Do Central Bank Liquidity Operations Affect Interbank Lending Rates?

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Abstract

In response to the severe global credit market dislocations that started in August 2007, central banks around the world injected extraordinary amounts of liquidity into the financial system. Using an empirical arbitrage-free term structure model, we investigate the effectiveness of these actions in reducing dollar-denominated interbank lending rates. Our model accounts for fluctuations in the nominal U.S. Treasury yield curve and in the term structure of risk in financial corporate bond yields and term interbank lending rates. Our estimates suggest that central bank liquidity operations did help lower term interbank lending rates.

The views in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Federal Reserve Bank of San Francisco or the Board of Governors of the Federal Reserve System.

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1 Introduction

In August 2007, as problems associated with U.S. mortgage-backed securities and other forms of structured credit continued to accumulate, international money markets started to exhibit unusual signs of stress. Short-term funding rates in the interbank market jumped significantly relative to yields on comparable-maturity government securities. For example, the three-month U.S. dollar London interbank offered rate (Libor) averaged about 20 basis points higher than the three-month U.S. Treasury yield during the first seven months of 2007, but that spread jumped to over 110 basis points during the final five months of the year. This increase was not only remarkable for its magnitude but also for its duration, which persisted throughout 2008. Libor rates are widely used as reference rates in financial instruments such as variable-rate home mortgages and corporate notes; indeed, the worldwide value of Libor-linked financial products has been estimated at around $150 trillion.[1] Therefore, the unusually high Libor rates in 2007 and 2008 appeared likely to have widespread adverse financial and macroeconomic repercussions.

In part to lower term Libor rates and ease strains in term interbank funding markets, central banks around the world conducted an extraordinary series of policy operations aimed at increasing financial market liquidity, especially at maturities of a few months or more. Central banks typically focus their monetary policy operations on a very short-term interbank lending rate; for example, the Federal Reserve, in its normal operations, tries to hit a daily target for the federal funds rate, which is the overnight interest rate for interbank lending of bank reserves. However, faced with elevated one- and three-month Libor spreads, several central banks, including the Federal Reserve, the European Central Bank, the Bank of England, and the Bank of Canada, increased lending at comparable maturities. For example, the Federal Reserve created a new Term Auction Facility, or TAF, to provide depository institutions with a source of term funding. The TAF and similar term lending operations were not monetary policy actions as traditionally defined. They were not intended to alter the current level or the expected future path for the overnight risk-free rate or the overall level of bank reserves (i.e., the term lending was sterilized by sales of Treasury securities). Instead, these central bank actions were meant to improve the distribution of reserves and liquidity by targeting a narrow market-specific funding problem. The press release introducing the TAF described its purpose in this way: "By allowing the Federal Reserve to inject term funds through a broader range of counterparties and against a broader range of collateral than open market operations, this facility could help promote the efficient dissemination of liquidity when the

[1] Although it is a redundant terminology, we follow the literature in referring to "Libor rates."

unsecured interbank markets are under stress." (Federal Reserve Board, December 12, 2007).

This paper assesses the effect of central bank liquidity operations on the interbank lending market and, in particular, on term Libor rate spreads over comparable-maturity Treasury yields. In theory, the provision of central bank liquidity could reduce the liquidity premium paid on private debt generally and on interbank debt in particular. This reduction could partly reflect the fact that lenders are more willing to provide funding to banks that have easy and dependable access to funds for repayment. In essence, with a liquidity backstop from the central bank in place, lenders should have a greater reassurance of timely repayment. However, assessing the efficacy of the liquidity facilities in lowering the liquidity premium—and hence term Libor spreads—during late 2007 and into 2008 is complicated by the facts that the credit risk premium was also rising during this period and that both risk premiums are unobserved.

Two earlier research papers, Taylor and Williams (2009) and McAndrews, Sarkar, and Wang (2008), have examined the effects of the TAF by controlling for movements in credit risk as measured by credit default swap (CDS) prices for the borrowing banks. Both analyses use standard event-study regressions to determine the effect of TAF lending operations or announcements on a Libor spread; unfortunately, the studies come to opposite conclusions based on small differences in the specifications of their regressions. Therefore, instead of event-study regressions, we employ a very different methodology. We analyze the effectiveness of central bank liquidity operations using a complete dynamic model of the term structure of interest rates and bank credit risk.

The model we employ is an affine arbitrage-free term structure representation of U.S. Treasury yields, the yields on bonds issued by financial institutions, and term Libor rates, and it is estimated using weekly data from 1995 to midyear 2008. The model uses the arbitrage-free Nelson-Siegel (AFNS) structure introduced by Christensen, Diebold, and Rudebusch (CDR, 2007). CDR show that a three-factor AFNS model fits and forecasts the Treasury yield curve very well. In this paper, we extend this model to incorporate three additional factors: two factors that capture bank debt risk dynamics, following the work of Christensen and Lopez (2008), and a third factor that is specific to Libor rates, as in Feldhütter and Lando (2006). The resulting six-factor representation provides arbitrage-free joint pricing of Treasury yields, financial corporate bond yields, and Libor rates. This structure allows us to decompose movements in Libor rates into changes in bank debt risk premiums and changes in factors specific to the interbank market, which includes a liquidity premium. We can also conduct hypothesis testing and counterfactual analysis related to the introduction of the central bank liquidity operations. Our results provide support for the view that central bank liquidity operations starting in December 2007 lowered Libor rates.

The remainder of the paper is structured as follows. The next section describes our interest rate data from the markets for Treasury securities, bank debt, and interbank lending. Section 3 motivates the use of an AFNS six-factor term structure model and details its structure. Section 4 presents our estimation method and our model estimates. Section 5 focuses on the financial crisis that started in 2007. It briefly describes the central bank liquidity operations taken in response to that crisis and examines recent interest rate movements through the lens of our estimated model. Section 6 concludes.

2 Three Financial Markets

In this section, we describe the three financial markets of interest and the associated data on Treasury yields, financial corporate debt yields, and Libor rates. These three markets are interrelated but differ in terms of the relative amounts of credit and liquidity risk. Our empirical model will account for these relative movements in interest rates.

Treasury bonds are generally considered to be free from credit risk and are the most liquid debt instruments available. We use zero-coupon Treasury yields with the maturities of 3, 6, 12, 24, 36, 60, 84, and 120 months (in order to match the maturity spectrum for our corporate bond yield data), which are constructed using the method described by Gürkaynak, Sack and Wright (2007). Our data sample consists of weekly observations (Fridays) from January 6, 1995 to July 25, 2008, which corresponds to 708 weekly observations. The starting point for our sample is also determined by the availability of our corporate bond yield data.

The price for unsecured lending of U.S. dollars between banks is given by the Libor rate, which is determined each day by a British Bankers' Association (BBA) survey of a fixed panel of 16 large banks at around 11 a.m. GMT.[2] The BBA discards the four highest and four lowest quotes and takes the average of the remaining eight quotes, which becomes the Libor rate for that specific term deposit on that day. Our Libor rate data consist of the three-month, six-month, and twelve-month Libor rates observed weekly (Fridays) over the sample period. In the credit risk literature, the Libor rate is often considered on par with a AA-rated financial institution since the panel of banks is reviewed and revised as necessary to maintain credit quality. (The appendix describes the conversion of the quoted Libor rates

[2] As of the time of this writing, the 16 banks in the U.S. dollar Libor panel include: Bank of America, Bank of Tokyo-Mitsubishi UFJ Ltd, Barclays Bank plc, Citibank NA, Credit Suisse, Deutsche Bank AG, HBOS, HSBC, JP Morgan Chase, Lloyds TSB Bank plc, Rabobank, Royal Bank of Canada, The Norinchukin Bank, The Royal Bank of Scotland Group, UBS AG, and West LB AG.

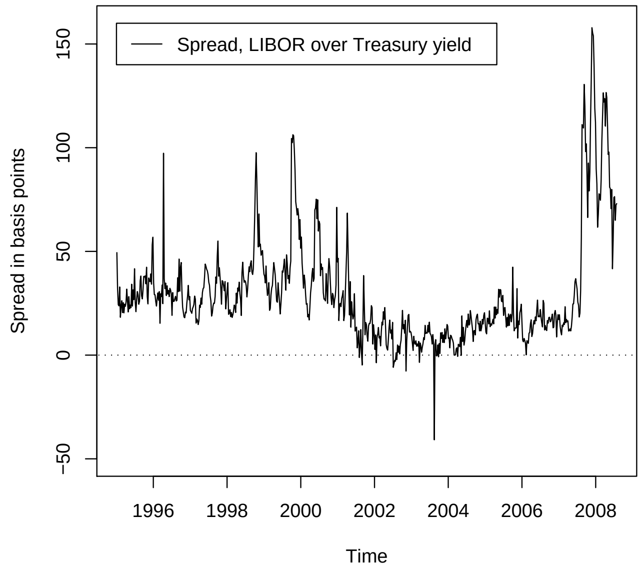


Figure 1: **Spread of Three-Month Libor rates over Treasury Yields.**
Illustration of the weekly spread of the three-month Libor rate over the three-month Treasury bond yield from January 6, 1995 to July 25, 2008.

into continuously compounded yields.)

Figure 1 illustrates the spread of the three-month Libor rate over the three-month Treasury rate. Fluctuations in this spread are commonly attributed to movements in credit and liquidity risk premiums. In simple terms, the credit risk premium is considered compensation for the possibility that a borrowing bank will become insolvent and default on repayment. (Note that the Libor rate refers to unsecured deposits with no collateral, although typically even in default there is partial repayment with some recovery rate of the principal.) In contrast, a liquidity risk premium is considered compensation for the possibility of delayed payment from an otherwise sound bank that is temporarily unable to supply the funds agreed upon.[3]

Both the size and duration of the elevated level of the Libor-Treasury spread in 2007 and 2008 clearly stand out as exceptional. The mean spread in our sample prior to August 10, 2007, is about 25 basis points, while after that date, the mean spread is 98 basis points.[4] A

[3]The Libor-Treasury spread is also affected by changes in the demand for Treasury securities, such as so-called flight-to-quality movements. Swap rates, which are also essentially free from credit and liquidity risk, have been used as an alternative benchmark that is free from idiosyncratic movements in the Treasury market. However, because the focus of this paper is on the dynamic interactions between bank bond yields and Libor rates, the measurement of the risk-free rate is not a critical element of our analysis.

[4]Data on the Libor-Treasury spread and on a very similar spread, the well-known eurodollar to Treasury (or

key question that has preoccupied many—including central bankers around the world—is the extent to which this widening spread represents credit or liquidity risk. To shed some light on this issue, our empirical model will employ additional information on the risk associated with lending to U.S. financial institutions based on the yields of their unsecured bonds, which are also traded in secondary markets. We obtain zero-coupon yields on the bond debt of U.S. banks and financial corporations from Bloomberg at the eight maturities listed above. Our empirical model will estimate the amount of risk associated with this financial debt by pooling across five different categories: A-rated and AA-rated financial corporate debt and BBB-, A-, and AA-rated bank debt.[5] Yields for the first four types of debt are available for our entire 1995-2008 sample, while yields on AA-rated bank debt are only available after September 2001. (The appendix describes the conversion of the reported interest rates into continuously compounded yields.)

At comparable maturities, the Libor rate and the yields on AA-rated bank debt should be very close, because both represent the cost of lending unsecured funds to similar institutions. Indeed, for much of our sample, these rates are almost identical. As shown in Figure 2, the spread of the three-month AA-rated bank debt yield over the three-month Libor rate and the spread of three-month AA-rated financial corporate debt yield over three-month Libor rate are very close to zero during our sample. While there are some large deviations in 2001 and 2002, they were short-lived, which indicates that over much of our sample, financial bond debt and interbank loans had very similar credit and liquidity risk characteristics.

3 A Model of Treasury, Bank, and Libor Yields

In this section, we introduce an affine arbitrage-free (AF) joint model of Treasury yields, financial bond yields, and Libor rates. Affine AF term structure models specify the risk-neutral evolution of the underlying yield-curve factors as well as the dynamics of risk premiums under the key theoretical restriction that there are no residual opportunities for riskless arbitrage across maturities and over time. Following Duffie and Kan (1996), such models have been very popular, especially because yields are convenient linear functions of underlying latent factors (i.e., state variables that are unobserved by the econometrician) with factor loadings that can be calculated from a system of ordinary differential equations. Unfortunately, affine

TED) yield spread, can be obtained earlier than the 1995 start of our estimation sample (which is determined by the availability of bank debt rates). Even in comparison to these earlier periods, the recent episode stands out as extraordinary.

[5] Banks consist of chartered bank holding companies that are subject to federal bank supervision. The category of financial corporations contains firms that are not bank holding companies, such as investment banks, though currently this distinction is becoming quite weak.

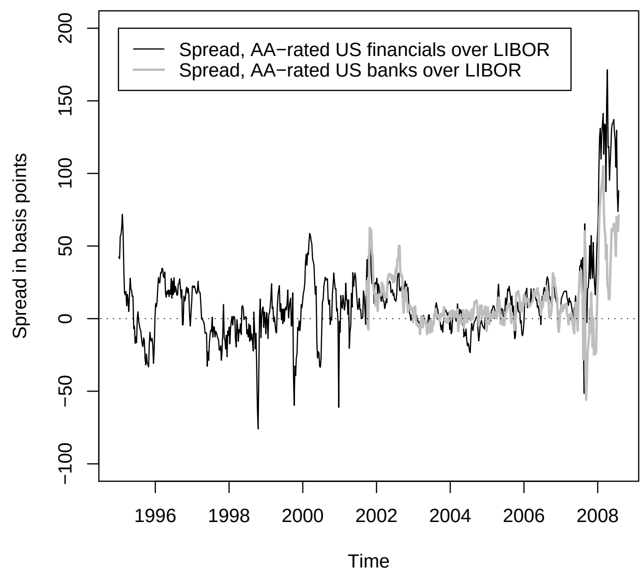


Figure 2: **Spreads of Three-Month Bank Debt Yields over Libor Rates.** Illustration of the weekly spreads of yields on three-month bonds issued by AA-rated U.S. banks and by AA-rated U.S. financial firms over the three-month Libor rate. The data cover the period from January 6, 1995, to July 25, 2008. The data for the U.S. banks start on September 11, 2001.

AF models can exhibit very poor empirical time-series performance, especially when forecasting future yields (Duffee, 2002). In addition, there are many technical difficulties involved with the estimation of these models, which tend to be overparameterized and have numerous likelihood maxima that have essentially identical fit to the data but very different implications for economic behavior (Kim and Orphanides, 2005, and Duffee, 2008). Researchers have employed a variety of techniques to facilitate estimation including the imposition of additional model structure.[6] Notably, CDR impose general level, slope, and curvature factor loadings that are derived from the popular Nelson and Siegel (1987) yield curve to obtain an AFNS model. CDR show that such a model can closely fit and forecast the term structure of Treasury yields quite well over time and can be estimated in a straightforward and robust fashion.[7]

In this paper, we show that an AFNS model can be readily estimated for a joint repre-

[6] For example, many researchers simply restrict parameters with small t-statistics in the first round of estimation to zero. Duffee (2008) describes the difficulties associated with the canonical model that require "a fairly elaborate hands-on estimation procedure."

[7] In related work, Christensen, Lopez, and Rudebusch (2008) show that a four-factor AFNS model provides a tractable and robust joint empirical model of nominal and real Treasury yield curves.

sentation of Treasury, bank bond, and Libor yields. Researchers have typically found that three factors—typically associated with level, slope, and curvature—are sufficient to model the time-variation in the cross-section of nominal Treasury bond yields (e.g., Litterman and Scheinkman, 1991). These results motivate our use of a three-factor representation for Treasury yields. The most general joint model would add three more factors for the bank bond yield curve and another three for the Libor rates of various maturities. However, this nine-factor model is unlikely to be the most parsimonious empirical representation, for as noted in the previous section, movements in Treasury, bank bond, and Libor rates all share common elements.

Some preliminary evidence on the number of factors required in addition to the three Treasury factors to capture variation in the financial bond yields can be obtained from a simple principal component analysis. We first subtract the bond yields for the four categories of debt that are available for our complete sample (i.e., A-rated and AA-rated financial corporate debt and BBB- and A-rated bank debt) from comparable-maturity Treasury yields. Then we calculate the first three principal components for these 32 yield spreads (i.e., four rating-industry categories times eight maturities). The first three principal components account for 85.5%, 8.8%, and 1.6%, respectively, of the total variation in the spreads, so just two factors may be able to account for almost 95% of the observed variation in the bank debt spreads. The associated 32 factor loadings for the first two principal components are reported in Table 1. The first principal component is a level factor since its loadings are of similar magnitude across various maturities. The loadings of the second principal component monotonically increase with maturity, suggesting that this factor describes the slope of the spread curves. Based on this and related evidence in Driessen (2005) and Christensen and Lopez (2008), we include two spread factors in our model to account for differences between bank debt yields and Treasuries. Finally, given the usually small deviations between Libor rates and bank debt yields, a single idiosyncratic Libor factor can likely capture the variation in the Libor rate given the other five factors (which is consistent with the model of Feldhütter and Lando, 2006).

Therefore, we use six factors for a joint representation: a standard three-factor AFNS model of nominal Treasury bond yields, two additional factors for financial bond rate spreads, and finally, a sixth factor to capture idiosyncratic variation in Libor rates. Treasury yields depend on a state vector of the three nominal AFNS model factors (i.e., level, slope, and

Maturity in months	U.S. Financials				U.S. Banks			
	A		AA		BBB		A	
	PC1	PC2	PC1	PC2	PC1	PC2	PC1	PC2
3	-0.16	-0.31	-0.16	-0.37	-0.18	-0.26	-0.14	-0.27
6	-0.15	-0.24	-0.14	-0.30	-0.17	-0.22	-0.15	-0.24
12	-0.14	-0.06	-0.13	-0.13	-0.16	-0.05	-0.15	-0.10
24	-0.17	0.01	-0.16	-0.08	-0.20	0.06	-0.18	-0.02
36	-0.20	0.08	-0.19	-0.01	-0.21	0.15	-0.19	0.11
60	-0.20	0.15	-0.18	0.05	-0.21	0.16	-0.20	0.09
84	-0.17	0.19	-0.17	0.12	-0.22	0.30	-0.20	0.19
120	-0.16	0.16	-0.15	0.10	-0.22	0.13	-0.19	0.07

Table 1: **Loadings on the First Two Principal Components of Credit Spreads**
The loadings of each maturity on the first (PC1) and second (PC2) principal components in the weekly zero-coupon credit spreads for A- and AA-rated U.S. financial firms and BBB- and A-rated U.S. banks covering the period from January 6, 1995, to July 25, 2008. The analysis is based on 32 time series, each with 708 weekly observations.

curvature) denoted as $X_t^T = (L_t^T, S_t^T, C_t^T)$. The instantaneous risk-free rate is given by

$$r_t^T = L_t^T + S_t^T,$$

while the dynamics of the three state variables under the risk-neutral (or Q) pricing measure are given by[8]

$$\begin{pmatrix} dL_t^T \\ dS_t^T \\ dC_t^T \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\lambda^T & \lambda^T \\ 0 & 0 & -\lambda^T \end{pmatrix} \begin{pmatrix} L_t^T \\ S_t^T \\ C_t^T \end{pmatrix} dt + \begin{pmatrix} \sigma_{L^T} & 0 & 0 \\ 0 & \sigma_{S^T} & 0 \\ 0 & 0 & \sigma_{C^T} \end{pmatrix} \begin{pmatrix} dW_t^{Q,L^T} \\ dW_t^{Q,S^T} \\ dW_t^{Q,C^T} \end{pmatrix},$$

where W^Q is a standard Brownian motion in $\mathbf{R}^3$. Given this affine framework, CDR show that the yield on a zero-coupon Treasury bond with maturity τ at time t, $y_t^T(\tau)$, is given by

$$y_t^T(\tau) = L_t^T + \left(\frac{1 - e^{-\lambda^T \tau}}{\lambda^T \tau}\right) S_t^T + \left(\frac{1 - e^{-\lambda^T \tau}}{\lambda^T \tau} - e^{-\lambda^T \tau}\right) C_t^T + \frac{A^T(\tau)}{\tau}.$$

That is, the three factors are given exactly the same level, slope, and curvature factor loadings as in the Nelson-Siegel (1987) yield curve. A shock to L_t^T affects yields at all maturities uniformly; a shock to S_t^T affects yields at short maturities more than long ones; and a shock to C_t^T affects mid-range maturities most.[9] The yield function also contains a yield-adjustment term,

[8]Here, we have fixed the mean under the Q-measure at zero, i.e. $\theta^Q = 0$. CDR (2007) show that this identification comes at no loss of generality.

[9]Again, it is this identification of the general *role* of each factor, even though the factors themselves remain

$\frac{A^T(\tau)}{\tau}$, that is time-invariant and only depends on the maturity of the bond. CDR provide an analytical formula for this term, which under our identification scheme is entirely determined by the volatility matrix. CDR find that allowing for a maximally flexible parameterization of the volatility matrix diminishes out-of-sample forecast performance, so we restrict it to be diagonal.

To add the bond yields for U.S. banks and financial firms to this model, we use the structure introduced by Christensen and Lopez (2008). In their representation, the instantaneous discount rate for corporate bonds from industry i (bank or financial corporation) with rating c (BBB, A, or AA) is assumed to be

$$r_t^{i,c} = \alpha_0^{i,c} + \left(1 + \alpha_{L^T}^{i,c}\right) L_t^T + \left(1 + \alpha_{S^T}^{i,c}\right) S_t^T + \left(\alpha_{L^S}^{i,c}\right) L_t^S + \left(\alpha_{S^S}^{i,c}\right) S_t^S,$$

where $\left(L_t^T, S_t^T\right)$ are the Treasury factors described above and $\left(L_t^S, S_t^S\right)$ are two bank debt yield spread factors. The instantaneous credit spread over the instantaneous risk-free Treasury rate becomes

$$\begin{aligned} s_t^{i,c} &\equiv r_t^{i,c} - r_t^T \\ &= \alpha_0^{i,c} + \left(\alpha_{L^T}^{i,c}\right) L_t^T + \left(\alpha_{S^T}^{i,c}\right) S_t^T + \left(\alpha_{L^S}^{i,c}\right) L_t^S + \left(\alpha_{S^S}^{i,c}\right) S_t^S. \end{aligned}$$

Note that the sensitivity of these risk factors can be adjusted by varying the $\alpha^{i,c}$ parameters, which is consistent with the pattern we observed in the principal component analysis of the corporate bond credit spreads in Table 1.[10]

To obtain the desired Nelson-Siegel factor-loading structure of a level and a slope factor for the two common credit risk factors, their dynamics under the pricing measure must be assumed to be given by the solution to

$$\begin{pmatrix} dL_t^S \\ dS_t^S \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & -\lambda^S \end{pmatrix} \begin{pmatrix} L_t^S \\ S_t^S \end{pmatrix} dt + \begin{pmatrix} \sigma_{L^S} & 0 \\ 0 & \sigma_{S^S} \end{pmatrix} \begin{pmatrix} dW_t^{Q,L^S} \\ dW_t^{Q,S^S} \end{pmatrix}.$$

Under the Q measure, the two common credit risk factors are assumed to be independent

unobserved and the precise factor loadings depend on the estimated λ, that ensures the estimation of the AFNS model is straightforward and robust—unlike the maximally flexible affine arbitrage-free model.

[10] Note that for each rating category, we do not take rating transitions into consideration. This is a theoretical inconsistency of our approach, but the model will extract common risk factors across rating categories and business sectors if they are present in the data. Taking the rating transitions into consideration will not change our results in a significant way. The model framework does allow for such extensions; for example, the method used by Feldhütter and Lando (2006) can be applied in this setting under the restriction that each rating category has the same factor loading on the two common credit risk factors. We leave this for future research.

of the three factors determining the risk-free rate. Thus, the entire system of stochastic differential equations under the Q measure is now given by

$$\begin{pmatrix} dL_t^S \\ dS_t^S \\ dL_t^T \\ dS_t^T \\ dC_t^T \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda^S & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\lambda^T & \lambda^T \\ 0 & 0 & 0 & 0 & -\lambda^T \end{pmatrix} \begin{pmatrix} L_t^S \\ S_t^S \\ L_t^T \\ S_t^T \\ C_t^T \end{pmatrix} dt + \Sigma^S \begin{pmatrix} dW_t^{Q,L^S} \\ dW_t^{Q,S^S} \\ dW_t^{Q,L^T} \\ dW_t^{Q,S^T} \\ dW_t^{Q,C^T} \end{pmatrix},$$

where Σ^S is a diagonal matrix. This structure delivers the desired Nelson-Siegel factor loadings for all five factors in the corporate bond yield function. As a result, the yield on a corporate zero-coupon bond from industry i with rating c and maturity τ is given by

$$\begin{aligned} y_t^{i,c}(\tau) \quad = \quad & \Big(1+\alpha_{L^T}^{i,c}\Big)L_t^T + \Big(1+\alpha_{S^T}^{i,c}\Big)\left(\frac{1-e^{-\lambda^T\tau}}{\lambda^T\tau}\right)S_t^T + \Big(1+\alpha_{S^T}^{i,c}\Big)\left(\frac{1-e^{-\lambda^T\tau}}{\lambda^T\tau} - e^{-\lambda^T\tau}\right)C_t^T \\ & +\alpha_0^{i,c} + \Big(\alpha_{L^S}^{i,c}\Big)L_t^S + \Big(\alpha_{S^S}^{i,c}\Big)\left(\frac{1-e^{-\lambda^S\tau}}{\lambda^S\tau}\right)S_t^S + \frac{A^{i,c}(\tau)}{\tau}, \end{aligned}$$

where the yield-adjustment term $\frac{A^{i,c}(\tau)}{\tau}$ is time-invariant and depends only on the maturity of the bond.

Finally, we include Libor rates in the model using a separate factor specific to the Libor rates (as in Feldhütter and Lando 2006). In theory, the U.S. dollar Libor rates should be nearly identical to the corporate bond yields paid by AA-rated U.S. financial institutions. However, to account for idiosyncratic developments in the London interbank market, especially since August 2007, and the fact that it is a survey-based measure, we include a sixth factor for modeling the discount rate applied to term loans in the dollar-based London interbank market. This instantaneous discount rate is given by

$$r_t^{Lib} = r_t^{Fin,AA} + \alpha^{Lib} + X_t^{Lib},$$

where the Q dynamics of the Libor-specific factor are assumed to be given by

$$dX_t^{Lib} = -\kappa_{Lib}^Q X_t^{Lib}dt + \sigma_{Lib}dW_t^{Q,Lib}.$$

This factor is assumed to be independent of the other five factors under the pricing measure. Thus, let the full state vector be denoted as $X_t = (L_t^S, S_t^S, L_t^T, S_t^T, C_t^T, X_t^{Lib})$, then the assumed Q-dynamics of the six-factor model are

$$dX_t = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda^S & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\lambda^T & \lambda^T & 0 \\ 0 & 0 & 0 & 0 & -\lambda^T & 0 \\ 0 & 0 & 0 & 0 & 0 & -\kappa^Q_{Lib} \end{pmatrix} X_t dt + \Sigma^{Lib} \begin{pmatrix} dW_t^{Q,L^S} \\ dW_t^{Q,S^S} \\ dW_t^{Q,L^T} \\ dW_t^{Q,S^T} \\ dW_t^{Q,C^T} \\ dW_t^{Q,Lib} \end{pmatrix},$$

where Σ^{Lib} is a diagonal matrix. The discount rate to be applied to Libor contracts is then

$$\begin{aligned} r_t^{Lib} &= r_t^{Fin,AA} + \alpha^{Lib} + X_t^{Lib} \\ &= \alpha_0^{Fin,AA} + \Big(1 + \alpha_{L^T}^{Fin,AA}\Big) L_t^T + \Big(1 + \alpha_{S^T}^{Fin,AA}\Big) S_t^T + \Big(\alpha_{L^S}^{Fin,AA}\Big) L_t^S + \Big(\alpha_{S^S}^{Fin,AA}\Big) S_t^S + \alpha^{Lib} + X_t^{Lib}. \end{aligned}$$

The continuously compounded yield is

$$\begin{aligned} y_t^{Lib}(\tau) &= \alpha_0^{Fin,AA} + \alpha^{Lib} \\ &\quad + \Big(1 + \alpha_{L^T}^{Fin,AA}\Big) L_t^T + \Big(1 + \alpha_{S^T}^{Fin,AA}\Big) \Big(\frac{1 - e^{-\lambda^T \tau}}{\lambda^T \tau}\Big) S_t^T + \Big(1 + \alpha_{S^T}^{Fin,AA}\Big) \Big(\frac{1 - e^{-\lambda^T \tau}}{\lambda^T \tau} - e^{-\lambda^T \tau}\Big) C_t^T \\ &\quad + \Big(\alpha_{L^S}^{Fin,AA}\Big) L_t^S + \Big(\alpha_{S^S}^{Fin,AA}\Big) \Big(\frac{1 - e^{-\lambda^S \tau}}{\lambda^S \tau}\Big) S_t^S + \Big(\frac{1 - e^{-\kappa^Q_{Lib} \tau}}{\kappa^Q_{Lib} \tau}\Big) X_t^{Lib} + \frac{A^{Lib}(\tau)}{\tau}, \end{aligned}$$

where the yield-adjustment term is

$$\begin{aligned} \frac{A^{Lib}(\tau)}{\tau} &= -\frac{\sigma_{L^T}^2 (1 + \alpha_{L^T}^{Fin,AA})^2}{6} \tau^2 - \sigma_{S^T}^2 (1 + \alpha_{S^T}^{Fin,AA})^2 \Big(\frac{1}{2(\lambda^T)^2} - \frac{1}{(\lambda^T)^3} \frac{1 - e^{-\lambda^T \tau}}{\tau} + \frac{1}{4(\lambda^T)^3} \frac{1 - e^{-2\lambda^T \tau}}{\tau}\Big) \\ &\quad - \sigma_{C^T}^2 (1 + \alpha_{S^T}^{Fin,AA})^2 \Big(\frac{1}{2(\lambda^T)^2} + \frac{1}{(\lambda^T)^2} e^{-\lambda^T \tau} - \frac{1}{4\lambda^T} \tau e^{-2\lambda^T \tau} - \frac{3}{4(\lambda^T)^2} e^{-2\lambda^T \tau}\Big) \\ &\quad - \sigma_{C^T}^2 (1 + \alpha_{S^T}^{Fin,AA})^2 \Big(\frac{5}{8(\lambda^T)^3} \frac{1 - e^{-2\lambda^T \tau}}{\tau} - \frac{2}{(\lambda^T)^3} \frac{1 - e^{-\lambda^T \tau}}{\tau}\Big) \\ &\quad - \frac{\sigma_{L^S}^2 (\alpha_{L^S}^{Fin,AA})^2}{6} \tau^2 - \sigma_{S^S}^2 (\alpha_{S^S}^{Fin,AA})^2 \Big(\frac{1}{2(\lambda^S)^2} - \frac{1}{(\lambda^S)^3} \frac{1 - e^{-\lambda^S \tau}}{\tau} + \frac{1}{4(\lambda^S)^3} \frac{1 - e^{-2\lambda^S \tau}}{\tau}\Big) \\ &\quad - \sigma_{Lib}^2 \Big(\frac{1}{2(\kappa^Q_{Lib})^2} - \frac{1}{(\kappa^Q_{Lib})^3} \frac{1 - e^{-\kappa^Q_{Lib} \tau}}{\tau} + \frac{1}{4(\kappa^Q_{Lib})^3} \frac{1 - e^{-2\kappa^Q_{Lib} \tau}}{\tau}\Big). \end{aligned}$$

The description so far has detailed the dynamics under the pricing measure and, by implication, determined the functions that we are going to fit to the observed yields. However, to have a complete model, we need to detail the risk premium specification that generates the connection to the dynamics under the real-world P-measure. An important point is that there are no restrictions on the dynamic drift components under the empirical P-measure. Therefore, beyond the requirement of constant volatility, we are free to choose the dynamics under the P-measure. To facilitate the empirical implementation, we limit our focus to the essentially affine risk premium specification introduced in Duffee (2002). In the Gaussian framework, this specification implies that the risk premiums Γ_t depend on the state variables; that is,

$$\Gamma_t = \gamma^0 + \gamma^1 X_t,$$

where $\gamma^0 \in \mathbf{R}^6$ and $\gamma^1 \in \mathbf{R}^{6\times 6}$ contain unrestricted parameters with n denoting the number of state variables. The relationship between real-world yield curve dynamics under the P-measure and risk-neutral dynamics under the Q-measure is given by

$$dW_t^Q = dW_t^P + \Gamma_t d_t.$$

Thus, we can write the P-dynamics of the state variables as

$$dX_t = K^P(\theta^P - X_t)dt + \Sigma dW_t^P,$$

where both K^P and θ^P are allowed to vary freely relative to their counterparts under the Q-measure.

4 Model estimation

This section first describes our Kalman filter estimation procedure for the AF joint model of Treasury, bank debt, and Libor rates and then provides estimation results.

4.1 Estimation procedure

We estimate the six-factor AFNS model using the Kalman filter, which is an efficient and consistent estimator for our Gaussian model. In addition, the Kalman filter requires a minimum of assumptions about the observed data and easily handles missing data.

The measurement equation for estimation is

$$y_t = \begin{pmatrix} y_t^c \\ y_t^T \\ y_t^{Lib} \end{pmatrix} = \begin{pmatrix} A^c \\ A^T \\ A^{Lib} \end{pmatrix} + \begin{pmatrix} B^c \\ B^T \\ B^{Lib} \end{pmatrix} X_t + \varepsilon_t.$$

The data vector y_t is a 51x1 vector consisting of y_t^c with 40 financial bond rates, y_t^T with the eight Treasury yields, and y_t^{Lib} with the three Libor yields.[11] Correspondingly, the constant term consists of a 40x1 vector A^c, an 8x1 vector A^T, and a 3x1 vector A^{Lib}. The factor-loading matrix for our six factors consists of a 40x6 matrix B^c, an 8x6 matrix B^T, and a 3x6 matrix B^{Lib}. Note that the λ parameters are included in these parameter matrices.

For identification, we choose the A-rated bond yields to be the benchmark for the financial corporate sector. That is, we set the constant $\alpha_0^{Fin,A}$ equal to zero, and let the factor loadings on the two spread factors have unit sensitivity, i.e., $\alpha_L^{Fin,A} = 1$ and $\alpha_S^{Fin,A} = 1$. This choice is motivated by the fact that the A-rating category is represented by a full sample of data for both banks and financial firms, but beyond that this choice is without consequences. It simply implies that the sensitivities to changes in the two spread factors are measured relative to those of the A-rated financial firms and the estimated values of those factors represent the absolute sensitivity of the benchmark A-rated financial corporate bond yields.

For continuous-time Gaussian models, the conditional mean vector and the conditional covariance matrix are given by

$$\begin{aligned} E^P[X_T|\mathcal{F}_t] &= (I - \exp(-K^P\Delta t))\mu^P + \exp(-K^P\Delta t)X_t, \\ V^P[X_T|\mathcal{F}_t] &= \int_0^{\Delta t} e^{-K^P s}\Sigma\Sigma' e^{-(K^P)'s} ds, \end{aligned}$$

where $\Delta t = T - t$ and $\exp(-K^P\Delta t)$ is a matrix exponential. Stationarity of the system under the P-measure is ensured provided the real component of all the eigenvalues of K^P are positive. This is imposed in all estimations. For this reason, we can start the Kalman filter at the unconditional mean and covariance matrix

$$\widehat{X}_0 = \mu^P \quad \text{and} \quad \widehat{\Sigma}_0 = \int_0^{\infty} e^{-K^P s}\Sigma\Sigma' e^{-(K^P)'s} ds.$$

The transition state equation for the Kalman filter is given by

$$X_{t_i} = \Phi^0_{\Delta t_i} + \Phi^1_{\Delta t_i} X_{t_{i-1}} + \eta_{t_i},$$

where $\Delta t_i = t_i - t_{i-1}$ and

$$\Phi^0_{\Delta t_i} = (I - \exp(-K^P\Delta t_i))\mu^P, \quad \Phi^1_{\Delta t_i} = \exp(-K^P\Delta t_i), \quad \text{and} \quad \eta_{t_i} \sim N\Big(0, \int_0^{\Delta t_i} e^{-K^P s}\Sigma\Sigma' e^{-(K^P)'s} ds\Big).$$

[11] Note that y_t^c contains 40 rates across our five (industry, rating) categories after September 11, 2001. Before that date, when yields for AA-rated bonds issued by U.S. banks are unavailable, y_t^c contains 32 series across the four categories.

In the Kalman filter estimation, all measurement errors are assumed to be independent and identically distributed white noise. Thus, the error structure is in general given by

$$\begin{pmatrix} \eta_t \\ \varepsilon_t \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix}\right].$$

Each maturity of the Treasury bond yields has its own measurement error standard deviation. For parsimony, the measurement errors for the corporate bond yields are assumed to have a uniform standard deviation across all ratings and maturities. Furthermore, we include a separate standard deviation parameter for each of the three maturities in the Libor rate data.

4.2 Estimation results

We now provide estimates of our six-factor model. A key element in estimation is the specification of the P-dynamics of the state variables. As in Feldhütter and Lando (2006), we assume that the Libor risk factor is independent of the other five factors in the model, because we are interested in the temporary deviations between Libor rates and the credit spreads on AA-rated U.S. financial corporate bonds of similar maturities.[12] Hence, we impose the restriction that the Libor-specific factor is independent of the others. Thus, the K^P matrix is specified as

$$K^P = \begin{pmatrix} \kappa_{11}^P & \kappa_{12}^P & \kappa_{13}^P & \kappa_{14}^P & \kappa_{15}^P & 0 \\ \kappa_{21}^P & \kappa_{22}^P & \kappa_{23}^P & \kappa_{24}^P & \kappa_{25}^P & 0 \\ \kappa_{31}^P & \kappa_{32}^P & \kappa_{33}^P & \kappa_{34}^P & \kappa_{35}^P & 0 \\ \kappa_{41}^P & \kappa_{42}^P & \kappa_{43}^P & \kappa_{44}^P & \kappa_{45}^P & 0 \\ \kappa_{51}^P & \kappa_{52}^P & \kappa_{53}^P & \kappa_{54}^P & \kappa_{55}^P & 0 \\ 0 & 0 & 0 & 0 & 0 & \kappa_{66}^P \end{pmatrix}.$$

We denote this model by "Full (5×5) upper K^P, X_t^{Lib} independent".

While this specification provides maximum flexibility in terms of the factor dynamics, we are interested in crafting a more parsimonious one. To do so, we draw on earlier work by Christensen and Lopez (2008) that finds reasonable parameter restrictions based on in-sample specification tests. In particular, we impose three sets of parameter restrictions. First, we

[12]Feldhütter and Lando (2006) incorporate Libor rates in a six-factor AF model of Treasury, swap, and corporate yields with two factors to describe Treasury yields, two factors for credit spreads of financial corporate bonds, a factor for Libor rates, and a factor for swap rates. However, all six factors are assumed independent in their model, while we allow dynamic interactions among our factors. We also include more maturities in estimation.

Model	Six-factor Libor rate models			
	Max log likelihood	D.f.	LR	p-value
Full (5×5) upper K^P, X_t^{Lib} independent	180,128.91	–	–	–
Preferred (5×5) upper K^P, X_t^{Lib} independent	180,125.11	10	7.60	0.6678

Table 2: **Maximum Log Likelihood Values for the Six-Factor Libor Rate Models.** The maximum log likelihood values obtained for different specifications of the six-factor Libor rate models estimated using corporate bond yields for U.S. banks and U.S. financial firms, Libor rates and U.S. Treasury bond yields. The data sample is weekly, covering the period from January 6, 1995, to July 25, 2008. The column labeled D.f. refers to the number of degrees of freedom in the likelihood ratio test, and the column labeled LR refers to the value of the likelihood ratio statistic.

impose the restrictions that the Treasury level and curvature factors impact the drift of the Treasury slope factor, but not each other, and the Treasury slope factor has no impact on the drift of the Treasury level and curvature factors. Second, we assume no dynamic interactions between the Treasury level factor and the two credit spread risk factors. Third, we impose the restrictions that there is no feedback from the credit risk level factor onto the Treasury curvature factor ($\kappa_{51}^P = 0$) nor from the credit risk slope factor to the Treasury slope factor ($\kappa_{42}^P = 0$); that is,

$$K^P = \begin{pmatrix} \kappa_{11}^P & \kappa_{12}^P & 0 & \kappa_{14}^P & \kappa_{15}^P & 0 \\ \kappa_{21}^P & \kappa_{22}^P & 0 & \kappa_{24}^P & \kappa_{25}^P & 0 \\ 0 & 0 & \kappa_{33}^P & 0 & 0 & 0 \\ \kappa_{41}^P & 0 & \kappa_{43}^P & \kappa_{44}^P & \kappa_{45}^P & 0 \\ 0 & \kappa_{52}^P & 0 & 0 & \kappa_{55}^P & 0 \\ 0 & 0 & 0 & 0 & 0 & \kappa_{66}^P \end{pmatrix}.$$

We denote this as the preferred specification.

Since the preferred specification is nested within the complete K^P specification, we test the validity of those parameter restrictions with standard likelihood ratio tests. The results presented in Table 2 show that the parameter restrictions are not rejected by the likelihhod ratio (LR) test, suggesting that they are reasonable assumptions.

Table 3 reports the estimated dynamic parameters for this specification.[13] The significance of the off-diagonal elements in K^P reported in Table 3 indicate that the assumption of independence between all the state variables applied in Feldhütter and Lando (2006) is

[13] The results shown here for the six-factor model are not much different from the parameter estimates of the five-factor model described in Section 3. The intuition here is that X_t^{Lib} only impacts the Libor rates and does not play a role in terms of fitting the other bond yields. Accordingly, the fitted errors of the Treasury and corporate bond yields are hardly affected by the inclusion of the Libor rates in the estimation.

K^P	$K^P_{\cdot,1}$	$K^P_{\cdot,2}$	$K^P_{\cdot,3}$	$K^P_{\cdot,4}$	$K^P_{\cdot,5}$	$K^P_{\cdot,6}$	θ^P	Σ
$K^P_{1,\cdot}$	-0.867	-1.127	0	-0.073	0.072	0	0.0108	0.001873
	(0.171)	(0.205)		(0.053)	(0.045)		(0.0184)	(0.000116)
$K^P_{2,\cdot}$	1.192	0.9368	0	-0.0626	0.163	0	-0.007317	0.002059
	(0.215)	(0.247)		(0.0588)	(0.0557)		(0.0164)	(0.000184)
$K^P_{3,\cdot}$	0	0	0.03388	0	0	0	0.06932	0.004759
			(0.171)				(0.0387)	(0.000124)
$K^P_{4,\cdot}$	1.142	0	1.444	0.9059	-1.040	0	-0.02938	0.008236
	(0.612)		(0.577)	(0.180)	(0.184)		(0.0185)	(0.000247)
$K^P_{5,\cdot}$	0	-3.324	0	0	1.437	0	-0.01449	0.02677
		(2.13)			(0.490)		(0.0374)	(0.000587)
$K^P_{6,\cdot}$	0	0	0	0	0	2.386	0.05550	0.004570
						(0.655)	(0.0114)	(0.000207)

Table 3: **Parameter Estimates for the Preferred Specification of the Six-Factor Libor Rate Model.**
The estimated parameters of the K^P-matrix, the θ^P-vector, and the Σ volatility matrix for the joint six-factor Libor rate model. The data used are weekly covering the period from January 6, 1995, to July 25, 2008. λ^T is estimated at 0.6403 (0.0033), λ^S is estimated at 0.3936 (0.0101), and κ^Q_{Lib} is estimated at 0.0367 (0.0764). Finally, the constant α^{Lib} is estimated at -0.05656 (0.114). The maximum log-likelihood value is 180,125.11. The numbers in parentheses are the estimated standard deviations of the parameter estimates.

not supported by the data. Looking at these parameter estimates, we highlight three results. First, the persistence in the diagonal elements is generally quite high, although much less so for the Libor-specific factor X^{Lib}_t. It has a mean-reversion rate of approximately 2.4, which means that it is normally a quickly mean-reverting process with a half-life of $\frac{\ln 2}{2.4} = 0.289$ years. By implication, any deviations between the credit spreads of AA-rated U.S. financial firms and the term Libor rates should normally be eliminated by way of arbitrage relatively quickly and not persist for long. The estimated volatility of the Libor-specific factor is also relatively modest, another sign that deviations between short-term corporate bond spreads and term Libor rates normally move in a structured way rather than fluctuating erratically.

Second, the effect of the Treasury factors on the credit risk factors seems to be limited, with only the Treasury curvature factor having a statistically significant effect on the credit risk slope factor. Third, as highlighted further in Christensen and Lopez (2008), the credit risk factors do influence the Treasury slope and curvature factors, suggesting that information from the credit markets, perhaps such as changes in default probabilities, may provide insights into the dynamics of the Treasury yield curve. Further research into this finding is ongoing.

Table 4 reports the pairwise correlations between the six factors in this model. Focusing on the Libor-specific factor, we observe large positive correlations with the Treasury slope

Levels	L_t^S	S_t^S	L_t^T	S_t^T	C_t^T	X_t^{lib}
L_t^S	1	-0.711	-0.205	-0.291	-0.483	-0.446
S_t^S		1	0.263	0.436	0.438	-0.063
L_t^T			1	-0.201	0.212	-0.098
S_t^T				1	0.531	0.308
C_t^T					1	0.264
X_t^{lib}						1

Diff.	ΔL_t^S	ΔS_t^S	ΔL_t^T	ΔS_t^T	ΔC_t^T	ΔX_t^{lib}
ΔL_t^S	1	-0.368	0.046	-0.162	0.114	-0.105
ΔS_t^S		1	0.043	0.050	-0.286	-0.177
ΔL_t^T			1	-0.683	0.312	-0.248
ΔS_t^T				1	-0.256	-0.023
ΔC_t^T					1	-0.259
ΔX_t^{lib}						1

Table 4: **Pairwise Correlations Between the Risk Factors in the Joint Six-Factor Libor Rate Model with the Preferred Specification of K^P.**
The pairwise correlation coefficients between the six risk factors in the joint six-factor Libor rate model with the preferred specification of K^P. The data used in the estimation are weekly data covering the period from January 6, 1995 to July 25, 2008.

and curvature factors as well as a high negative correlation with the credit risk level factor. This result suggests that a high value for this credit risk factor tends to push down X_t^{Lib}, leading to a widening of the spread between Libor rates and comparable unsecured AA-rated U.S. financial bond yields. This is an interesting finding, but since varying the K^P-matrix does not change the filtered paths of the factors in the model, we leave it for future research to detail these interactions more accurately.

Table 5 reports the estimated factor loadings of the state variables in the corporate bond yield function for each rating category represented in the data sample. Note that for both U.S. banks and financial firms, lower credit quality tends to imply higher sensitivities to the two common credit risk factors. The exception is the sensitivity of AA-rated U.S. financials to the common credit risk slope factor, which is marginally higher than the value observed for A-rated U.S. financials. Generally speaking, this implies that the credit spreads of bonds issued by firms with lower credit quality tend to have higher and steeper credit spread curves.

Furthermore, we can compare the risk sensitivities for U.S. banks and financial firms. For the benchmark A-rating category, we see that bonds with this rating have nearly identical risk sensitivities across the two sectors. For the AA-rating category, we see greater sensitivities in financial bonds than AA-rated bonds issued by U.S. banks. A partial explanation for this difference is the different data sample periods, where yields for AA-rated U.S. banks do not

Rating	U.S. Financials				
	α_0^C	α_{LT}^C	α_{LS}^C	α_L^C	α_S^C
A	0	-0.01862	-0.05997	1	1
		(0.0234)	(0.00573)		
AA	0.003369	-0.07536	-0.06674	0.9031	1.039
	(0.000245)	(0.0211)	(0.00624)	(0.00566)	(0.0112)
Rating	**U.S. Banks**				
	α_0^C	α_{LT}^C	α_{ST}^C	α_L^C	α_S^C
BBB	0.0002008	-0.02540	-0.06864	1.153	1.072
	(0.000261)	(0.0267)	(0.00624)	(0.00516)	(0.00953)
A	-0.00002398	-0.02505	-0.05198	1.032	1.024
	(0.00294)	(0.0238)	(0.00668)	(0.00538)	(0.00933)
AA	-0.0002434	-0.002739	-0.03696	0.8221	0.8725
	(0.000487)	(0.0198)	(0.00701)	(0.00863)	(0.0120)

Table 5: **Estimated Factor Loadings in the Corporate Bond Yield Function across Rating Categories in the Libor Rate Model with the Preferred Specification of K^P.**
The estimated factor loadings for each of the rating categories in the two sectors: U.S. Financial firms and U.S. Banks. The model used is the preferred six-factor Libor rate model estimated with Treasury bond yields and corporate bond yields for U.S. banks and U.S. financial firms in addition to Libor rates. The data used are weekly, covering the period from January 6, 1995 to July 25, 2008. The numbers in parentheses are the estimated standard deviations of the parameter estimates.

enter the sample until September 2001. Thus, the previous downturn in the credit cycle is only partially represented for banks, while the very calm period from mid-2003 until mid-2007 is fully represented.

We end the description of the estimation results by detailing the fit of the model. Table 6 reports the summary statistics for the fitted errors of the Treasury yields in the preferred Libor model. Here, we note a very good fit for the maturities from six months up to seven years, while the fit of the shortest and the longest maturity in the data set is slightly less satisfactory. However, importantly, the fit is only slightly worse than that reported by other studies using the AFNS models for Treasury yields only (see for example CDR, 2007) despite the fact that we are making a simultaneous modeling of Treasury, corporate bond, and Libor yields.

Table 7 reports the summary statistics of the fitted errors for the corporate bond yields. The reported root mean squared errors (RMSEs) are in line with the estimated standard deviation for the fitted errors that we obtain from the Kalman filter, which is estimated at $\widehat{\sigma}_{\varepsilon^c} = 11.3$ basis points. Overall, this model fit is slightly worse than the fit reported by Feldhütter and Lando (2006) for their corporate bond yields. One possible explanation is

Maturity in months	Preferred model	
	Mean	RMSE
3	-5.53	15.71
6	-3.53	6.62
12	0.28	3.17
24	2.05	2.64
36	-0.29	1.69
60	-3.21	3.90
84	0.48	3.52
120	12.58	14.82

Table 6: **Summary Statistics for the Fitted Errors of the Treasury Bond Yields.** The mean and root mean squared error of the fitted errors across the 8 different maturities in the Treasury bond yield data set. All numbers are measured in basis points. The data set used is weekly data covering the period from January 6, 1995 to July 25, 2008.

Maturity in months	U.S. Financials				U.S. Banks					
	A		AA		BBB		A		AA	
	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
3	-1.50	11.68	0.85	12.79	-6.35	13.20	-4.38	12.25	-3.52	11.13
6	3.28	10.21	5.05	12.05	3.90	10.12	5.16	10.53	0.65	11.63
12	-1.18	10.65	0.89	9.46	0.18	11.48	0.75	9.27	1.66	11.03
24	-1.30	6.53	0.18	7.86	1.29	11.22	0.05	7.39	1.55	8.10
36	-0.81	8.61	-1.31	9.67	-1.14	12.06	-0.07	8.96	1.62	12.55
60	-3.38	8.89	-3.99	9.87	-3.62	11.59	-6.39	10.01	-9.23	13.29
84	1.42	8.76	-1.31	8.86	5.11	14.51	3.37	9.57	8.24	14.94
120	3.31	12.76	-0.47	12.83	0.51	14.61	1.37	14.13	-1.17	13.54
No. obs	708		708		708		708		358	

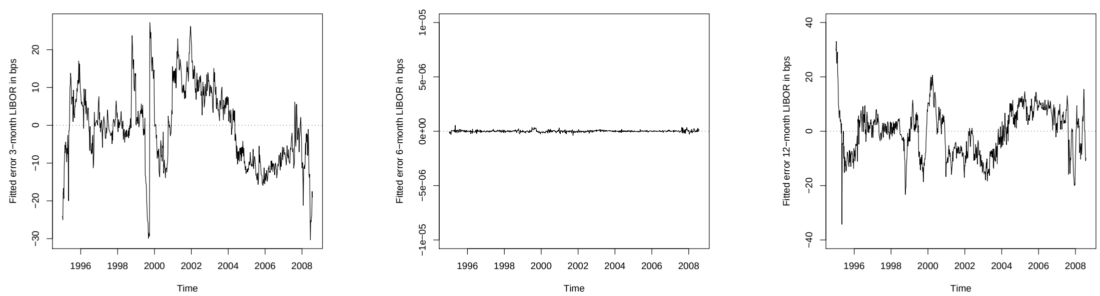
Table 7: **Summary Statistics for the Fitted Errors of the Corporate Bond Yields.** The mean and the root mean squared errors for the fit of the corporate zero-coupon bond yields for U.S. financial firms rated A and AA and U.S. banks rated BBB, A, and AA, respectively, for the eight fixed maturities covering the period from January 6, 1995, to July 25, 2008. The model used is the preferred six-factor Libor model. All numbers are measured in basis points.

that they do not include the 3- and 6-month yields in their analysis. However, given the fact that we are fitting the 48 time series of Treasury and corporate bond yields jointly with only five state variables, we are quite satisfied with the achieved fit of the model.

Focusing on the overall fit of the Libor rates in the six-factor Libor rate model, Table 8 presents the fitted errors of the three Libor rates for the preferred specification analyzed here. The model fits the six-month Libor rate perfectly, while there are fitted errors of some magnitude for the two other maturities. However, their size is on par with the fit of the other yields included in the estimation. Thus, the fit of the Libor rates is well within the range considered acceptable when it comes to regular Treasury bond yield term structure models.

Maturity in months	Preferred model	
	Mean	RMSE
3	-0.51	10.18
6	0.00	0.00
12	-0.15	8.68

Table 8: **Summary Statistics for the Fitted Errors of the Libor Rates in the Preferred Six-Factor Libor Rate Model.**
The mean and root mean squared error of the fitted errors of the Libor rates for the three different maturities in the Libor rate data set. All numbers are measured in basis points. The data set used is weekly data covering the period from January 6, 1995, to July 25, 2008.



(a) Three-month Libor. (b) Six-month Libor. (c) Twelve-month Libor.

Figure 3: **Fitted Errors of the Libor Rates in the Six-Factor Libor Rate Model with the Preferred Specification of K^P.**
The fitted errors of the Libor rates in the joint six-factor Libor rate model with the preferred specification of K^P. The bond yields and Libor rates used in the estimation are weekly data covering the period from January 6, 1995, to July 25, 2008.

Figure 3 illustrates the time series of the fitted errors for each of the three term Libor rates in the data set. Note that we only observe a marginal deterioration in the model's ability to fit the Libor rates from the fall of 2007 through the end of the sample. Thus, the model is flexible enough to handle the turmoil in the Libor market.

5 The financial crisis and central bank actions

In this section, we use the estimated six-factor model to assess the effect on Libor rates of the financial crisis that started in mid-2007 and the resulting central bank liquidity operations. Figure 4 focuses on the movements in the spread between the three-month Libor rate and the three-month Treasury yield from the beginning of 2007 through July 25, 2008, which is the

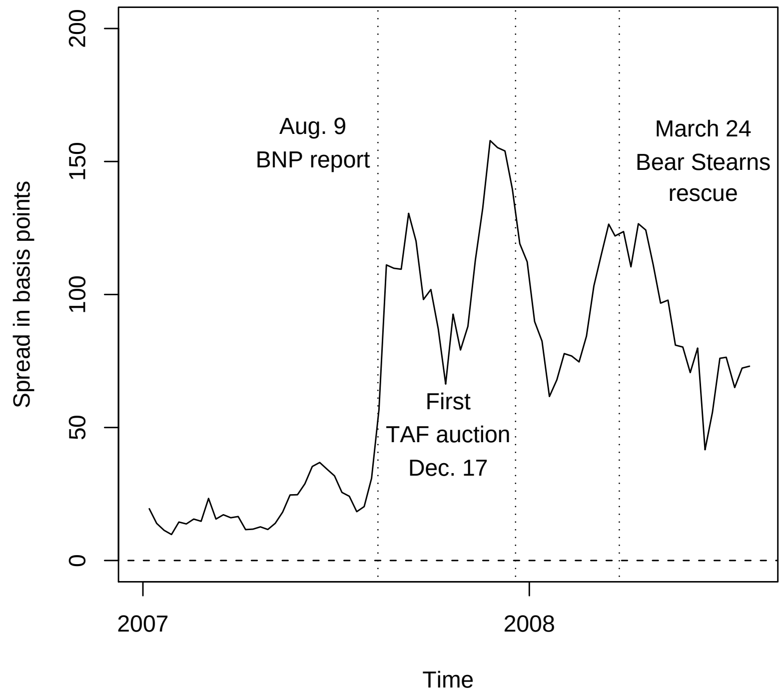


Figure 4: **Spread of the Three-Month Libor Rate over the Three-Month Treasury Bond Yield.**
The spread of the three-month Libor rate over the three-month Treasury bond yield since the beginning of 2007.

end of our estimation sample.[14] There are two key dates during this period. The first marks the start of the turmoil in financial markets. As evident in Figure 4, Libor rates jumped on August 9, 2007. An important trigger for this tightening of interbank money markets was the announcement by the global bank BNP Paribas that it would suspend redemptions from three of its investment funds.[15]

The second key date came in mid-December, when the Federal Reserve and other central banks made a more forceful commitment to improve liquidity and functioning in the

[14]We limit our analysis to the first year of the financial crisis for two reasons. During this period, the Fed's liquidity operations were being sterilized, so they altered the composition and not the size of the Fed's balance sheet. Also, after the end of our sample, there were additional policy actions, such as government insurance for bank debt and interbank loans, that have potentially significant implications for bank credit and liquidity risk but do not involve direct injections of liquidity.

[15]The BNP Paribas press release stated that "the complete evaporation of liquidity in certain market segments of the U.S. securitisation market has made it impossible to value certain assets fairly regardless of their quality or credit rating ... during these exceptional times, BNP Paribas has decided to temporarily suspend the calculation of the net asset value as well as subscriptions/redemptions."

interbank market. Of course, the Fed has been at the center of various initiatives to provide dollar-denominated liquidity to markets. However, the initial response of the Fed to the dislocations in the interbank lending market in the fall of 2007 was to promote and enhance the availability of its discount window as a source of funding.[16] However, through the end of 2007, discount window borrowing remained relatively low and interbank lending rates remained quite elevated. Therefore, on December 12, 2007, the Fed announced the creation of the Term Auction Facility (TAF), which consisted of periodic auctions of fixed quantities of term funding to sound depository institutions. The first TAF auction occurred on December 17 for $20 billion in 28-day credit and was greatly oversubscribed. On December 12, 2007, the Fed also announced coordinated dollar liquidity actions with the European Central Bank and the Swiss National Bank. These involved the reciprocal foreign exchange swap lines, in which dollars were passed through to term lending by foreign central bank lending in dollars abroad. The TAF and the swap lines were meant to alleviate the dollar liquidity risk by making cash loans to banks that were secured by those banks' illiquid but sound assets.[17] Both the TAF and the swap lines were scaled up in size during 2008; however, arguably the Fed's initial actions in mid-December 2007 were the key events signalling a change in the bank liquidity regime. Even though the liquidity programs would both grow over time, the initial announcement of these programs was accompanied by a widespread realization that the Fed and other central banks would provide forceful and innovative responses to bank liquidity needs going forward.

After the central bank actions in December 2007, the Libor-Treasury spread did fall, but not permanently, and it did not revert to its original level. Therefore, it is unclear to what extent the central bank liquidity operations alleviated stress in the dollar interbank market. To address this issue, we use our six-factor model to describe the relationship between the yields of financial corporate bonds and term Libor rates during this period.

Figure 5 shows the estimated path of our sixth factor, which is specific to the Libor market. The interpretation of this factor is that deviations from its mean (shown by a horizontal dashed line) indicate the direction (and also approximately the size) of the difference between the yield on AA-rated U.S. financial bonds and the term Libor rates with the same maturity. Until December 2007, this factor moved within a fairly close range to its mean. However, following the introduction of the TAF it dropped rapidly below its historical mean and remained low through the end of the sample. Based on the figure and the factor's summary statistics, it

[16] In particular, the Federal Reserve reduced the spread between the discount rate (or primary credit rate) and the target federal funds rate.

[17] The Federal Reserve has also established several other liquidity facilities that provide loans to financial institutions other than banks, such as the Primary Dealer Credit Facility.

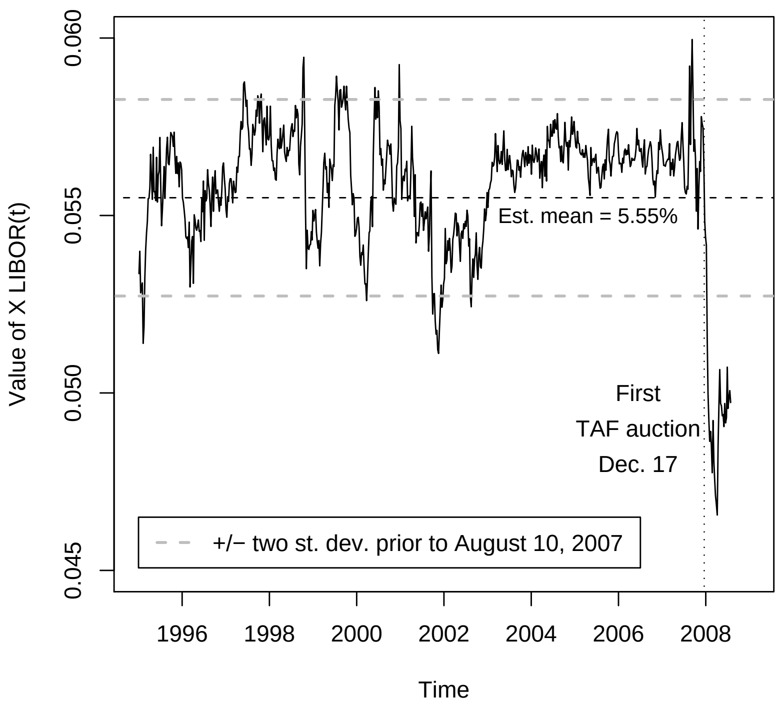


Figure 5: **The Estimated Libor Factor from the Preferred Six-Factor Libor Rate Model.**
The estimated Libor-specific factor from the preferred six-factor Libor rate model. The bond yields and Libor rates used in the estimation are weekly data covering the period from January 6, 1995 to July 25, 2008.

appears that there might have been a regime change in the dynamic behavior of X_t^{Lib} following the introduction of the TAF and other central bank liquidity operations.

The simplest way to analyze whether the dynamic properties of X_t^{Lib} have changed is to assume that its parameters prior to the first TAF auction, denoted $\psi_{Lib}^{pre} = (\kappa_{66}^P, \theta_{Lib}^P, \sigma_{Lib}, \kappa_{Lib}^Q, \alpha^{Lib})$, changed to a new set of parameters, denoted $\psi_{Lib}^{post} = (\widetilde{\kappa}_{66}^P, \widetilde{\theta}_{Lib}^P, \widetilde{\sigma}_{Lib}, \widetilde{\kappa}_{Lib}^Q, \widetilde{\alpha}^{Lib})$. As the Kalman filter can handle time-varying parameters, we can test this hypothesis. Since the data we use are weekly (Fridays), the first observation after the first TAF auction is December 21, 2007. As of that observation date, the five parameters in ψ_{Lib}^{pre} are assumed to have permanently changed to the new ψ_{Lib}^{post} values. All remaining parameters in the model are unchanged. As a consequence, for observation dates up to and including December 14, 2007,

K^P	$K^P_{\cdot,1}$	$K^P_{\cdot,2}$	$K^P_{\cdot,3}$	$K^P_{\cdot,4}$	$K^P_{\cdot,5}$	θ^P	Σ
$K^P_{1,\cdot}$	-0.8712	-1.126	0	-0.07313	0.07063	0.01204	0.001874
	(0.174)	(0.207)		(0.0530)	(0.0449)	(0.0105)	(0.000118)
$K^P_{2,\cdot}$	1.190	0.9351	0	-0.06381	0.1639	-0.008402	0.002059
	(0.234)	(0.267)		(0.0593)	(0.0573)	(0.00923)	(0.000188)
$K^P_{3,\cdot}$	0	0	0.06106	0	0	0.06629	0.004761
			(0.194)			(0.0208)	(0.000126)
$K^P_{4,\cdot}$	1.148	0	1.448	0.9092	-1.040	-0.02899	0.008241
	(0.615)		(0.608)	(0.182)	(0.188)	(0.0163)	(0.000257)
$K^P_{5,\cdot}$	0	-3.277	0	0	1.432	-0.01698	0.02677
		(2.18)			(0.510)	(0.0214)	(0.000597)

Table 9: **Parameter Estimates for the Preferred Specification of the Six-Factor Libor Rate Model With a Regime Switch for X_t^{Lib} following the TAF Introduction.** The estimated parameters of the K^P matrix, the θ^P vector, and the Σ volatility matrix for the preferred joint six-factor Libor rate model. The data used are weekly, covering the period from January 6, 1995, to July 25, 2008. λ^T is estimated at 0.6408 (0.00341), λ^S is estimated at 0.3935 (0.0102). The maximum log-likelihood value is 180,151.18. The numbers in parentheses are the estimated standard deviations of the parameter estimates.

the prediction step in the Kalman filter is calculated using

$$\widehat{X}_{t|t-1} = \Phi^0(\psi^{pre}) + \Phi^1(\psi^{pre})\widehat{X}_{t-1}.$$

After then, the prediction step is calculated with

$$\widehat{X}_{t|t-1} = \Phi^0(\psi^{post}) + \Phi^1(\psi^{post})\widehat{X}_{t-1}.$$

The same procedure is applied to the pricing of the Libor rates.

The estimated dynamic parameters for the non-Libor factors in the estimation of our preferred specification with a regime switch are not meaningfully different from before, as shown in Table 9. Table 10 reports the estimated parameters for the Libor-specific factor and compares them to those for the model without a regime switch. The likelihood ratio test of the hypothesis that no regime switch has taken place is

$$LR = 2[180,151.18 - 180,125.11] = 52.14 \sim \chi^2(5),$$

which is highly significant. This test suggests that the hypothesis should be rejected and that a regime switch is supported by the data. In line with the LR-test result, several of the parameter estimates reported in Table 10 are significantly different across the two regimes; in

Parameter	Full sample	Regime switch	
		pre TAF	post TAF
κ_{66}^{P}	2.386	4.912	14.84
	(0.655)	(0.998)	(4.898)
θ_{Lib}^{P}	0.05550	0.05718	0.05008
	(0.0114)	(0.136)	(0.137)
σ_{Lib}	0.004570	0.004444	0.006747
	(0.000207)	(0.000230)	(0.00204)
κ_{Lib}^{Q}	0.03673	0.03686	0.01166
	(0.0764)	(0.0887)	(0.0366)
α^{Lib}	-0.05656	-0.05771	-0.05802
	(0.114)	(0.136)	(0.137)

Table 10: **Estimated Dynamic Parameters for the Libor Factor in the Preferred Six-Factor Libor Rate Model with TAF Regime Switch.**
The estimated dynamic parameters for the Libor-specific factor with and without a regime switch included following the introduction of the TAF. The model used is the preferred six-factor Libor rate model estimated with Treasury bond yields and corporate bond yields for U.S. banks and U.S. financial firms in addition to Libor rates. The data used are weekly covering the period from January 6, 199,5 to July 25, 2008. The numbers in parentheses are the estimated standard deviations of the parameter estimates.

particular, the mean-reversion rate for the Libor factor κ_{66}^{P} and its volatility σ_{Lib}. This result suggests that the mean-reversion rate of the Libor-specific factor and its volatility increased after central bank liquidity operations began.

To quantify the impact that the introduction of the TAF had, we use a counterfactual analysis of what would have happened had the TAF *not* been introduced. We use the full-sample model without the regime switch to generate a counterfactual path for the 3-month Libor rate that suggests what that rate *might* have been if it had been priced in accordance with prevailing conditions in the Treasury and corporate bond markets for U.S. financial firms. To quantify this effect, we "turn off" the Libor-specific factor by fixing it at its mean and leaving the remaining factors unchanged. Thus, the counterfactual path provides a Libor rate consistent with the credit factors reflected in the bonds issued by AA-rated U.S. financial institutions.

Figure 6 illustrates the effect of the counterfactual path on the three-month Libor spread over the three-month Treasury rate since the beginning of 2007. Note that the model-implied three-month Libor spread is close to the observed spread over this period. Until December 2007, the counterfactual spread was tracking the observed spread relatively closely. However, by the end of 2007, a significant wedge developed between the two. As of the end of our sample on July 25,2008, the difference between the counterfactual spread and the observed

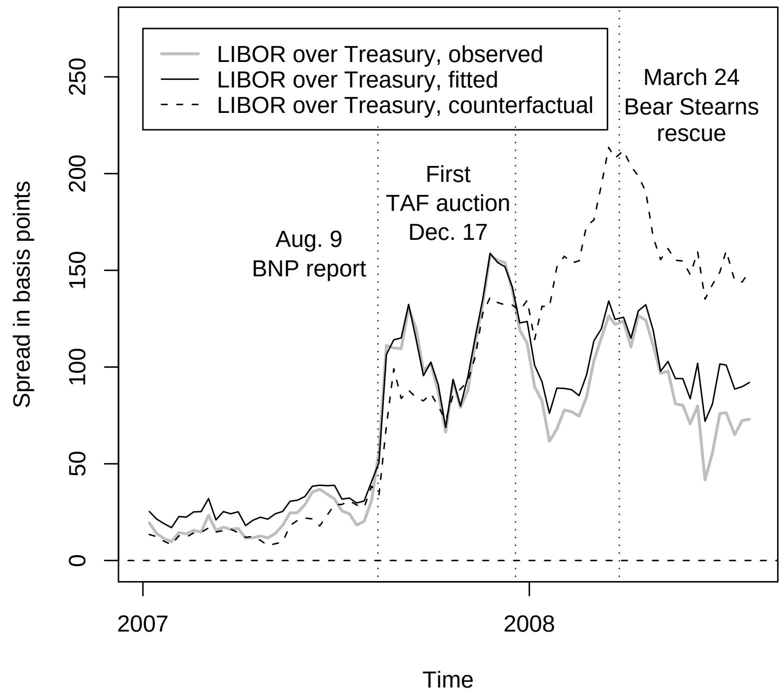


Figure 6: **Spread of the Three-Month Libor Rate over the Three-Month Treasury Bond Yield Since the Beginning of 2007.**
The spread of the observed and fitted three-month Libor rate over the three-month Treasury bond yield in the preferred six-factor Libor rate model. The figure also illustrates the spread of the fitted three-month Libor rate when the Libor-specific factor is fixed at its mean, in effect neutralizing the idiosyncratic effects in the Libor market. The bond yields and Libor rates used in the estimation are weekly data covering the period from January 6, 1995 to July 25, 2008.

three-month Libor spread was 77 basis points.

Figure 7 brings the difference between the observed three-month Libor rate and its counterfactual path into sharper focus. From the start of the financial crisis in August 2007 until mid-December 2007, the difference was +14.8 basis points on average and occasionally quite large. These were signs of distress in the interbank market that ultimately led to the introduction of the TAF. Following the introduction and expansion of the TAF through January 2008, the difference between the observed three-month Libor rate and the counterfactual rate quickly turned negative and reached a level of approximately -75 basis points, where it stayed for the remainder of the sample. The average value from December 21, 2007, through July 25, 2008 was -71.2 basis points. This analysis suggests that, according to our model, the

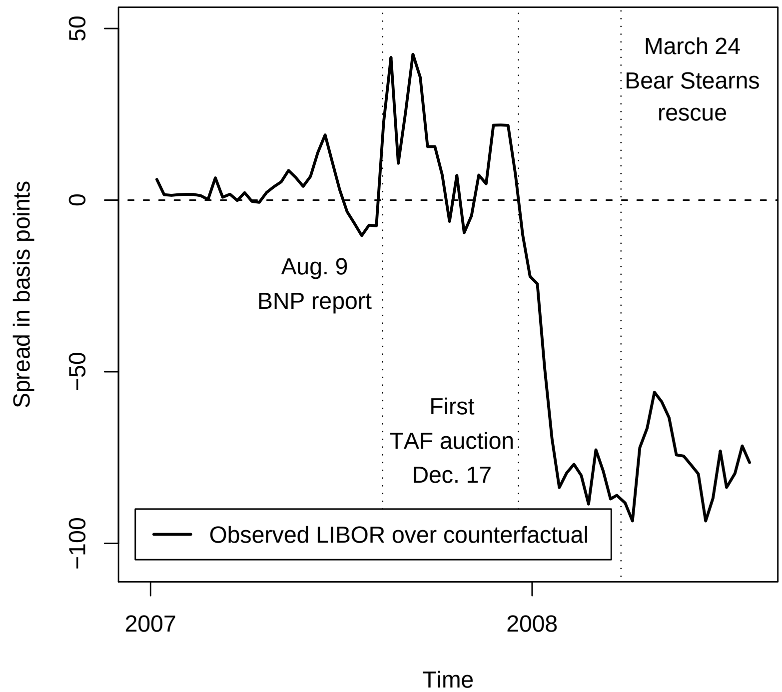


Figure 7: **Difference Between the Three-Month Libor Rate and Its Counterfactual Since the Beginning of 2007.**
Illustration of the difference between the observed three-month Libor rate and the counterfactual path generated by fixing the Libor-specific factor at its mean, in effect neutralizing the idiosyncratic effects in the Libor market. The bond yields and Libor rates used in the estimation are weekly data covering the period from January 6, 1995, to July 25, 2008.

three-month Libor rate would have been higher in the absence of the TAF.[18]

The most straightforward interpretation of Figure 7 is that the liquidity measures provided by the Fed and other central banks have had a significant impact on conditions in the term-funding market. However, as Figure 4 clearly demonstrates, these measures have not been able to bring the three-month Libor spread over the three-month Treasury yield back to the levels observed prior to the current crisis.

We can also speculate further on the nature of our result. During the recent financial crisis, there has been an abnormally large and persistent spread between bank debt and

[18]In mid-April 2008, there were news reports that the 16 banks surveyed as part of the daily fixing of the Libor rates on U.S. dollar-denominated term deposits were underreporting their actual borrowing costs. In that case, the distress in the interbank market would be more severe than perceived by the market. Still, while it appears that the story about the underreporting in the Libor market temporarily reduced the difference between the observed three-month Libor rate and the counterfactual rate, this effect soon vanished.

Libor yields. We believe that this new deviation does not reflect a change in the relative risk characteristics of the bank debt and interbank loans; that is, the relative default probabilities and payment delay probabilities likely have changed little across the two market sectors.[19] However, while the relative quantities of credit and liquidity risk have likely remained the same, the relative prices that investors have attached to the latter risk may have changed. Specifically, the unusual deviation between bank bond rates and interbank lending rates may reflect their varied market microstructures and, specifically, the very different lender classes in these two markets. The Libor rate and interbank market are based on banks providing other banks with short-term funding. In contrast, the financial corporate yields are derived from debt obligations issued to a broader class of investors that includes nonbank institutions. These two classes of lenders likely attach similar prices to credit risk; i.e., they likely have a similar perception of and tolerance to default. However, they likely have different tolerances to liquidity problems. In particular, because of the financial interconnectedness of all banks, any liquidity event in a financial crisis has the potential to adversely affect all banks, whether borrowing or lending in the interbank market, much more than nonbank investors. That is, occasions when borrowing banks may have trouble funding their immediate debt obligations are also occasions when all banks find themselves under balance sheet pressure to cover their own debt obligations. Therefore, consistent with what many financial market participants believe, the interbank lending market may be much more sensitive to liquidity pressures than the bank debt market.

6 Conclusion

In this paper, we address the question of whether interbank lending rates have responded to central bank liquidity operations by using a six-factor AFNS model that encompasses Treasury rates, financial corporate debt rates, and Libor rates. Our results provide support for the view that these operations, such as the introduction of the TAF, did lower Libor rates starting in December 2007 and through the end of our sample in July 2008. We find that the parameters governing the Libor factor in our model appear to change after the introduction of the liquidity facilities; i.e., the hypothesis of constant parameters over the full sample period

[19] An unsecured deposit is more senior in the liability structure of a bank than senior unsecured debt. McAndrews, Sarkar, and Wang (2008) mention a recovery rate of 91.25% for unsecured deposits at banks with assets larger than \$5 billion, as per the work of Kuritzkes, Schuermann, and Weiner (2005). On the other hand, the data provider Markit typically works with a recovery rate as low as 40% in its pricing of credit default swap contracts. It is not clear why this difference in recovery rates would have changed dramatically in December 2007. Similarly, changes in the relative amount of credit risk between the Libor panel of AA-rated banks and the Bloomberg AA-rated bank bond panel are also hard to date to that period.

is rejected. This result suggests that the behavior of this factor, and thus of the Libor market, was directly affected by these central bank liquidity operations.

To quantify this effect, we use the model to construct a counterfactual path for the three-month Libor rate by assuming that the Libor-specific factor remained constant at its historical average after the introduction of the liquidity facilities. Our analysis suggests that the counterfactual 3-month Libor rate averaged significantly higher than the observed rate from December 2007 into midyear 2008. This result suggests that, if the central bank liquidity operations had not occurred, the three-month Libor spread would have been even higher than the observed historical spread.

Appendix: Conversion of interest rate data

We convert the Bloomberg data for financial corporate bond rates into continuously compounded yields. The n-year yield at time t, $r_t(n)$, the corresponding zero-coupon bond price, $P_t(n)$, and the continuously compounded yield, $y_t(n)$, are related by

$$P_t(n) = \frac{1}{(1+r_t(n))^n} = e^{-y_t(n)n} \iff y_t(n) = -\frac{1}{n}\ln\frac{1}{(1+r_t(n))^n} = \ln(1+r_t(n)).$$

For maturities shorter than one year, we assume the standard convention of linear interest rates. For example, the zero-coupon bond price corresponding to the six-month yield is calculated as

$$P_t(6m) = \frac{1}{1+0.5r_t(6m)} = e^{-0.5y_t(6m)},$$

and the corresponding continuously compounded yield as

$$y_t(6m) = -2\ln\frac{1}{1+0.5r_t(6m)} = 2\ln(1+0.5r_t(6m)).$$

We also convert the Libor rates into continuously compounded yields, as in Feldhütter and Lando (2006). To facilitate this conversion, we approximate the day count ratio assuming that the Libor curve is smooth. Therefore, the net present value of the three-month Libor contract is

$$NPV_t^{Lib} = \frac{1}{1+\frac{1}{4}L(t,t+0.25)} = e^{-0.25y^{Lib}(t,t+0.25)},$$

where $L(t, t+0.25)$ denotes the quoted three-month Libor rate. The continuously compounded equivalent to the quoted three-month Libor rates is then

$$y^{Lib}(t,t+0.25) = -4\log\left[\frac{1}{1+\frac{1}{4}L(t,t+0.25)}\right] = 4\log(1+0.25L(t,t+0.25)).$$

Similarly, the six-month and twelve-month Libor rates can be converted into continuously compounded zero-coupon yields by the following formulas:

$$\begin{aligned} y^{Lib}(t,t+0.5) &= 2\log(1+0.5L(t,t+0.5)), \\ y^{Lib}(t,t+1) &= \log(1+L(t,t+1)). \end{aligned}$$

References

[1] Christensen, Jens H. E., Francis X. Diebold, and Glenn D. Rudebusch, "The Affine Arbitrage-free Class of Nelson-Siegel Term Structure Models," unpublished working paper, Federal Reserve Bank of San Francisco, September 2007.

[2] Christensen, Jens H. E., and Jose A. Lopez, "Common Risk Factors in the U.S. Treasury and Corporate Bond Markets: An Arbitrage-free Dynamic Nelson-Siegel Modeling Approach," unpublished working paper, Federal Reserve Bank of San Francisco, October 2008.

[3] Christensen, Jens H. E., Jose A. Lopez, and Glenn D. Rudebusch, "Inflation expectations and risk premiums in an arbitrage-free model of nominal and real bond yields," Working Paper Series 2008-34, Federal Reserve Bank of San Francisco, 2008.

[4] Driessen, Joost, "Is Default Event Risk Priced in Corporate Bonds?," Review of Financial Studies, Vol. 18, No. 1, 165-195, January 2005.

[5] Duffee, Gregory R., "Term Premia and Interest Rate Forecasts in Affine Models," Journal of Finance, Vol. 57, 405-443, 2002.

[6] Duffee, Gregory R., "Forecasting with the Term Structure: The Role of No-Arbitrage," manuscript, Johns Hopkins University, 2008.

[7] Duffie, Darrell, and Rui Kan, "A Yield-Factor Model of Interest Rates," Mathematical Finance, Vol. 6, 379-406, 1996.

[8] Federal Reserve Board of Governors, Press release. December 12, 2007.

[9] Feldhütter, Peter, and David Lando, "Decomposing Swap Spreads," forthcoming Journal of Financial Economics, May 2006.

[10] Gürkaynak, Refet S., Brian Sack, and Jonathan H. Wright, "The U.S. Treasury Yield Curve: 1961 to the Present," Journal of Monetary Economics, Vol. 54, 2291-2304, 2007.

[11] Kim, Don H., and Athanasios Orphanides, "Term Structure Estimation with Survey Data on Interest Rate Forecasts," Finance and Economics Discussion Series, No. 2005-48, Board of Governors of the Federal Reserve System, 2005.

[12] Kuritzkes, Andrew, Til Schuermann, and Scott M. Weiner, "Deposit Insurance and Risk Management of the U.S. Banking System: What is the Loss Distribution Faced by the FDIC?," Journal of Financial Services Research, Vol. 27, No. 3, 217-242, 2005.

[13] Litterman, R., and J. A. Scheinkman, "Common Factors Affecting Bond Returns," Journal of Fixed Income, Vol. 1, 62-74, 1991.

[14] McAndrews, James, Asani Sarkar, and Zhenyu Wang, "The Effect of the Term Auction Facility on the London Inter-Bank Offered Rate," Staff Report No. 335, Federal Reserve bank of New York, July 2008.

[15] Nelson, Charles R., and Andrew F. Siegel, "Parsimonious Modeling of Yield Curves," Journal of Business, Vol. 60, No. 4, 473-489, 1987.

[16] Taylor, John B., and John C. Williams, "A Black Swan in the Money Market," American Economic Journal: Macroeconomics, 1(1): 58-83, 2009.