Black-Markets for Currency, Hoarding Activity and Policy Reforms

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Abstract

In emerging market economies, under-development of financial markets leads to goods hoarding and foreign currency accumulation as forms of investment. In this paper, an asset market model, supplemented by explicit treatment of smuggling and second-economy activity, is used for studying the paths of black-market exchange rates, second-economy prices, hoarding stocks, and privately held dollar balances following policy reforms. We discuss conditions for overshooting and related dynamics of exchange rates and prices following: official exchange-rate adjustments, price reforms, and altered risks of monetary confiscation or currency reforms. The saddlepath trajectory of financial adjustment is simulated using data from recent Russian experience.

Keywords: Black Markets, Exchange Rate, Hoarding, Overshooting, Emerging Markets, Transition

JEL Classification Codes: F3, F31, F41,O17, P21

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1. Introduction

In emerging markets and some developing economies, restrictions still hamper free-market transactions in some goods and financial assets. Financial markets remain under-developed and few investment assets and outlets for savings are available to individuals. The existence of pervasive shortages, rigidities, and the vestiges of central planning and controlled prices, lead second-economy or black-market production structures to operate in tandem with secondary markets for fully-convertible foreign currencies. In this context, characterized by a paucity of financial instruments and uncertainty regarding future consumption-goods availability, one form of investment and intertemporal transfer of wealth has been through purchases of storable consumption goods.[1]

In the present paper, these features of the transition economy are shown to bear upon the dynamic effects of various policy initiatives, such as official price reforms, official exchange rate devaluations, and monetary reforms. With investment opportunities available to the consumer/ investor consisting of hoarded goods, accumulated hard currency, and domestic-currency savings, we trace the dynamics of adjustment of the stocks and prices of domestic currency assets and foreign exchange. An explicit treatment of overshooting is provided. These dynamics of exchange rates and prices depend on events in both financial and goods markets. An important implication is that demand for second-economy goods arises from <u>both</u> investment and consumption motives, rather than purely from consumption motives. Thus, a broader range of reform initiatives have significantly richer effects on exchange rates and prices than those predicted by more conventional approaches. Moreover, we highlight a role for hoarding activity beyond that typically cited as important in discussions of policy credibility and "big bang" approaches to reform during transition [van Wijnbergen (1992)].

The key theoretical contribution of the present paper arises from the simultaneous treatment of exchange rate *and* price dynamics, coupled with an explicit treatment of smuggling

[1]Weitzman (1991) also studies the importance of goods as a store of value.

activities. While the methodology and dynamic treatment of the problem per se are not novel, the specific application is: its solution provides useful insights for transition and developing economies. By explicitly studying the likelihood of overshooting of exchange rates and prices, we provide insights into the observed pattern of relatively high volatility of nominal and real exchange rates in emerging markets. Our results support criticisms of the use of black market exchange rates as a guide to equilibrium fixed exchange rates in transition economies. Both real and nominal black market exchange rates can overshoot in response to goods market events, in addition to overshooting in response to actual or pending foreign exchange market reforms.

The theory that we present complements two other important approaches to modeling exchange rate and price dynamics. First, we contribute to the theory of black-market exchange rates. This prior literature was motivated mainly by the experiences of Latin American economies, wherein black-market currency demands are treated as arising from "portfolio" motives: changes in the risk, return and stocks of various financial assets drive the currency price.[2] Recent applications include Pinto's (1991) analysis of the fiscal ramifications of unification of black and official market exchange rates, and Lane's (1992) study of Polish household behavior in money markets during the 1980s.[3]

Our approach also is closely related to the literature on the dynamics of exchange rate adjustment which stemmed from the seminal Dornbush exchange-rate overshooting framework. Such models typically have two distinct markets: the market for financial assets and the market

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[2]Calvo and Rodriguez (1977) provide an early model of currency substitution. Branson and Henderson (1985) provide a comprehensive presentation of portfolio theory applied to the asset market approach to exchange rate determination. In the context of black market exchange rate determination, the existing literature, as in de Macedo (1982, 1987) and Dornbush et al. (1983), models dollar holdings by individuals arising as a means of diversifying a "portfolio" of assets held to maximize expected returns on "invested" wealth and minimize the variance of these returns.

[3]Lane (1992) considers portfolio demands for two assets: domestic currency and foreign exchange.

for domestic goods.[4] As such, these models are useful for analyzing the dynamic effects of changes in the financial and goods market structures when financial market structures are fully developed and when goods prices are sticky.[5] However, when there are underdeveloped financial and goods markets the theory requires modification. Useful steps in this direction are taken by Calvo and Frenkel (1991) in their analysis of the implications of financial market reforms, and by Agenor and Flood (1992) and Goldberg (1995) in their analyses of exchange rate unification. However, unlike the present paper, all of these papers maintain the assumption of distinct dynamics in foreign exchange and goods markets.

Finally, our analysis provides insights into the meaning of "monetary overhang". According to the traditional view for socialist and in emerging market economies, money holdings are, in part, involuntary.[6] Instead, we maintain that, in an environment with under underdeveloped financial markets, money balances are held as part of an optimizing decision by households. Therein, individuals take into account the variances and covariances among the returns on alternative savings vehicles, in addition to considering the pure expected values of these returns. In this treatment, the risk-adjusted returns on money may not be dominated by those adjusted returns on alternative investment assets.[7] The theoretically "optimal" demand for money balances are distinct from those generated by current transactions motives and are not related exclusively to set-asides for stochastic future consumption opportunities.

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[4]See Obstfeld and Rogoff (1995) for an important reworking of the sticky price framework to incorporate monopolistic competition and intertemporal issues.

[4]Frenkel and Rodriguez (1982) present a two-asset model which relaxes the assumption that goods prices are sticky. However, in that paper prices still are determined in goods markets, rather than alongside investment vehicles in assets markets. In contrast to our approach, goods are held purely for consumption purposes, and smuggling and hoarding activities are not considered. This treatment of real goods as an investment asset, with the smuggling linkages, further distinguishes our work from simple three-asset models of the portfolio approach to exchange rate determination.

[6]For example, see Birman (1980) and Cottarelli and Blejer (1992).

[7]This type of treatment also is provided by Lane (1992).

The organization of the paper is as follows. First, Section II develops the model of the black-market for foreign exchange and hoarding of real goods. Section III analyzes the adjustment paths of black-market exchange rates, second-economy goods prices, dollar balances in private portfolios, and hoarding stocks, in response to a set of announced and unanticipated policy initiatives. Section IV concludes.

II. The Model: Black Markets for Foreign Currencies and Goods Hoarding

Two aspects of the demand and supply for (black) foreign exchange and for second-economy consumer goods are modeled. The first aspect is derived from "financial" or investment motives for holding foreign currency and hoarding goods, and is subsumed under the heading of "portfolio" or speculative currency-demands. The second aspect is related to current consumption needs and is subsumed under the heading of transaction demands with smuggling.

A. Portfolio Demands in the Black Market

The consumer/investor chooses an optimal division of his wealth among domestic savings, foreign savings, and hoarding of goods. The investment assets have some trend rates of return, and each of these returns have a stochastic element (represented by a Brownian-motion diffusion term). Given the choices available to consumer, we derive their optimal portfolio allocations,[8] and later use these mean /variance computations to motivate a more general asset market model. Specific formulas resulting from the portfolio problem are meant to be suggestive, since asset prices may not precisely follow the Brownian motion processes as specified in our derivations.

Domestic savings, denoted by B, have an expected nominal return of i_b equal to the interest paid on domestic-currency savings deposits. The risk associated with this form of savings, σ_b, arises due to the possibility that savings accounts can be confiscated in a monetary

[8] The technical details of the derivation of optimal demands are provided in the Appendix.

reform or, alternatively, that savings are the main officially-recognized assets in the event of a new issue of notes.[9] Example of such events abound in emerging market economies, and include: the treatment of savings by East German households savings in the process of German unification; the January 1991 Russian invalidation of cash holdings outside of the savings accounts; and the July 1992 Russian announcement that ruble notes issued prior to 1993 were to be invalidated.

The second asset available to the consumer/investor is foreign-currency (i.e. dollars) accumulated through the black market. For our exposition, the foreign currency is called dollars and the domestic currency is called rubles. The domestic-currency value of these dollars is the product of the quantity of dollars held, F, multiplied by the black-market exchange rate S, where the exchange rate is defined as units of domestic currency per unit of foreign exchange. Domestic residents are prohibited from opening interest-bearing foreign-currency accounts, so foreign-exchange holdings do not yield a nominal return**.**[10] The valuation and rate-of-return on these dollar stocks depends on the expected evolution of the short-run black-market exchange rate. The expected rate of depreciation of the ruble against the dollar is given by ϕ and the standard deviation of this forecast is denoted by σ_s.

The third investment opportunity is via the accumulation of storable goods, G, valued at second-economy prices, P. The expected rate of return on the hoarding of goods is the expected rate of increase in durable-goods prices, π, (i.e. the inflation rate on domestic goods) which is adjusted for the cost c of obtaining or storing these goods over the same interval of time for

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[9]This risk enters through the standard deviation of the diffusion process on returns on domestic currency assets. It also is possible to model this risk as a Poisson process. Analogous interpretations will apply to other risk terms introduced.

[10] This assumption is not important for our results and easily could be relaxed. In Poland, domestic residents are permitted to maintain both domestic currency and foreign exchange accounts. Also, one could easily introduce expected costs of obtaining and later liquidating black-market foreign exchange, as consumer/investors pay a range of fees to money dealers. Also, it is possible to introduce a parameter reflecting the risks associated with holding foreign exchange, for example, those due to the likelihood of being caught by authorities and penalized.

which the inflation series is measured. This cost can be interpreted as the opportunity cost of waiting in queues or searching for the goods.[11] The uncertainty in the forecast of future goods availability and future inflation is subsumed within the index σ_g.

These investments are made out of available nominal wealth stock *W*, defined by:

$$W = B + PG + SF \tag{1}$$

and the portfolio shares allocated to the respective assets are:

$$\lambda_1 = \frac{PG}{W}, \lambda_2 = \frac{SF}{W}, \text{ and } \lambda_3 = 1 - \lambda_1 - \lambda_2 = \frac{B}{W} \tag{2}$$

with λ_1 the share of wealth invested in hoarded goods, λ_2 the share of wealth held in foreign currency assets, and λ_3 the share of wealth in domestic currency assets.[12] The objective of the consumer/investor is to choose portfolio shares optimally in order to maximize the expected real return on his portfolio while minimizing its variability.

$$\text{maximize}\Big|_{\lambda_1,\lambda_2} \quad U = E\left(\frac{dw}{w}\right) - \frac{1}{2} R \cdot Var\left(\frac{dw}{w}\right) \tag{3}$$

where *R* is the coefficient of relative risk aversion, $w = W/Q$ is real wealth, and *Q* is the price index of the consumption basket.

Consumption occurs over two types of goods, domestic and imported, which enter the price index with weights α and $(1-\alpha)$, respectively. We assume that importers are price-takers in the market for foreign products and normalize so that the foreign currency price equals one. Thus, the domestic currency prices of imported goods follow the black-market exchange rate.

The optimal-portfolio demands for each asset, provided in the Appendix, are divided among minimum-variance and speculative portfolios. The minimum-variance portfolio demands precisely equal their shares in the future consumption baskets on individuals: αW is

[11]See Stahl and Alexeev (1985) and Weitzman (1991).

[12]In our model the demand for domestic currency assets arises out of investment motives. This system also could be supplemented by a transactions demand for money motive, as in Lane's (1992) study of Poland, but this would add to the complexity of the model and would not qualitatively change our results.

invested in real goods, while (1-α)W is invested in foreign exchange. A short position is taken in domestic currency assets to offset the long minimum-variance position in the other two investment alternatives. However, this does not imply that negative stocks and demands for domestic currency (i.e. net borrowing) will prevail. For positive domestic asset holdings, the speculative portfolio demands for domestic-currency assets would be required to be positive and at least as great as the negative minimum-variance demands. This leads to the following proposition:

> **Proposition 1**: For positive holdings of domestic currency assets, domestic interest rates must be at least as large as the rates of return on each alternative asset, net of a linear combination of the expected variances and covariances of those returns.
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> Proof: See the Appendix for conditions required for $\lambda_3 > 0$.

For conciseness and brevity, in this main body of the paper and in Table 1 we present only the signs of the asset-demand elasticities with respect to the parameters of financial markets, relegating to the appendix the full set of equations and results on optimal portfolio shares. Asset demands are normal, so that $\lambda_1^{\pi} > 0$ and $\lambda_1^{\phi} > 0$: the subscripts here refer to the specific asset of the portfolio; the superscripts refer to the derivative of this portfolio share with respect to durable goods prices or expected currency depreciation. Increased expected hoarded good inflation (π) raises the current demand for durable goods (λ_1). This responsiveness is decreasing in the relative-risk-aversion of the investor, since the source of demand for durables is the investor's attempts at minimizing the variance of his investment portfolio. The responsiveness of portfolio demands to changes in π is influenced by the variances and covariances of the returns on alternative assets, but not by the expected returns on those assets.

Table 1 *Asset Demand Functions: Signs of Demand Elasticities*

effect on demand for \		σ_b^2	σ_g^2	i_b	c	ϕ	π
goods-hoarding	λ_1	+ 1	- 2	-	-	-	+
dollars	λ_2	+ 1	+ 3	-	+	+	-
home-assets	λ_3	- 1	+ 3	+	+	-	-

1: Unambiguously satisfied if $\lambda_3 > 0$

2: Unambiguously satisfied if $(\lambda_1 - \alpha) + \frac{\alpha}{R} > 0$

3: Unambiguously satisfied if $(\lambda_2 - (1-\alpha)) < \frac{\alpha}{R} + \frac{1}{R}\left(\frac{\phi - i_b - \sigma_s^2}{\sigma_s^2 + \sigma_b^2}\right)$ where

$A = (\sigma_b^2 + \sigma_g^2)(\sigma_b^2 + \sigma_s^2) - (\sigma_b^2 + \rho_{gs})^2$ and ρ_{gs} is the correlation between shocks to black market exchange rates and to second economy goods prices.

Optimal portfolio demands also shift with changes in the expected return on domestic investments, in the costs of holding or acquiring hoarded goods, and in the uncertainty surrounding the expected returns on the investments. The signs of these effects are presented in Table 1. Increased goods supply uncertainty (σ_g^2) reduces the hoarding demand for goods and increases the demand for foreign and domestic currency.[13] This risk also has a second-order effect: it alters the responsiveness of asset demands to other policy and process changes. For example, as σ_g^2 rises, the expansionary effect on goods hoarding of increasing π is reduced. Likewise, an increased risk of monetary reform (σ_b^2) generally increases the portfolio demands for hoarding and foreign currency.

These optimal portfolio demands for assets are summarized by the reduced form equations (4) through (6). All demand influences other than ϕ and π are collected into the vector *Z*. The total amount of each asset demanded is the product of the optimal portfolio share and the investable wealth. Stock equilibriums in the asset markets are given by:

[13] The theoretical possibility of reversing the sign of those elasticities is likely to occur only if $\lambda_3 < 0$.

$$\lambda_1\left(\overset{-}{\phi},\overset{+}{\pi},Z\right)(B+PG+SF)=PG \tag{4}$$

$$\lambda_2\left(\overset{+}{\phi},\overset{-}{\pi},Z\right)(B+PG+SF)=SF \tag{5}$$

$$\lambda_3\left(\overset{-}{\phi},\overset{-}{\pi},Z\right)(B+PG+SF)=B \tag{6}$$

where $Z=\left(\sigma_b^2,\sigma_g^2,i_b,c\right)$ and $\lambda_3\cdot(B+PG+SF)=\left(1-\lambda_1-\lambda_2\right)\cdot(B+PG+SF)$

Equations (4) and (5) provide two equations in two endogenous unknowns, *S* and *P*. Both the black-market exchange rate and the second-economy price of goods are assumed to be perfectly flexible. Note that this treatment of goods-price flexibility is more general than the conventional approaches and also is appropriate if goods are traded as assets in the short run. Also, observe that the values of the black market exchange rate and the second economy price of goods depend on agents forecasts of expected rates of depreciation and expected inflation, ϕ and π. We will assume that investors have rational expectations: they correctly anticipate the values of *S* and *P* that will result after the market responds to some initial anticipated or implemented disturbances.

Equilibria in asset markets are assumed to be maintained by the instantaneous adjustment of all asset prices. In the short and medium run, flow disequilibrium can exist in the second-economy goods markets and in the black market "trade balances". These markets respectively provide adjustments in quantities of hoardable goods and dollars accumulated or decumulated through illicit cross-border trades. In the long-run, asset markets, second-economy goods markets, and the illicit trade balance are in equilibrium. Specifically, in the long-run there is balance both in the current account (i.e. balanced smuggling), $\dot{F}=0$; in the second-economy goods market (i.e. no additions or reductions of hoarding stocks), $\dot{G}=0$; and $\phi = \pi = 0$.[14]

[14] We use $\phi = \pi = 0$ as the steady-state condition of our model. However, in practice these inflation and depreciation rates will return to some "normal" rates determined by forces outside of the scope of the model, which need not equal zero. For example, these forces would include the dynamic path of money creation. Any discussion herein of

B. Stock Adjustments on Black Markets

To complete the specification of markets for foreign exchange and storable goods, it is necessary to introduce non-portfolio-related flows. In this respect, the emerging market economies are not fundamentally different from other countries with active parallel-markets. In the foreign-exchange market, illegal foreign-currency flows arise from under-invoicing or smuggling of exports, over-invoicing of imports and from exchanges made by tourists.[15] Surpluses or deficits in the second-economy trade balance, generated by changes in the net supply of private dollars, are represented by equation (7).[16]

$$\dot{F} = f\left(S/P, S/S^{0}\right) \text{ where } f_1 > 0, f_2 > 0 \qquad (7)$$

A surplus or accumulation of dollars in the second economy occurs when the transactions or consumption demands for foreign goods and foreign exchange is less than the foreign demand for goods smuggled out of the domestic economy. This second economy trade balance is a function of the relative price of smuggled imports and the black market exchange rate premium. As *S/P*, the relative price of imports, rises, the demand for domestic goods rises and demand for foreign goods falls. This leads to an improvement in the second-economy trade balance and accumulation of dollars, so that $f_1 = f_{S/P} > 0$. Increases in the black-market exchange-rate premium, S/S^o, where S^o is the official exchange rate offered to tourists or

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short-run and long-run activity clearly is limited to the specific forces triggered by the events analyzed within this paper.

[15]For example, see Pitt (1984) and Kamin (1992).

[16]Our specification of foreign asset creation assumes that foreign asset stocks change due to cross-border activities. An alternative approach would be to model the banking sector's ability to create foreign liabilities by indexing household deposits to foreign currency returns. We opt for the former approach as a more accurate representation of activities outside of the official sector. By contrast, the latter approach may be more appropriate for a country such as Poland that has implemented more radical banking sector reforms.

would-be emigrants, improve the second-economy trade balance by increasing the transactions supply of dollars, so that $f_2 = f_{S/S^0} > 0$.[17]

Equation (8) shows the determinants of changes in the availability of hoardable goods in the second economy. Goods availability is a function of the relative price of imported and domestic goods, as well as the premium from smuggling goods out of the official market and into the second economy.

$$\dot{G} = g\left(S/P, P/P^0\right) \quad \text{where } g_1 = g_{\frac{S}{P}} < 0 \text{ and } g_2 = g_{\frac{P}{P^o}} > 0 \tag{8}$$

Consumption demand for second-economy goods is increasing (net supply is decreasing) in the relative price of imported to second-economy prices of domestic goods, *S*/*P*, so that fewer goods are available for hoarding and $g_1 = g_{S/P} < 0$. Also, the net supply of goods is increasing in P/P^0, the ratio of the free-market to the government-controlled price of goods. In the second-economy market for goods, demand decreases and supply increases in *P*. This supply response is attributed to the motives for increased theft of goods from the official sector. Thus, an increase in *P* unambiguously raises hoarded goods availability. By contrast, the impact on goods availability of changes in government-controlled prices is less clear-cut. First, an increase in official prices can lead to a reduction in the supply diverted to secondary markets, since the premium to theft is reduced. In the official sector, more goods are available but at higher prices. Hence, demand in the second-economy declines as P^0 increases. The supply effects dominate, so that $g_2 = g_{P/P^o} > 0$.

III. Dynamic Effects of Reform Initiatives

Using our asset market model supplemented by hoarding and smuggling activity, in this section we analyze the dynamic effects of: *i*) official/ tourist exchange-rate devaluation (an

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[17] See Dornbusch et al. (1983). Note also that the function representing the net supply of dollars easily can be modified to account for the considerable obstacles that limit East-West trade. Historically, these obstacles have included prohibitive border-taxes, tariffs, and more subtle costs of illicit activity.

increase in S^0); *ii*) relaxation of the state-controlled prices of goods, either through one-shot price adjustment or liberalization (increase in P^0); *iii*) a sale of government bonds to the domestic population; *iv*) changes in the risk of currency confiscation, and *v*) changes in the degree of uncertainty surrounding future goods supply.

The general principles behind the adjustment dynamics are similar to those observed in all asset market models. Unlike the basic two-asset models usually studied, our system of equations has *two* prices (the price of goods and the black-market exchange rate) adjusting instantaneously to yield equilibrium in the markets for three assets. Thus, in our model *both* the real and the nominal black-market exchange rates have well-specified (and not identical) dynamics in the very short run. The dynamics of G and F (i.e accumulation or decumulation of foreign exchange holdings and hoarding activity) also are well-specified along the path to the steady-state equilibrium.

Analysis and simulation of the dynamic system shows that this system has a saddlepath solution, i.e. it is characterized by two positive and two negative eigenvalues. Below, we discuss the properties of this saddlepath and the implications of constraining adjustment along this path. In conjunction with an analysis of the steady-state values of each variable following policy initiatives, the saddlepath results are used to address the issue of price and exchange rate overshooting. Before turning to the results of the analysis, the next section provides a description of the parameters used in the simulations of our model.

Parameterization using data from Russia: Data from Russia in the early 1990s are directly relevant for our experiment and simulations because, in addition to having experienced extensive smuggling and hoarding, various discrete policy announcements led to overshooting and sharp short-term speculative movements in both exchange rates and prices. Computation of the linearized version of the system of dynamic equations (i.e. equations 4,5,7 and 8) requires initial parameter values for: household portfolio shares λ_1, λ_2; elasticities of these portfolio shares with respect to changes in expected inflation and the expected rate of black market exchange rate depreciation, $\lambda_1^{\pi}, \lambda_1^{\phi}, \lambda_2^{\pi}, \lambda_2^{\phi}$; secondary market premium on foreign exchange and durable goods, $S/S^0, P/P^0$; and partial derivatives of the flow equations for foreign exchange and durable goods accumulations, i.e. f_1, f_2 , g_1, g_2 .[18]

The true allocations of liquid household wealth in reforming economies generally are unknown and elude survey efforts.[19] In general, we expect that at least 50 percent of household wealth is held in domestic currency assets. Thus, $\lambda_3 = 1 - \lambda_1 - \lambda_2 \geq .50$, and the sum of portfolio shares toward foreign currency and hoarded assets will be at most 0.50. Our grid of values allows $\lambda_1 \in (.1, .4)$ and $\lambda_2 \in (.1, .4)$, subject to $\lambda_1 + \lambda_2 < .50$.

Since we also lack reasonable data on the price elasticities of the portfolio allocations, we turn to the portfolio model presented in Section IIA for intuition. The model shows that $\lambda_1^{\pi} > \left|\lambda_1^{\phi}\right|, \lambda_2^{\phi} > \left|\lambda_2^{\pi}\right|$, and $\lambda_1^{\phi} = \lambda_2^{\pi} < 0$. The ranking of λ_1^{π} and λ_2^{ϕ} depends on the relative size of exchange rate variance about its trend, compared with inflation variance about its trend. Data from the Russian economy in 1992 and 1993 show that black market exchange rate volatility generally exceeds by threefold detrended inflation volatility. Moreover, since positive stocks of all three assets are held by the population, *Proposition 1* implies that the riskiness of domestic

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[18]For further discussion of the Russian foreign exchange system and the implications of exchange rate reforms in this economy, see Goldberg (1993, 1995). Goldberg and Tenorio (1995a, 1995b) discuss the link between the foreign exchange market microstructure in Russia and its outcomes. Goldberg and Karimov (1995) consider the implications of price and official sector foreign exchange market reforms for producer decisions regarding foreign exchange auction participation and for their allocations of production across domestic versus external destinations.

[19]We have undertaken extensive efforts to determine the actual allocation of liquid household wealth in Russia. Even those detailed household surveys that have been conducted under the auspices of international organizations and the Soviet Interview Project have not been able to have households respond adequately to questions about liquid wealth. The main available data related to our portfolio allocation issue are household deposits in savings accounts and estimates of currency in circulation. But even this data most likely misses significant forms of domestic currency savings, as well as foreign exchange holdings, and holdings of storable goods. We also consider anecdotal evidence on this issue unreliable.

currency denominated assets is far inferior to the riskiness of the other two assets.[20] Finally, the data show that the covariances between these series are small relative to their variances. The rankings of volatilities which emerge from this appeal to the portfolio approach suggest that $\lambda_1^\pi \geq 3\lambda_2^\phi$. Our simulations use $\lambda_1^\pi \in (0.1,0.3)$, $\lambda_2^\phi \in (0.05,0.2)$, and $\lambda_1^\phi = \lambda_2^\pi \in (-0.1,-0.02)$.

Finally, we assign values to the initial: black market exchange rate premia; ratio of free to controlled prices of goods; and elasticities of response of hoarded goods stocks and foreign exchange accumulation in the second economy with respect to their respective arguments. These values are: $S/S^0 \in (1,2)$, $P/P^0 \in (1,2)$, $g_1 \in (-0.1,-0.5)$, $g_2 \in (0.1,0.5)$, $f_1 \in (0.1,0.5)$, and $f_2 \in (0.1,0.5)$.

The vast majority of parameter combinations yielded an eigenvalue sign pattern of: $\theta_1 > 0, \theta_2 < 0, \theta_3 < 0, \theta_4 > 0$, where the θ_i represent the respective eigenvalues of the model. A distant second pattern of eigenvalues (Case #2) is $\theta_1 > 0, \theta_2 < 0, \theta_3 > 0, \theta_4 < 0$. This case appeared infrequently, and was associated with λ_1 at the low end of its range (around 0.10) and λ_2 was at the high end of its range (around 0.30): in these initial conditions hoarding activity is nearly absent.

Short-Run Overshooting and Steady-State Responses to Policy Initiatives. For our exposition of the overshooting results we will use eigenvalues and eigenvectors generated by

[20]The black market exchange rate data are weekly series reported in *Commersant* and compiled by the authors. *Commersant* is a Russian business and news publication. The price data also are weekly, and are drawn from *Russian Economic Trends*. We use two alternative measures of consumer prices: an aggregate consumer price index (cpi) and a nonfood consumer price index. It is difficult to vouch for the quality of the data. However, we were convinced that the data are satisfactory by a range of checks for consistency with monthly series on consumer and retail prices. Both of these measures exclude the prices of services. For computing our benchmark results on portfolio share elasticities, we assume that the variance of returns on holding domestic currency assets is at most one half of the variance of the return on holding real goods. We also assume that the coefficient of relative risk aversion can take on benchmark values of one and two.

two sets of parameter combinations and corresponding to each of the benchmark cases.[21] The parameter combinations, corresponding eigenvalues and eigenvectors, and resulting equations (after working through the algebra) for exchange rate overshooting are presented in Table 2.

Table 2 Eigenvalues, eigenvectors, and overshooting

Benchmark Case 1:

parameters: $\lambda_1 = 0.30\ \lambda_2 = 0.10\ \lambda_1^{\phi} = \lambda_2^{\pi} = -0.05\ \lambda_1^{\pi} = 0.30\ \lambda_2^{\phi} = 0.10$
$S/S^0 = 1.25\ P/P^0 = 1.25\ g_1 = -0.25\ \ g_2 = 0.50\ \ f_1 = 0.25\ \ f_2 = 0.50$

negative eigenvalues (real parts) with eigenvectors [...]:

-4.8934 [2.8401, -3.8289,-0.2868, 1]
-0.1472 [-0.1167, -0.1910, 1.2792, 1]

Equations for instantaneous adjustment of exchange rates and prices:

$dS_0 - d\overline{S} = 1.8881 \cdot d\overline{F} - 2.2986 \cdot d\overline{G}$; $dP_0 - d\overline{P} = -2.3231 \cdot d\overline{F} + 3.1626 \cdot d\overline{G}$

Benchmark Case 2:

parameters: $\lambda_1 = 0.10\ \lambda_2 = 0.10\ \lambda_1^{\phi} = \lambda_2^{\pi} = -0.05\ \lambda_1^{\pi} = 0.30\ \lambda_2^{\phi} = 0.10$
$S/S^0 = 1.25\ P/P^0 = 1.25\ g_1 = -0.10\ \ g_2 = 0.50\ \ f_1 = 0.10\ \ f_2 = 0.50$

negative eigenvalues (real parts) with eigenvectors [...]:

-2.6767 [2.0803, -3.0513,-0.4314, 1]
-0.1079 [-0.1054, -0.1596, 0.9799, 1]

Equations for instantaneous adjustment of exchange rates and prices:

$dS_0 - d\overline{S} = 1.5487 \cdot d\overline{F} - 1.4122 \cdot d\overline{G}$; $dP_0 - d\overline{P} = -2.0490 \cdot d\overline{F} + 2.1674 \cdot d\overline{G}$

The equations presented in Table 2 show the exchange rates and prices which prevail immediately after a policy initiative places the economy on the saddlepath toward the new

[21] We follow the generally accepted approach to rational expectations models which emphasizes the saddlepath solution instead of non-saddlepath adjustments.

steady state. This initial jump occurs at time t=0. Using the dynamic equations of exchange rates and prices, overshooting occurs if the instantaneous jumps in these variables exceed their steady-state adjustments.[22]

Steady State and Overshooting Results: The overshooting possibilities and subsequent dynamics of exchange rates and price depend on the steady-state results of the model. These steady-state effects are summarized in Table 3 for: (*i*) an increase in the official price of controlled goods; (*ii*) a devaluation of the official exchange rate; (*iii*) a government bond sale; (*iv*) an increase in the uncertainty on government bond yields; and (*v*) an increase in uncertainty about the future availability of real durable goods.[23]

A devaluation of the official exchange rate leads to a steady-state increase in the price of hoarded goods and a steady state depreciation of the black market exchange rate. The intuition is straight-forward. Since the official devaluation reduces the attractiveness to tourists of supplying foreign currency in the second economy, in the long run there are lower F stocks in black markets. The steady-state effect on asset markets is a nominal and real depreciation of the black-market exchange rate. Moreover, since imported goods are relatively more expensive, there is a shift toward greater consumption of storable goods so that the supply of goods available for hoarding or accumulation contracts. Thus, in the steady state we observe both lower F and lower G, and an increase in the price of hoarded goods and in the price of foreign currency.

[22] Maury Obstfeld provided exceptionally useful suggestions for addressing the overshooting problem.

[23] To compute the actual overshooting results, the long-run comparative statics results for each shock, provided in the Appendix, are used in the overshooting equations for each benchmark case. The resulting conditions then are evaluated at the initial parameterizations of the respective benchmark cases. The interested reader may refer to these equations to generate more detailed information about the extent of exchange rate and price overshooting across the respective cases, and with respect to the particular policy experiments.

Table 3 Sign of Steady State Implications of Policy Initiatives

Policy Initiative \\	Steady-State Effects on S	P	F	G
Price Reform (ΔP^o)	increase	increase	decrease	decrease
Official Devalu. (ΔS^O)	increase	increase	decrease	decrease
Bond sale (ΔB)	no effect	no effect	increase	increase
Confiscat. threat ($\Delta\sigma_b^2$)	no effect	no effect	increase	increase
Supply uncertainty($\Delta\sigma_g^2$)	no effect	no effect	increase[1]	decrease[1]

1: See Table 1 for necessary and sufficient conditions on this partial derivative.

In the very short run, black market exchange rates will depreciate. Nominal exchange rate overshooting is possible but depends on the initial parameterization of the economy. In particular, overshooting requires real exchange rates to be "high" initially. Real exchange rates in transition economies are likely to satisfy this overshooting criteria, since the domestic currency price of foreign goods generally is greater than the price of domestic goods. Thus, in the context of transition economies, a devaluation of the official exchange rate will cause a large initial jump depreciation of the black market exchange rate, followed by a period of corrective (but partial) nominal appreciation. Immediately after the foreign exchange reform, the black market exchange rate premium will be at much higher levels than at the new steady-state.

Overshooting of nominal prices is unlikely following the official exchange rate devaluation. Prices jump instantaneously and continue to rise with gradually declining rate until the steady state is attained. Together with the nominal exchange rate depreciation results, the model shows that in the presence of hoarding and black markets for foreign exchange, there will be both real and nominal exchange rate overshooting following this type of official foreign market reform.

An official sector price reform, represented as an increase in the price of a controlled price good, has the same qualitative effect on asset stocks and prices in the steady state as did the official exchange rate devaluation. The implications of the two policy initiatives differ, however, in the relative size of the resulting increases in S and P, and the resulting declines in G and F. In comparison with the effects of an official exchange rate devaluation, an official sector price reform has smaller steady-state proportional effects on S and F than on P and G, respectively. Also, the price reform ultimately leads to a real appreciation of the black market exchange rates, rather than a real depreciation: the proportionate increase in prices exceeds the proportionate depreciation of the nominal exchange rates. Compared with an official devaluation, the price reform leads to larger reductions in hoarding stocks and smaller reductions in foreign exchange holdings by the population of consumer/investors.

In the very short run, second economy goods prices do not overshoot in response to the increase in prices in the official sector. The nominal exchange rate bears the brunt of the instantaneous adjustment to the price reform, with likely immediate overshooting followed by a period of gradual and partial recovery of value. Again, the exchange rate overshoots both in nominal and real terms, and exhibits greater volatility than prices.

Next, consider the effects of an increase in the riskiness of the returns on holding domestic currency assets, perhaps due to changes in the perceived threat of savings or monetary confiscation. The direct effect is to reduce the attractiveness of domestic currency assets and increase portfolio demands for goods and foreign exchange. The immediate jump by exchange rates and prices onto the saddlepath implies a jump appreciation of the nominal exchange rate and an immediate increase in the price of hoarded goods. In the scenarios captured by our benchmark cases, the immediate increase in both nominal and real black market exchange rates implies overshooting behavior in the transition economy. As hoarding stocks and foreign exchange availability adjust, the exchange rates will gradually depreciate and return to their real and nominal levels. The initial inflation surge will be followed by a period of deflation. In the steady state, individuals will hold larger stocks of foreign exchange and of hoarded goods in their investment portfolios.

The other policy initiatives or circumstances which we analyze using the portfolio and asset accumulation model are an increase in the supply of domestic bonds (or domestic currency assets in general), and an increase in the uncertainty surrounding the availability of real goods in the future. The steady-state implications of these measures are summarized in Table 3. Detailed formulas for these steady state results are provided in the Appendix.

IV. Concluding Remarks

This paper has documented the behavior of black-market exchange rates, second-economy prices, hoarding activity and foreign currency holdings in an economy with under-developed financial markets and smuggling activity. In contrast to previous studies, we use a framework that: (i) introduces hoarding activity and smuggling into an asset market approach, and (ii) assumes that goods prices are determined in asset markets along with the prices of financial assets that are used stores of value. This set of innovations enables us to more realistically integrate the dynamics of the real and financial sectors of an emerging market economy.

The key point is that <u>both</u> exchange rates and prices reflect speculative and transitory demands for currency and for goods which arise from optimizing behavior: goods prices are driven by the same type of speculative forces that are observed in financial markets. The speculative influence on goods prices and exchange rates injects increased volatility into these variables in transition economies. The existence of black markets and smuggling means that such volatility cannot be regulated away using, for example, capital controls or taxation on official transactions.

The results of our paper are especially important for economies in transition or in the process of economic stabilization. We show that observed sharp increases in spreads between black market and official exchange rates need not be indicative of unsuccessful reforms. Instead, observed black market premia may be purely transitory overshooting phenomena, reflecting inter-temporal speculation and the channels for clearing asset markets. Analogously, prices (and the spread between free market prices and controlled prices) may rise at the outset of specific reforms, but this type of inflationary burst also can be purely transitory.

Adding fuel to the skepticism voiced in Edwards (1989), Kamin (1993), and Montiel and Ostry (1994), our results show that parallel market premia in both exchange rates and prices can be misleading indicators of misalignment in developing economies. The biases (relative to long-run equilibria) implicit in black to official premia depend on the structure of announced and implemented policy initiatives operating in the economy in transition. Moreover, the adjustment paths for black-market exchange rates and prices, and for smuggling and hoarding activity, are dependent on observable features of the reforming economies, including real exchange rates and initial portfolio allocations. These conclusions are especially pressing for transition economies, wherein the frequency of policy changes adds to the likelihood that exchange rates, at any moment in time, are still adjusting to previously implemented or announced policy initiatives.

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Appendix:

The problem of the consumer/ investor is captured by the objective function:

$$\text{maximize}\Big|_{\lambda_1,\lambda_2} \quad U = E\left(\frac{dw}{w}\right) - \frac{1}{2} R \cdot Var\left(\frac{dw}{w}\right) \tag{a1}$$

where $w= W/Q$ is real wealth, Q is the price index of the consumption basket, and R is his coefficient of relative risk aversion. Nominal wealth, W, is the sum of stock holdings of domestic savings, B, foreign currency holdings, SF, and domestic goods hoarded, PG. The risks and returns of the various processes are given by

$$\frac{dB}{B} = i_b dt + \sigma_b dZ_b \tag{a2}$$

$$\frac{dS}{S} = \phi dt + \sigma_s dZ_s \tag{a3}$$

$$\frac{dPG}{PG} = (\pi - c)dt + \sigma_g dZ_g \tag{a4}$$

so that the portfolio shares allocated to the respective assets are given by:

$$\lambda_1 = \frac{PG}{W}, \lambda_2 = \frac{SF}{W}, \lambda_3 = 1 - \lambda_1 - \lambda_2 = \frac{B}{W} \tag{a5}$$

The process driving the price index is:

$$\frac{dQ}{Q} = \pi_q dt + \sigma_q dZ_q \tag{a6}$$

Moreover, we assume that the domestic aggregate price index is a weighted average of the unofficial price of domestic goods P and the price of imported-goods price P_T .[24] The black-market exchange rate is used to convert the foreign price (hereafter set equal to unity) into domestic-currency values. The domestic aggregate price index is:

$$Q = P^{\alpha} S^{1-\alpha} \tag{a7}$$

where α is the weight of the price of domestic goods and (1-α) is the weight of the imported-goods price in the consumer price index. Applying Ito's Lemma, assuming the risks associated with domestic currency savings accounts are uncorrelated with other stochastic processes, we derive the optimal portfolio shares:

[24]This price index could be modified to include expenditure on goods supplied at fixed official prices. However, we view the unofficial price of goods as the relevant marginal cost of these products.

$$\lambda_1 = \alpha + \frac{1}{AR}\begin{pmatrix} \left(\pi - i_b - c - \alpha\sigma_g^2 - (1-\alpha)\rho_{gs}\right)\left(\sigma_b^2 + \sigma_s^2\right) \\ -\left(\phi - i_b - \alpha\rho_{sg} - (1-\alpha)\sigma_s^2\right)\left(\sigma_b^2 + \rho_{sg}\right) \end{pmatrix} \tag{a8}$$

$$\lambda_2 = (1-\alpha) + \frac{1}{AR}\begin{pmatrix} -\left(\pi - i_b - c - \alpha\sigma_g^2 - (1-\alpha)\rho_{gs}\right)\left(\sigma_b^2 + \rho_{gs}\right) \\ +\left(\phi - i_b - \alpha\rho_{sg} - (1-\alpha)\sigma_s^2\right)\left(\sigma_b^2 + \sigma_g^2\right) \end{pmatrix} \tag{a9}$$

$$\lambda_3 = -\frac{1}{AR}\begin{pmatrix} \left(\pi - c - i_b - \rho_{gs}\right)\left(\sigma_s^2 - \rho_{gs}\right) \\ +\left(\phi - i_b - \sigma_s^2\right)\left(\sigma_g^2 - \rho_{gs}\right) \end{pmatrix} \tag{a10}$$

where $A = \left(\sigma_b^2 + \sigma_s^2\right)\left(\sigma_b^2 + \sigma_g^2\right) - \left(\sigma_b^2 + \rho_{gs}\right)^2$.

Proposition 1: For positive holdings of domestic currency assets, domestic interest rates must be at least as large as the returns on each alternative asset, net of a linear combination of the expected variances and covariances of those returns.

Proof :
From (a10) positive holdings of domestic currency assets require that i_b is large enough so that $\lambda_3 > 0$. $i_b > \left(\pi - c - \rho_{gs}\right)\dfrac{\left(\sigma_s^2 - \rho_{gs}\right)}{\left(\sigma_s^2 + \sigma_g^2 - 2\rho_{gs}\right)} + \left(\phi - \sigma_s^2\right)\dfrac{\left(\sigma_g^2 - \rho_{gs}\right)}{\left(\sigma_s^2 + \sigma_g^2 - 2\rho_{gs}\right)}$.

Table A1: Short-Run Comparative Statics Results for Asset-Demand Equations

Sensitivity of Hoarding Demands

$$\frac{\partial\lambda_1}{\partial\pi} = \frac{1}{AR}\left(\sigma_b^2 + \sigma_s^2\right) > 0 \qquad \frac{\partial\lambda_1}{\partial\phi} = -\frac{1}{AR}\left(\sigma_b^2 + \rho_{gs}\right) < 0$$

$$\frac{\partial\lambda_1}{\partial\sigma_g^2} = -\frac{1}{A}\left(\sigma_b^2 + \sigma_s^2\right)\left(\frac{\alpha}{R} + \left(\lambda_1 - \alpha\right)\right) \qquad \frac{\partial\lambda_1}{\partial\sigma_b^2} = \frac{1}{A}\left(\sigma_s^2 - \rho_{gs}\right)\cdot\lambda_3 > 0$$

$$\frac{\partial\lambda_1}{\partial c} = -\frac{1}{AR}\left(\sigma_b^2 + \sigma_s^2\right) < 0 \qquad \frac{\partial\lambda_1}{\partial i_b} = -\frac{1}{AR}\left(\sigma_b^2 - \rho_{gs}\right) < 0$$

Sensitivity of Portfolio Demands for Black-Market Dollars

$$\frac{\partial\lambda_2}{\partial\pi} = -\frac{1}{AR}\left(\sigma_b^2 + \rho_{gs}\right) < 0 \qquad \frac{\partial\lambda_2}{\partial\phi} = \frac{1}{AR}\left(\sigma_b^2 + \sigma_g^2\right) > 0$$

$$\frac{\partial\lambda_2}{\partial\sigma_g^2} = -\frac{1}{A}\left(\sigma_s^2 + \sigma_b^2\right)\left(\left(\lambda_2 - (1-\alpha)\right) - \frac{\alpha}{R} - \frac{1}{R}\left(\frac{\phi - i_b - \sigma_s^2}{\sigma_s^2 + \sigma_b^2}\right)\right) \qquad \frac{\partial\lambda_2}{\partial i_b} = -\frac{1}{AR}\left(\sigma_b^2 - \rho_{gs}\right) < 0$$

$$\frac{\partial\lambda_2}{\partial\sigma_b^2} = \frac{1}{A}\left(\sigma_g^2 - \rho_{gs}\right)\cdot\lambda_3 > 0 \qquad \frac{\partial\lambda_2}{\partial c} = \frac{1}{AR}\left(\sigma_b^2 + \rho_{gs}\right) > 0$$

where $A = \left(\sigma_b^2 + \sigma_g^2\right)\left(\sigma_b^2 + \sigma_s^2\right) - \left(\sigma_b^2 + \rho_{gs}\right)^2 > 0$ and demands are normal.

Determining the Dynamic Adjustment of Prices and Stocks to Policy Changes

I. **Long-Run Equilibrium: Comparative-Statics Matrix**

$$\begin{bmatrix} \left(f_1 (P)^{-1} + f_2 (S^o)^{-1}\right) & -f_1 S(P)^{-2} & 0 & 0 \\ g_1 (P)^{-1} & g_2 (P^o)^{-1} - g_1 S(P)^{-2} & 0 & 0 \\ -\lambda_1 F & (1-\lambda_1)G & -\lambda_1 S & (1-\lambda_1)P \\ (1-\lambda_2)F & -\lambda_2 G & (1-\lambda_2)S & -\lambda_2 P \end{bmatrix} \begin{bmatrix} \partial S \\ \partial P \\ \partial F \\ \partial G \end{bmatrix}$$

$$= \begin{bmatrix} f_2 S(S^o)^{-2} & 0 & 0 & 0 \\ 0 & g_2 P(P^o)^{-2} & 0 & 0 \\ 0 & 0 & W\lambda_1^Z & \lambda_1 \\ 0 & 0 & W\lambda_2^Z & \lambda_2 \end{bmatrix} \begin{bmatrix} \partial S^o \\ \partial P^o \\ \partial Z \\ \partial B \end{bmatrix}$$

Long-run Comparative Statics Results

For the examples of official exchange rate devaluation, official sector price adjustment, and increased risk of holding domestic currency assets, the long-run comparative statics results are:

$$\begin{bmatrix} \partial \overline{S} \\ \partial \overline{P} \\ \partial \overline{F} \\ \partial \overline{G} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & 0 & 0 \\ c_{21} & c_{22} & 0 & 0 \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{bmatrix} \begin{bmatrix} \partial S^o \\ \partial P^o \\ \partial \sigma_b^2 \\ \partial B \end{bmatrix}$$

where

$$c_{11} = \frac{Sf_2}{\kappa S^{o2}}\left(\frac{g_2}{P^o} - \frac{g_1 S}{P^2}\right) > 0, \quad c_{12} = \frac{Sg_2}{\kappa P^{o2}}\left(\frac{f_1}{P}\right) > 0, \; c_{21} = \frac{Pf_2}{\kappa S^{o2}}\left(-\frac{g_1 S}{P^2}\right) > 0$$

$$c_{22} = \frac{Pg_2}{\kappa P^{o2}}\left(\frac{f_1}{P} + \frac{f_2}{S^o}\right) > 0, \; c_{31} = \frac{-f_2 F}{\kappa S^{o2}}\left(\frac{g_2}{P^o} - \frac{g_1 S}{P^2}\right) < 0, \; c_{32} = \frac{-f_1 F}{\kappa}\left(\frac{g_2}{P^{o2} P}\right) < 0$$

$$c_{33} = \frac{W}{SA}\left(\left(\sigma_s^2 - \rho_{gs}\right)\lambda_2 + \left(\sigma_g^2 - \rho_{gs}\right)(1-\lambda_1)\right) > 0 \;, \; c_{34} = \frac{\lambda_2}{\lambda_3 S}, \; c_{41} = \frac{g_1 f_2 SG}{\kappa S^{o2} P^2} < 0,$$

$$c_{42} = \frac{-Gg_2}{\kappa P^{o2}}\left(\frac{f_1}{P} + \frac{f_2}{S^o}\right) < 0, \; c_{43} = \frac{W}{PA}\left(\left(\sigma_s^2 - \rho_{gs}\right)(1-\lambda_2) + \left(\sigma_g^2 - \rho_{gs}\right)\lambda_1\right) > 0, \; c_{44} = \frac{\lambda_1}{\lambda_3 P}$$

where $\kappa = \dfrac{f_1 g_2 P S^o + f_2 g_2 P^2 - f_2 g_1 P^o S}{S^o P^o P^2} > 0$.

Table 2 Eigenvalues and Eigenvectors of the Dynamic System

Benchmark case #1

parameters:

$\lambda_1 = 0.30\ \lambda_2 = 0.10\ \lambda_1^{\phi} = \lambda_2^{\pi} = -0.05\ \lambda_1^{\pi} = 0.30\ \lambda_2^{\phi} = 0.10$

$S/S^0 = 1.25\ P/P^0 = 1.25\ g_1 = -0.25\ \ g_2 = 0.50\ \ t_1 = 0.25\ \ t_2 = 0.50$

eigenvalues (real parts):

8.3344	-4.8934	-0.1472	0.1244

eigenvectors (real parts):

-0.3035	0.4050	-0.0499	-0.0901
-0.7666	-0.5460	-0.0817	0.0122
-0.0709	-0.0409	0.5471	-0.6880
-0.0783	0.1426	0.4277	0.3696

Benchmark case #2

parameters:

$\lambda_1 = 0.10\ \lambda_2 = 0.10\ \lambda_1^{\phi} = \lambda_2^{\pi} = -0.05\ \lambda_1^{\pi} = 0.30\ \lambda_2^{\phi} = 0.10$

$S/S^0 = 1.25\ P/P^0 = 1.25\ g_1 = -0.10\ \ g_2 = 0.50\ \ t_1 = 0.10\ \ t_2 = 0.50$

eigenvalues (real parts):

5.2769	-2.6767	0.1190	-0.1079

eigenvectors (real parts):

-0.3465	0.4017	0.1272	-0.0703
-0.7122	-0.5892	-0.0860	-0.1064
-0.0711	-0.0833	0.7200	0.6533
-0.0947	0.1931	-0.7202	0.6667