Heterogeneous Expectations, Learning and European Inflation Dynamics*

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October 2010

Abstract

This paper provides a first attempt to investigate how different learning rules perform in explaining survey data on inflation expectations of households and professional forecasters in five core European economies (France, Germany, Italy, Netherlands and Spain). It is shown that adaptive learning algorithms with constant gain perform well in out-of-sample forecasting and that households in countries with a history of high inflation use higher constant gain parameters to predict inflation than those in countries with low inflation. They are hence able to pick up structural changes faster. Professional forecasters update their information sets more frequently than households. Furthermore, household expectations in the Euro Area have not converged to the inflation objective of the ECB, which is to keep inflation below but close to 2% in the medium term. This contrasts with the findings for experts, which seem to be more inclined to incorporate the implications of monetary union for the convergence in inflation rates into their expectations.

JEL Classification: E31, E37, D84

Key Words: Monetary Policy, Heterogeneous Expectations, Adaptive Learning, Survey Expectations

*I am grateful to Sandra Eickmeier, Christina Gerberding and Chryssi Giannitsarou for many helpful discussions. I thank Olivier Basdevant, Petra Geraats, Stefan Gerlach, Seppo Honkapohja, Christopher Kent, Albert Marcet, Demosthenes Tambakis, participants at the 6th Norges Bank Monetary Policy Conference on 'Inflation targeting twenty years on', June 11-12, 2009; and seminar participants at the Cambridge Macroeconomics Workshop, the Deutsche Bundesbank, the Swedish Riksbank and the Oesterreichische Nationalbank for helpful comments. Part of this paper was written whilst I was visiting the Deutsche Bundesbank, which I would like to thank for its hospitality.The views expressed in this paper are those of the author and do not necessarily represent those of the International Monetary Fund. The paper was written prior to joining the Fund.

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1 Introduction

Most central banks nowadays gear monetary policy directly towards maintaining a low and stable level of inflation (IMF, 2005, chapter 4). There are of course important differences between central banks. Whilst some are explicit inflation targeters, others, such as the European Central Bank, have a numerical definition of price stability as the overriding objective of monetary policy (Gerlach and Schnabel 2000). However, in either case, an understanding of how the public forms inflation expectations is of crucial importance for policymakers.

From the 1970s onwards the idea that expectations are rational has dominated much of the literature. Lately a new view on expectations has emerged, which views economic agents as econometricians when forecasting (an extensive overview of this literature is provided by Evans and Honkapohja 2001). This approach, referred to as adaptive learning, assumes that economic agents are boundedly rational but employ statistical forecasting techniques, which allow for the possibility of a rational expectations equilibrium to be learnt in the long run. One important insight from the adaptive learning literature is that policies which may be optimal under rational expectations are not optimal when individuals use a learning process (Orphanides and Williams 2005). Orphanides and Williams (2005) show that the optimal monetary policy under a learning process should respond more aggressively to inflation and become more focused on inflation stability than if expectations were rational, since tight inflation control can facilitate learning and provide better guidance for the formation of inflation expectations.

The contribution of this paper is twofold: First, it investigates whether learning by economic agents is a plausible assumption for the Euro Area. The second contribution is to analyse whether the learning process of economic agents converges towards equilibrium and specifically whether households and professional forecasters are able to learn the inflation objective of the European Central Bank (ECB), which is to maintain inflation close to but below 2% in the medium term. Thus, the paper focuses on expectations in the Euro Area. One reason this is interesting is that there are several member countries and thus different sets of inflation expectations. Hence, it is possible to investigate whether there are differences in inflation expectations between countries and between households and professional forecasters. Thereby it is also examined to what extent the learning behaviour of economic agents is determined by past inflation rates.

In order to analyse whether expectations result from a learning process, the performance of different forecasting models with time varying parameters in terms of their ability to fit actual data on inflation and inflation expectations is assessed. Data on household and expert expectations for Germany, Spain, France, Italy and the Netherlands are used. The paper finds evidence that inflation expectations result from a learning process and that a simple constant gain algorithm, which is used widely in the learning literature, performs best in fitting data on inflation and inflation expectations. These findings for the Euro Area confirm the results by Branch and Evans (2006) for the US. Branch and Evans (2006) show that a simple recursive forecasting model with constant gain learning forecasts well out of sample and also provides the best fit with the Survey of Professional Forecasters.

The results show furthermore that professional forecasters use higher constant gain parameters than households. They hence update their information sets more frequently and are able to pick up structural changes faster. A possible explanation is that households find it more costly to update their information sets than professional forecasters. This empirical finding is related to Carroll's (2003, a,b) theoretical model, which assumes that households update their information sets only sporadically by reading newspapers and thus learn from professional forecasters. It is supported by the literature on sticky information, which emphasizes that agents only sporadically update their information sets and that they incur a cost in doing so (Mankiw and Reis 2007). The paper also shows that economic agents update their information sets more frequently in countries with higher inflation. A possible explanation is provided by Sims' theory of 'Rational Inattention'. Sims (2003, 2006) models economic agents as having a limited capacity to absorb information. They therefore need to decide how much to pay attention and which pieces of news to look at. Sims (2003, 2006) argues that when inflation is high, agents will pay more attention to new information as their opportunity cost of being inattentive is significantly higher during these periods.

It is also crucial to investigate whether the learning process converges to equilibrium and whether expectations are anchored at the inflation objective of the ECB. It has often been argued that economic agents should understand the implications of monetary union and hence conclude that inflation differentials cannot last in the medium to long run (ECB, 2003). Empirical evidence typically finds large persistent inflation differentials between European countries (Rogers 2001, Berk and Swank 2002 and Ortega 2003). Angeloni and Ehrmann (2007) show that after converging sharply in the 1990s, national inflation rates started to diverge again around 1999. They find that although recently the differentials have closed somewhat, inflation differentials in the Euro Area are larger and more persistent than, for example, in the United States. However, if actual inflation rates are influenced by inflation expectations through wage and price setting behaviour, then convergence in inflation expectations should ultimately lead to convergence in inflation rates across countries. Thus, analysing the convergence of inflation expectations of households and professional forecasters gives us some indication of the likely convergence of future inflation rates. The results show that professional forecasters are more inclined to incorporate the implications of monetary union into their expectations than households. These findings correspond to those by Arnold and Lemmen (2006) who use a growth theory type of model and also find that the expectations of professional forecasters demonstrate more convergence than exists among the public.

The remainder of the paper is organised as follows: Section 2 gives an overview of the data. Section 3 discusses the general model. Section 4 analyses the fit of simple learning rules with Euro Area data. Section 5 tests for convergence of expectations to equilibrium. Section 6 concludes.

2 Data

Data sources

The paper uses data on household inflation expectations derived from the European Commission's Consumer Survey as well as expectations of professional forecasters extracted from Consensus Economics. Data for Germany, France, Netherlands, Italy and Spain are used. The paper also uses Euro Area inflation and inflation expectations. These data are compiled by aggregating the individual country data using weights based on each country's share in total Euro Area private domestic consumption expenditure.[1]

The EC Consumer Survey asks approximately 20000 consumers in the Euro Area for information regarding their expectations of future and past price developments. The survey is conducted on a monthly basis and consumers are asked about their expectations of inflation 12 months ahead. Questions and response categories of the survey are shown in Table 2.1:[2]

Q1: How do you think that consumer prices have developed over the last 12 months? They have...	**Q2: By comparison with the past twelve months, how do you expect consumer prices to develop over the next twelve months? They will...**
Fallen	Fall
Stayed about the same	Stay about the same
Risen slightly	Increase at a slower rate
Risen moderately	Increase at the same rate
Risen a lot	Increase more rapidly
Don't know	Don't know

Table 2.1: The EC Consumer Survey

The data derived from the EC Consumer Survey are hence qualitative in nature and need to be quantified. This paper uses data, which have been quantified by Gerberding (2006).

[1] The most recent weights that are assigned to each country are published by Eurostat with the release of the January data each year under HICP country weights (http://sdw.ecb.int/reports.do?currentNodeId=100000298)

[2] This table is adapted from Gerberding (2006).

Gerberding (2006) follows the probability method of Carlson and Parkin (1975), which was extended to the five-category case by Batchelor and Orr (1988). Due to the wording of Question 2 (see above), the procedure requires the specification of a variable that captures the perception of respondents of the rate of inflation over the past 12 months. Gerberding (2006) follows Berk (1999) in estimating the perceived rate of inflation on the basis of the results from the question on price developments in the past 12 months in the EC Consumer Survey (Question 1 in the above table). A detailed overview of the method used to quantify the qualitative data in this paper is provided by Gerberding (2001, 2006) and Nielsen (2003).

The data on experts' expectations are provided by Consensus Economics, a London based firm. Every quarter, more than 700 professional forecasters from major banks, economic research institutes and investment firms are asked to provide quantitative forecasts on key macro variables, including consumer prices. These forecasts are available for each of the following six quarters. Simple arithmetic means of these quarterly forecasts are then published for each country. In order for expert expectations to be comparable to the inflation expectations of households derived from the EC Consumer Survey, this paper uses expectations of professional forecasters on consumer prices for four quarters ahead.

Further details on the data sources including those sources used to construct time series of actual inflation can be found in Table A.1 in the Appendix.

It has to emphasised that there are limits to data compatibility in this paper. First, observations for households are monthly whilst the data on expectations of professional forecasters are quarterly. Second, household expectations have to be quantified whilst expert expectations are an average of quantitative forecasts. In addition, there are limitations to the probability method. These include the rather strict assumption of normality of the underlying aggregate distribution function. This assumption has been criticized by Carlson (1975) and Pesaran (1987) who find non-normal features of the aggregate distribution function. However, as noted by Nielsen (2003) and Berk (1999) alternatives to the normal distribution make little difference to the derived expectations series.

The probability approach is widely used in the literature and an important advantage of this approach is that it does not impose unbiasedness as an a priori property of the measure of future expectations of inflation. This is crucial as this paper tests whether households are boundedly rational. Nevertheless, the limitations of the probability approach have to be taken into account when evaluating the results. In particular, it should be noted that survey data and therefore the quantified proxies constructed from them are only an approximation of unknown economic agents' expectations (Nardo, 2003). Thus survey data approximations of unobservable expectations necessarily entail a measurement error. This error can be due both to sampling and aggregation error and to the general uncertainty attached to survey figures. Depending on the quality of the approximation, the performance of the quantified proxy in fitting true inflation might be poor even if economic agents were perfectly rational. Thus, the findings that economic agents are boundedly rational and use adaptive learning to form expectations might be severely affected. In this light, the paper also computes quarterly averages of the constructed series for household inflation expectations and assesses the robustness of the key results. If the measurement error is unbiased, then using quarterly household inflation expectations data helps to average out this error.

So far, it has been explained how the data are obtained and some of the shortcomings of the probability method were pointed out. The next section provides a more detailed analysis of the data. It will examine whether monthly household expectations and quarterly averages of those expectations differ and thereby assess to what extent the measurement error that is likely to be present in the quantified household expectations data might affect the analysis in this paper. Rationality tests are also conducted for household and expert inflation expectations.

A preliminary look at the data

Figure A.1 in the Appendix shows monthly data for actual inflation as well as household expectations from 1990 to 2006 for the different countries investigated in this paper.[3] Consensus forecasts and actual inflation are also plotted for 1990-2006. These series are shown in Figure A.2. The expectations series are dated back twelve months for households and four quarters for experts. Hence, the vertical differences between the series in each figure measure the forecast errors of households and professional forecasters.

For household expectations data, the mean forecast errors and mean squared forecast errors are shown in Table 2.2. Table 2.2 also shows the mean forecast errors and mean squared forecast errors of quarterly averages of household expectations.

The mean squared forecast errors of households differ depending on whether monthly data or quarterly averages of household inflation expectations are used. These differences might partly be due to the measurement error that arises when quantifying the qualitative household expectations data. However, they will also reflect differences between quarterly and monthly measures of inflation.

In order to compare the mean squared forecast errors of expert and household expectations, quarterly averages of household expectations are used.[4] The mean forecast errors and

[3] There were some missing observations in the quanitifed consumer expectations series, which reflects the fact that the quantification method breaks down when the share of respondents in one category is equal to zero (Berk, 1999). To deal with these gaps, the consumer expectations series were interpolated using the cubic spline function in Matlab. This was needed for some of the computations conducted in this paper.

[4] In order to test for equal forecast accuracy between households and professional forecasters, both series need to have the same frequency and number of observations. Data for the expectations of professional forecasters are available from 1990Q4 to 2006Q3 for Germany, France and Italy and from 1995Q4 to 2006Q3

	Monthly data		Quarterly averages	
	ME HH	MSE HH	ME HH	MSE HH
Germany	0.2166	1.1565	0.3280	1.6106
France	0.3941	0.8140	0.3576	0.6910
Italy	-0.1008	1.3209	-0.1070	1.1650
Netherlands	0.6052	1.1442	0.4962	1.0975
Spain	0.7776	3.3285	0.7130	2.9854

Note: ME denotes the mean forecast error, whereas MSE denotes the mean squared forecast error. HH denotes household inflation expectations. Monthly data for inflation and household expectations from 1990M1-2006M9 are used. Quarterly data for inflation and averaged household expectations from 1990Q1-2006Q3 are used.

Table 2.2: Mean and mean squared forecast errors, households.

mean squared forecast errors of the averaged household inflation expectations and expert expectations are shown in Table 2.3. The results illustrate that the mean squared forecast errors are larger for households than for professional forecasters. It is possible to test whether these differences in mean squared errors are significant. Equal forecast accuracy can be tested using the method proposed by Diebold and Mariano (1995), with the small sample correction for the Diebold/Mariano statistic as introduced by Harvey et al. (1997). The mean forecast errors as well as the mean squared forecast errors and the Diebold Mariano test results are shown in Table 2.3.

	ME HH	ME Exp.	MSE HH	MSE Exp.	DM	Std. Error	P-value
Germany	0.3376	0.0537	1.6105	0.7604	0.8501	0.450979	0.0640
France	0.3102	-0.1531	0.6335	0.4446	0.1888	0.173606	0.2809
Italy	-0.1340	0.1120	1.1950	0.6921	0.5029	0.291471	0.0894
Netherlands	0.1730	0.0799	0.8899	0.3684	0.5215	0.303481	0.0929
Spain	0.0341	0.2594	2.0311	0.8659	1.1654	0.764674	0.1350

Note: ME denotes the mean forecast error, whereas MSE denotes the mean squared forecast error. HH denotes household inflation expectations. DM denotes the modified Diebold Mariano test statistic, Std. Error denotes the standard errors. For expert expectations in Spain and the Netherlands data are available for 1995Q4-2006Q3. For the other countries, data for 1990Q4-2006Q3 are available.

Table 2.3: Mean and mean squared forecast errors; Modified Diebold/Mariano tests.

Table 2.3 shows that with the exception of France and Spain, the differences between the mean squared errors of professional forecasters and households are significant at the 10% level. Thus, there is evidence that professional forecasters are on average better at forecasting inflation than households.

for the Netherlands and Spain.

Several studies have investigated whether expectations of households and professional forecasters are unbiased. This paper follows Forsells and Kenny (2004) and investigates the rationality of monthly household and quarterly expert expectations by running the following regression:

$$\pi_t = \alpha + \beta \pi_t^e + \varepsilon_t \tag{2.1}$$

where π_t denotes the actual inflation rate in period t and π_t^e denotes the expected inflation rate formed in $t-12$ by households and $t-4$ by professional forecasters where the data frequency is monthly and quarterly respectively. If the joint null hypothesis $H_0 : (\alpha, \beta) = (0, 1)$ cannot be rejected and ε_t exhibits no evidence of autocorrelation, then it follows that expectations are unbiased in a statistical sense. The above rationality tests are conducted for both data on household and expert inflation expectations by ordinary least squares using covariance matrix corrections suggested by Newey and West (1987). Tables A.2 and A.3 show the estimation results for households and professional forecasters respectively. The results illustrate that for household expectations the null hypothesis, $H_0 : (\alpha, \beta) = (0, 1)$, can be rejected at the 1% and 5% level for each country and the Euro Area as a whole. For expert expectations, the null hypothesis, $H_0 : (\alpha, \beta) = (0, 1)$, can be rejected at the 1% and 5% levels for most countries and for the Euro Area with the exception of Germany and the Netherlands. However, the Durbin-Watson statistic shows evidence of significant autocorrelation for both households and experts and in each country, which is inconsistent with rationality.

As Holden and Peel (1990) have shown, if the null hypothesis in equation (2.1) cannot be rejected this is sufficient for rationality but not necessary. Holden and Peel (1990) suggest regressing the forecast error on a constant instead and testing whether the constant is significantly different from zero:

$$\pi_t - \pi_t^e = \alpha + \varepsilon_t \tag{2.2}$$

If ε_t is i.i.d., then it can be shown that the condition $\alpha = 0$ is both necessary and sufficient for rationality. The test is conducted for household and expert expectations. Table A.4 shows the estimation results for households and professional forecasters. For households, the null hypothesis, $H_0 : \alpha = 0$, can be rejected at the 1% and 5% level for each country and the Euro Area with the exception of Italy. For experts, the null hypothesis, $H_0 : \alpha = 0$, can be rejected for Italy, Spain and the Euro Area as a whole at the 1% and 5% level. Again, the Durbin-Watson statistic shows evidence of significant autocorrelation for both households and experts and each country. This is inconsistent with rationality.[5]

[5] In order to test the robustness of the results for the monthly household inflation expectations, both rationality tests are also conducted for quarterly averages. The results generally confirm the findings for monthly expectations. The only exception is Germany, for which the null hypothesis, $H_0 : \alpha = 0$ in equation (2.2), cannot be rejected at the 5% level. However, for all countries, there is evidence of significant autocorrelation, which is inconsistent with rationality. Test statistics and results of key diagnostic tests are available from

So far it is has been illustrated that there is little evidence that the inflation expectations of households and professional forecasters are rational. This raises the question of whether expectations can be better explained with theories of adaptive learning. The next section will introduce a general model that is used to examine the fit of simple recursive forecasting rules with data on actual inflation and inflation expectations.

3 The model

This section follows Branch and Evans (2006) and Basdevant (2005) and outlines a general state space forecasting model that nests alternative models.

Let π_t denote inflation in period t. It is assumed that the reduced form that economic agents use in order to form expectations of inflation is given by

$$\pi_t = \mathbf{b}_t'\mathbf{x}_t + \varepsilon_t \tag{3.1}$$

where $\mathbf{b}_t = (b_{1t}, b_{2t}, b_{3t}, ..., b_{(n+1)t})'$ and $\mathbf{x}_t = (1, \mathbf{y}_{t-1})'$. Furthermore ε_t is a serially uncorrelated disturbance with mean zero and variance H_t, that is $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = H_t$.

Let $\mathbf{y}_t$ with dimension nx1 denote a vector of variables of interest. Thus n is the number of independent variables in our model. These could be lagged values of inflation, output growth or changes in interest rates for example. It is hence assumed that economic agents view inflation in period t as a function of a constant and lagged variables of general interest. Furthermore economic agents are seen as forming their expectations for inflation for the next period using the current values of variables of interest such as inflation and output growth.

Together with the assumption that

$$\mathbf{b}_t = \mathbf{b}_{t-1} + \boldsymbol{\eta}_t \tag{3.2}$$

where $E(\boldsymbol{\eta}_t) = 0$ and $E(\boldsymbol{\eta}_t\boldsymbol{\eta}_t') = \mathbf{Q}_t$, the above corresponds to a general state space model with $\mathbf{b}_t$ being the state.

Conditional forecasts of π_t are given by $\widehat{\pi}_{t|t-1} = \widehat{\mathbf{b}}_{t-1}'\mathbf{x}_t$.

The parameter vector $\mathbf{b}_t$ can be estimated using the Kalman filter.[6] The recursion can be written as follows:

$$\widehat{\mathbf{b}}_t = \widehat{\mathbf{b}}_{t-1} + \mathbf{k}_t(\pi_t - \widehat{\mathbf{b}}_{t-1}'\mathbf{x}_t) \tag{3.3}$$

the author upon request.

[6]For an explanation of the basic Kalman filtering procedure, see Hamilton (1994).

where the Kalman gain, $\mathbf{k}_t$, is given by

$$\mathbf{k}_t = \frac{(\mathbf{P}_{t-1} + \mathbf{Q}_t)\,\mathbf{x}_t}{H_t + \mathbf{x}_t'(\mathbf{P}_{t-1} + \mathbf{Q}_t)\mathbf{x}_t} \tag{3.4}$$

and

$$\mathbf{P}_t = \mathbf{P}_{t-1} - \frac{(\mathbf{P}_{t-1} + \mathbf{Q}_t)\,\mathbf{x}_t\mathbf{x}_t'\,(\mathbf{P}_{t-1} + \mathbf{Q}_t)}{H_t + \mathbf{x}_t'\,(\mathbf{P}_{t-1} + \mathbf{Q}_t)\,\mathbf{x}_t} + \mathbf{Q}_t \tag{3.5}$$

where $\mathbf{P}_t = E(\mathbf{b}_t - \widehat{\mathbf{b}}_t)(\mathbf{b}_t - \widehat{\mathbf{b}}_t)'$.

As shown by Marcet and Sargent (1989a,b) the learning process converges to equilibrium only when the law of motion of the parameters is time invariant.[7] In other words, convergence requires $\mathbf{Q}_t = 0$. Within the Kalman filter framework it is hence possible to test whether learning is perpetual or whether it converges to equilibrium by examining whether the variance of the state variables is significantly different from zero.

If $\mathbf{Q}_t = 0$ and $H_t = 1$, the Kalman filter recursions, (3.3)-(3.5), become equivalent to recursive least squares (RLS) as shown by Sargent (1999). The system can then be written as

$$\widehat{\mathbf{b}}_t = \widehat{\mathbf{b}}_{t-1} + \gamma_t \mathbf{R}_t^{-1}\mathbf{x}_t(\pi_t - \widehat{\mathbf{b}}_{t-1}'\mathbf{x}_t) \tag{3.6}$$

$$\mathbf{R}_t = \mathbf{R}_{t-1} + \gamma_t(\mathbf{x}_t\mathbf{x}_t' - \mathbf{R}_{t-1}) \tag{3.7}$$

where $\gamma_t = t^{-1}$ and $\mathbf{R}_t$ is the matrix of second moments of $\mathbf{x}_t$. The gain, γ_t, will approach zero as $t \to \infty$. Thus, the above algorithm corresponds to the recursive formulation of ordinary least squares. As shown by Evans and Honkapohja (2001), when economic agents use recursive least squares to update their parameter estimates, these estimates will eventually converge to their rational expectations values.

If $\mathbf{Q}_t = \frac{\gamma}{1-\gamma}\mathbf{P}_{t-1}$ and $H_t = 1 - \gamma$, the system becomes equivalent to the constant gain version of recursive least squares (Sargent 1999), so that $\gamma_t = \gamma$ in equations (3.6) and (3.7). Using a constant gain algorithm implies that more weight is placed on recent observations. This algorithm is equivalent to applying weighted least squares where the weights decline geometrically with the distance in time between the observation being weighted and the most recent observation. Thus, the constant gain learning algorithm resembles estimation by ordinary least squares, but with a rolling window of data where the sample size is approximately $\frac{1}{\gamma}$. Past observations are discounted at a geometric rate of $1 - \gamma$. Hence constant gain least squares learning (CGLS) is more robust to structural change than recursive least

[7] According to Basdevant (2005), the Kalman filter framework allows one to test whether expectations converge towards the rational expectations equilibrium. However, this assumes that agents use the correct model of the economy. If the model used for forecasting is incorrect, expectations may converge towards a so called 'restricted perceptions equilibrium' (Evans and Honkapohja 2001).

squares learning. Evans and Honkapohja (2001) provide a more detailed explanation of both learning algorithms.

4 Simple learning rules

This section compares the performance of alternative recursive forecasting models. It assesses the ability of different simple learning models to fit data on actual inflation and inflation expectations. It is thereby examined whether learning is a plausible description of household and professional forecaster behaviour. It will also be investigated to what extent recursive least squares and constant gain least squares, which are the two most commonly used learning mechanisms described in the theoretical literature, provide a good description of forecaster behaviour. Estimates of the constant gain parameters are provided for each country and it is analysed whether there is country heterogeneity with respect to learning. Heterogeneity between households and professional forecasters is also examined. It will then be assessed to what extent the results are plausible and specifically whether they agree with other economic theories, such as Sims's theory of 'Rational Inattention'.

Estimation procedure

The paper follows Branch and Evans (2006) and divides the sample for each country in three parts: a pre-forecasting period in which prior beliefs are formed by estimating (3.1); an in-sample period in which optimal gain parameters are determined for the case of constant gain least squares, while for recursive least squares learning the gain sequence continues to be updated as t^{-1}; and finally, an out-of-sample forecasting period.

For household expectations, a fairly long pre-forecasting period, 1981M1-1989M12 is chosen in order to avoid over-sensitivity of the initial estimates. The in-sample period is 1990M1-1998M4. The out-of-sample period is hence 1998M5-2006M9.[8] Given the monthly frequency of the data, the independent variable vector $\mathbf{x}_t$ is defined as $(1, \mathbf{y}_{t-12})'$. The inflation expectation by households in period $t-12$ for period t is hence given by

$$\pi_{t|t-12} = \widehat{\mathbf{b}}'_{t-12}\mathbf{x}_t \tag{4.1}$$

When economic agents form expectations, the best estimate of the coefficients in period $t-12$ is used. As new data become available agents update their estimates according to either constant gain least squares learning or recursive least squares learning. The formulae

[8] This sample period was chosen so that the in- and out-of-sample periods correspond to the period for which household expectations are available. The period from 1990M1-2006M9 was then split in half to generate the in- and out-of-sample periods.

for this updating process under recursive least squares learning are given by equations (3.6) and (3.7). Under constant gain least squares learning, γ_t in those recursions is replaced by the constant gain, γ. It should be noted that in order to form inflation expectations in period $t-12$ for period t, π_{t-12} is used as an explanatory variable. Therefore, the forecast error, $\pi_t - \pi_{t|t-12}$, will be serially correlated due to overlapping forecast errors. If the exact nature of this serial correlation is known, in principle, the serial correlation can be incorporated into the Kalman filtering framework by specifying additional measurement and state equations. In our simple learning models, this means that, when forming expectations of inflation and updating the coefficients $\widehat{\mathbf{b}}_t$, economic agents could make use of those past forecast errors. Following the analysis of Branch and Evans (2006), this paper does not incorporate the serial correlation into the Kalman filtering framework or the updating process of coefficients under learning. Instead, a very simple model is used in which individuals predict inflation using relevant explanatory variables such as inflation and output growth. This means that the results of this paper may understate the case for learning, as using past values of forecast errors could improve the coefficient estimates and thus improve the fit of the simulated inflation expectations with the true inflation expectations by households.

To calculate the optimal in-sample constant gain parameters, the in-sample mean square forecast error

$$MSE_{IN}(\pi) = \frac{1}{T}\sum_{t=t_0}^{T}(\pi_t - \widehat{\pi}_t)^2$$

is minimised by searching over all $\gamma \in (0,1)$ with t_0 =1990M1 and T =1998M4. The distances between grids are set at 0.0001. $\widehat{\pi}_t$ denotes the forecast made in period $t-12$ for t. This forecast is generated by starting the recursions, equations (3.6) and (3.7), with $\gamma_t = \gamma$ where the initial values are calculated from the pre-sample period, and then using these recursive equations to calculate $\widehat{\mathbf{b}}_t$. The fact that $\widehat{\pi}_t = \widehat{\mathbf{b}}'_{t-12}\mathbf{x}_t$ is then used to generate values for $\widehat{\pi}_t$. When using recursive least squares to update estimates of $\widehat{\mathbf{b}}_t$, there is no need to compute an optimal gain parameter as $\gamma = t^{-1}$. However, the mean square errors can be computed by updating the sequence for $\widehat{\mathbf{b}}_t$ with t^{-1} and then using the fact that $\widehat{\pi}_t = \widehat{\mathbf{b}}'_{t-12}\mathbf{x}_t$ to generate values for $\widehat{\pi}_t$. These values can then be used as before in order to calculate in-sample mean square errors.

Having determined the optimal in-sample values of the constant gain, out of sample MSEs can be computed for each country as

$$MSE_{OUT}(\pi) = \frac{1}{T}\sum_{t=1}^{T}(\pi_t - \widehat{\pi}_t)^2$$

where t ranges from 1998M5 to 2006M9.

It is also possible to find best fitting constant gain parameters for households. These are computed by minimising the in-sample mean square comparison error

$$MSCE_{IN}(\pi) = \frac{1}{T}\sum_{t=t_0}^{T}(\pi_t^F - \widehat{\pi}_t)^2$$

by searching over all $\gamma \in (0,1)$ with t_0 =1990M1 and T =1998M4. π_t^F denotes household expectations for period t. The distances between grids are set at 0.0001. Best fitting constant gain parameters are computed to determine whether the best fitting gains that are needed to fit household expectations are equivalent to those needed to fit actual data on inflation in the in-sample period. This is important to investigate as Branch and Evans (2006) find that for explaining the forecasts of professional forecasters in the US the best fitting gain is substantially below the optimal gain for fitting data on actual inflation. As before, using the best fitting gains for household expectations, the out-of-sample mean square comparison forecast error is determined. This is given by

$$MSCE_{OUT}(\pi) = \frac{1}{T}\sum_{t=1}^{T}(\pi_t^F - \widehat{\pi}_t)^2$$

where t ranges from 1998M5 to 2006M9.

For RLS learning, the in-sample and out-of sample MSCEs are calculated as above. The recursive equations (3.6) and (3.7) are updated with t^{-1}.

In addition to absolute mean square comparison errors, relative MSCEs for each country for the model that yields the smallest mean square comparison forecast error are also calculated. These are computed out-of-sample relative to the variance of the series that the paper is trying to predict, i.e. household inflation expectations. This follows Forni et al (2003) and Schumacher (2007). Computing relative MSCEs is related to the concept of predictability of a series (see for example Diebold and Kilian, 2001). It could be the case that household expectations are more predictable in some countries, which results in lower MSCEs for those countries. Computing the variances of these series gives us some indication about how predictable the different series are.

For professional forecasters the method is identical to that described above with the exception that the data are quarterly. Forecasts of experts for four quarters ahead are used in order to make results comparable between households and professional forecasters.[9] The sample is divided as follows. Data on inflation from 1961Q1 to 1975Q4 are used as the

[9]Household expectations are averaged so that rather than having monthly data we get quarterly data for household expectations as well. Results for households are derived using the same methods as for experts. They are provided together with the results for professional forecasters for direct comparison purposes.

pre-sample period. The in-sample period consists of data from 1976Q1 to 1990Q3. The out-of-sample period was chosen so that it corresponds to the sample of professional forecasters: 1990Q4-2006Q3. Given the quarterly frequency of the data, the independent variable vector $\mathbf{x}_t$ is now defined as $(1, \mathbf{y}_{t-4})'$. The inflation expectation by professional forecasters in period $t-4$ for period t is hence given by

$$\pi_{t|t-4} = \widehat{\mathbf{b}}_{t-4}' \mathbf{x}_t \tag{4.2}$$

It should be noted that because there are relatively few observations for expert expectations, it is only possible to determine in-sample best fitting gains and in-sample mean square comparison errors for quarterly data.

Four different models are estimated. Model 1 is a simple AR(1) model where the independent variables are a constant and the lagged value of inflation. Model 2 is a simple AR(2) model with a constant and lagged values of inflation.[10] Model 3 includes a constant, lagged inflation and lagged output growth, which is approximated by growth in industrial production.[11] Model 4 includes changes in interest rates in addition to the variables in Model 3 . Models 1-4 for households can thus be written as follows:

$$\pi_{t+12|t} = b_{1t} + b_{2t}\pi_t + \varepsilon_t \tag{Model 1}$$

$$\pi_{t+12|t} = b_{1t} + b_{2t}\pi_t + b_{3t}\pi_{t-1} + \varepsilon_t \tag{Model 2}$$

$$\pi_{t+12|t} = b_{1t} + b_{2t}\pi_t + b_{3t}z_t + \varepsilon_t \tag{Model 3}$$

$$\pi_{t+12|t} = b_{1t} + b_{2t}\pi_t + b_{3t}z_t + b_{4t}w_t + \varepsilon_t \tag{Model 4}$$

where z_t denotes industrial production growth and w_t denotes changes in interest rates. The interest rate used in the models is the three-month interbank lending rate. Since the introduction of the European single currency, this rate is known as Euribor (Euro Interbank Offered Rate). For quarterly data, models 1-4 are identical except for the fact that the dependent variable is now denoted as $\pi_{t+4|t}$. In addition, for quarterly data, data on GDP are available and hence it is not necessary to approximate output growth by industrial production.

[10] Results for higher order AR models were also computed but it was found that the AR(1) and AR(2) models outperformed higher order models. The AR(1) and AR(2) models led to both smaller out-of-sample MSEs and smaller out-of-sample MSCEs for each country. This is true for both households and professional forecasters.

[11] This paper follows Branch and Evans (2006) in using output growth as one of the explanatory variables. Conventional New Keynesian Phillips curve estimations typically use the output gap instead. Results using the output gap (defined as $y = \ln(Y) - \ln(Y^*)$ where Y is GDP seasonally adjusted and Y^* is potential output estimated as the HP filtered Y) instead of output growth were also computed and found to be very similar.

Results

'Households: Learning matters'

This section examines the ability of simple linear recursive forecasting rules to explain actual data on inflation and inflation expectations. It is also examined whether there exists heterogeneity between households in different countries.

In order to assess whether it is possible to fit actual inflation with a learning model, the optimal constant gains that minimise the MSE for the in-sample period are first computed for different countries. These are shown in Table 4.1.

1990M1-1998M4		γ		
	Model 1	Model 2	Model 3	Model 4
Germany	0.1400	0.0960	0.1740	0.1300
France	0.1870	0.1280	0.1700	0.1360
Netherlands	0.2410	0.1580	0.1420	0.1150
Italy	0.1790	0.1490	0.0950	0.0670
Spain	0.1750	0.1480	0.1752	0.1090

Table 4.1: Optimal constant gain parameters, monthly data

These optimal constant gain parameters are significantly higher than those typically found for the US. For the US, Orphanides and Williams (2007) suggest estimates of around 0.01-0.04, Branch and Evans (2006) find values of the gain of around 0.06 and Milani (2007) finds values between 0.02 and 0.12 using quarterly data and depending on the time period used.[12] A higher gain coefficient for the Euro Area implies that agents should optimally use fewer years of data to form a prediction of inflation. A possible explanation for this might be that inflation in European countries was subject to more frequent structural breaks. Constant gain least squares learning discounts past observations geometrically and hence if there are more structural breaks, fewer years of data should optimally be used to generate forecasts.

The ability of different models to fit inflation is also assessed and it is thereby examined whether RLS or CGLS generates better predictions of actual inflation. Table 4.2 shows out-of-sample mean square forecast errors using both constant gain and recursive least squares learning.

It can be seen that constant gain clearly dominates recursive least squares learning in terms of forecast accuracy.[13] No single model fits best for all countries though. However,

[12] If the gain is denoted by γ, then this gain implies that agents use $(1/\gamma)/f$ years of data, where f denotes the data frequency: $f = 1$ for yearly data, $f = 4$ for quarterly data and $f = 12$ for monthly data.

[13] We performed modified Diebold/Mariano tests with the null of equal forecast accuracy to test whether the differences in MSEs between RLS and CGLS are significant. We test whether the difference between the largest MSE under CGLS and the smallest MSE under RLS is significant. It is found that the null hypothesis of equal forecast accuracy can be rejected at the 5% level of significance for each country. P-values and modified Diebold/Mariano statistics can be provided by the author upon request.

Out-of-Sample Period: 1998M5-2006M9								
	RLS				Constant Gain			
	Model 1	Model 2	Model 3	Model 4	Model 1	Model 2	Model 3	Model 4
Germany	0.4929	0.4859	0.4864	0.5350	**0.0720**	0.0879	0.1220	0.4129
France	0.3269	0.3200	0.3217	0.3520	0.0457	0.0613	0.1648	**0.0430**
Netherlands	0.7602	0.4580	0.7584	0.4349	**0.0440**	0.0784	0.0806	0.0670
Italy	0.2153	0.2243	0.2147	0.2170	**0.0198**	0.0260	0.0535	0.0346
Spain	0.7727	0.7680	0.7631	0.8599	0.0664	**0.0611**	0.1397	0.0688

Note: bold entries correspond to the model that yields the smallest MSE.

Table 4.2: Mean square forecast errors, monthly data

the simple model with constant gain learning and just lagged inflation and a constant as the independent variables does well for all countries. Figure A.3 shows actual inflation together with forecasts generated using the optimal gain and model for the different economies. This figure highlights the fact that constant gain least squares learning performs well in fitting actual inflation.

It is also important to analyse which model can best explain data on inflation expectations. Best fitting gains are computed by minimising the in-sample mean square comparison errors. Hence, it is possible to assess whether there is heterogeneity regarding the best fitting constant gain parameters between countries. These gains are shown in Table 4.3 for each country and model.

1990M1-1998M4	γ			
	Model 1	Model 2	Model 3	Model 4
Germany	0.0010	0.0020	0.0010	0.0010
France	0.0002	0.0082	0.0001	0.0051
Netherlands	0.0010	0.0010	0.0210	0.0010
Italy	0.0270	0.0280	0.0260	0.0240
Spain	0.0530	0.0510	0.0640	0.0460

Table 4.3: Best fitting constant gain parameters, households, monthly data

From Table 4.3, it can be seen that best fitting gains are much smaller than the optimal constant gains. Best fitting gains for the European economies in our sample range from 0.0001 to 0.064. These results roughly correspond with results found for the US (Pfajfar and Santoro 2006 find best fitting constant gains between 0.0008 and 0.001 for monthly data). The fact that best fitting constant gains are well below optimal constant gains might imply that households are possibly unaware of some of the structural breaks in the data and use a larger number of past observations to form an expectation of inflation than would be optimal.

Results from Table 4.3 suggest that households in ‘high inflation’ countries such as Spain and Italy use higher constant gains than those in ‘low inflation’ countries and hence detect structural changes faster. A possible explanation for why households in 'high inflation' countries 'learn faster' is provided by Sims (2003, 2006), who argues that economic agents will pay more attention to new information coming available as their cost of being inattentive is significantly higher during periods of high inflation. It is also found that higher constant gains are needed to explain the data on inflation expectations of professional forecasters than for households. This could be caused by a greater awareness of the presence of structural breaks by professional forecasters but it could also be the case that professional forecasters are more willing to incur the costs of updating their information sets than households, which update their information sets less frequently (Carroll 2003 a,b; Döpke et al. 2005).[14] Theories of sticky information also emphasise that households update their information sets infrequently because of the substantial costs incurred in this updating process (Mankiw and Reis 2007).

Table 4.3 also shows that the average best-fitting constant gains for monthly data across the four models for Germany and Spain equal 0.00125 and 0.054 respectively. This implies that for Germany if an observation in September 2006 gets a weight of one, we have to go back as far as July 1960 before seeing an observation that receives a weight of 1/2.[15] The unconditional inflation expectation of German households is thus based on a long history of inflation. In the case of Spain, in order to get the same result, we only need to go back about one year. A sensible interpretation of this finding lies in the potential consequences of the introduction of the European Monetary Union. There was a large change towards lower and more stable inflation in Spain and Italy whereas the changes in inflation observed in Germany and the Netherlands were much smaller. In particular, the inflation process itself in Spain and Italy was evolving more sharply over time. The constant gain learning algorithm is robust to these structural changes. When structural breaks are present, a higher constant gain and thus fewer years of data are used to form expectations of inflation. Thus, the higher constant gains that are observed in Italy and Spain compared to Germany and the Netherlands may be a result of these structural changes and not of the high level of inflation that was observed in those countries itself. Therefore, the results are not easily comparable across countries and one has to be careful in interpreting the differences in constant gains between 'low inflation' countries and 'high inflation' countries as evidence for Sims' theory of rational inattention. Ideally, with longer data series on household expectations, one could evaluate whether the constant gain that households use to form expectations in a particular country is larger in periods in which there is a high level of inflation.

Mean square comparison forecast errors are then computed for household expectations

[14] Papers by Carroll (2003a,b) and Doepke (2005) are based on a model in which households only update their information sets sporadically by reading newspapers and thus learn from professional forecasters. Unfortunately, the data sample is too short to test for such behaviour in this paper.

[15] The time (in months) it takes for the weight given to an observation to fall to 1/2 is given by the following formula: $t_{half} = \frac{\ln 2}{\gamma}$.

using both data generated with the RLS algorithm as well as data generated using the CGLS algorithm with the best fitting constant gains. Hence, it is possible to examine whether learning matters for inflation expectations formation of households and which dependent variables households use when predicting inflation. We can also assess whether recursive least squares or constant gain learning provides a better description of household behaviour and whether there is country heterogeneity with respect to learning. The results are found in Table 4.4.

Out-of-Sample Period: 1998M5-2006M9

	RLS				Constant Gain			
	Model 1	Model 2	Model 3	Model 4	Model 1	Model 2	Model 3	Model 4
Germany	0.5589	0.5508	0.5609	0.6502	**0.5349**	0.5631	0.5360	0.6858
France	0.3226	0.3096	0.3229	0.3549	0.4491	0.3532	0.3812	**0.2958**
Netherlands	0.5278	0.3320	0.5325	0.3657	0.4500	0.5753	0.6906	**0.2774**
Italy	0.3781	0.3785	0.3805	0.3229	0.3095	0.2991	0.3082	**0.2402**
Spain	1.7622	**1.7565**	1.7661	1.9075	1.9083	1.9885	2.0407	2.1847

Note: bold entries correspond to the model that yields the smallest MSCE.

Table 4.4: Mean square comparison errors, households, monthly data

Table 4.4 shows that expectations in France, the Netherlands and Italy can be fitted better with our simple models than expectations in Germany and Spain. Specifically Model 4 seems to perform well in those countries, which suggests that agents use more complicated models than those simply including lagged inflation. In the case of Spain, however, given the large forecast errors, there is little evidence that agents are using any of the simple linear forecasting models employed by this paper. Furthermore, it can be seen with the exception of Spain, constant gain dominates recursive least squares learning in terms of forecast accuracy.

The relative MSCEs for the model that yields the smallest mean square comparison error are also computed for each country and are shown in Table 4.5. In order to compute relative MSCEs, the out-of-sample mean square comparison errors are divided by the variance of the household expectations series. It could be the case that household expectations are more predictable in some countries, which results in lower MSCEs. Computing the variances of these series gives us some indication about how predictable the different series are.

The relative MSCE is still smallest for Italy, meaning that the model is able to fit expectations in Italy best. The difference between the relative MSCE for the best fitting model for Italy and the relative MSCE corresponding to the best fitting models for France and Netherlands is now larger than was the case with absolute MSCEs. There is hence evidence, that our simple learning model does significantly better in predicting household expectations in Italy than in predicting expectations in other countries.

Figure A.4 shows actual household inflation expectations and the generated series for ex-

Out-of-Sample Period: 1998M5-2006M9	
	Relative MSCE
Germany	0.7865
France	0.5096
Netherlands	0.5660
Italy	0.0619
Spain	0.9494

Table 4.5: Relative mean square comparison forecast errors, households, monthly data

pectations of inflation using the optimal model and best fitting constant gain for each country. Whilst the direction of inflation expectations can be predicted well (even for Spain), expectations are somewhat more volatile than our generated series. A possible explanation may be that whilst households use simple linear forecasting models, there are certain stochastic shocks and events to which households react and which also influence their expectations.

So far it has been demonstrated that constant gain least squares learning performs well in explaining actual inflation and monthly household inflation expectations. Furthermore, households in 'high inflation' countries use higher constant gain parameters than households in 'low inflation' countries. This raises the issue whether the same is true for experts and for quarterly averages of household expectations, which we turn to next.

'Professional forecasters use higher constant gain parameters than households'

This section assesses the extent to which simple learning rules can explain survey data on inflation expectations by professional forecasters. It is also investigated whether there exists heterogeneity between experts and households.

First, it is assessed whether a simple learning model can fit actual data on inflation. Optimal gains for each model are shown in Table 4.6. Results are only shown for three countries, because there are data constraints for the Netherlands and Spain.[16]

1976Q1-1990Q3		γ		
	Model 1	Model 2	Model 3	Model 4
Germany	0.1380	0.1120	0.1780	0.1110
France	0.2160	0.1050	0.1230	0.1020
Italy	0.3000	0.2000	0.1570	N/A

Table 4.6: Optimal constant gain parameters, quarterly data

[16] Data on expert expectations for the Netherlands and Spain are available for 1995Q4-2006Q3. Data on output growth are available from 1977Q2 for the Netherlands and from 1970Q2 for Spain. Data on interest rates are available from 1986Q2 for the Netherlands and 1977Q2 for Spain. These series would have been too short for our purposes. We could have tested the ability of the simple recursive forecasting model to fit averaged household expectations in the Netherlands and Spain but given that the purpose of this section is a comparison between households and experts, these results are not reported here.

As was the case in the previous section, optimal constant gains are again higher than those found by empirical studies for the US.

The out of sample forecast errors for actual inflation are shown in Table 4.7. It can be seen that constant gain least squares learning again dominates recursive least squares learning and that the simplest model does well in explaining actual inflation.[17] This is illustrated in Figure A.5, which shows actual and predicted inflation using the optimal model and gain parameter for each country.

Out-of-Sample Period: 1990Q4-2006Q3

	RLS				Constant Gain			
	Model 1	Model 2	Model 3	Model 4	Model 1	Model 2	Model 3	Model 4
Germany	0.9801	1.0137	0.8508	0.8734	0.2356	0.3864	**0.2142**	0.3888
France	0.2986	0.3226	0.3043	0.4526	**0.0721**	0.1203	0.1742	0.2296
Italy	1.1611	1.3113	0.9977	N/A	**0.0658**	0.1011	0.2647	N/A

Note: bold entries correspond to the model that yields the smallest MSE.

Table 4.7: Mean square forecast errors, quarterly data

Table 4.8 shows the best fitting constant gains, which can be used to examine whether there is heterogeneity between professional forecasters and households. As indicated before, data on household expectations, which have monthly frequency, are averaged to convert them into quarterly data and then the same estimations are performed with household expectations as with expert expectations in order to have a direct comparison between expectations of households and professional forecasters.

In-Sample Period: 1990Q4-2006Q3

	γ							
	Model 1		Model 2		Model 3		Model 4	
	Experts	HH	Experts	HH	Experts	HH	Experts	HH
Germany	0.1380	0.0018	0.1000	0.0010	0.1080	0.0010	0.0460	0.0012
France	0.0200	0.0080	0.0240	0.0142	0.0130	0.0060	0.0410	0.0070
Italy	0.1780	0.0720	0.1380	0.0720	0.1370	0.0930	N/A	N/A

Note: 'HH' denotes households.

Table 4.8: Best fitting constant gain parameters, households and experts, quarterly data

Experts seem to update their information sets more frequently than households. This could be due to the fact that households find it more costly to update their information sets than professional forecasters.

[17] Modified Diebold/Mariano tests are computed to test the hypothesis of equal forecast accuracy between the model yielding the largest MSE under CGLS and the model yielding the smallest MSE under RLS. The hypothesis of equal forecast accuracy can be rejected at the 5% level of significance for each country. Test statistics and P-values are available from the author upon request.

Tables 4.9 and 4.10 show mean square comparison errors for households and experts. These suggest that there is not a single model that fits best across all three countries. There is some evidence that households are more inclined to use simpler models with just lagged values of inflation compared to professional forecasters who use a larger variety of variables to predict inflation. However, this does not correspond to the findings for the monthly data. This apparent contradiction between the results for household expectations for monthly and quarterly data could be due to the fact that using quarterly averages of household data helps to average out the measurement error in monthly expectations and this could affect the results.

In-Sample Period: 1990Q4-2006Q3

	RLS				Constant Gain			
	Model 1	Model 2	Model 3	Model 4	Model 1	Model 2	Model 3	Model 4
Germany	0.3419	0.4805	0.2930	0.3704	0.4068	0.2268	**0.2046**	0.2664
France	0.2752	0.2910	0.2765	0.4613	0.2780	0.2439	0.2707	**0.2194**
Italy	0.8475	1.0138	0.8242	N/A	**0.4300**	0.4865	0.4926	N/A

Note: bold entries correspond to the model that yields the smallest MSCE.

Table 4.9: Mean square comparison errors, experts, quarterly data

In-Sample Period: 1990Q4-2006Q3

	RLS				Constant Gain			
	Model 1	Model 2	Model 3	Model 4	Model 1	Model 2	Model 3	Model 4
Germany	0.7816	0.9610	0.7913	0.9064	0.7197	**0.6912**	0.7113	0.9762
France	0.7233	0.7918	0.7439	0.9859	**0.3897**	0.6250	0.5403	0.4757
Italy	0.8662	0.9625	0.9711	N/A	0.6062	**0.5811**	0.8091	N/A

Note: bold entries correspond to the model that yields the smallest MSCE.

Table 4.10: Mean square comparison errors, households, quarterly data

Again, it is possible to compute relative mean square comparison forecast errors. The best fitting model is used for each country. Relative MSCEs for households and experts are shown in Table 4.11.

In-Sample Period: 1990Q4-2006Q3	Relative MSCEs	
	HH	Experts
Germany	1.0710	0.2794
France	0.6938	0.4705
Italy	0.1510	0.1679

Note: 'HH' denotes households

Table 4.11: Relative mean square forecast comparison errors, households and experts, quarterly data

From Table 4.11 it can be seen that according to the relative MSCEs the simple recursive forecasting model is able to fit expectations in Italy best. This is different to the conclusions drawn from Tables 4.9 and 4.10. It highlights the fact that expectations in Germany and France may be more predictable than in Italy.

It seems to be the case that our simple models fit expectations of professional forecasters somewhat better than those of households. It can be tested whether the differences in mean squared comparison errors are significant using a modified Diebold/Mariano (1995) test with the small sample correction proposed by Harvey et al (1997). It is possible to compare the mean square comparison errors of the optimal model for each country, i.e the model that yields the smallest absolute MSCE. For example, for Germany, Model 3 is used for experts and Model 2 for households. The results of the modified Diebold/Mariano tests are shown in Table 4.12.

	mod. DM statistic	P-value
Germany	2.0921	0.0487
France	1.3768	0.1906
Italy	1.1567	0.2706

Table 4.12: Modified Diebold/Mariano tests for equal forecast accuracy of households and experts, quarterly data

With the exception of Germany, the null hypothesis of equal forecast accuracy cannot be rejected at the 1% and 5% level. There is hence evidence that for France and Italy the model is able to predict expectations of households and experts equally well. Figure A.6 shows expert expectations and our generated series for inflation forecasts. It can be seen that the general direction of expectations can be predicted well with our model. This is also the case for fitting household expectations, which Figure A.7 illustrates.

Thus, the findings of this section provide support for adaptive learning as a description of actual forecaster behaviour. This raises the issue whether the learning processes of households and professional forecasters converge to equilibrium, which we turn to next.

5 Testing for convergence

Estimation procedure

This section investigates whether expectations converge to equilibrium. It is also investigated whether agents are able to learn the inflation objective of the ECB, which is to maintain inflation close to but below 2% in the medium term. As explained above this can be tested within a Kalman filtering framework by investigating whether the variance of the hyper-parameters is significantly different from 0. Time-varying parameters are estimated using the model outlined in equations (3.1)-(3.5). Given that the simplest model of inflation performs quite well for all countries, it is assumed that inflation expectations are derived from the following rule:

$$\pi_{t+12|t} = b_{1t} + b_{2t}\pi_t + \varepsilon_t \tag{5.1}$$

for households and

$$\pi_{t+4|t} = b_{1t} + b_{2t}\pi_t + \varepsilon_t \tag{5.2}$$

for professional forecasters.

Furthermore the following assumptions are made:

$$b_{i,t} = b_{i,t-1} + \eta_{i,t} \tag{5.3}$$

and

$$\varepsilon_t \sim N(0, \sigma^2) \text{ and } \eta_{i,t} \sim N(0, (Q_t^i)^2).$$

It is hence assumed that the variance on the measurement equation is constant while the variance of the hyper-parameters may be time dependent. The variance of the measurement equation is assumed to be constant in order to restrict the number of free parameters that have to be estimated within the Kalman filter. To test for convergence, it is investigated whether the variance of the state decreases over time, which would imply that the learning process is converging towards least squares estimates. Following Basdevant (2005) who uses the methods discussed in Hall et al. (1997) to test for convergence, Q_t is modelled as follows

$$Q_{i,t} = \lambda^2 Q_{i,t-1} \tag{5.4}$$

for $i = 1, 2$.

As shown by Hall et al. (1997) and Hall and St. Aubyn (1995), if $0 \leq \lambda < 1$ convergence in expectations holds. The null hypothesis $H_0 : \lambda = 1$ is tested against the alternative $H_1 : \lambda < 1$. In order to obtain the distribution of some function of λ under the null, this paper follows Basdevant (2005) in constructing the test statistic proposed by Hall and St. Aubyn (1995) and St. Aubyn (1999). This is given by

$$HSA = \frac{\widehat{\lambda} - 1}{\widehat{\sigma}(\widehat{\lambda})}$$

where $\widehat{\sigma}(\widehat{\lambda})$ is the estimated standard error of λ. Hall and St. Aubyn (1995) and St. Aubyn (1999) calculate critical values for the HSA statistic. These are -3.479 at the 1% level, -2.479 at the 5% level and -1.970 at the 10% level.

To test for convergence EViews is used in order to set up a state space model. As EViews cannot estimate equation (5.4) in its present form, the equation is rewritten as $Q_{i,t} = \lambda^{2t} Q_{i,0}$ where t is a time trend. In order to impose values for $Q_{i,0}$, equations (5.1) and (5.2) are estimated using OLS and the squared standard deviations of the coefficients are used as estimates of the initial variances. For household expectations, initial values of the variances are determined using data for 1981M1-1989M12, and for experts initial values are determined using data for 1961Q1-1990Q3.

5.1 Results

Household expectations

The results are shown in Tables 5.1 and 5.2.

	λ	**Std. Error**	**HSA**
Germany	0.996640	0.000387	-8.6820***
France	0.998199	0.000525	-3.4302**
Italy	0.995096	0.000667	-7.3522***
Netherlands	0.998652	0.000579	-2.3274*
Spain	0.998010	0.000505	-3.9406***
Euro Area	0.991442	0.000543	-15.7510***

* "No convergence" rejected at 10% confidence level
** "No convergence" rejected at 5% confidence level
*** "No convergence" rejected at 1% confidence level

Table 5.1: Households: Testing for convergence

		Final State	**Root MSE**	**P-value**
Germany	$\widehat{b}_1$	1.4536	0.3550	0.0000
	$\widehat{b}_2$	-0.0584	0.2934	0.8422
France	$\widehat{b}_1$	2.3013	0.4103	0.0000
	$\widehat{b}_2$	0.2106	0.1934	0.2759
Italy	$\widehat{b}_1$	3.0022	0.734328	0.0000
	$\widehat{b}_2$	-0.7352	0.3493	0.0353
Netherlands	$\widehat{b}_1$	1.1782	0.4746	0.0131
	$\widehat{b}_2$	0.1214	0.1172	0.3002
Spain	$\widehat{b}_1$	4.4108	1.2780	0.0006
	$\widehat{b}_2$	-0.1406	0.2512	0.5755
Euro Area	$\widehat{b}_1$	1.7892	0.3176	0.0000
	$\widehat{b}_2$	0.2662	0.1455	0.0673

Table 5.2: Households: Testing for convergence: Final state estimates

There is evidence of convergence to equilibrium for all countries. However, the values found for λ are extremely close to 1 and hence convergence is very slow. From equation (5.1), the steady state rate of inflation is given by $\frac{b_{1t}}{1-b_{2t}}$. It can be seen that the weights on lagged inflation converge to zero in Germany, France, the Netherlands and Spain. In the Euro Area and Italy they remain significant at the 5% level. This suggests that inflation expectations are becoming more anchored. However, the coefficients on the constant in equation (5.1) do not converge to just below 2, which would imply that economic agents have learned the inflation goal of the ECB correctly. Instead, households in Spain consistently over-estimate the inflation goal and households in Germany consistently under-estimate the inflation goal. For the Euro Area as a whole, the steady state rate of inflation is more in line with the goal of the ECB. Thus household expectations in European economies do not seem to have converged to the inflation goal of the ECB. If there is a link between actual inflation and expected rates of inflation, via a New Keynesian Phillips curve relationship for example, this implies that it is likely that there will remain persistent differences in inflation rates between Euro Area countries even though the average inflation rate will be on target.

In an integrated market such as the Euro Area, inflation differentials across countries arise as a part of catching up and adjustment mechanisms to shocks. At least part of the difference in inflation rates reflects different rates of productivity across countries, with the process of convergence driving up wages and hence prices of non-traded goods and services. This is just the Balassa-Samuelson effect at work (Rogers, 2001). However, if the inflation differentials between countries are more than just temporary deviations from the Euro Area average, they could be harmful in a monetary union. As Angeloni and Ehrmann (2007) argue, in a monetary union countries share the same nominal interest rates and thus a high-inflation country tends to have a lower real interest rate. A lower real interest rate discourages saving and stimulates consumption and investment, thereby amplifying the inflation differentials. This effect may be further strengthened by wealth effects, as low real interest rates may inflate share and real estate prices. Whilst a high inflation country tends to lose price competitiveness within the currency area, something that dampens demand and output at home and thus inflation, this effect is likely to operate only at a slow pace (Arnold and Lemmen, 2006).

Figure A.8 shows smoothed state estimates. The estimates for the constant in equation (5.1) rise substantially around 2002 and then fall again in Germany and the Netherlands but stay at elevated levels in Italy and Spain. In 2002 there was the cash changeover, when Euro notes and coins came into circulation and this had a large effect on the perceived inflation rate of households. Berk and Hebbink (2006) also conclude that this event had a significant effect on perceived inflation. They argue that this effect is due to a relative price increase of the most visible expenditure items in the period before the cash changeover. The fact that household expectations are affected so substantially means that one has to be cautious in interpreting the results in Tables 5.1 and 5.2. Even though the final state estimates for the constant in Table 5.2 are highly significant, it could be the case that as a result of the developments in 2002 our estimates for the coefficients may not have converged to their final values. A longer data period after the events of 2002 would enable us to be more confident in the conclusions drawn from Tables 5.1 and 5.2.

Expectations of professional forecasters

It is also investigated whether the expectations of professional forecasters converge towards equilibrium. Tables 5.3 and 5.4 show the results of convergence tests for the expectations of professional forecasters for 1990Q4-2006Q3.

	λ	**Std. Error**	**HSA**
Germany	0.998366	0.000199	-8.2094***
France	0.998787	0.000341	-3.5580***
Italy	0.994084	0.000368	-16.0773***
Netherlands	0.996944	0.000314	-9.7319***
Spain	0.998939	0.000394	-2.6927**
Euro Area	0.992691	0.000515	-14.1916***

* "No convergence" rejected at 10% confidence level
** "No convergence" rejected at 5% confidence level
*** "No convergence" rejected at 1% confidence level

Table 5.3: Experts: Testing for convergence

		Final State	**Root MSE**	**P-value**
Germany	$\widehat{b}_1$	1.6322	0.2622	0.0000
	$\widehat{b}_2$	0.3248	0.1644	0.0482
France	$\widehat{b}_1$	1.7068	0.1753	0.0000
	$\widehat{b}_2$	-0.0021	0.0510	0.9716
Italy	$\widehat{b}_1$	1.6705	0.1825	0.0000
	$\widehat{b}_2$	0.0591	0.0872	0.4980
Netherlands	$\widehat{b}_1$	1.7160	0.1622	0.0000
	$\widehat{b}_2$	-0.0050	0.0534	0.9260
Spain	$\widehat{b}_1$	2.9048	0.3512	0.0000
	$\widehat{b}_2$	0.1007	0.0455	0.0270
Euro Area	$\widehat{b}_1$	1.7463	0.2636	0.0000
	$\widehat{b}_2$	0.1548	0.1156	0.1806

Table 5.4: Experts: Testing for convergence: Final state estimates

The results indicate that the null hypothesis of 'no convergence' can be rejected at the 5% level of significance for all countries in our sample. However, λ is very close to 1, which implies that convergence takes a long time. It is again interesting to note that with the exception of Spain and Germany the weight on lagged inflation converges to zero and expectations become anchored to a constant. The coefficients on this constant seem to be more in line with the goal of the ECB. This contrasts the findings for the inflation expectations of households. Only professional forecasters' expectations for Spain now somewhat overestimate inflation. Hence, professional forecasters' expectations of inflation seem to be more anchored to the inflation goal of the ECB than inflation expectations of households. Figure A.9 shows smoothed state estimates for the constant and lagged inflation and suggests that expectations have not been affected by the introduction of the Euro currency. The graphs give further evidence that coefficients have converged to the values given in Tables 5.3 and 5.4.

The results suggest that professional forecasters are more inclined to incorporate the implications of monetary union for convergence in inflation rates into their expectations than ordinary consumers. Unfortunately, given that the EC Consumer Survey only asks households for expectations of inflation 12 months ahead, it is not possible to test whether our results hold for longer expectation horizons (for instance expectations 2 years ahead). It should be noted that our findings correspond to those by Arnold and Lemmen (2006) who use a growth theory type of model to test for convergence and also find that Consensus Economics data on the inflation expectations of professional forecasters demonstrate more convergence than exists among the public.

The paper illustrates that there is heterogeneity between households and professional forecasters. Professional forecasters use higher constant gains than households and thus seem to be more aware of structural changes. Their expectations also display more evidence of convergence. This raises the issue of which type of agent might matter more for the process of setting wages and prices. If household expectations have a stronger impact on wage and price setting, then the results of this paper provide some explanation for the empirical finding that there are sizeable persistent inflation differentials between Euro Area countries. Analysing whose expectations are more important thus represents an interesting direction for future research.

6 Conclusion

It is of crucial importance for central banks to understand how inflation expectations are formed. This is true for all central banks that gear monetary policy directly towards maintaining a low and stable level of inflation. Against this background this paper provides a first attempt to assess whether adaptive learning behaviour of economic agents is a reasonable assumption for the Euro Area.

Overall, the paper provides further support for constant gain algorithms as a description of actual forecaster behaviour. Heterogeneity in expectations is found between different Euro Area economies and between households and professional forecasters. Households in so-called 'high inflation' countries use higher constant gain parameters and hence update their information sets more frequently than households in 'low inflation' countries. Professional forecasters update their information sets more frequently than households. It is furthermore shown that the inflation expectations by households and experts converge to equilibrium but at a slow rate. Household expectations have not converged to the inflation objective of the ECB, which contrasts with the findings for professional forecasters, which are more inclined to incorporate the implications of monetary union into their expectations.

Some useful directions for further research should be noted. First, it would be interesting to evaluate more complicated forecasting models. Data on expectations of output are available for professional forecasters and with these data it would be possible to use vector autoregressive forecasting models in order to predict inflation-output vectors. Recently, there has been a growing literature on estimating DSGE models under learning, particularly for the US (e.g. Milani 2007, Slobodyan and Wouters 2008). Assessing to what extent these models can be used to explain survey based expectations would be an area of research worth pursuing as in those models, rather than taking inflation as given, households can affect the dynamics of inflation. Furthermore, the inflation dynamics of more countries could be assessed. The UK would be an interesting example, as it is not part of the monetary union and has had an independent central bank since 1997 with an explicit inflation target. One could then investigate whether different institutional setups of central banks affect the learning behaviour of agents. Once longer data sets on expectations are available it would be possible to test whether optimal gains stay constant over time and to analyse whether learning is faster in periods of high inflation than in periods of low inflation, a finding, which would give further support to theories of rational inattention. Additionally, with longer data sets, it would be possible to test whether agents exhibit switching behaviour as outlined by Marcet and Nicolini (2003) in which they switch between constant gain least squares and recursive least squares learning. It would be interesting to investigate whether recursive least squares learning outperforms constant gain least squares learning in periods with very stable inflation, such as have been observed during the past decade. These questions are left to be explored in future research.

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A Appendix: Tables and Figures

Variable	Source	Frequency	Data period
Household Expectations for Inflation in $t+12$	European Commission Consumer survey (DG ECFIN)	Monthly	1990M1-2006M9
Professional Experts Expectations for Inflation in $t+4$	Consensus Economics	Quarterly	1990Q1-2006Q3
Consumer Price Index (HICP)	Eurostat-Indices of Consumer Prices	Monthly	1981M1-2006M9
Consumer Price Index-All Items	OECD-Main Economic Indicators	Quarterly	1961Q1-2006Q3
Industrial Production-All Items, Seasonally adjusted	Bank for International Settlements (BIS)	Monthly	1981M1-2006M9
GPP in real terms, Seasonally adjusted	BIS	Quarterly	1961Q1-2006Q3
3-month interbank interest rate	BIS and ECB	Monthly	1981M1-2006M9
3-month interbank interest rate	BIS and ECB	Quarterly	1961Q1-2006Q3

Table A.1: Data sources

	α	β	R^2	χ^2 for H_0	DW
Germany	1.2275	0.4349	0.1228	24.142	0.0991
	(0.2535)	(0.1530)		[0.0000]	
France	1.1943	0.4669	0.2022	35.754	0.1852
	(0.2115)	(0.1317)		[0.0000]	
Italy	1.1347	0.6430	0.6224	43.485	0.2464
	(0.1872)	(0.0569)		[0.0000]	
Netherlands	1.9934	0.2511	0.0352	61.891	0.1180
	(0.2742)	(0.1425)		[0.0000]	
Spain	2.8414	0.3343	0.0960	43.418	0.0696
	(0.4807)	(0.1609)		[0.0000]	
Euro Area	-0.1384	1.2290	0.4528	11.509	0.1509
	(0.3448)	(0.1717)		[0.0032]	

Notes: Figures in parentheses are standard errors. Figures in brackets are P-values. Chi-squared statistics pertain to null hypothesis $H_0 : (\alpha, \beta) = (0, 1)$ where $\pi_t = \alpha + \beta\pi_t^e + \varepsilon_t$. DW denotes the Durbin-Watson statistic. 5% significance points of the lower and upper values, dL and dU are 1.65 and 1.69 (>100 observations) respectively. Equations are estimated by OLS using covariance matrix corrections suggested by Newey and West (1987).

Table A.2: Tests for unbiasedness, households

	α	β	R^2	χ^2 for H_0	DW
Germany	-0.4072	1.2129	0.5970	0.8689	0.3641
	(0.4801)	(0.2351)		[0.6476]	
France	0.7303	0.5497	0.2971	7.7406	0.3226
	(0.3993)	(0.1865)		[0.0209]	
Italy	0.7366	0.8052	0.7368	6.6533	0.2873
	(0.2856)	(0.0913)		[0.0359]	
Netherlands	0.1197	0.9815	0.4776	0.5463	0.4135
	(0.4922)	(0.2724)		[0.7610]	
Spain	2.2672	0.2717	0.0583	17.683	0.4552
	(0.5824)	(0.1736)		[0.0001]	
Euro Area	1.7781	0.2655	0.0275	83.607	1.3329
	(0.3706)	(0.2226)		[0.0000]	

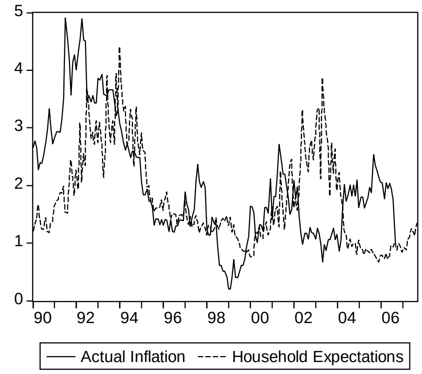
Notes: Figures in parentheses are standard errors. Figures in brackets are P-values. Chi-squared statistics pertain to null hypothesis $H_0 : (\alpha, \beta) = (0, 1)$ where $\pi_t = \alpha + \beta\pi_t^e + \varepsilon_t$. DW denotes the Durbin-Watson statistic. 5% significance points of the lower and upper values, dL and dU are 1.55 and 1.62 ($\approx$ 60 observations) respectively. Equations are estimated by OLS using covariance matrix corrections suggested by Newey and West (1987).

Table A.3: Tests for unbiasedness, experts

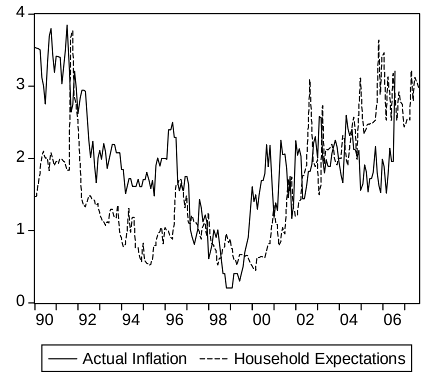
	Households			Experts		
	α	t-statistic	DW	α	t-statistic	DW
Germany	0.2173	1.3870	0.1791	0.0537	0.2786	0.3359
	(0.1567)	[0.1670]		(0.1926)	[0.7814]	
France	0.4062	3.6573	0.2531	-0.1531	-1.0151	0.2942
	(0.1111)	[0.0003]		(0.1509)	[0.3140]	
Italy	-0.1208	-0.7403	0.3741	0.3675	3.6520	0.5652
	(0.1631)	[0.4600]		(0.1006)	[0.0005]	
Netherlands	0.6116	4.7273	0.1861	0.0799	0.5065	0.4189
	(0.1294)	[0.0000]		(0.1578)	[0.6151]	
Spain	0.7823	3.2869	0.2028	0.3710	2.7511	0.7135
	(0.2380)	[0.0012]		(0.1348)	[0.0084]	
Euro Area	0.3818	3.2634	0.1163	0.4939	9.7549	1.6063
	(0.1170)	[0.0013]		(0.0618)	[0.0000]	

Notes: Figures in parentheses are standard errors. Figures in brackets are P-values. t-statistics pertain to null hypothesis $H_0 : \alpha = 0$ where $\pi_t - \pi_t^e = \alpha + \varepsilon_t$. DW denotes the Durbin-Watson statistic. Equations are estimated by OLS using covariance matrix corrections suggested by Newey and West (1987).

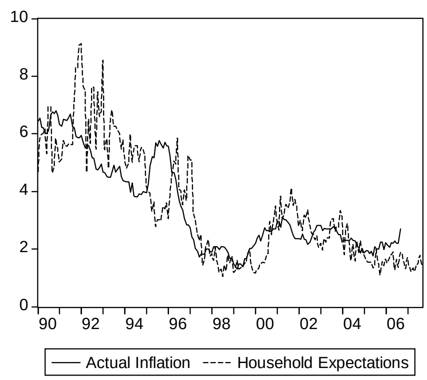
Table A.4: Tests for unbiasedness, following Holden and Peel (1990)



Germany



France



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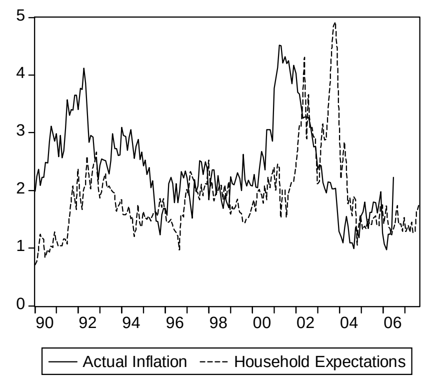

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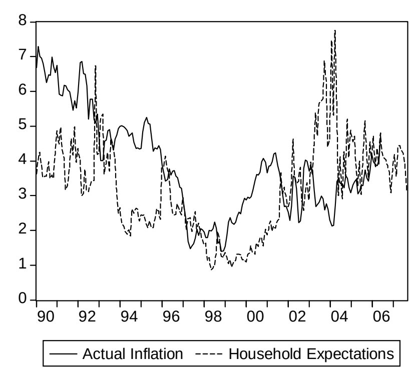


Spain

Figure A.1: Actual inflation and household expected inflation from t-12 for t.

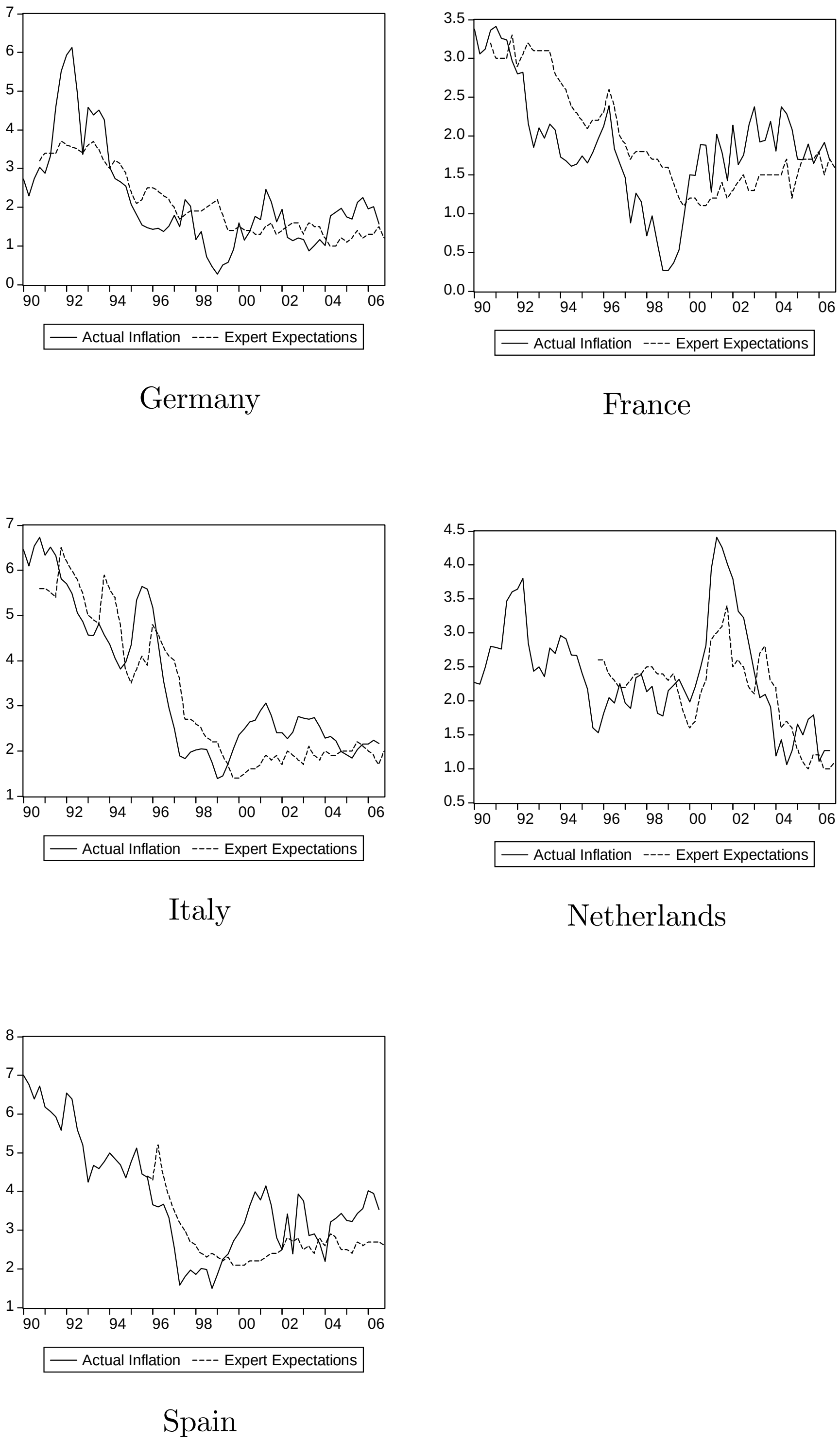
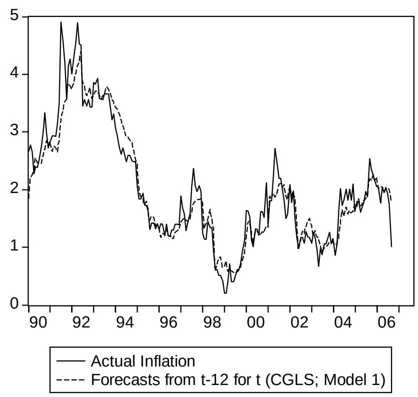
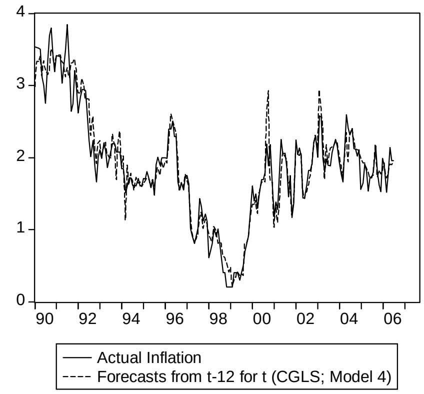


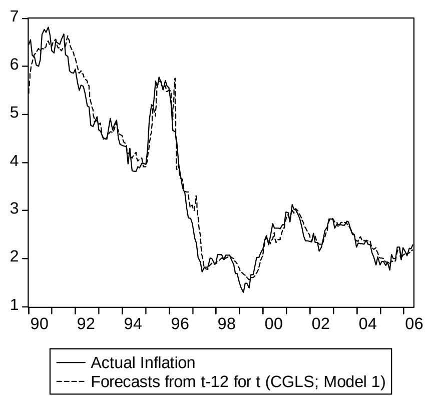
Figure A.2: Actual inflation and consensus forecasts from t-4 for t



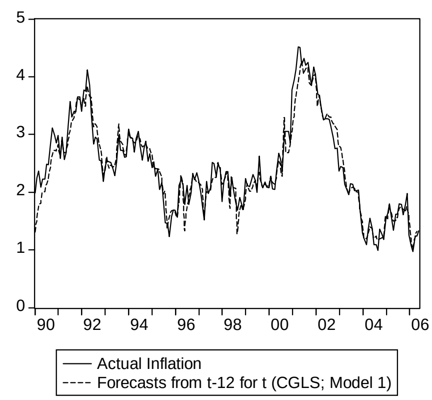
Germany



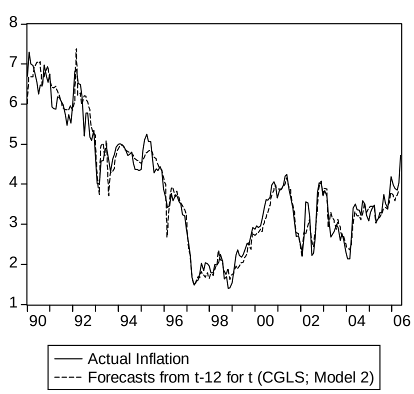
France



Italy

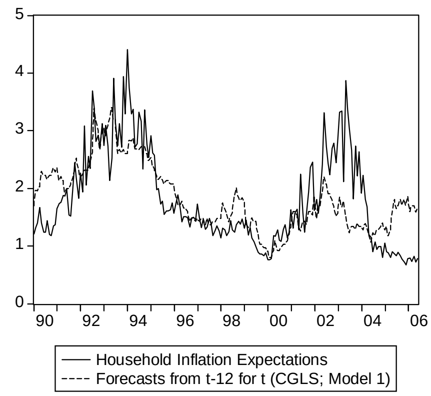


Netherlands

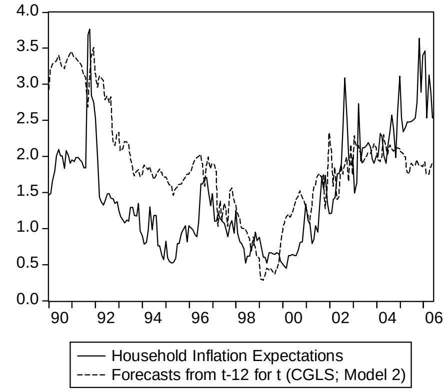


Spain

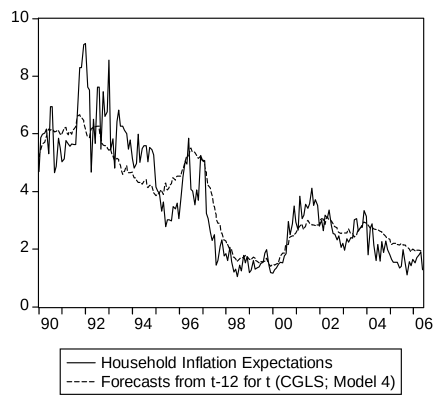
Figure A.3: Actual inflation and generated forecasts from t-12 for t using the optimal constant gain and model



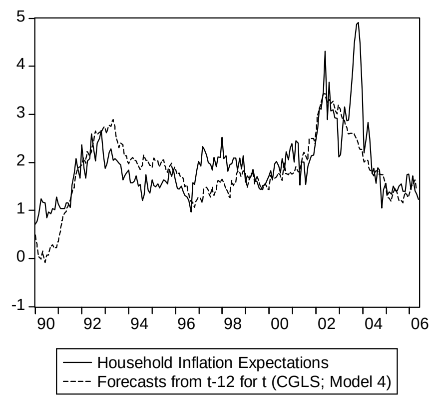
Germany



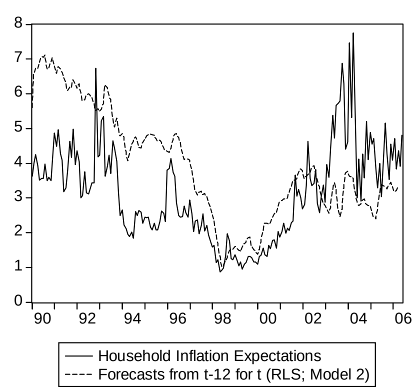
France



Italy

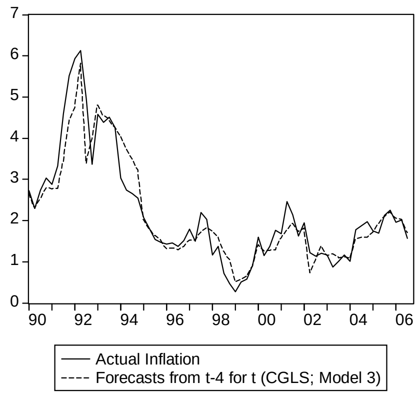


Netherlands

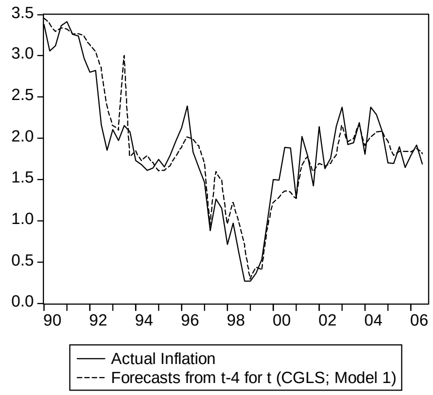


Spain

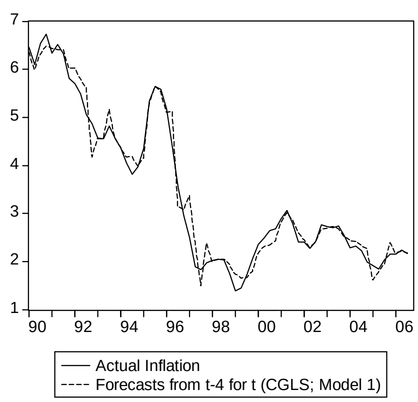
Figure A.4: Household expectations from t-12 for t and generated forecasts using the best-fitting constant gain and model



Germany

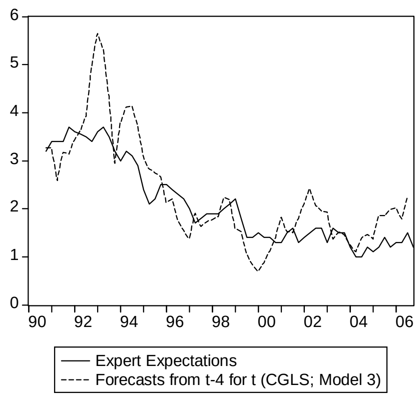


France



Italy

Figure A.5: Actual inflation and generated forecasts using the optimal constant gain and model from t-4 for t



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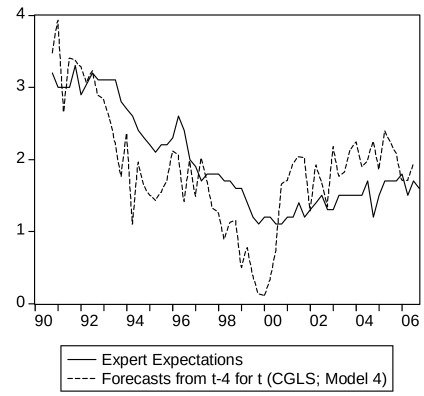

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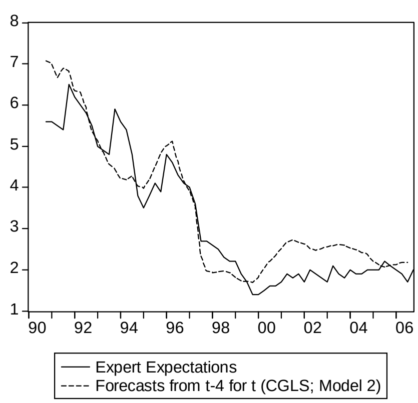
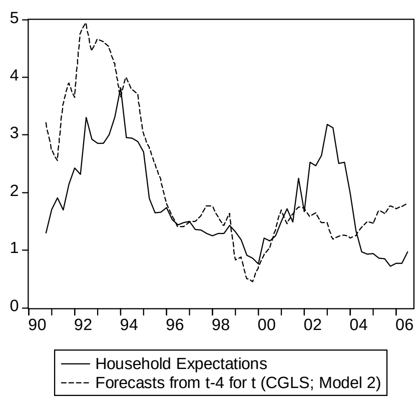


Italy

Figure A.6: Consensus forecasts from t-4 for t and generated forecasts using the best-fitting constant gain and model



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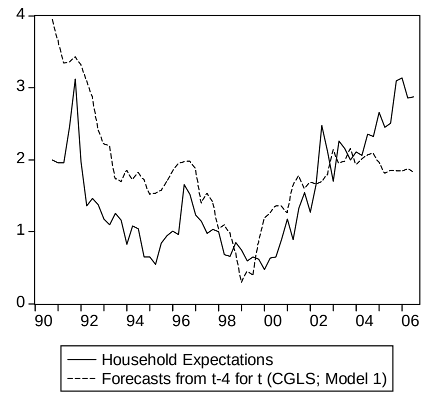

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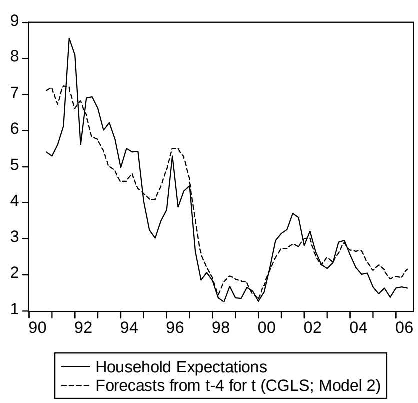
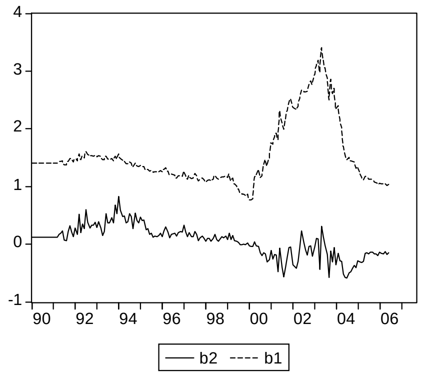


Italy

Figure A.7: Household expectations from t-4 for t and generated forecasts using the best-fitting constant gain and model



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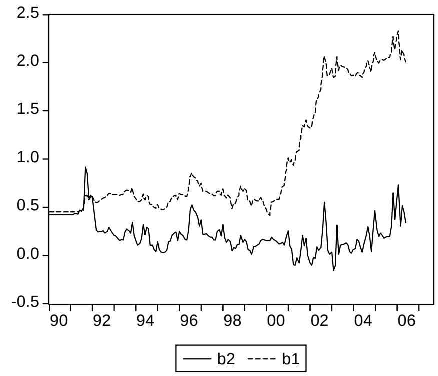

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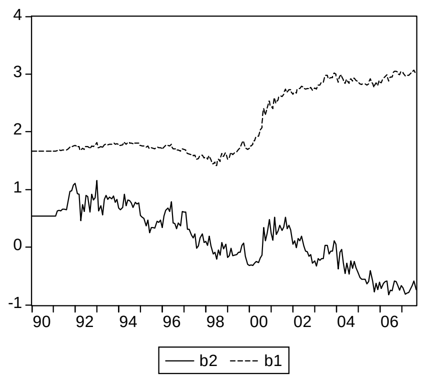

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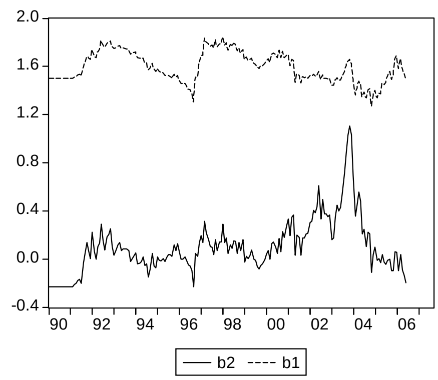
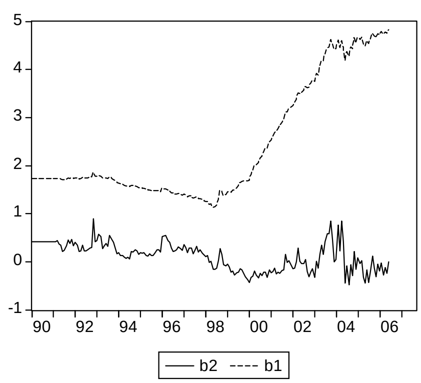
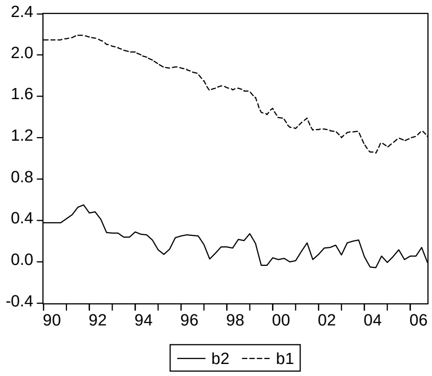


Netherlands



Spain

Figure A.8: Smoothed state estimates, Household expectations



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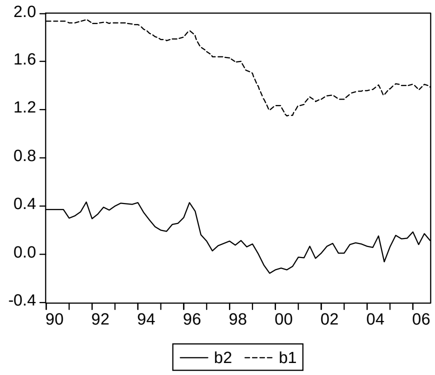

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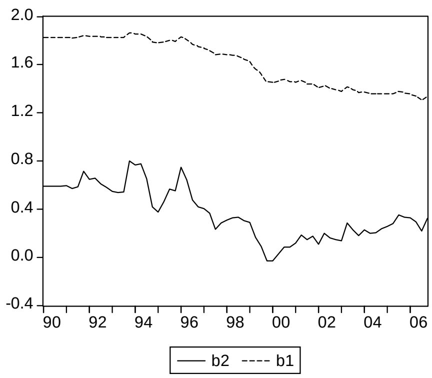

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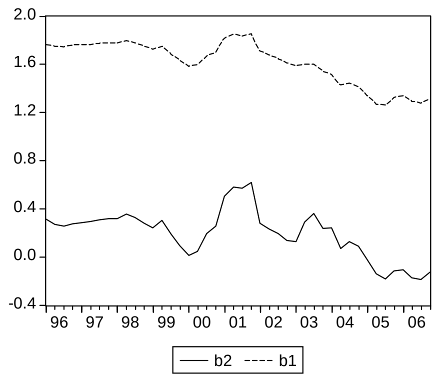

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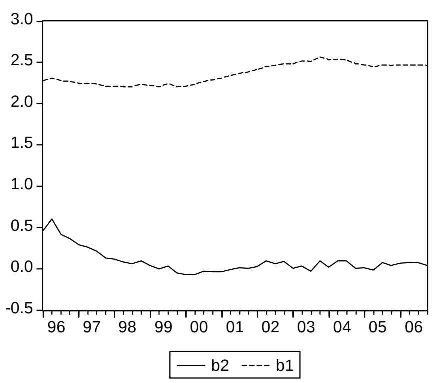

Spain

Figure A.9: Smoothed State Estimates, Consensus forecasts