^l = \frac{1}{L} M_9^l = \frac{1}{L}\sum_{l=1}^{L} m_9^l$. Substituting for m_9^l from (7) and simplifying,

$$m_6^l - p_6^l - p_6^k - f_6^k + w_6^l = \frac{1}{L}(M_6^l - P_6^l + W_6^l).$$

Substituting for $w_6^l = \frac{1}{L} W_6^l$ and solving for f_6^k gives

$$f_6^k = -\frac{1}{L}(M_6^l - P_6^l) + m_6^l - p_6^l - p_6^k,$$

to complete bank l's optimization at $t = 6$.

For bank s,

$$\pi^s = (b_1^s + m_1^s - m_3^s)R_1^b - R_6^w w_6^s + R_3^s f_3^s.$$

Bank s chooses only discount window borrowing. Constraints (4) and (5) imply that bank s chooses

$$w_6^s = \max\{0, -m_3^s + p_3^s + f_3^s + p_6^s\}.$$

■

Proof of Lemma 2. The first order conditions for f_3^l and f_3^k are

$$R_3^s = \frac{d}{df_3^l} E_3[R_6^w w_6^l - R_6^k f_6^k - R_3^k f_3^k] \tag{38}$$

$$R_3^k = \frac{d}{df_3^k} E_3[R_6^w w_6^l - R_6^k f_6^k - R_3^s f_3^l], \tag{39}$$

respectively. For solutions such that constraint (3) does not bind, $f_3^l < 0$ implies $R_3^k = R_3^s$. To show this, suppose $R_3^k < R_3^s$. Bank l would borrow infinitely from small banks to lend to other large banks, implying $f_3^k = \infty$. In aggregate, $F_3^k = \sum_{l=1}^{L} f_3^k = \infty$, a contradiction to the equilibrium condition of $F_3^k = 0$. Suppose instead $R_3^s > R_3^k$. Bank l would demand to borrow from other large banks and not from small banks, implying $f_3^l(R_3^s) = 0$ for all l, a contradiction to $f_3^l < 0$.

Since $R_3^k = R_3^s$, bank l is indifferent between lending to large or small banks, so its choice between f_3^l and f_3^k is arbitrary. We assume for simplicity that all large banks borrow equally from small banks according to $f_3^l = \frac{F_3^l}{L}$ and then redistribute funds among themselves. This structure would also correspond to a model of small banks lending in a correspondent banking relationship to large banks, which then relend the funds on the interbank market.

Net borrowing at $t = 6$ is

$$R_6^w w_6^l - R_6^k f_6^k = \{ \begin{array}{ll} 0 & \text{if } W_6^l = 0 \\ R_6^w(-m_6^l + p_6^l + p_6^k) & \text{if } W_6^l > 0, \end{array} \tag{40}$$

found by substituting into the left-hand side of (40) for w_6^l from (35), and for R_6^k from (36), noting that $w_6^l > 0$ if and only if $W_6^l > 0$.

Expected net borrowing at $t = 6$ is

$$\begin{aligned} E_3[R_6^w w_6^l - R_6^k f_6^k] &= R_6^w \int_{-\overline{P}}^{\overline{P}} \int_{-\overline{p}^l}^{\overline{p}^l} \int_{-\overline{p}^k}^{\overline{p}^k} (-m_6^l + p_6^l + p_6^k) \mathbf{1}_{W_6^l > 0} \psi(p_6^k, p_6^l, P_6^l) dp_6^k dp_6^l dP_6^l \\ &= R_6^w \int_{-\overline{P}}^{\overline{P}} \int_{-\overline{p}^l}^{\overline{p}^l} \int_{-\overline{p}^k}^{\overline{p}^k} (-m_6^l + p_6^l + p_6^k) \mathbf{1}_{P_6^l > M_6^l} \psi(p_6^k, p_6^l, P_6^l) dp_6^k dp_6^l dP_6^l \\ &= R_6^w \int_{M_6^l}^{\overline{P}} \int_{-\overline{p}^l}^{\overline{p}^l} \int_{-\overline{p}^k}^{\overline{p}^k} (-m_6^l + p_6^l + p_6^k) \psi(p_6^k, p_6^l, P_6^l) dp_6^k dp_6^l dP_6^l, \end{aligned} \tag{41}$$

where $\psi(\cdot)$ is a uniform (joint where appropriate) p.d.f. Substituting the right-hand side for the left-hand side of (41) into (38), substituting for m_6^l from (6), noting $R_3^k = R_3^s$ and simplifying gives

$$\begin{aligned}
R_3^s &= (1+\frac{df_3^k}{df_3^l})R_6^w \int_{M_6^l}^{\overline{P}} \int_{-\overline{p}^l}^{\overline{p}^l} \int_{-\overline{p}^k}^{\overline{p}^k} \psi(p_6^k, p_6^l, P_6^l) dp_6^k dp_6^l dP_6^l - R_3^s \frac{df_3^k}{df_3^l} \\
&= R_6^w \int_{M_6^l}^{\overline{P}} \int_{-\overline{p}^l}^{\overline{p}^l} \int_{-\overline{p}^k}^{\overline{p}^k} \psi(p_6^k | p_6^l, P_6^l)\psi(p_6^l|P_6^l)\psi(P_6^l) dp_6^k dp_6^l dP_6^l \\
&= R_6^w \int_{M_6^l}^{\overline{P}} \frac{1}{2\overline{P}} dP_6^l \\
&= \frac{R_6^w(\overline{P} - M_6^l)}{2\overline{P}}.
\end{aligned}$$

Substituting similarly as above into (39) and simplifying gives the same solution:

$$\begin{aligned}
R_3^s &= (1+\frac{df_3^l}{df_3^k})R_6^w \int_{M_6^l}^{\overline{P}} \int_{-\overline{p}^l}^{\overline{p}^l} \int_{-\overline{p}^k}^{\overline{p}^k} \psi(p_6^k, p_6^l, P_6^l) dp_6^k dp_6^l dP_6^l - R_3^s \frac{df_3^l}{df_3^k} \\
&= \frac{R_6^w(\overline{P} - M_6^l)}{2\overline{P}}.
\end{aligned}$$

Substituting for M_6^l from (37) gives

$$R_3^s = R_6^w \frac{(\overline{P} + P_3^l + F_3^l - M_3^l)}{2\overline{P}}. \tag{42}$$

Solving for $-F_3^l$ gives (11). Finally,

$$\begin{aligned}
E_3[R_6^k] &= R_6^w E[\mathbf{1}_{W^C > 0}] \\
&= R_6^w \int_{M_6^l}^{\overline{P}} \frac{1}{2\overline{P}} dP_6^l \\
&= R_3^s,
\end{aligned}$$

where we substitute for R_6^s on the left-hand side from (36). Since $E_3[R_6^k] = R_3$ and (42) are independent of f_3^l and f_3^k, bank l is indifferent to borrowing/lending at $t = 3$ versus at $t = 6$. ■

Proof of Lemma 3. For bank s, the first order condition for f_3^s is

$$R_3^s = R_6^w \frac{d}{df_3^s} E_3[w_6^s],$$

where

$$\begin{aligned} E[w_6^s] &= E[w_6^s | p_6^s > m_6^s] \Pr(p_6^s > m_6^s) \\ &= \left(\frac{\overline{p}^s - m_6^s}{2\overline{p}^s}\right)\left(\frac{\overline{p}^s - m_6^s}{2}\right). \end{aligned}$$

In the second line, the first factor is the probability of being overdraft, and the second factor is the expected discount window borrowing given that the bank is overdraft. Taking the derivative with respect to f_3^s gives

$$\begin{aligned} E_3[w_6^s] &= \int_{-\overline{p}^s}^{\overline{p}^s} (p_3^s + p_6^s + f_3^s - m_3^s) \mathbf{1}_{p_6^s > m_3^s - p_3^s - f_3^s} \psi(p_6^s) dp_6^s \\ &= \int_{m_3^s - p_3^s - f_3^s}^{\overline{p}^s} (p_3^s + p_6^s + f_3^s - m_3^s) \psi(p_6^s) dp_6^s \\ &= \frac{(p_3^s + f_3^s - m_3^s + \overline{p}^s)^2}{4\overline{p}^s}, \end{aligned} \tag{43}$$

giving

$$R_3^s = R_6^w \left[\frac{\overline{p}^s - (m_3^s - p_3^s - f_3^s)}{2\overline{p}^s}\right].$$

Solving for f_3^s gives (15). ■

Proof of Proposition 1. By equations (36), (37) and (42), (22) and (23) hold since $w_6^l = \frac{1}{L} W_6^l$. At $t = 1$, bank i chooses m_3^i by buying Δb_1^i bonds according to their first order condition for m_3^i. For bank l, this is

$$R_1^b = \frac{d}{dm_3^l} E_1[-R_6^w w_6^l + R_6^k f_6^k + R_3^s f_3^l + R_3^k f_3^k].$$

Substituting for R_3^k with R_3^s, for $-R_6^w w_6^l + R_6^k f_6^k$ from (40), for f_3^k from (14) and simplifying gives

$$\begin{aligned} R_1^b &= \frac{d}{dm_3^l} E_1[R_6^w (\frac{M_6^l}{L} - p_6^l - p_6^k) \mathbf{1}_{W_6^l > 0} - R_3^s (\frac{M_6^l}{L} - m_3^l + p_3^l + p_3^k)] \\ &= E_1[R_3^s] = R_3^s. \end{aligned}$$

For bank s, the first order condition is

$$\begin{aligned} R_1^b &= \frac{d}{dm_3^s} E_1[-R_6^w w_6^s + R_3^s f_3^s] \\ &= \frac{d}{dm_3^s} E_1[E_3[-R_6^w w_6^s + R_3^s f_3^s]] \end{aligned}$$

Substituting for w_6^s from (43) and for f_3^s from (15) and simplifying gives the same result,

$$\begin{aligned} R_1^b &= \frac{d}{dm_3^s} E_1[-R_6^w \overline{p}^s (\frac{R_3^s}{R_6^w} + 1)^2 + R_3^s [2\overline{p}^s \frac{R_3^s}{R_6^w} - p_3^s + m_3^s - \overline{p}^s]] \\ &= E_1[R_3^s] = R_3^s. \end{aligned}$$

■

Proof of Proposition 2. The net amount that bank l lends at $t = 3$ is

$$f_3^k + f_3^l = -m_3^l + \frac{P_3^l}{L} + \frac{F_3^l}{L} + m_3^l - p_3^l - p_3^k \tag{44}$$
$$= m_3^l - p_3^l - p_3^k - \frac{\overline{P}}{L}(1 - \frac{2R_1^b}{R_6^w}), \tag{45}$$

which is found by substituting on the right-hand side of (44) for $\frac{F_3^l}{L} = f_3^l$ from (19), solving for M_3^s in (24) and substituting for it, then simplifying. The reserve balances that bank l has available to lend at $t = 3$ are

$$m_3^l - p_3^l - p_3^k. \tag{46}$$

The net amount that bank s lends at $t = 3$ is

$$f_3^s = m_3^s - p_3^s - \overline{p}^s(1 - \frac{2R_1^b}{R_6^w}), \tag{47}$$

which is found by solving for M_3^l in (24) and substituting for it in (20). The reserve balances that bank s has available to lend at $t = 3$ are

$$m_3^s - p_3^s. \tag{48a}$$

To compare lending percentage between bank l and s when their scaled bank balances are equal, set the right-hand side of (46) divided by $\overline{p}^l + \overline{p}^k$ equal to the right-hand side of (48a) divided $\overline{p}^s$:

$$\frac{m_3^l - p_3^l - p_3^k}{\overline{p}^l + \overline{p}^k} = \frac{m_3^s - p_3^s}{\overline{p}^s}. \tag{49}$$

We want to show that bank l lends a greater percentage of available balances at $t = 3$ than bank s:

$$\frac{m_3^l - p_3^l - p_3^k - \frac{\overline{P}}{L}(1 - \frac{2R_1^b}{R_6^w})}{m_3^l - p_3^l - p_3^k} > \frac{m_3^s - p_3^s - \overline{p}^s(1 - \frac{2R_1^b}{R_6^w})}{m_3^s - p_3^s}, \tag{50}$$

where the percentage of balances lent by bank l is on the left-hand side and by bank s is on the right-hand side.

With positive available reserve balances, substituting from (49) and for $\overline{P} = \gamma^l \overline{p}^l$ and simplifying gives the inequality condition (50) as

$$L > \frac{\overline{p}^l}{\overline{p}^l + \overline{p}^k}\gamma^l,$$

which always holds. ■

Figure 1(a): Aggregate lending of large banks by time of day and percentile of own balance

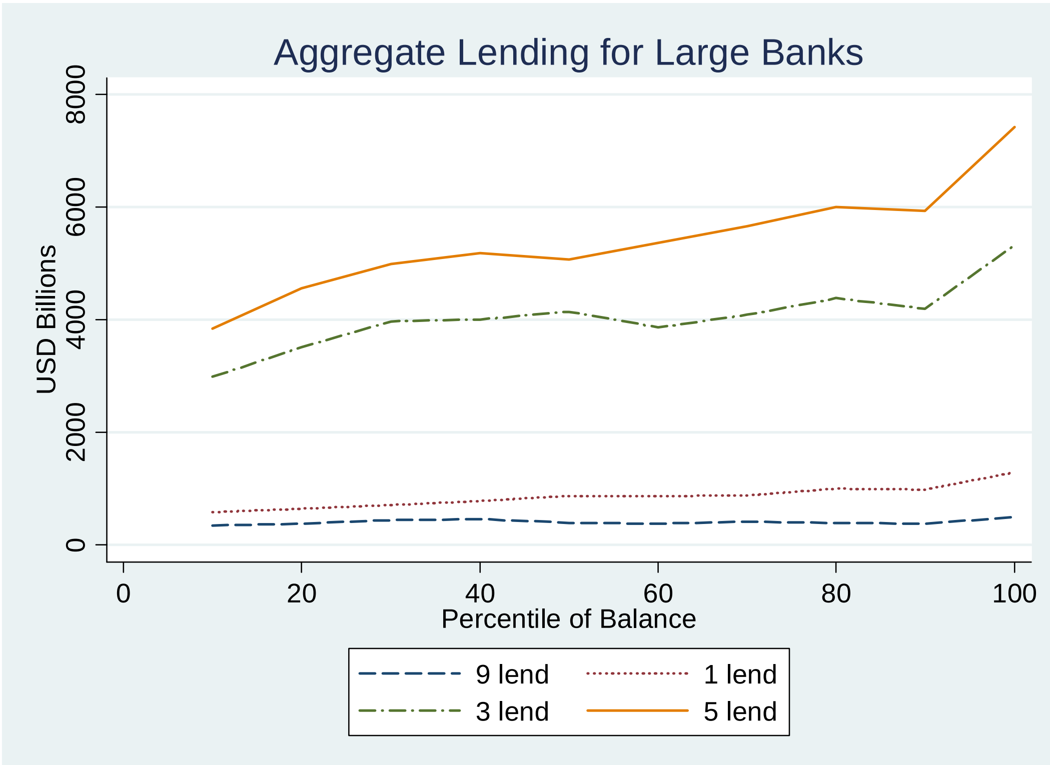


Figure 1(b): Aggregate borrowing of large banks by time of day and percentile of own balance

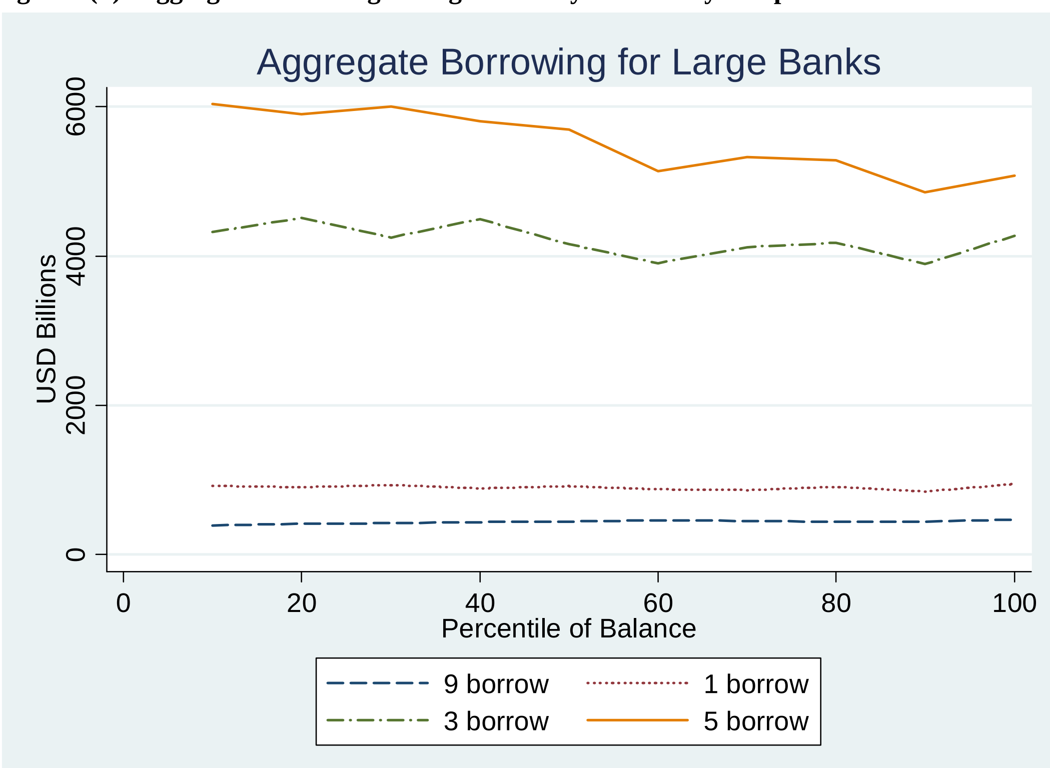


Figure 2(a): Aggregate lending of small banks by time of day and percentile of own balance

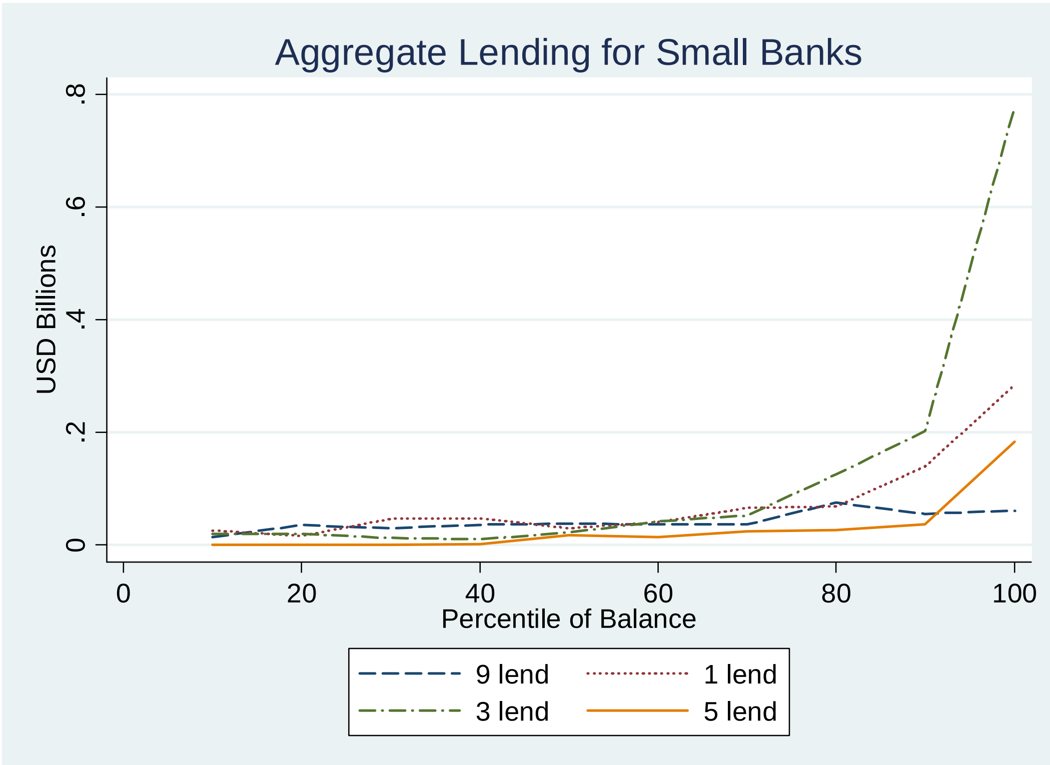


Figure 2(b): Aggregate borrowing of small banks by time of day and percentile of own balance

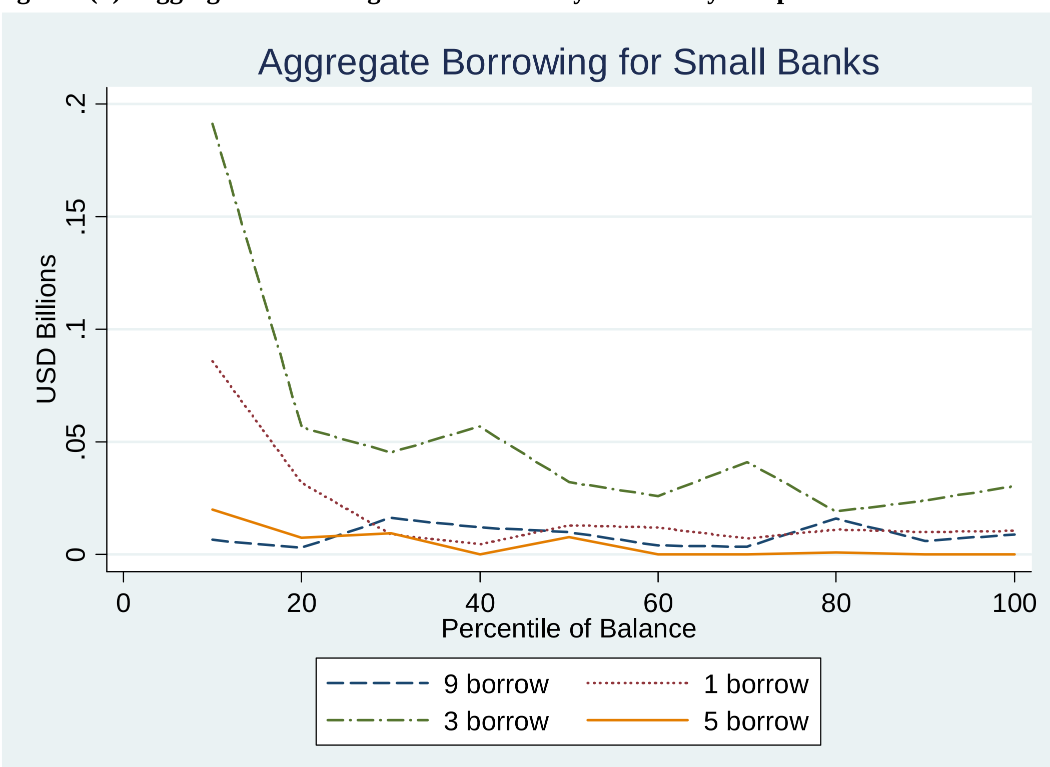


Figure 3: Box-cox plot of median normalized balance for large banks over last 90 minutes of day

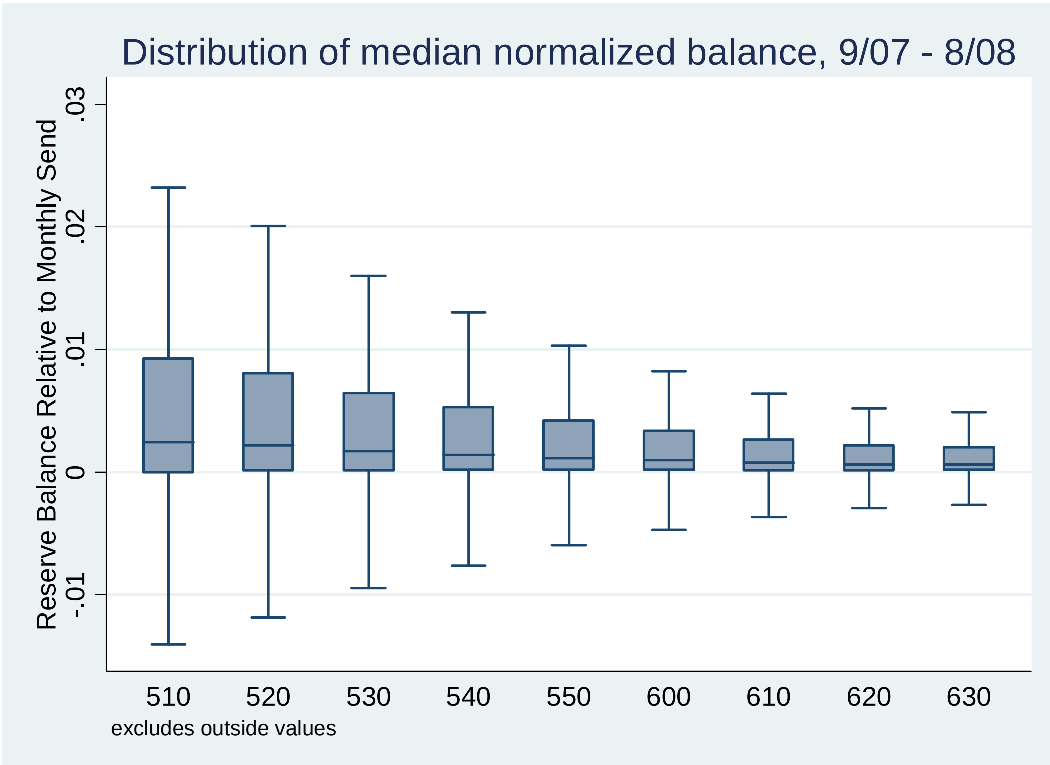


Figure 4: Box-cox plot of federal funds rate volatility over last 90 minutes of the business day

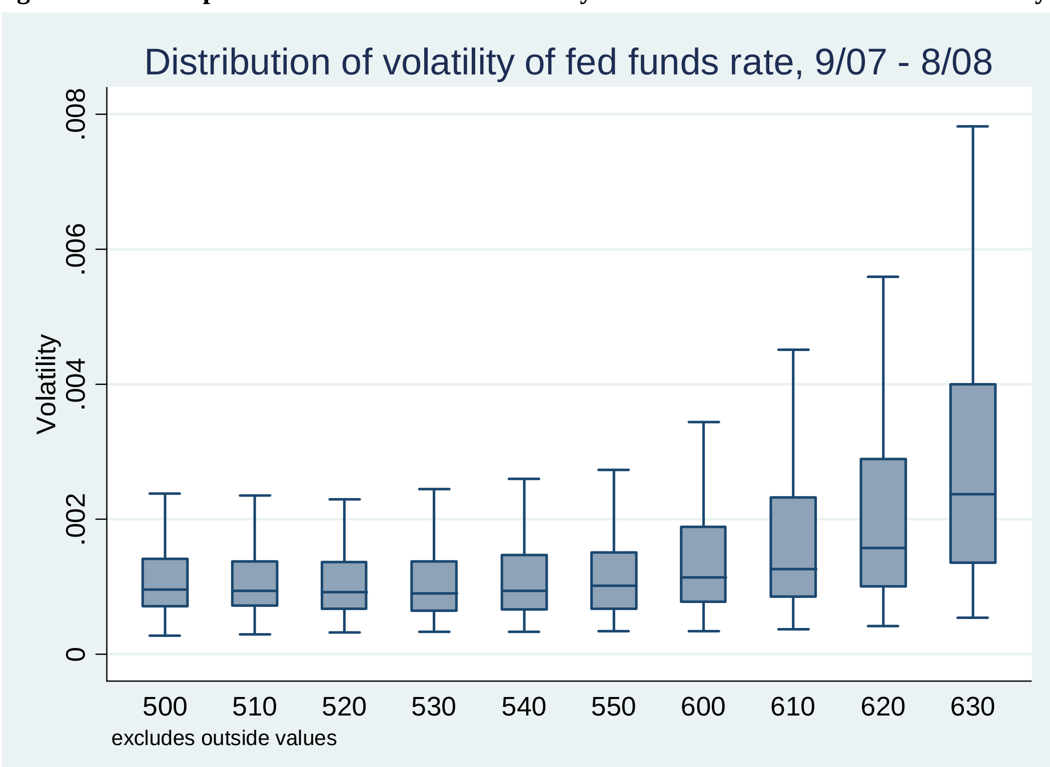
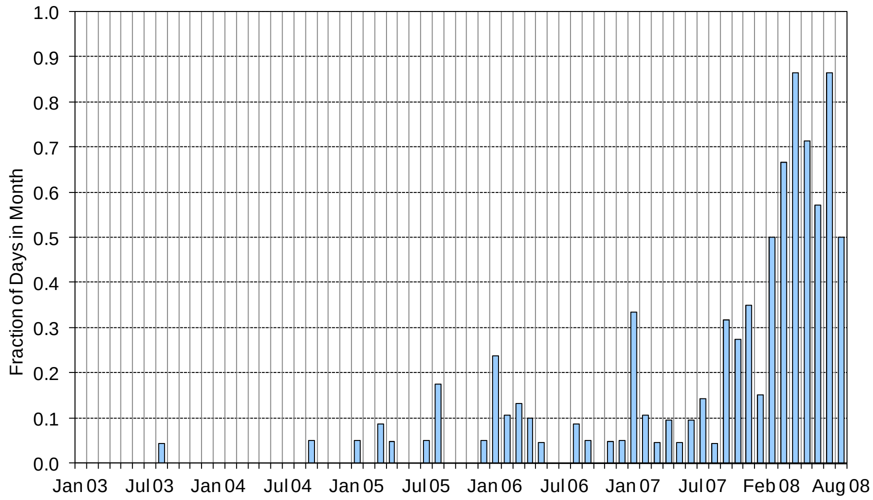
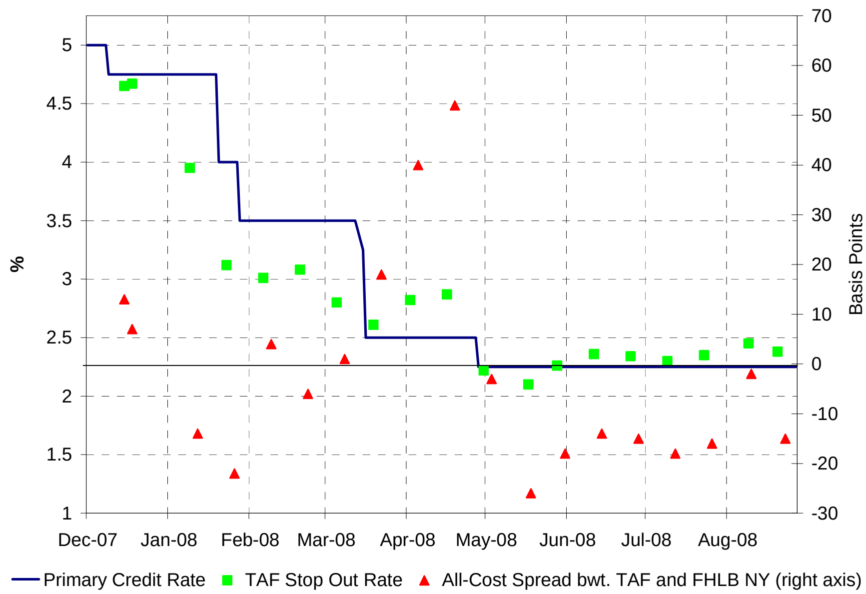


Figure 5(a): The Fraction of Days Where Federal Funds Intraday High Exceeds Primary Credit Rate



Source: Federal Reserve Bank of New York
Reproduced from Ashcraft, Bech, and Frame (2008)

Figure 5(b): Primary Credit Rate, TAF Stop Out Rate and All-in Cost Spread bwt. TAF and FHLB Advance



Source: Federal Reserve Bank of New York, Bloomberg, Authors calculations
Reproduced from Ashcraft, Bech, and Frame (2008)

Figure 6(a): Fraction of largest banks lending by time of day and percentile of own balance

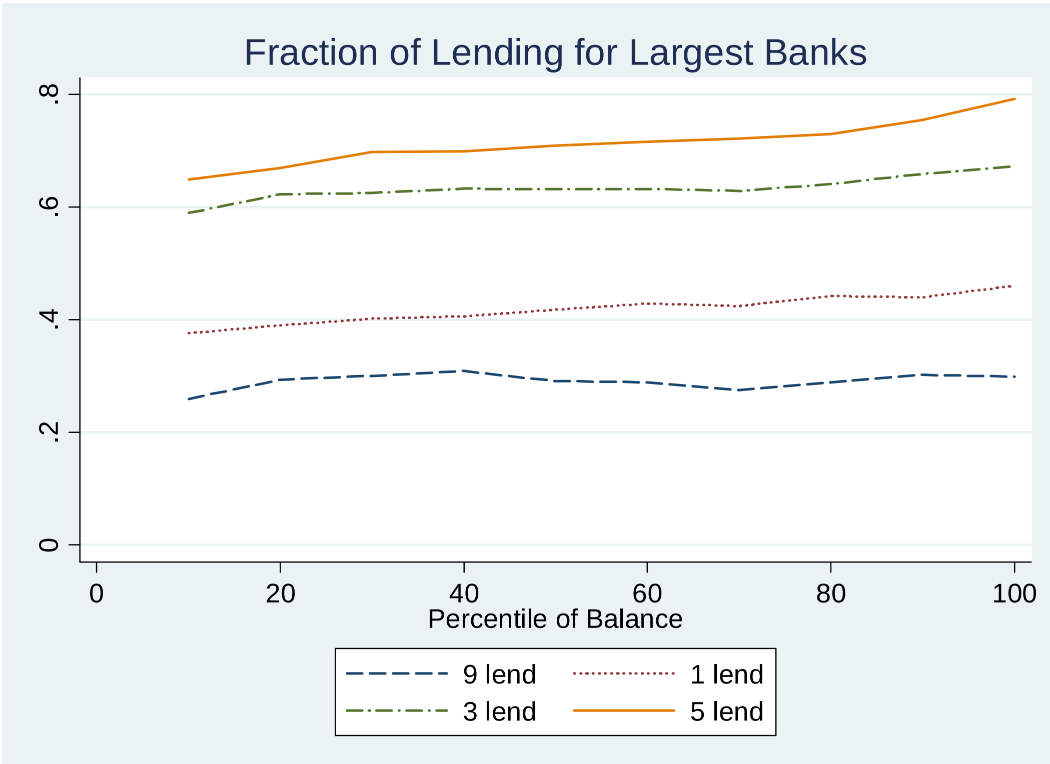


Figure 6(b): Fraction of largest banks borrowing by time of day and percentile of own balance

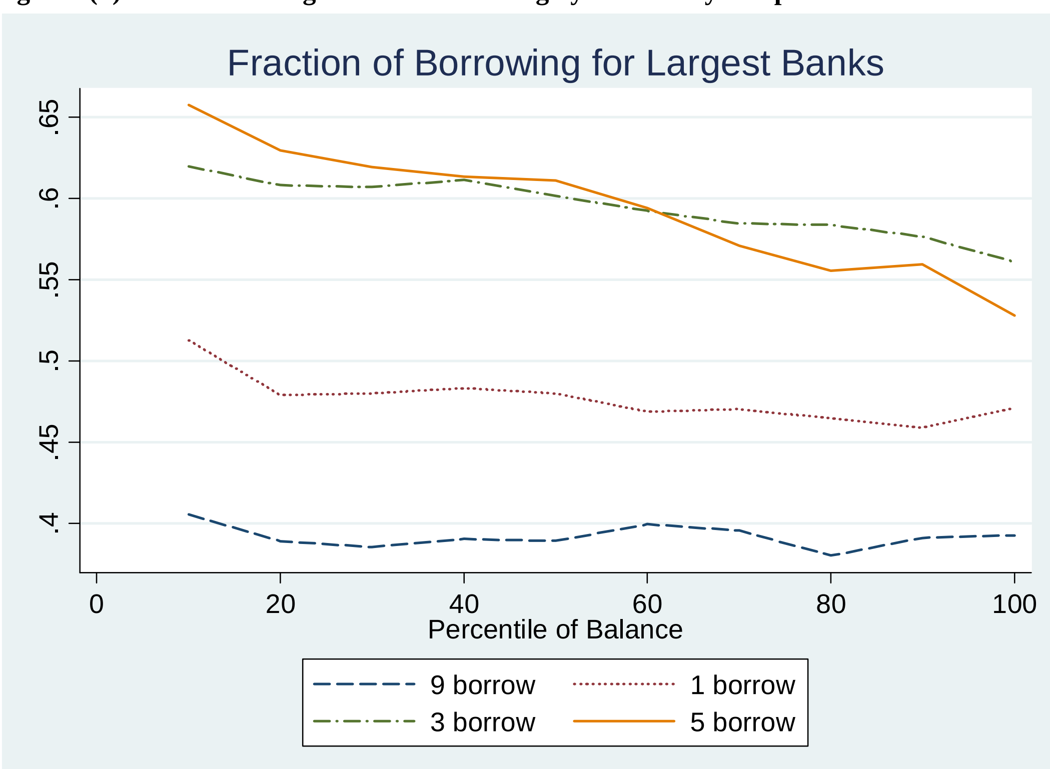


Figure 7(a): Fraction of smallest banks lending by time of day and percentile of own balance

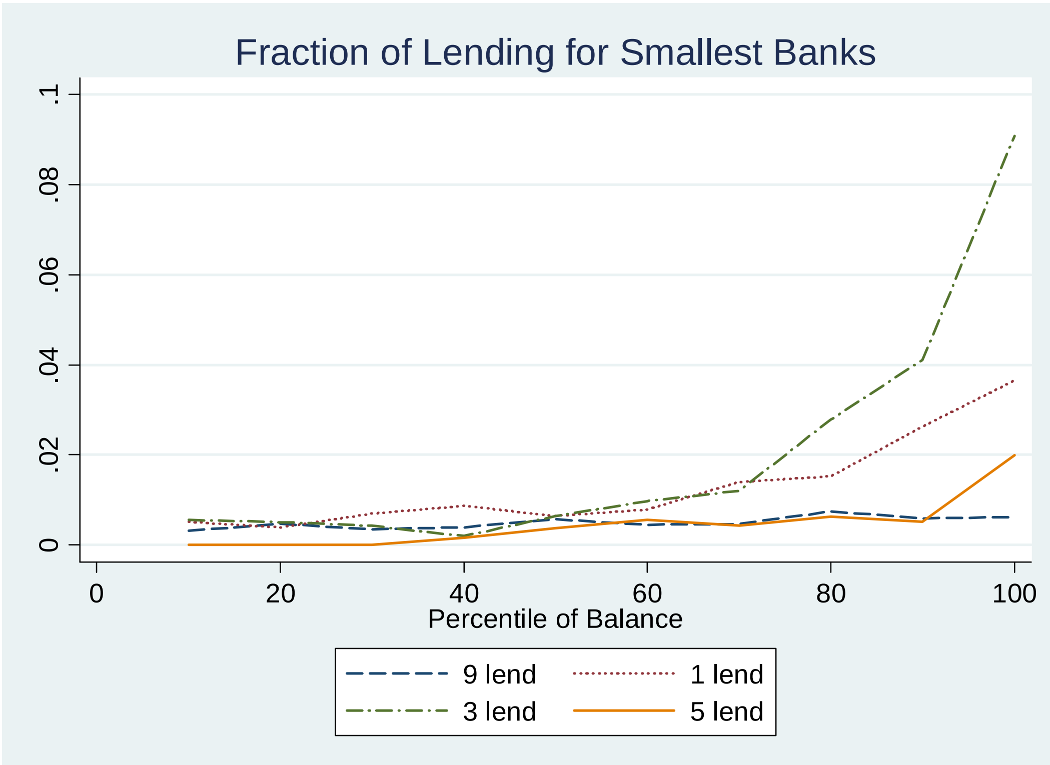


Figure 7(b): Fraction of smallest banks borrowing by time of day and percentile of own balance

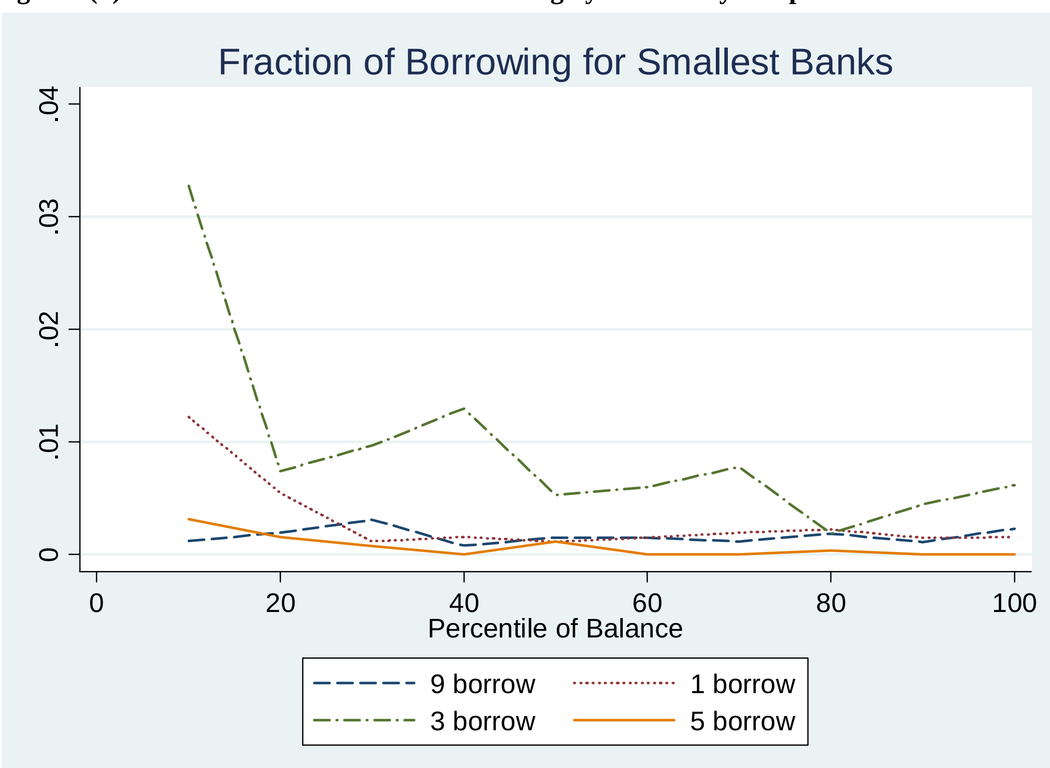


Figure 7(c): Fraction of banks borrowing and lending by decile of bank size (average daily Fedwire send)

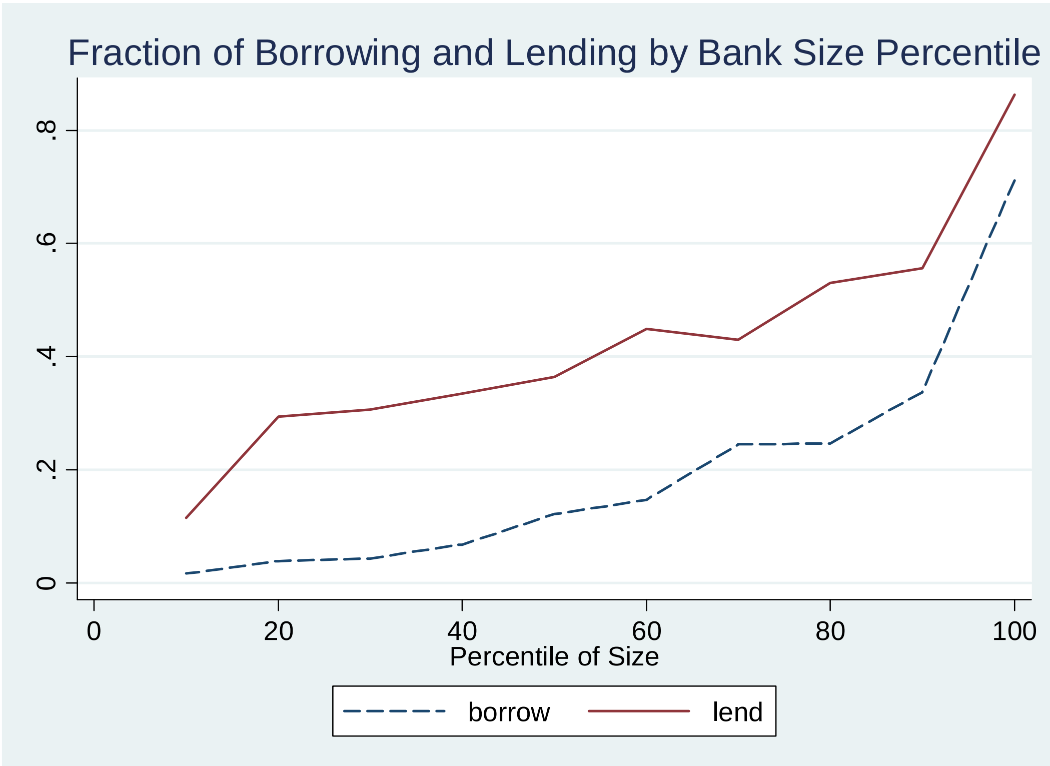


Figure 8(a): Marginal density of 3pm balances scaled by standard deviation of net sends

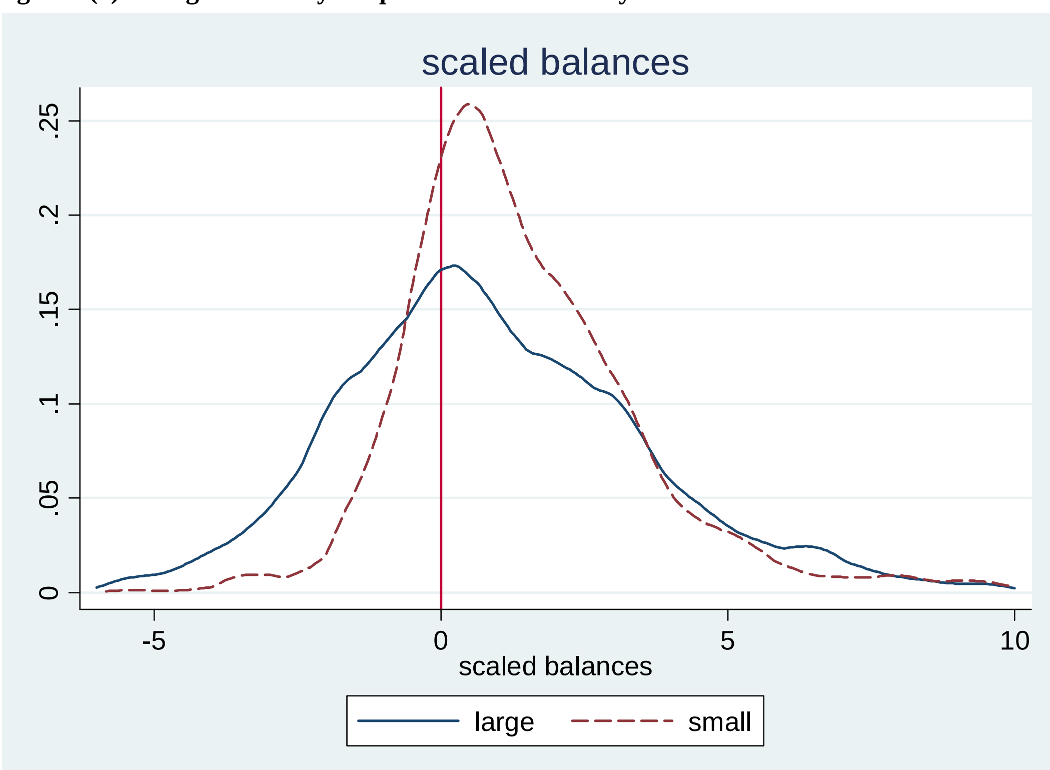


Figure 8(b): Cumulative density of 3pm balances scaled by standard deviation of net sends

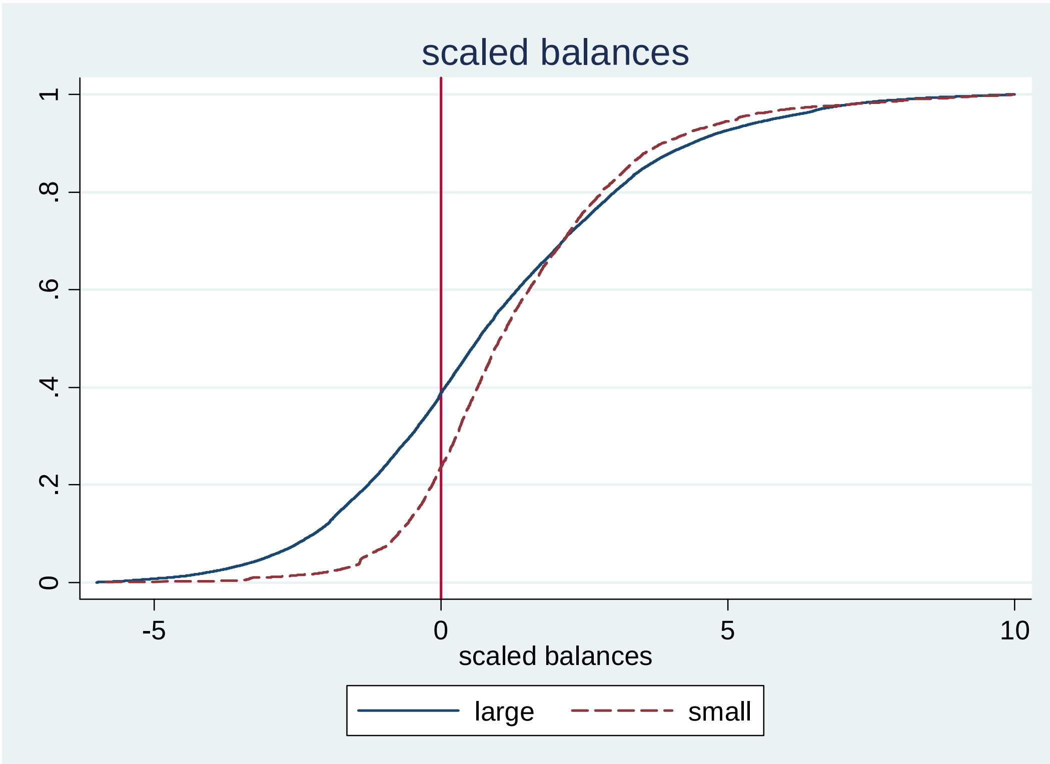


Figure 9(a): Marginal density of 3-5pm net lending scaled by standard deviation of net sends

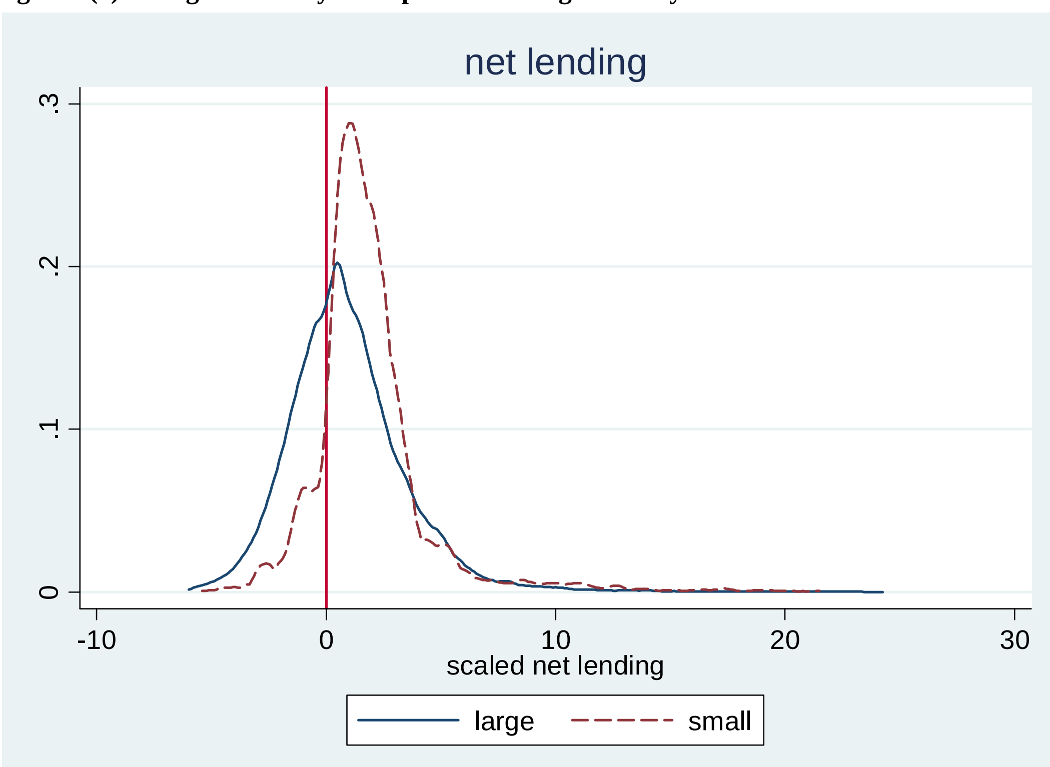


Figure 9(b): Cumulative density of 3-5pm net lending scaled by standard deviation of net sends

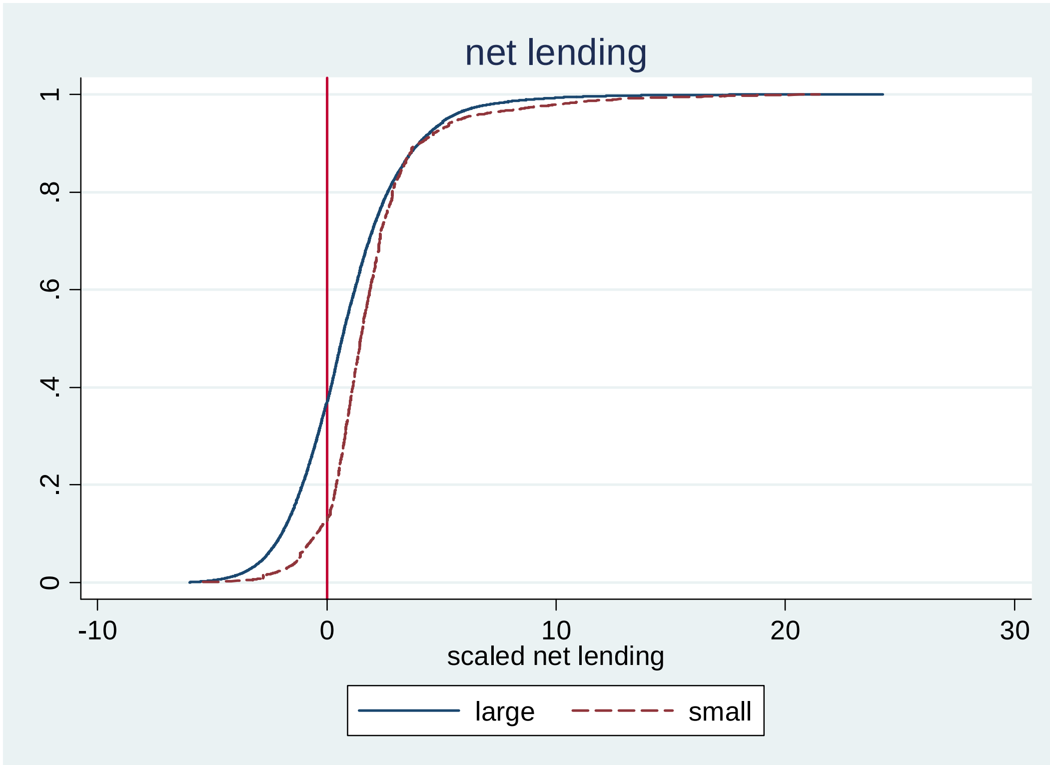


Figure 10(a): Marginal density of end of day balance ex federal funds activity

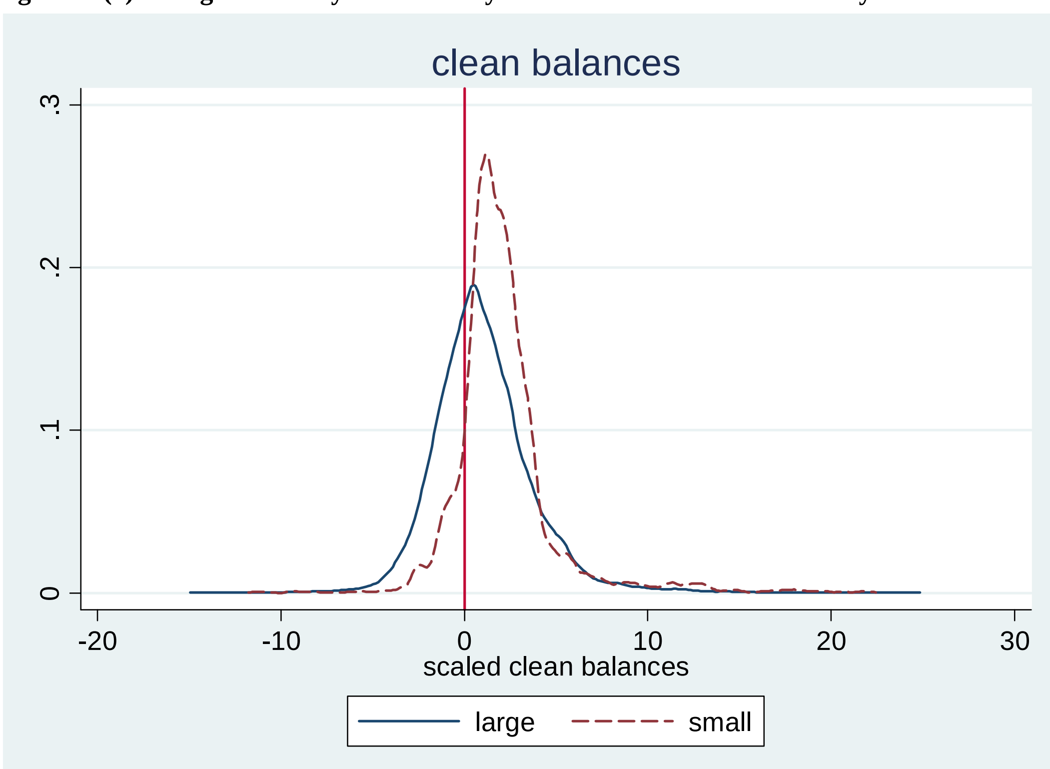


Figure 10(b): Cumulative density of end of day balance ex federal funds activity

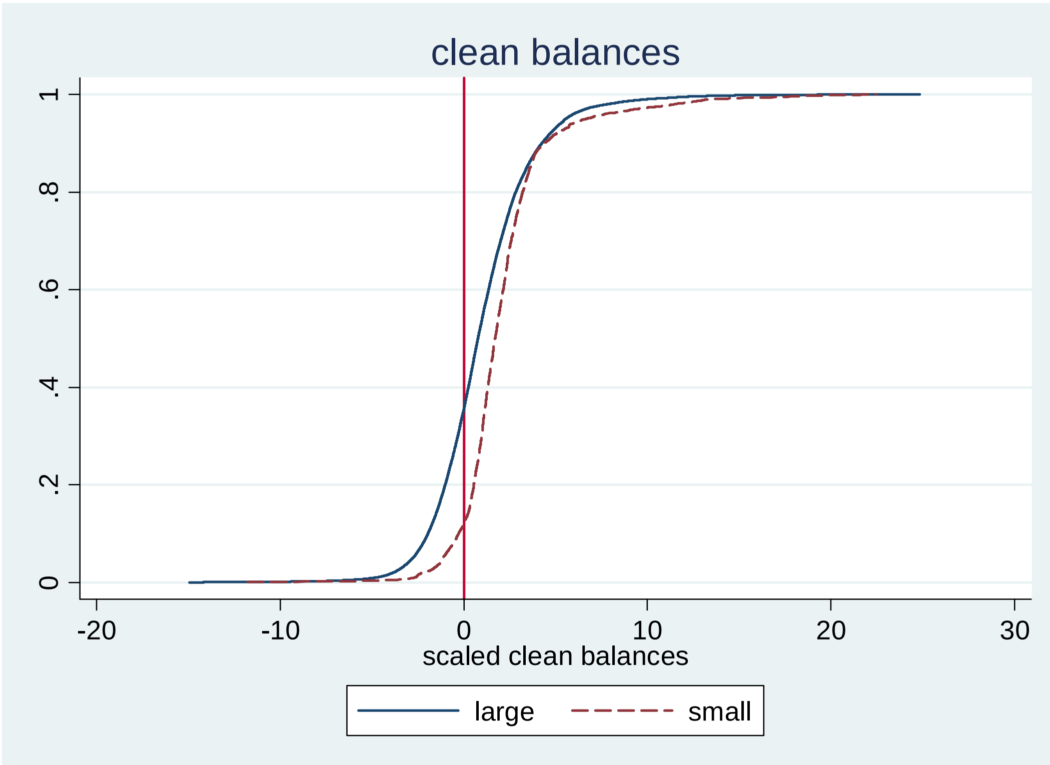


Figure 11(a): Banks' excess reserve balances

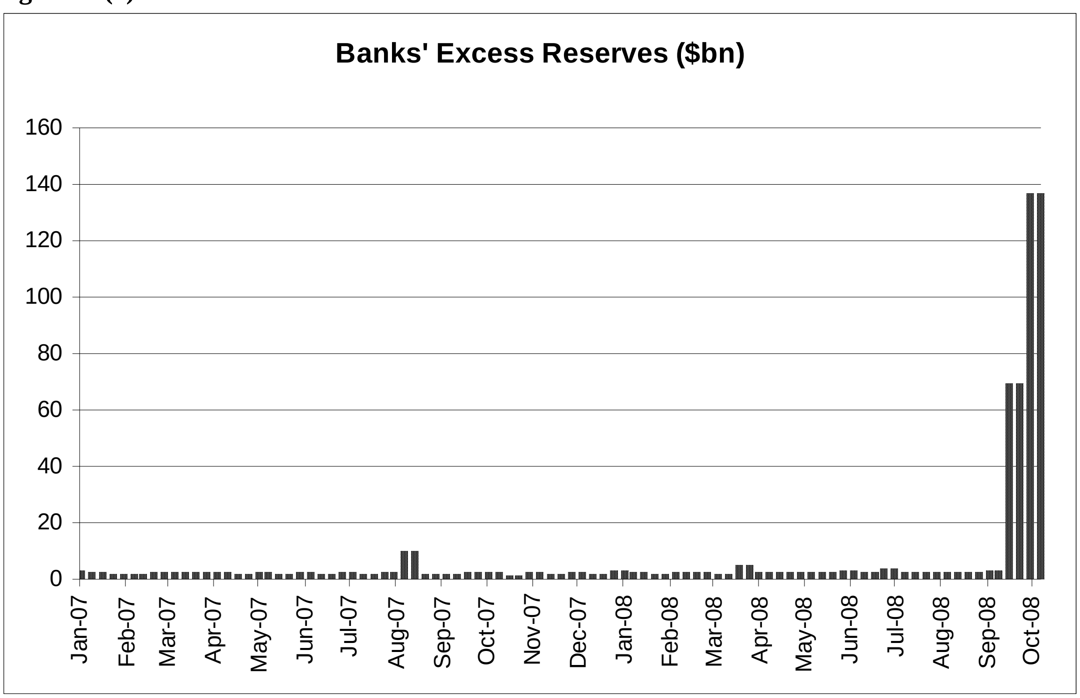


Figure 11(b): Effective fed funds rate and target fed funds rate

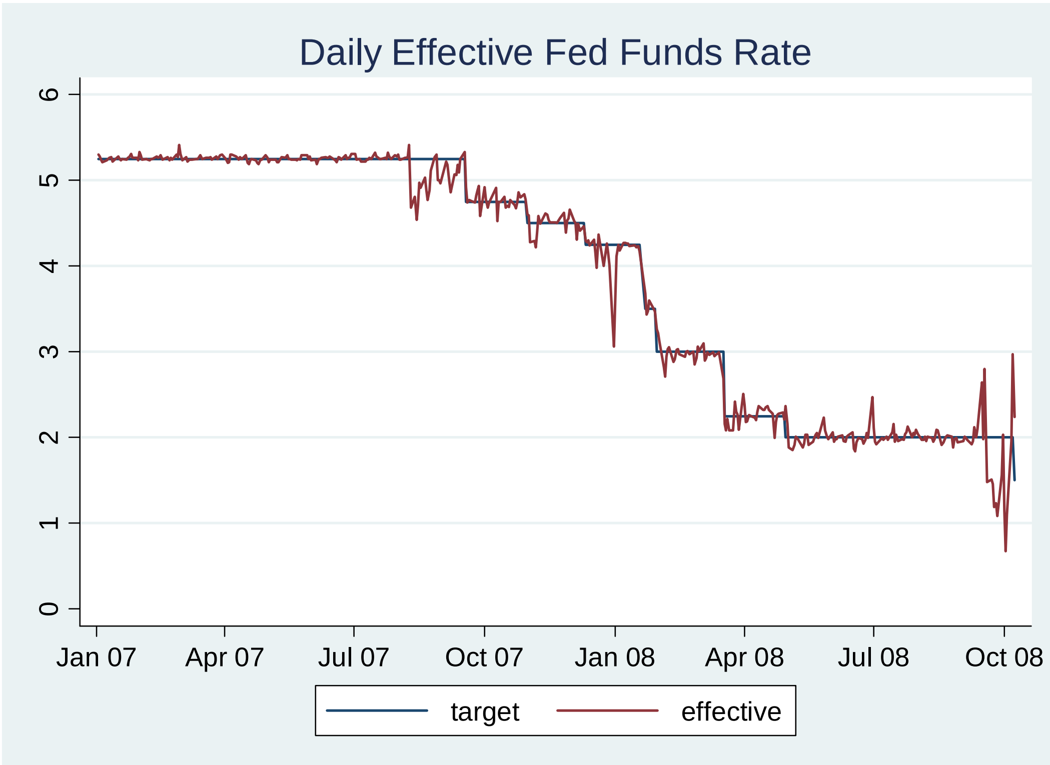


Figure 11(c): Effective federal funds rate, relative to target funds rate

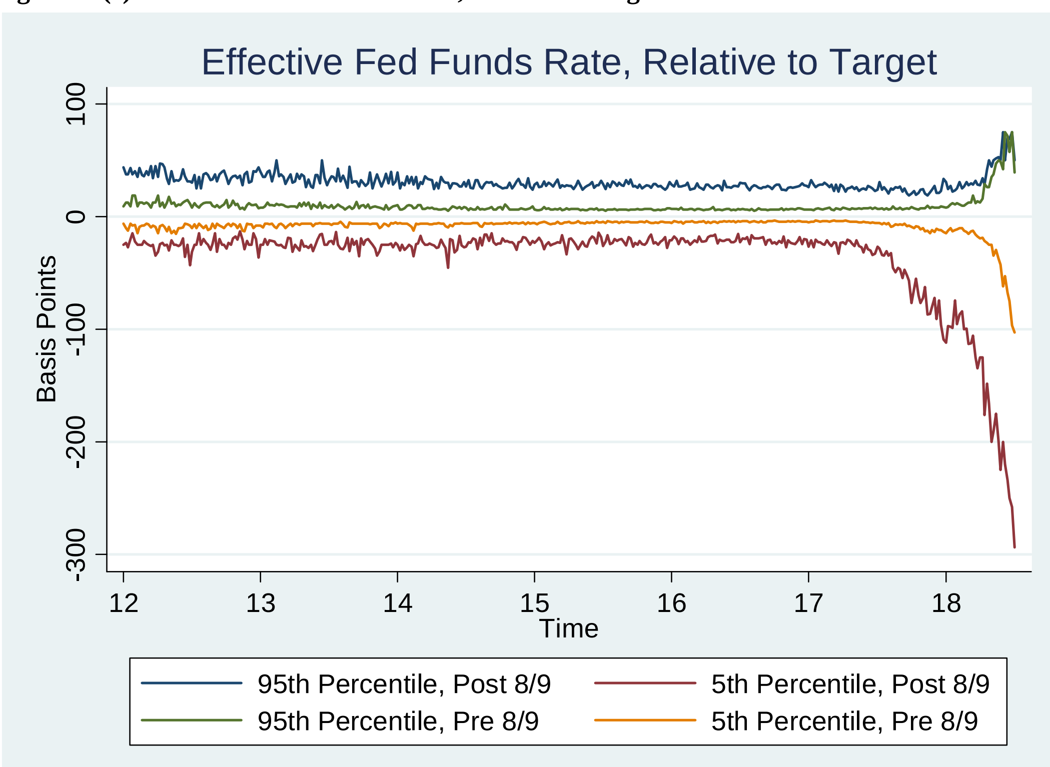


Figure 12: Overnight federal funds loans, by dollar volume

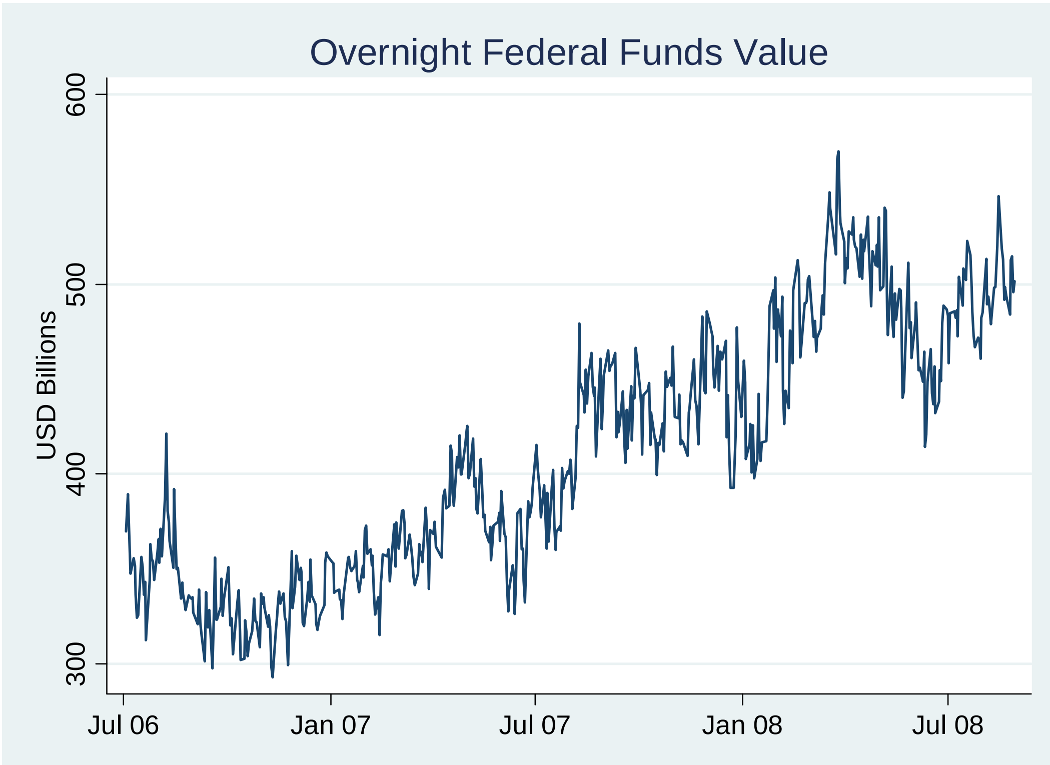


Figure 13: Standard deviation of payment shocks

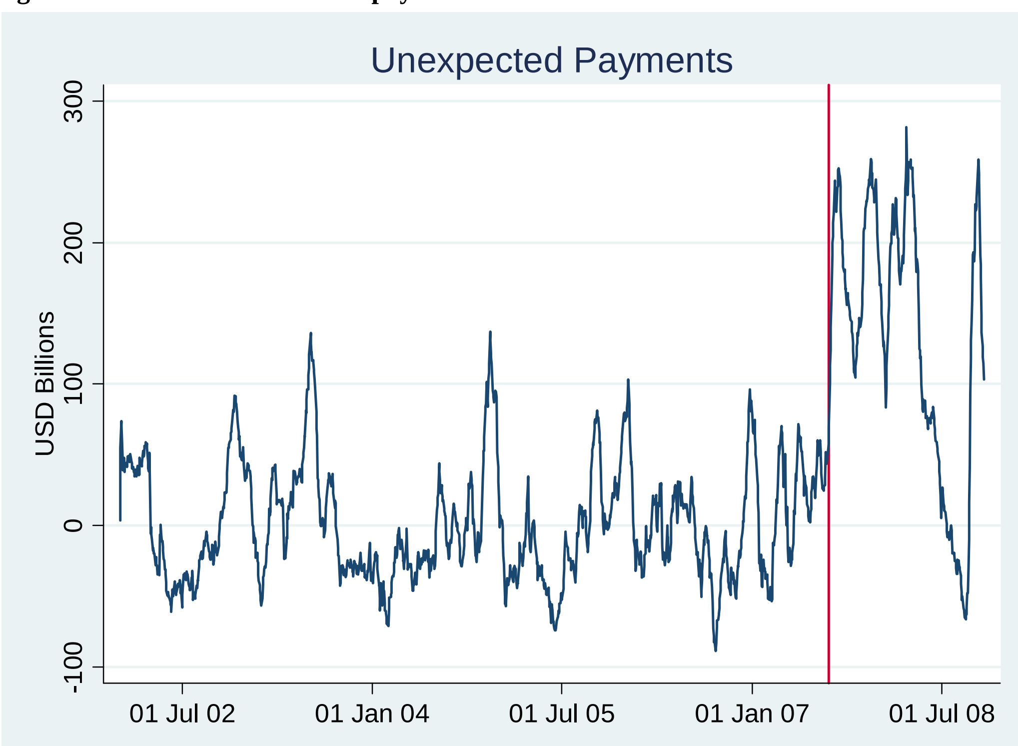


Figure 14(a): Reluctance to lend and desperation to borrow

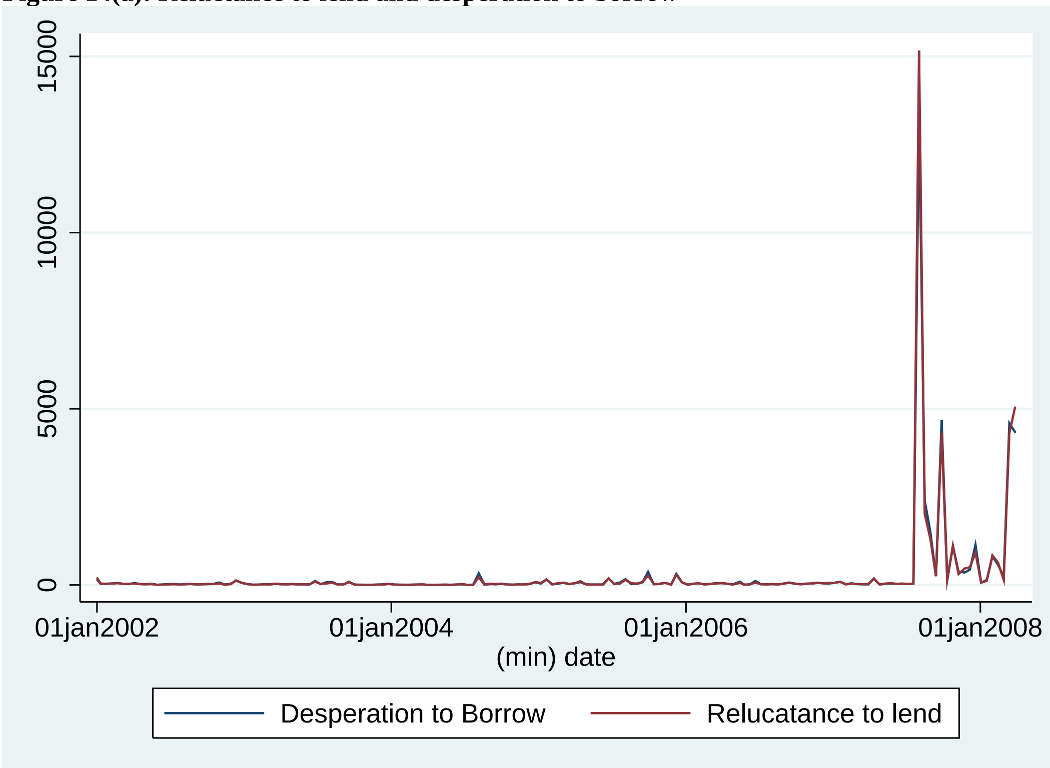


Figure 14(b): Number of banks borrowed from

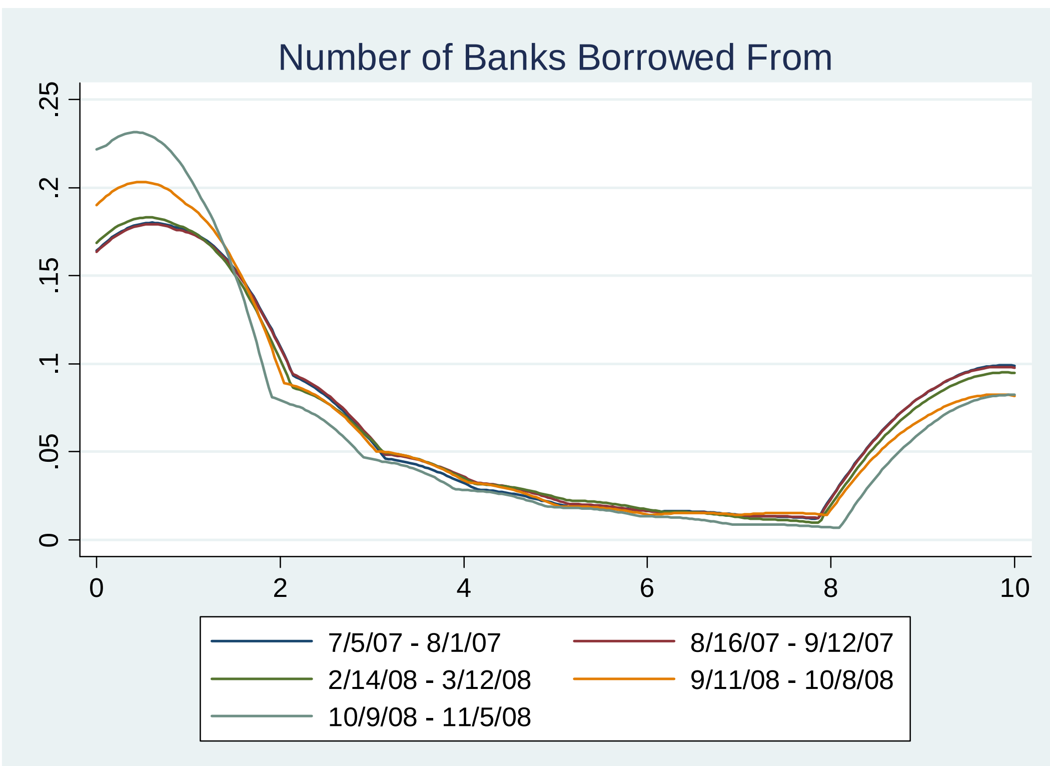


Figure 14(c): Number of banks lent to