Economic Discontinuities at Borders: Evidence from Satellite Data on Lights at Night*

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Abstract

Using a new methodology for the computation of standard errors in a regression discontinuity design with infill asymptotics, I document the existence of discontinuities in the levels and growth of the amount of satellite-recorded light per capita across national borders. Both the amount of lights per capita and its growth rate are shown to increase discontinuously upon crossing a border from a poorer (or lower-growing) into a richer (or higher-growing) country. I argue that these discontinuities form lower bounds for discontinuities in economic activity across borders, which suggest the importance of national-level variables such as institutions and culture relative to local-level variables such as geography for the determination of income and growth. I find that institutions of private property are helpful in explaining differences in growth between two countries at the border, while contracting institutions, local and national levels of public goods, as well as education and cultural variables, are not.

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1 Introduction

Does political economy matter for economic growth? If yes, what are the channels through which it matters? A critical difficulty in answering this question is the endogeneity of political economy and politically determined variables such as institutions, public goods provision, macroeconomic policies, education and others: they may be correlated with unobserved variables that also affect growth. In particular, they may be correlated with geographic variation: countries that are more favorably endowed by geography may have better institutions and a better-functioning government. A large and fruitful literature has endeavored to resolve the endogeneity problem by using instrumental variables (La Porta et al. (1998), Acemoglu et al. (2001)) and found large effects of institutions.

This paper presents an alternative approach to measuring the impact of political economy, as opposed to geography, on growth: exploiting spatial discontinuities created by national borders. While borders are obviously determined endogenously (e.g. through war or national reunification), their precise location is often arbitrary, following a river or a line of latitude or longitude, and without regard to the characteristics of localities within 30 or 50 kilometers of the proposed line. Therefore, it may be expected that nearby locations separated by national borders should be similar in terms of geography and other local variables, but different in terms of national-level variables including political economy. Moreover, if these localities are small enough, it is overwhelmingly likely that while they are affected by national-level variables of the country that they are part of, they do not affect these variables themselves. Hence, we can view locations near the border as subjected to a natural experiment, in which they are randomly assigned to different national-level institutions, and in particular, to political economy.

While many potential determinants of growth change discontinuously at national borders, I argue that border discontinuities can be used to assess the impact of political economy on economic activity because these determinants are produced by government activity. Institutions, such as existence of the rule of law, protection of property rights and political freedom, are perhaps a classic example of such a "spillover" determinant of growth.[1] Some public goods, such as a nationwide infrastructure grid, or a program of universal education provision, may also exhibit such spillovers through decreasing transaction costs and creating human capital externalities. Finally, culture and the level of trust may be affected by national-level shocks through centralized television programming and the presence of a common language. However, all of the spillover effects described above are mediated by the one economically relevant variable that necessarily changes at a national border: the identity of the national government. Governments choose (or perpetuate) institutions of private property and manage the national stocks of public goods. To the extent that culture is affected by national-level shocks, it tends to be shaped by the actions of the government, such as the creation of a government television or radio channel that is accessible in all parts of the country, or a policy of cultural and linguistic homogenization. Therefore, discontinuities in economic activity at borders should be interpreted as estimates of the importance of government activity (short-run policies or long-run institutions and culture) for the level and growth of the economy, for good or for ill. The presence of large border discontinuities in economic activity suggests that actions taken by governments (again, over the short or the long run) have powerful effects on income per capita, whereas their absence indicates that economic shocks, rather than political actions, account for differences in the wealth of nations.

An alternative view of borders could be that they are discontinuities in the level of transaction costs in the purely private economy. The fact that language and culture are different on different sides of borders, as well as the existence of explicit tariffs and subsidies that discourage trade, imply that the economies on the two sides of the border are less connected by trade than two economically comparable regions within the same country. Therefore, a shock to the private economy in a region of one country (such as a poor harvest,

[1] In developing countries where the central government is weak, institutions may be nonuniform across a country, as hypothesized by Acemoglu and Dell [2010]

an influx of immigrants, a commodity bubble, a discovery of natural resources or a labor dispute) that spills over into neighboring regions in terms of changes in wages, prices and demand, may fail to spill across a national border because of the discontinuous increase in transaction costs along that border. Therefore, border discontinuities in economic activity may exist without being mediated by government activity. However, this explanation relies entirely on trade as a transmission channel of economic disturbances, and therefore, is easy to confirm or rule out. In Section 5, I present evidence that the (normalized) amount of trade between two countries does not explain the size of the economic discontinuity at their border or its relationship to key variables that are determined by governments.

To compute GDP and growth in narrow bands around national borders, I use satellite data on lights at night collected by the NOAA in the DMSP-OLS satellite program. The night lights dataset has been first described by Henderson et al. (2011), who have shown a strong correlation between the amount of light emitted from a country and its GDP, both for levels as well as for growth rates. Night lights are an ideal and indispensable data source for this project because they are one of the few indicators of economic activity that exist at a sufficiently fine resolution to allow the analysis of narrow neighborhoods around national borders, as well as because the method of their collection is continuous across national borders. National accounts data is fundamentally unsuitable for such a project because it is available, at best, only at a regional level, and does not permit considering regions other than large political subdivisions, thus making it inappropriate for a regression discontinuity analysis. Survey data may overcome this problem as it samples individuals or villages rather than geographical units, but it would introduce an artificial discontinuity at borders because each survey is conducted within a single country. Hence, errors attributable to the questionnaire or to the performance of different survey teams would be different on different sides of national borders. On the other hand, even if there is important spatial heterogeneity in the way that satellites record lights data, the remote sensing process should be continuous, and therefore, roughly stable within a neighborhood around a given border.[2]

This paper is part of a large literature on the effects of political economy on growth that includes Dell (2010) on the impact of colonial forced labor in Peru, Banerjee and Iyer (2005) on the present-day influence of taxation systems in British India, Nunn (2008) on the persistent effects of the African slave trade, and Larreguy (2011) on the persistence of colonial institutions in Nigeria. This paper is closest to Michalopoulos and Papaioannou (2011a), who use night lights data to look at the (non)importance of institutions for economic activity within African ethnicities split by national borders. The innovation of this paper is 1) its much wider scope in considering the universe of borders around the globe rather than an institution in a particular country, 2) its use of regression discontinuity neighborhoods rather than existing regions, and 3) its development of an econometric theory to accommodate the special nature of the data used.

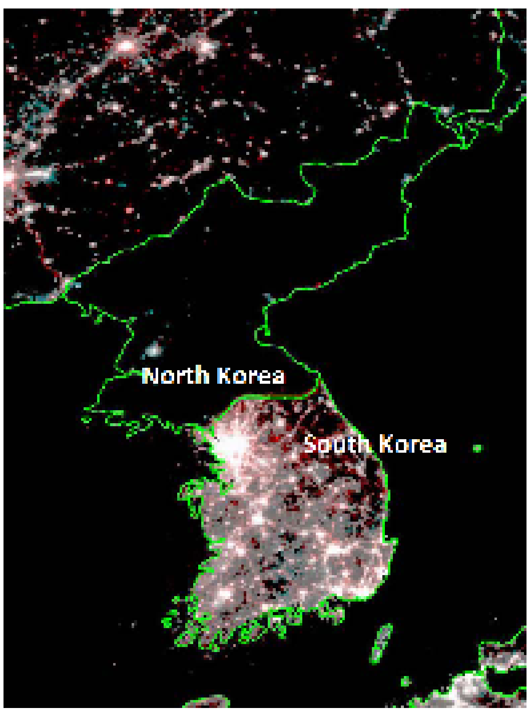
Using the amount of lights per capita as a proxy for economic activity around borders, I document a strong and highly significant relationship between national GDP and GDP at the border. As one moves from a poorer to a richer country sharing a border, the amount of light per capita (calibrated to be comparable to GDP per capita) rises on average by 40 log points (50%). Moreover, for every 1% difference in GDP per capita between the two bordering countries, there is a 0.63% difference between the amount of light per capita at their borders. More surprisingly, there also exists a relationship between differentials in growth of lights per capita across a border and differences in growth in the bordering countries over a 20-year period from 1990 to 2010.[3] As one moves from a slower-growing to a faster-growing country, the 20-year growth rate of light per capita rises on average by 2.6 percentage points, and for every 1 percentage point difference in the growth rates of GDP per capita of two bordering

[2]The recorded brightness of lights may depend on cloud cover, humidity and other atmospheric conditions in a region, but it is implausible that national borders consistently conform to atmospheric fronts. Robustness checks with controls for temprerature, precipitation, altitude and slope on both sides of borders do not alter the results.

[3]Strictly speaking, this is an 18-year growth rate because the lights data does not start until 1992. However, high-resolution population data is not available for 1992 but is available for 1990, so that is the data I use to compute per capita growth rates. I will refer to this measure as a 20-year growth rate throughout.

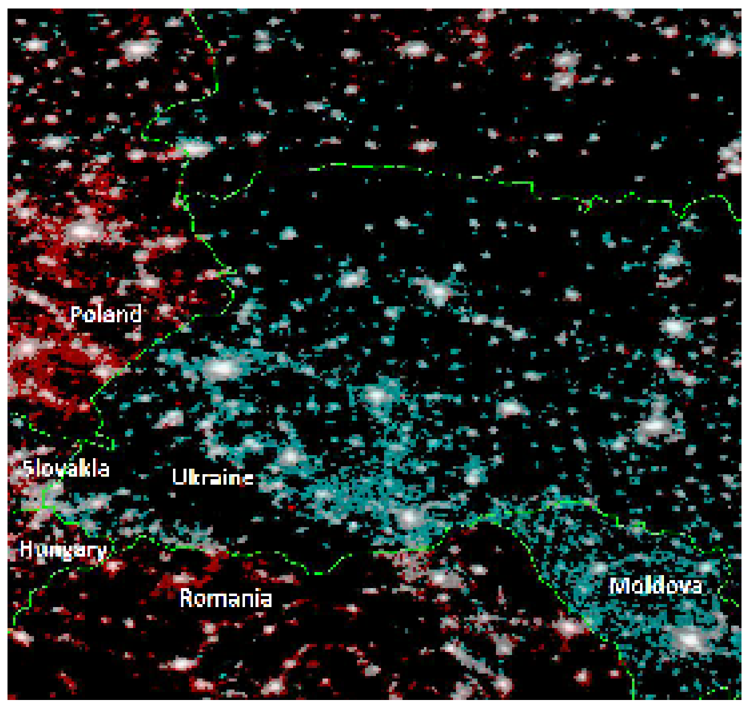
countries, there is a 0.88 percentage point difference in the growth rate of lights per capita of these countries at their mutual border.This finding is unexpected because while differentials in levels of income (the world distribution of income) tends to be persistent, growth rates are much more volatile, both across time and within a single country. Therefore, an association between differences in national growth rates and differences in border growth rates suggests that border discontinuities represent not only accumulated effects of large historical events in the past, but that they represent factors that promote or stymie current economic activity and the ability of people to take advantage of or overcome their past. If discontinuities at the border can be attributed to political economy, this finding shows a substantial effect of suitable government activity for growth in a country over a short period of time.

Figure 1 (1)



Korean Peninsula, 2000

(1)



Ukraine, 1992-2000

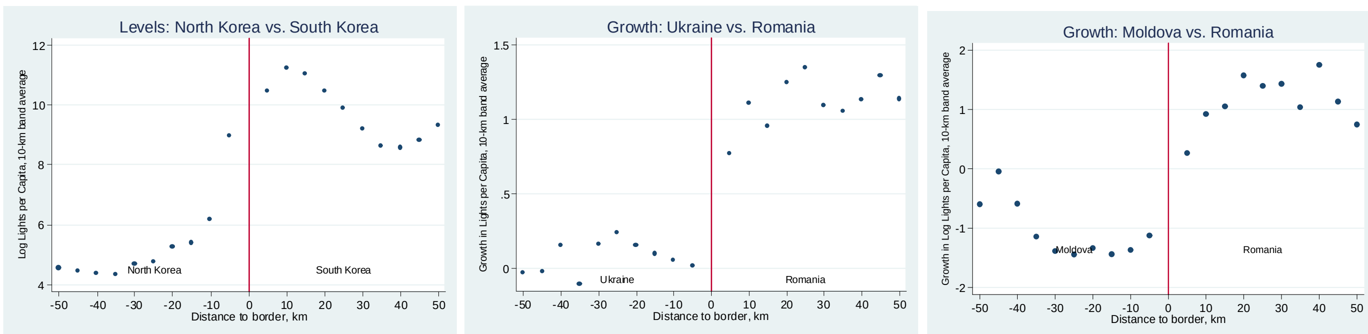
(2)

The finding can be highlighted in two pictures, both from Elvidge (2003). The first, Panel 1 in Figure 1 is a satellite photo of North Korea and South Korea, the former covered in darkness, the latter lit up, with the light beginning right at their common boundary. The second, Panel 2, is a comparison of two satellite photos of Ukraine and its neighbors: the first taken in 1992 and the second taken in 2000. Areas that gained light are represented in red, whereas areas that lost light are represented in blue. The comparison reflects the obvious fact that during the transition from communism in the 1990s, Ukraine (and its ex-Soviet southern neighbor Moldova) contracted much more severely than did Poland, Romania and Hungary, which had exceeded their 1992 GDP by 2000. What is striking about the picture is that there is a border discontinuity in growth in nighttime lights – there are virtually no red dots in Ukraine and Moldova and virtually no blue dots in Poland, Hungary and Romania (except for a few in the Carpathian mountains, which are not on a border). Almost all places in the former Soviet republics had contracted, and almost all places in Eastern European nations had expanded between 1992 and 2000, even those very close to the borders between these two sets of countries. Particularly striking is that the westernmost tip of Ukraine (Ruthenia) had been part of Ukraine for only 50 years before the time period in question and had been in a political union with Hungary, Slovakia and Romania for most of its prior history[4], and yet, it experienced a decline in lights as did the rest of Ukraine, while the neighboring parts of Hungary, Slovakia and Romania experienced growth in lights. Moreover, Moldova and Romania (southwest corner of picture) share the same language, religion and culture (although they have been politically separate for most of their modern history), but have had radically different growth experiences in the 1990s with Romania growing, Moldova shrinking, and the growth experience changing discontinuously at the border. Figure 2 formalizes Figure 1 by presenting local average lights per capita (and growth in local average lights per capita)

[4]Ruthenia had been a part of the Kingdom of Hungary since 1526, and of the Habsburg empire (which included Hungary, Slovakia, and the parts of Poland and Romania that are visible in this picture) since 1699. After the collapse of the Habsburg empire as a result of World War I, Ruthenia became a part of Czechoslovakia in 1918. Ruthenia was annexed by the Soviet Union in 1945 as a result of World War II, and attached to the Ukrainian Socialist Soviet Republic, which became the independent country of Ukraine in 1991.

for the places described. We can see very strong and very clean discontinuities at borders in all three graphs. My finding in this paper is that these pictures are not anomalies, but rather very stark depictions of a general pattern.

Figure 2 (2)



Graphs of levels and growth discontinuities in lights.

It is also intuitive that border discontinuities present lower bounds for the importance of government activity for the economy. First, borders are porous, which means that trade and migration may mitigate differences created by government activity on the different sides of the border. Second, and more fundamentally, border discontinuities are biased downward in the night lights dataset because of blooming: satellite-recorded light tends to spread away from its source, thus leading light generated on one side of the border to be seen on the other side of the border. In Section 5, I document that poorer countries tend to experience a rise in lights per capita relative to richer countries as one approaches their mutual border.

The above results are most straightforwardly obtained by considering lights in narrow neighborhoods around borders, which is a version of local constant regression discontinuity estimation and is known to be biased (with the bias going to zero asymptotically) if the derivative of the outcome variable with respect to the running variable at the border is large (if lights per capita converge rapidly very close to the border). Regression discontinuity estimates with better bias behavior can be obtained by using local polynomial estimation. A complication in using local polynomial estimation to calculate border discontinuities with nighttime lights data is that the data generating process does not obey the standard assumption of independently generated data with the number of observations at each site going to infinity. Instead, the nighttime lights data constitute a global census of visible nighttime lights, taken at a fixed resolution. Therefore, the asymptotics for the regression discontinuity estimator must be calculated as the resolution of the data goes to infinity (the pixel size goes to zero) rather than as the domain of the pixels expands to infinity. Such an asymptotic scheme is referred to as *infill asymptotics* in the spatial econometrics literature. A natural assumption for such data is that the errors from trend of neighboring data points are correlated. A contribution of this paper is to derive the properties of the local polynomial estimator under infill asymptotics with correlated errors. I prove that with very general assumptions on the covariance structure of the outcome variable the local polynomial estimator is consistent, and has a smaller asymptotic variance than it would if the errors from trend were independent.[5] Intuitively, the local polynomial estimator exploits the correlation in the errors, so that only their unpredictable component contributes to the asymptotic variance. In the special case that the error from trend is mean-square continuous (has no unpredictable component), the local polynomial estimator converges at a nonstandard rate of $1/\sqrt{h}$, where h is its bandwidth.

A further complication of using local linear regression is contamination of the night lights dataset. It is well known (Doll 2008) that the satellites recording nighttime light density tend to attribute light generated at a particular site to nearby sites as well. For example, the Pacific Ocean is lit up as far as 50 kilometers away from the California shore near Los Angeles. This phenomenon is known as overglow. While local linear regression is very important for recording the potential narrowing of differences in economic activity at borders because of cross-border trade, it also will pick up the convergence of nighttime lights density at borders because of overglow, which will complicate finding any discontinuities in economic activity that may exist. In this paper, I propose a novel correction for overglow by calibrating an

[5] This result is most closely related to Card and Lee (2008)

overglow function over territories on the borders of wastelands and using this function to correct nighttime lights values at borders. I implement this correction to improve my local linear estimates and demonstrate that while overglow can be a substantial problem for local linear analysis, it ceases to be a problem once the correction is made.

I then go beyond providing evidence of discontinuities at national borders, and hence of the importance of political economy to economic activity, and attempt to uncover which politically determined variables are useful in understanding and explaining border discontinuities. First, I show that richer sides of borders do not tend to have more public goods – specifically, roads, railroads and utilities – in narrow neighborhoods of the border than poorer sides of borders do. Therefore, one cannot explain border discontinuities through differences in local public good provision. However, public goods provision could still explain border discontinuities if it has large spillovers – for instance, good infrastructure in the country as a whole may benefit a region with worse infrastructure through endowing the region with richer trading partners from other regions.

Restricting myself to analyzing discontinuities in growth rates, I perform a correlational analysis to see whether they can be explained by several national-level variables frequently discussed in the cross-country growth literature. I show that when the extent of the World Bank measure of the rule of law (which considers the impartiality of the judicial system, the quality of contract enforcement, and the protection of private property against confiscation) is accounted for, the correlation between differences in national growth and differences in growth at the border falls substantially and becomes insignificant, while differences in the rule of law between two countries are associated with higher differences in subsequent growth at their borders. While there is similarly an association between differences in countries' initial levels of public goods provision (proxied by the fraction of roads paved) and differences in their subsequent growth rates at the border, controlling for public goods provision does not eliminate the association between differences in national growth and differences in growth at the border. Accounting both for the rule of law and for public goods provision, the association between differences in the rule of law and differences in border growth remains intact, while the association between differences in public goods and differences in border growth shrinks substantially. I further show that the rule of law retains its explanatory power when I control for contracting institutions (Acemoglu and Johnson [2005]), political freedom, the average amount of education, and a measure of interpersonal trust from the World Values Survey.

The paper is organized as follows: Section 2 describes the data. Section 3 discusses the efficiency improvement of the local polynomial estimator in an infill asymptotics setting with correlated errors. Section 4 provides baseline results on border discontinuities in GDP and growth rates as robustness checks to accounting for local climate and public goods variation. Section 5 explores the role of property rights protection in generating border discontinuities and provides evidence that border discontinuities do not arise because of the discontinuous barriers to trade that borders pose. Section 7 concludes.

2 Description of the Data

The Night Lights Dataset

Data on luminosity at night is collected by the DMSP-OLS satellite program and is maintained and processed by the NOAA. Satellites orbit the Earth every day between 20:30 and 22:00, sending images of every location between 65 degrees south latitude and 65 degrees north latitude at a resolution of 30 arcseconds (approximately 1 square km at the equator). The images are processed to remove cloud cover, snow and ephemeral lights (such as forest fires and gas flaring) to produce the final product available for download at

http://www.ngdc.noaa.gov/dmsp/downloadV4composites.html

Each pixel (1 square kilometer) in the luminosity data is assigned a digital number (DN) representing its luminosity. The DNs are integers ranging from 0 to 63, with the relationship between DN and luminosity being

$$\text{Luminosity} \propto \text{DN}^{3/2}$$

(Chen and Nordhaus [2010]). However, pixels with DN equal to 0 or 63 may be top- or bottom-censored. Another known problem with the lights data is the presence of overglow and blooming: light tends to travel to pixels outside of those in which it originates, and light tends to be magnified over certain terrain types such as water and snow cover. All of these problems tend to make nearby pixels more similarly lit than they should be, thus working against the hypothesis of this paper.

The night lights dataset has been extensively analyzed in the remote sensing literature for its utility in predicting economic activity; see Elvidge(1997), Sutton et al. (2007), Doll (2006). A comprehensive discussion of the collection, use and pitfalls of the night lights dataset is Doll (2008). Its pioneering use in the economics literature has been Henderson et al. (2011). Chen and Nordhaus (2010) discuss the limitations of the lights dataset; in particular, they argue that the relationship between luminosity density and output density becomes uninformative because of top-censoring and bottom-censoring at $DN = 63$ and $DN = 0$. Michalopoulos and Papaioannou (2011a and b) use the night lights dataset to construct a proxy for output per capita in African ethnic territories to assess the consequences of partitioning ethnicities during the Scramble for Africa.

Gridded Population of the World Data

The Gridded Population of the World (GPW) dataset is constructed and maintained by the Socioeconomic Data and Applications Center (SEDAC) at the Center for International Earth Science Information Network at the Earth Institute at Columbia University. The dataset compiles population information from national censuses for very small political units (municipalities, census tracts) in order to achieve its resolution. Within a political unit, population is distributed uniformly.

Other Data

In Sections 4 and 6 I use a number of geographic, political and social variables to validate the regression discontinuity and explore correlations between discontinuities in growth rates at national borders and politically affected determinants of economic growth for the bordering countries. I obtain data on the distributions of temperature and precipitation levels (means, medians, annual and mean daily ranges, coefficients of variation) at 30 arcsecond resolution from the dataset constructed by Hijmans et al. (2005) and distributed on the WorldClimate website: http://www.worldclim.org/. I obtain elevation at 30 arcsecond resolution from the USGS Shuttle Radar Topography Mission (SRTM), and use the data to compute slope in ArcGIS. Data on roads, railroads and utilities is from the US National Imagery and Mapping Agency, and originally from the Digital Chart of the World. I also use several country-wide covariates from standard sources in the literature. I obtain data on the presence of the rule of law and other governance indicators from the World Governance Database (WGI) sponsored by the World Bank.[6] Data on the fraction of roads paved and on the amount of time required to enforce a contract is obtained from the World Bank's World Development Indicators. Religious composition and legal origin of countries is obtained from La Porta et al. (1998). Political freedom is measured using the Freedom House Political Rights Index, from Acemoglu, Johnson, Robinson and Yared (2008). Average years of education are obtained from Barro and Lee (2010). A measure of trust is obtained from the World Values Survey via La Porta (2011). Bilateral national-level trade data is obtained from the IMF, Direction of Trade Statistics. Finally, I use national GDP data from the Penn World Tables, Mark 7.1.

[6] In results not reported, I also use average protection from expropriation risk between 1985 and 1995 from Political Risk Services via Acemoglu, Johnson and Robinson (2001), and an index of property rights protection from the Economic Freedom of the World database (variable 2C).

3 Regression Discontinuity under Infill Asymptotics

3.1 Discussion of Literature

The methodology for regression discontinuity has been extensively developed by Hahn, Todd and van der Klaauw (1999), Porter (2003) and Card and Lee (2008) and is reviewed in Imbens and Lemieux (2008) and Lee and Lemieux (2010). However, the asymptotics in these papers assume either that observations are independent or that the diameter of the domain from which observations are drawn expands to infinity as the number of observations tends to infinity. In the context of estimating border discontinuities from nighttime lights data, these assumptions are unsuitable because the nighttime lights represent a regular grid of observations in a fixed domain (a neighborhood of the border in question). Moreover, nearby values of lights are very likely to be correlated. In particular, it makes sense to think of asymptotics in the nighttime lights dataset as an improvement in the resolution of the regular grid rather than as an increase in the number of observations. Such an asymptotic analysis, while very uncommon in econometrics, is frequently performed in spatial statistics and geology, and is referred to as *infill asymptotics.* One contribution of this paper is to develop the asymptotic properties of the local polynomial estimator in an infill asymptotics setting.

There is an emerging literature on the econometrics of processes defined on two-dimensional surfaces rather than on a time axis (Conley 1999, Hansen et al. 2008, Robinson 2011), which extends insights from the time series literature in multiple dimensions. However, this literature concerns itself entirely with increasing-domain asymptotics: it is assumed that as the number of observations increases, the domain of the grid tends to infinity. There is substantial discussion of processes under infill asymptotics in the spatial statistics literature, most notably by Stein (1987, 1999), particularly in the context of kriging, or spatial interpolation and extrapolation. Stein's results are derived for covariance stationary Gaussian processes when the statistician has substantial prior information about the shape of the covariance function, such as the functional form. The results presented here will be valid for substantially more general processes without any functional form assumptions, and without the assumption of covariance stationarity. General references on spatial statistics are Cressie (1993) and Schabenberger and Gotway (2005).

This section is closest to Card and Lee (2008), who consider regression discontinuity estimation when the running variable is observed at a number of discrete sites, and perform asymptotic analysis as the number of these sites goes to infinity. However, Card and Lee (2008) assume that the errors between the assumed and the true functional forms of the relationship between the outcome variable and the running variable are independent. The analysis in this paper will relax this assumption, which will entail a substantially different analysis from that of Card and Lee.

Properties of the Local Polynomial Estimator under Infill Asymptotics

I consider the properties of the standard local polynomial estimator computed for an outcome variable y that is observed on a regular one-dimensional grid;[7] hence, for the sequence $\left\{y\left(\frac{u}{N}\right)\right\}_{u=1}^{N}$. As N goes to infinity, it is clear that y is never observed outside of $[0,1]$, but it is observed at an increasing frequency. The running variable x is distance: $x=\left\{\left(\frac{u}{N}\right)\right\}_{u=1}^{N}$The core result is that for estimation of a regression discontinuity at a single point, the local polynomial estimator is consistent, and its asymptotic variance is smaller than the probability limit of the traditional White estimator for heteroskedasticity based on the residuals of the local polynomial estimator. The intuition for this fact is that when the errors from the deterministic relationship between the outcome and the running variable are correlated, the weighting scheme of the local polynomial estimator exploits this correlation to predict the outcome at the discontinuity, which makes the effective magnitude of the error equal to that

[7] The mathematical results for a two-dimensional grid are straightforward extensions of the results presented in the Appendix. The empirical results are also very similar, but computationally more difficult to obtain.

component of it that is unpredictable. The White estimator, however, assumes that all errors are independent and computes the variance accordingly. Therefore, when facing infill data with correlated errors, the typical variance estimator is overly conservative.[8]

I further present an estimator that is consistent for the true asymptotic variance of the local polynomial estimator. Instead of being based on the squared residuals, it is based on squared differences of residuals from adjacent observations. This estimator filters out both the deterministic trend and the correlated component of the residual, regardless of their functional form and correlation structure, leaving only the idiosyncratic variability at each site to contribute to the estimated variance. Hence, the variability in the correlated component of the error term is not (mistakenly) attributed to the estimator.

The variance estimator I propose is given by

$$\hat{V}_{1,N} = \frac{1}{2} e_1' D_N^{-1} \left[\sum_{u=1}^{N} \hat{e}_{N,u}^2 \left(\frac{u}{N}\right) \frac{1}{Nh} k^2 \left(\frac{u}{Nh}\right) X \left(\frac{u}{Nh}\right) X \left(\frac{u}{Nh}\right)' \right] D_N^{-1} e_1 \tag{1}$$

where $k\left(\right)$ is a kernel, h is the bandwidth, N is the number of observations in the fixed interval under consideration, $X\left(\right)$ is a vector of polynomials in distance to the border, D_N is the denominator of the local polynomial estimator, e_1 is a vector with first component equal to 1 and all the others equal to zero, and $\hat{e}_{N,u}\left(\frac{u}{N}\right) = \hat{e}\left(\frac{u}{N}\right) - \hat{e}\left(\frac{u-1}{N}\right)$ is the difference between adjacent residuals obtained from local polynomial estimation. One should contrast this variance estimator with the traditional White estimator, which is given by

$$\hat{V}_N^{OLS} = e_1 D_N^{-1} \left(\sum_{u=1}^{N} \hat{e}^2 \left(\frac{u}{N}\right) \frac{1}{Nh} k^2 \left(\frac{u}{Nh}\right) X \left(\frac{u}{Nh}\right) X \left(\frac{u}{Nh}\right)' \right) D_N^{-1} e_1$$

It is also useful to note the relationship between the proposed variance estimator $\hat{V}_{1,N}$ to the classical estimator of the variogram proposed by Matheron (1962) (see also Schabenberger

[8] The only substantive assumption necessary to prove this is that the process of error terms can be decomposed into the sum of a process of independent random variables and a process of correlated random variables whose covariance function is sufficiently smooth. If there is no unpredictable component to the error term (the error process is mean-square continuous), the local polynomial estimator converges to its probability limit at a nonstandard rate of $1/\sqrt{h}$.

and Gotway 2005). Matheron's estimator of the variogram $\gamma(\tau) = E\left((e(u) - e(u-\tau))^2\right)$ is given by

$$S(\tau) = \frac{1}{|A(\tau)|} \sum_{A(\tau)} (e(s) - e(s+\tau))^2$$

where $A(\tau)$ is the set of pairs of points in the space that are τ apart.

The proposed estimator $\hat{V}_{1,N}$ can be thought of as estimating the limit of the variogram as τ goes to zero. The kernel density estimation ensures that the limit is computed for the variogram that holds at $x = 0$, and therefore allows for the variogram to be nonstationary and change over regions of space.

The estimator $\hat{V}_{1,N}$ converges in probability to a smaller value than the estimator $\hat{V}_N^{OLS}$, but in finite samples, $\hat{V}_N^{OLS}$ may be numerically smaller. Therefore, I use the minimum of the two estimators when computing the variance. I also consider data at a resolution of 1 km (the resolution at which the night lights data is available), which brings $\hat{V}_{1,N}$ closer to its probability limit. All propositions and proofs are relegated to the Appendix.

4 Baseline Results

4.1 Calibration

To convert lights data into a single quantity comparable to GDP, I assume a low-parameter function approximating the relationship between DN and output density, and calibrate its parameters using aggregate light density for countries and national accounts data on GDP per capita. Specifically, I estimate the parameters of the function using nonlinear least squares, in which I try to explain GDP density per unit area in a country with measures of light density for the country constructed using pixel digital numbers. The assumed relationship is

$$\ln(1+y_i) = c + \ln\left(1 + c_0 * v_{0,i} + c_b * \sum_j \left(j^d\right) * v_{j,i} + c_1 * v_{63,i}\right) + \varepsilon_i \tag{2}$$

where i indexes countries, y_i is GDP density of country i (obtained from the World Bank), $v_{j,i}$ is the fraction of pixels with digital number equal to j in country i, and ε_i is the error term. I use the transfer function $\ln(1+x)$ rather than $\ln(x)$ because the latter is not defined for $x = 0$ while the former is defined for all nonnegative x (and some negative values of x as well). This is not a problem in the estimation of the calibration equation, as the light density of no country is equal to zero; however, the light density at some borders does attain the value zero, which explains the need to use such a specification. For reasonable values of the output density y_i, $\ln(1+y_i)$ is indistinguishable from $\ln(y_i)$, while the parameters on the right hand-side allow full parametrization of the scale of the index of fractions of pixels relative to 1. Note that the measure of light density used by Henderson et al. (2011) would be equivalent to setting

$$c_0 = c_1 = 0, d = 1$$

and the measure used by Chen and Nordhaus (2010) would amount to

$$c_0 = c_1 = 0, d = 3/2$$

(The values c and c_b would be set sufficiently large in magnitude to make the 1 in the parentheses inconsequential). I estimate equation (2) for every satellite-year in the DMSP-OLS dataset, using the Chen and Nordhaus (2010) specification as my initial values. For multiple years (in particular 2000 and 2005), the estimates of the top-censoring and bottom-censoring coefficients c_0 and c_1 are equal to zero, suggesting that top-censoring and bottom-censoring is not a particularly important limitation of the data.

Descriptive Analysis and Graphs

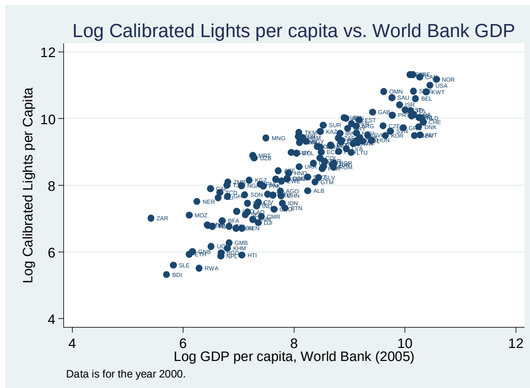
Computation of the Dependent Variable

Since national borders tend to differ substantially in length, I standardize them by dividing each border into pieces corresponding to its intersection with a 1-degree by 1-degree grid superimposed on the world map. I obtain 1352 border pieces, having started with 270 borders. In all my computations of standard errors, I cluster the standard errors by border (rather than border piece) so I have 270 clusters. Throughout the rest of the paper, I will refer to border pieces as borders to minimize terminology unless I need to make the distinction explicit.

To obtain a lights-based proxy for economic activity around a given border for a given year, I construct neighborhoods containing all points whose shortest distance to the border is less than X kilometers, and use an ArcGIS Python program to compute the fraction of pixels with each digital number for each side of the border within the neighborhood. I then use the calibrated values of c, c_0, c_1, c_b and d to compute the right-hand side of equation (2) for each side of the border, thus obtaining a proxy for the output density of the given country within X kilometers of the given border. Finally, I multiply by the area and divide by the population of this region (as discussed in Section II, I obtain population at 2.5 arcminute resolution from the Gridded Population of the World dataset) to obtain a lights-based estimate of GDP per capita for the given country within X kilometers of the given border.

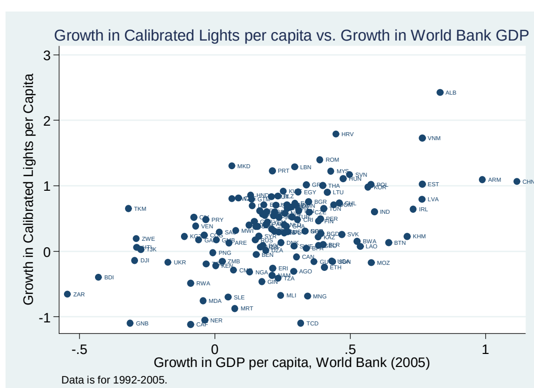
There are good reasons to expect that this calibration procedure creates a variable that is, on average, close to the true value of GDP per capita in the region of interest. Henderson et al. (2011) and Chen and Nordhaus (2010) document the tight association between GDP density and measures of light density that are similar to the one used in this paper (in fact, they are special cases of my measure). The fit of the selected specifications to the GDP data for countries is very good as Figure 3 below attests: the lights explain approximately 73% of the variation in GDP per capita, and the plot looks approximately linear. I also present the relationship between the growth of the calibrated lights series and the growth of GDP per capita in Figure 4. The fit is not as good (the lights explain only 23% of the variation in growth), but still quite strong, and the positive correlation is unmistakeable.

Figure 3 (3)



Quality of Calibration, GDP per Capita

Figure 4 (4)



Quality of Calibration, Growth

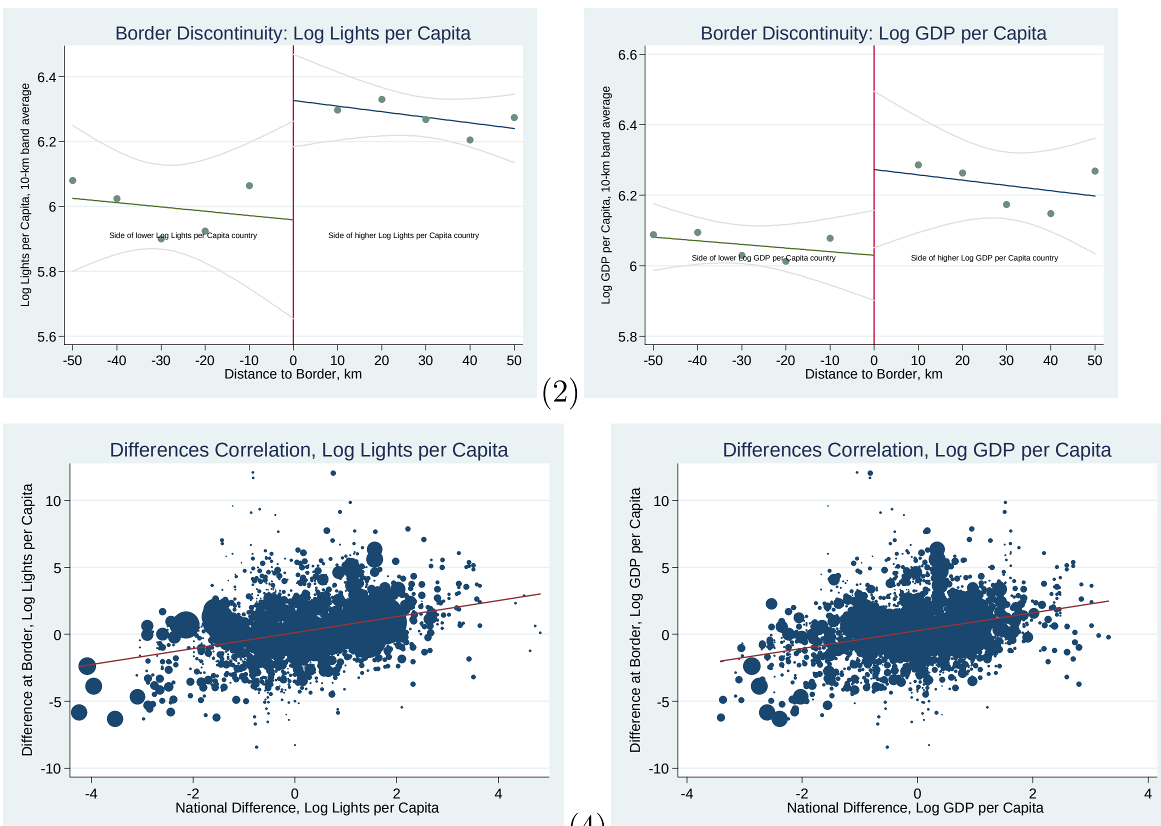
4.1.1 Descriptive Statistics

Table 1 in the Appendix provides descriptive statistics for the main variables of interest: log lights per capita at borders and their growth rate, log lights per capita and log GDP per capita nationwide, with their growth rates, and some covariates related to institutions and public goods. I present the mean and the standard deviation of each variable, as well as the mean and the standard deviation of each variable computed over the richer (or higher-growing) and poorer (or lower-growing) sides of borders exclusively. The descriptive statistics foreshadow more formal results. We immediately see that log lights per capita are higher on richer sides of borders than on poorer ones, and that the mean difference between the two is about half of the mean difference between nationwide log lights per capita (or nationwide log GDP per capita) of the bordering countries. We see even starker results for differences in growth rates of light per capita between higher-growing and lower-growing countries at their mutual border. In panel 2 of the table, where we look at institutions and public goods, we see that the rule of law of the higher-growing country at a border are much better on average than the rule of law of the lower-growing country at that border. Public goods (proxied by the fraction of roads paved) are also better in the higher-growing country at a border, but not by much, as is trust and the number of years of schooling. Interestingly, local public goods (log roads near the border) are very close to each other on the higher-growing and lower-growing sides of borders on average.

4.1.2 Elementary Discontinuity Plots and Correlations

I now present several elementary graphs that suggest large discontinuities in GDP per capita and its growth rate at national borders. Panel 1 of Figure 5 shows a discontinuity plot of lights per capita (predicted GDP per capita using lights) against distance from border in the direction of the richer country at the border as predicted by the lights calibration. To construct this plot, I identify the richer country and the poorer country at each border according to which one has the higher lights per capita. I then pool all points over the portions of the discontinuity neighborhoods that belong to the poorer countries, and compute the average lights per capita for 5-km intervals of distance to the border. I repeat the same procedure for the richer countries and plot the averages as a function of distance to the border either for the richer (on the right) or for the poorer (on the left) side.

Figure 5 (5)



(1) (2) (3) (4)

Discontinuity and Correlation Plots, Log Lights per Capita at Borders

It is apparent that there is a discontinuity at the border crossing point, with the richer (according to lights) side of the border having a GDP per capita at least 0.2 log points (about 22%) higher than the poorer side. The last point on the poorer side (at -5 km) is approaching the points on the richer side, but the other points on the poorer side are far removed from those on the richer side (by at least the 0.2 of the discontinuity). This can be explained by overglow in the data: light from the richer side of the border illuminates the poorer side, making it appear to be richer.

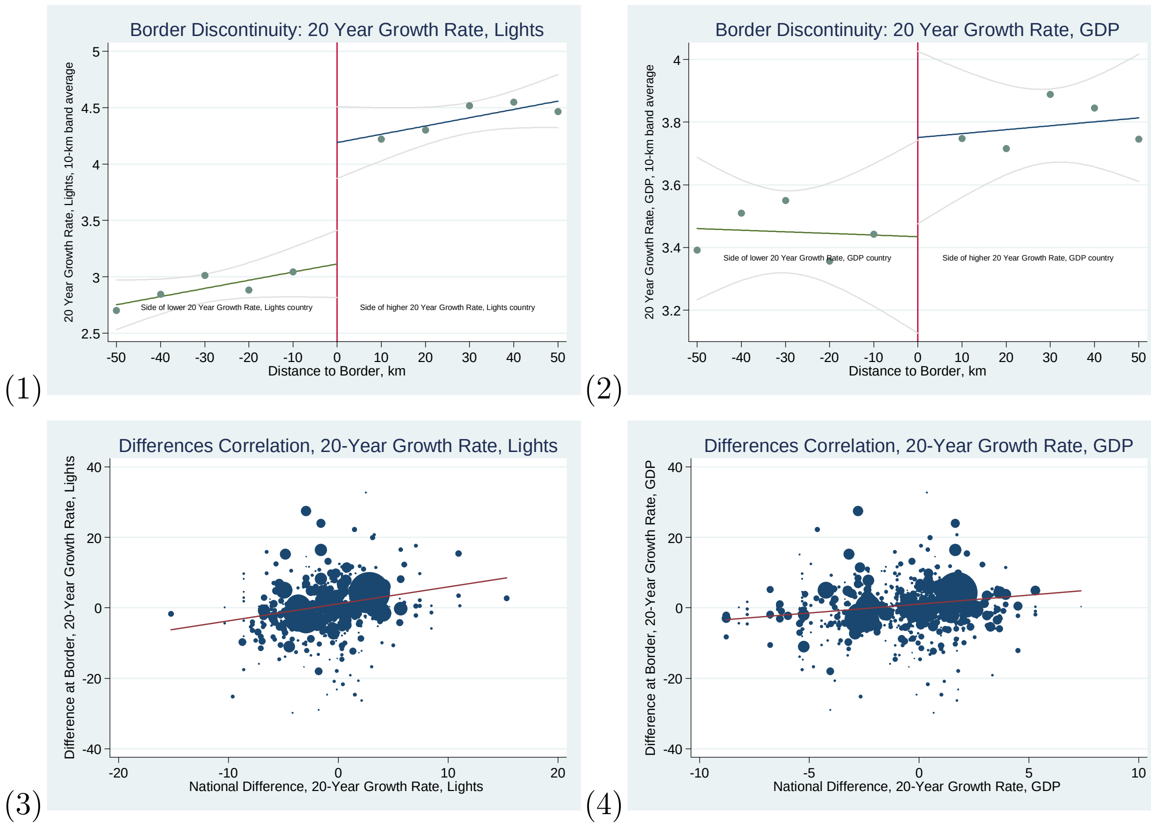
Panel 2 of Figure 5 presents the same discontinuity plot, but with countries categorized as rich or poor on the basis of GDP per capita from the Penn World Tables rather than on the basis of national lights per capita. Constructing this plot has both advantages and disadvantages compared with Panel 1: I use a real rather than calibrated output measure in this plot, but I also have to compare lights to GDP in this plot, which introduces measurement error. We again see a clear discontinuity at zero that is substantially larger than any other difference between consecutive points, with a somewhat smaller magnitude than before.

The discontinuity plots in Figure 5 elide the fact that countries are extremely heterogeneous and the difference in economic activity at the border between a poorer and a richer country may vary widely, even if it is large and positive on average. One would instead expect the difference at the border to be somehow related to the difference in economic activity between the two countries overall: countries with wide disparities in income per capita (like North Korea and South Korea) should have larger discontinuities at their common border than countries with similar incomes (like France and Germany). Panel 3 of Figure 5 presents a plot of differences in log lights per capita at a border against differences in log lights per capita in the bordering countries (each border difference being weighted by population). The positive correlation and its strength are manifest. Panel 4 shows that if differences in log lights per capita at the border are plotted against national differences in log GDP per capita (from the World Bank), the correlation is similar.

It is perhaps not very surprising that GDP per capita may be discontinuous at national borders since borders change infrequently, and in many cases remain stable for decades and even centuries, allowing a discontinuity to accumulate. However, just as there is a discontinuity in GDP per capita across borders, so there is one in GDP growth over relatively short periods of time. Panel 1 in Figure 6 presents a discontinuity plot similar to Panel 1 in Figure 5, but computing the average annualized 20-year growth rate in lights rather than the average amount of lights at each location, and comparing countries with higher growth in national lights per capita with countries with lower growth in national lights per capita rather than richer countries with poorer countries.

Figure 6

(6)



Discontinuity and Correlation Plots, Growth of Lights per Capita at Borders

There is an extremely prominent discontinuity in growth of lights per capita at the border, with the higher-growing country (according to lights) growing by over 1 percentage point more each year for 20 years, on average, than the lower-growing country. Thus, not only have borders contributed to (static) discontinuities in levels over the decades that they remain unchanged, but they also create (dynamic) discontinuities in growth rates over time periods as short as 13 years. Panel 2 of Figure 6 shows the same discontinuity plot with countries categorized as higher-growing or lower-growing based on national GDP per capita growth. The discontinuity is again prominent, but more modest in magnitude. Panels 3 and 4 show correlation plots between differences in growth at the border and differences in national growth in output per capita (measured with lights or GDP) similar to the same panels of Figure 5: the conclusions are the same.

4.2 Overglow Correction

A problem in the analysis of nighttime lights data, which is particularly severe for its use in a regression discontinuity design, is the presence of oveglow and blooming. The satellite sensor tends to observe light in territories that are devoid of human activity, but are close to densely settled areas. For example, Doll (2006) presents an illustration of light from Los Angeles being observed over the Pacific Ocean as far as 50 kilometers away the California shore. In a regression discontinuity analysis, overglow operates continuously across borders and transforms any discontinuities into nonlinearities. In Figure 2, we saw the profound effect of overglow at the border between the two Koreas, and in Figures 5 and 6 we see the discontinuity gaps shrinking as one approaches the border.

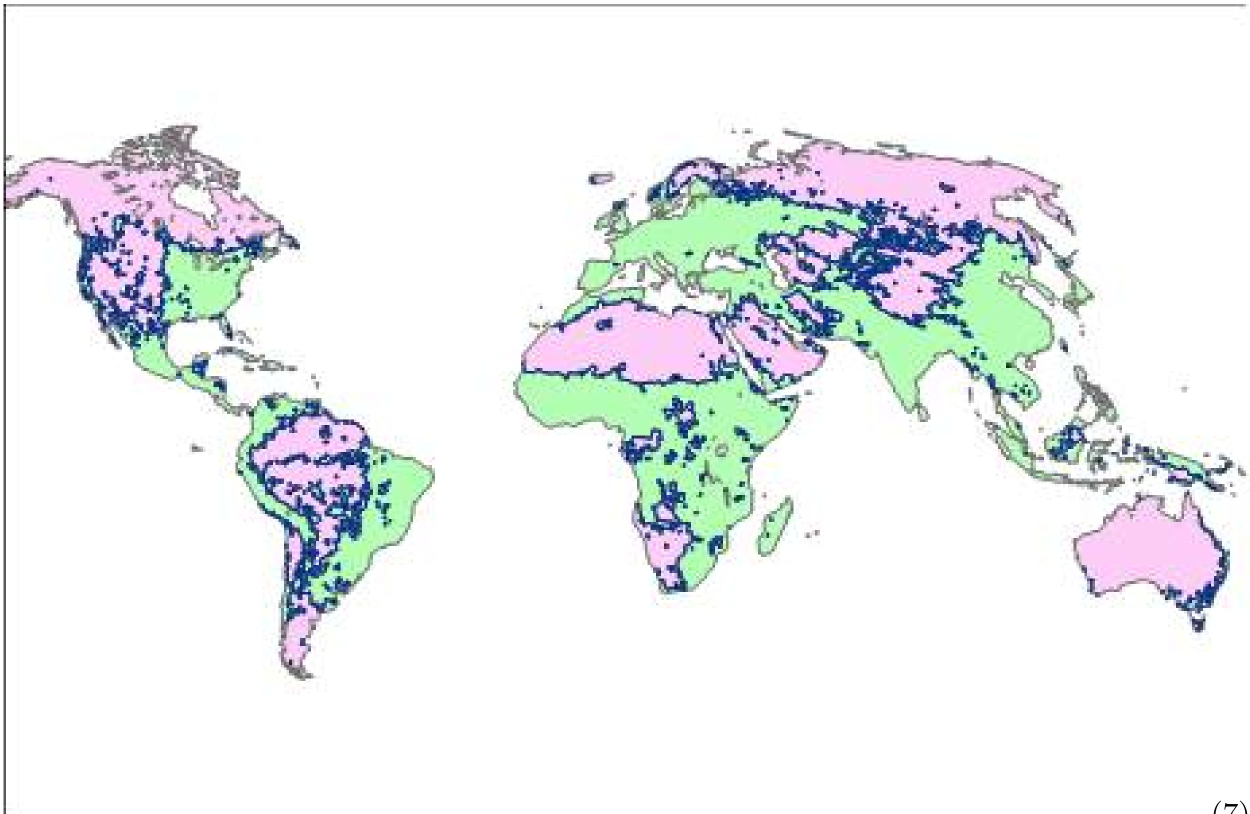
To alleviate the impact of overglow, I construct a parsimonious model for the overglow process, estimate the parameters of the model, and use it to correct my measures of nighttime light density. I assume a one-dimensional autoregressive model (to parallel my approach to estimating border discontinuities) in which light observed over one 10-km strip of land increases the amount of light observed in neighboring 10-km strips (the width again chosen to parallel the border discontinuity setting). Specifically, if I divide a tract of territory into square sites, subdivide it into 10-km strips of territory, and assume no overglow across sites (if the square sites are large enough), I posit

$$\hat{u}_{s,d} = u_{s,d} + \sum_{j=1}^{D} \rho_j \hat{u}_{s,d\pm j} + \varepsilon_{s,d} \tag{3}$$

where $u_{s,d}$ is the amount of light generated in site s and strip d, and $\hat{u}_{s,d}$ is the amount of light observed in site s and strip d. All strips of land in a site are ordered, so that strip 0 is adjacent to strips 1 and -1, which in turn are adjacent to strips 2 and -2 respectively, and so on.

The variable $u_{s,d}$ is unobserved, which prevents me from estimating equation 3 without additional assumptions. A straightforward approach would be to look for regions in the globe where I can construct strips such that $u_{s,d} = 0$ for some strips d. One such approach could be to use overglow into oceans; however, there is reason to believe that the reflective properties of light over oceans (and hence, the parameters of the overglow relationship in equation 3) are likely to be different from the reflective properties of light over land. Instead, I look at the edges between land subject to some kind of economic development and economically unexploited wasteland. Figure 7 shows the wasteland areas of the globe as defined by CIESIN in violet: they are mostly deserts (Sahara, Arabian peninsula, Kalahari, Central Asia, the Australian Outback), rain forests (Amazon, Congo basin), mountains (United States, Chile) and tundra (Siberia and the Canadian north).

Figure 7



(7)

Areas denoted as wasteland by CIESIN shown in violet.

I break up the world map into 1×1 degree squares and extract those that contain boundaries between wasteland and non-wasteland areas. I then break up each square into 10-km wide strips parallel to the wasteland boundary and estimate equation 3 for wasteland strips in each square, assuming that for these strips, $u_{s,d} = 0$. Table 2 presents my results from assuming various orders of autoregression in the overglow relationship captured by equation 3. We see that the first-order autoregression is large and statistically significant, while subsequent orders are smaller in magnitude and only marginally statistically significant if at all. In particular, I fail to reject the null hypothesis that all autoregressive coefficients beyond the first are joinly zero unless I use a 10% significance level, in which case I marginally reject this null hypothesis in one of the specifications. Hence, a reasonable assumption for the overglow process is that it is first-order autoregressive, with the autoregressive coefficient being about 0.23.

I implement my overglow correction by inverting equation 3 to recover the amounts of light generated at the strips of land near the borders. Specifically, I obtain

$$u_{b,d} = \max\left(\hat{u}_{b,d} - 0.23 * \left(\hat{u}_{b,d-1} + \hat{u}_{b,d+1}\right), M\right) \tag{4}$$

where M is the recorded minimum value of light density, the censoring done to prevent negative values of light density.

Baseline Results

I now present formal analysis to document economic discontinuities at borders. I first run regressions at each border piece in each year of the form

$$y_{i,b,t,d} = \tilde{y}_{i,b,t} + \tilde{\delta}_{i,b,t}d + \eta_{i,b,t,d}, \text{ weighted by } 1\left(d \leq h\right)$$

where $y_{i,b,y,d}$ is the value of the dependent variable (lights per capita, or growth of lights per capita) in country i, year t, border piece b and distance d away from border b. The parameters to be estimated are $\tilde{y}_{i,b,t}$, the value of y at the border in country i, and $\tilde{\delta}_{i,b,t}$, the slope of y at the border. I choose the bandwidth h to assign greater weight to observations close to the border using a cross-validation procedure that selects the bandwidth to maximize the accuracy of predictions 10 km away from the border for each regression I run.

Having obtained estimates of log lights per capita and its slope at borders, I run regressions of the form

$$\tilde{y}_{i,b,t} = \alpha^w_{b,t} + \beta^w u^w_{i,b,t} + \varepsilon^w_{i,b,t} \tag{5}$$

$$\tilde{y}_{i,b,t} = \alpha^w_{b,t} + \gamma^w y^w_{i,t} + \varepsilon^w_{i,b,t} \tag{6}$$

where $\tilde{y}_{i,b,t}$ is the local linear estimate of log lights per capita at border piece b in country i and year t, $y^w_{i,t}$ is a measure of log output per capita (using method w, lights or national accounts) for country i as a whole, $u^w_{i,b,t}$ is an indicator that country i has the larger log output per capita (measured by w) of the two countries at border piece b, $\alpha^w_{b,t}$ is a border piece-year fixed effect, and $\varepsilon^w_{i,b,t}$ is the error term. The parameters of interest to be estimated are β, the average percentage rise in output per capita as one crosses a border from a poorer to a richer country, and γ, the elasticity of the ratio of output per capita at the border to the ratio of output per capita of the bordering nations. I weigh all observations by the population in a 70-km neighborhood of the border, and I cluster all standard errors by border (not border piece).

The first four columns of table 3 presents estimates of β and γ for different choices of the national output series as well as robustness and placebo checks. The first row shows the baseline estimates, in which I have corrected the data for overglow before computing the local linear estimates. We see that when national output is measured by log lights, light density jumps on average by 0.58 log points or nearly 80%, upon crossing from a poorer country into a richer one. About 65% of the difference in log output per capita between two bordering countries persists up to their joint border. Both of these findings are significant at 5%. If we look at discontinuities in light density at borders when national output is measured by GDP per capita, the results are slightly muted: light density jumps on average by only 0.4 log points (49%, and this is significant only at 10%), but 63% of the difference in log output per capita between bordering countries persists to the border. The average bandwidth used to obtain these estimates is large, about 51 km (I restrict my analysis to a 70-km neighborhood of the border). To assess the sensitivity of my estimates to bandwidth choice, I provide results with a bandwidth of 30 km for all countries. The coefficients β in the specifications with indicator variables (5) decline and lose significance, but the coefficients γ in the elasticity specifications (6) remain significant at 5%, though their magnitude is somewhat smaller. While the sensitivity of my result for indicator variables to the bandwidth is concerning, it is intuitive that the elasticity specifications are more flexible in accounting for the fact that discontinuities at different borders are of different size than are the indicator variable specifications. Moreover, any residual overglow after the correction is a much larger problem for a smaller bandwidth than for a larger one.[9]

A natural falsification exercise for my results is to re-run my regressions using fake borders, for which I should not expect a discontinuity. I perform two such exercises: one in which I draw the fake borders at a distance of 30 km from the real borders into the interior of the richer country, and one in which I draw the fake borders at a distance of 30 km into the interior of the poorer country. All the discontinuity estimates at the fake borders are less than half the size of the baseline estimates and statistically insignificant (although some of the fake indicator estimates are close in magnitude to the indicator estimates for the 30-km bandwidth), which gives reassurance to the hypothesis that there is something meaningful about national borders that creates discontinuities at them.

In the remaining rows of Table 3 I demonstrate both the importance and the plausibility of the overglow correction. A telltale feature of overglow should be that the slope of light density on the poorer side of the border should be negative (because light density is rising towards the border through contamination from the richer country) and that the slope of light density on the richer side of the border is positive (because light density is falling towards the border as there is no reinforcement of light from the poorer country). Hence, the slope of the light density from the local linear estimation on a given side of that border

[9] In results not reported, I estimate equations 5 and 6 in which the weights are divided by the estimation variance from the discontinuity step for each observation in order to obtain feasible GLS estimates. For borders with very low estimation variance (e.g. borders with zero light density), I censor the reciprocal of the estimation variance at a fixed value, and about 27% of the observations (country-border piece-year) are so censored. The point estimates are similar to the ones reported (in fact, slightly larger) and the t-statistics of the point estimates increase, but the FGLS results cannot be distinguished from the baseline results by the Hausman test.

should be negatively associated with the output on that side. In the second row of Table 3 I run the regressions (5) and (6) that produce my baseline estimates, but I use the local linear slope rather than the intercept as the dependent variable (standardizing it for ease of interpretation). For my baseline estimates, the correlation between the local linear slope and output at each side of the border is insignificantly different from zero, and low in absolute value, though positive, suggesting some residual overglow. In rows 6-9, I present discontinuity estimates for log light density and its slope without the overglow correction, and with a cruder overglow correction in which I simply omit the last 10 km before the border from my analysis. The last correction is not preferred, because along with the overglow, it ignores any convergence in the level and growth rate of economic activity that might be going on over these last 10 km. We see that without an overglow correction, the discontinuities in log light density across borders are significant only at 10%, and substantially lower in magnitude than in my baseline results. However, the local linear slopes are radically higher on richer sides of borders – by as much as 0.3 standard deviations – which is consistent with substantial overglow. Correcting crudely for overglow by omitting the last 10 km yields discontinuity estimates much closer to the baseline, although the local linear slopes are still statistically significantly higher on richer sides of borders (though not by as much as without the overglow correction). Hence, we see that the data is consistent with the hypothesis that overglow is muting discontinuities in economic activity at borders, and that correcting for overglow in various ways yields estimates of similar magnitude.

Since national-level variables are expected to affect GDP per capita in some manner, and since national borders change very infrequently, it may not be very surprising that there is a discontinuity across borders in GDP per capita produced by a long-term and consistent operation of national-level variables. What is more surprising to find is that 20-year growth rates in GDP show a similar discontinuity. Given that differentials in growth rates between countries are much less persistent than are differentials in GDP per capita, such a result suggests that national variables affect output rapidly and profoundly enough for changes to be noticeable over short periods of time.The last four columns of Table 3 present similar discontinuity estimates for average annualized 20-year growth rates. Here, the parameter β measures the average percentage point difference in annualized 20-year growth rates between higher-growing and lower-growing countries at borders, and the parameter γ measures the fraction of the percentage point differences in nationwide growth rates of two bordering countries that persists to the border. We see that discontinuities in economic growth are, if anything, even stronger than discontinuities in the level of economic activity. Crossing a border from a lower-growing into a higher-growing country is associated with a 2-3 percentage point rise in the average annualized growth rate. The differential between the nationwide growth rates of two bordering countries, on average, persists up to the border almost completely, or actually increases near the border. Using a smaller bandwidth, if anything, strengthens these results, and the falsification exercises with the fake borders show no discontinuities away from the true national borders. Overglow appears to be a smaller problem for measuring discontinuities in growth rates than in levels: the crude overglow correction of dropping the last 10 km before the border produces insignificant differentials in the slopes of growth rates on higher- and lower-growing sides of borders just as the more comprehensive overglow correction discussed in the text.

4.3 Results by Continent

It is important to understand which borders contribute most to my baseline finding. Table 4 presents estimates of discontinuities in log light density and in the growth rate of light density at borders between OECD countries, between post-Communist countries and other European countries, in Asia, Africa and in the Americas.[10] It is apparent that the borders with the strongest discontinuity estimates are Asian borders and the borders of post-Communist

[10] The Americas are treated as a single continent because they are contiguous, and the number of borders in the Americas is comparable to that in Europe, Asia or Africa. Since the latter three continents form a continuous landmass, I define Russia, Turkey, Egypt and Israel to be part of two continents at once. I count the borders of these countries to belong to the continent of the bordering country: e.g. Russia's border with Estonia is a European border, but Russia's border with China is an Asian border. I count the Egypt-Israel border to be an Asian border.

European countries, followed by borders in the Americas and among OECD countries. One noticeable fact is that border discontinuities are much weaker in Africa than everywhere else. One reason why Africa may have much smaller border discontinuities is that as a result of Africa's colonial experience, borders of African nations tend to be relatively underdeveloped hinterlands, with most political and economic activity concentrated around capital cities. Therefore, African nations having heterogeneous government activity in their heartland may have similar government activity (i.e. none) at their borders, and therefore, they may fail to exhibit border discontinuities.

4.4 Robustness Check to Geographic Controls

One potential problem with the results may be if national borders tend to be drawn at discontinuous changes in geographic variables, such as altitude, slope of the terrain (Nunn and Puga's [2009] ruggedness), and climatic zones (because differences in altitude may lead to differences in soil type, temperature and precipitation). To address that problem, I calculate average values of altitude, slope, and statistics of temperature and precipitation (means, maximums, minimums, variability measures) on either side of every border, obtained from WorldClimate at 30 arcsecond resolution. I also calculate, on either side of every border, the fraction of land belonging to each of the 14 climatic zones defined by the International Geosphere Biosphere Programme, by using the Global Land Cover Characteristics Data available from USGS, also at 30 arcsecond resolution [11]. I include these covariates in the regressions (5) and (6) as control variables on the right-hand side. I present estimates of

[11] The climatic variables are: (1) Annual Mean Temperature, (2) Mean Diurnal Range, (3) Isothermality, (4) Temperature Seasonality, (5) Max Temperature of Warmest Month, (6) Min Temperature of Coldest Month, (7) Temperature Annual Range, (8) Mean Temperature of Wettest Quarter, (9) Mean Temperature of Driest Quarter, (10) Mean Temperature of Warmest Quarter, (11) Mean Temperature of Coldest Quarter, (12) Annual Precipitation, (13) Precipitation of Wettest Month, (14) Precipitation of Driest Month, (15) Precipitation Seasonality, (16) Precipitation of Wettest Quarter, (17) Precipitation of Driest Quarter, (18) Precipitation of Warmest Quarter, and (19) Precipitation of Coldest Quarter. The land cover categories are: (1) Evergreen Needleleaf Forest, (2) Evergreen Broadleaf Forest, (3) Deciduous Needleleaf Forest, (4) Deciduous Broadleaf Forest, (5) Mixed Forest, (6) Closed Shrublands, (7) Open Shrublands, (8) Woody Savannas, (9) Savannas, (10) Grasslands, (11) Permanent Wetlands, (12) Croplands, (13) Urban and Built-Up, (14) Cropland/Natural Vegetation Mosaic.

the main coefficients of interest from both the levels and growth regressions in the second row of Table 5. It is clear that including the covariates does not change the results, and, if anything, strengthens them slightly. I also rerun the specifications given by equation (5) replacing the dependent variable by each of the 35 geographic variables used. Out of 140 possible tests, exactly 8 reject at 5%, which is approximately the number that one would expect if none of the geographic variables were discontinuous. I present the local linear regressions for some geographic variables thought to be potentially important for economic growth in Table 6: population density, altitude, ruggedness, mean temperature, temperature standard deviation, mean precipitation and fraction of land that is cropland. All the variables are scaled to have mean zero and standard deviation equal to unity. We see that richer sides of borders may be more rugged than poorer sides are, which is counterintuitive because ruggedness is typically associated with lower economic activity (Nunn and Puga 2009). They also have a lower degree of seasonal temperature variation, which is more alarming as a more moderate climate may improve economic activity, but as the magnitudes of the coefficients show, this tendency is very low even though statistically significant. All the other important measures show no tendency to be different on richer sides of borders from poorer sides of borders. There is also no tendency of these measures to be different on higher-growing sides of borders and lower-growing sides of borders.

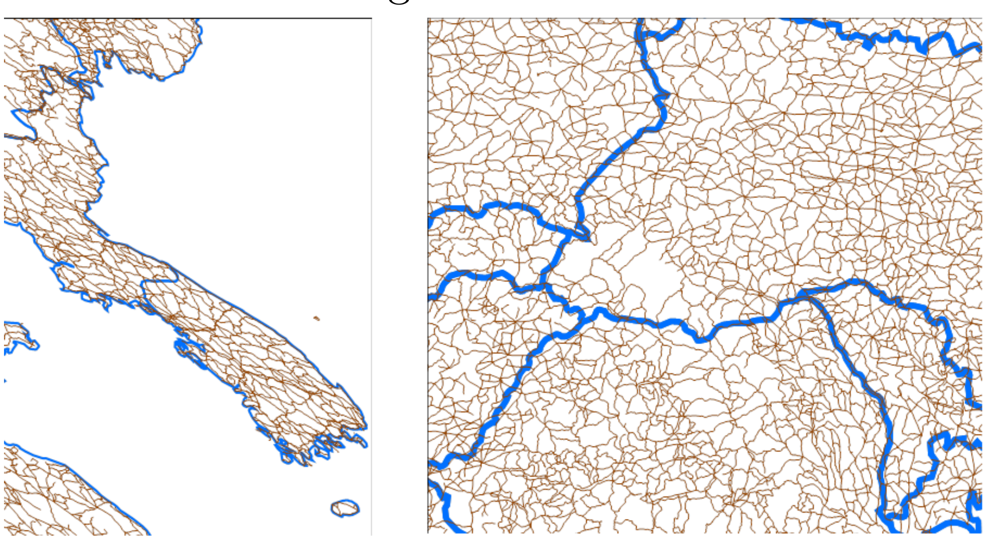
4.5 Local Variation in Public Goods

I now consider potential explanations for discontinuities in economic activity across borders. First, I check whether public goods, for which extensive geographic information exists, are discontinuous across borders. If public goods are more extensively provided on richer sides of borders than on poorer sides of borders, it is conceivable that the output and growth differentials between the two sides are explained by the local effects of public goods. An example of local public goods driving income differentials is found by Dell (2010), who documents that road density falls discontinuously as one crosses a border into a Peruvian region in which forced labor was practiced during colonial times, and provides anecdotal evidence that road quality in that region is critical for access to markets.

I obtain spatial data on roads, railroads and utility lines (power and telephone lines) from the Digital Chart of the World. In results not reported, I confirm that the national aggregates of roads and railroads match up well with national-level statistics presented by the World Bank, and that there is a strong positive association between the log total public goods in a fixed neighborhood of a border and the amount of lights per capita in that neighborhood. In Table 7, I obtain estimates of discontinuities in the densities of these public goods variables per capita across national borders, which I construct using the two-step local linear procedure described in Section 4. I normalize the dependent variable to have mean zero and unit variance in all specifications. We see that road density is not systematically higher on the sides of borders corresponding to richer- or higher-growing countries (the estimates of the difference are positive, as might be expected if higher road density is associated with higher economic growth or development, but they are quantitatively small and statistically insignificant). An important component of public goods provision not captured by the road density maps is road quality. The Digital Chart of the World classifies roads as "primary or secondary roads," "divided highways" and "paths or trails." Presumably, trails are of lower quality than are the other two types of roads, so the fraction of roads that are not trails should be a reasonable measure of road quality. However, as we see in column 2, the fraction of roads that are not trails does not vary discontinuously across borders. Column 3 of Table 7 presents the results for railroads. The estimates are larger, but not significant (except the estimate for differences in GDP growth rates, which is significant at 10%). Finally, column 4 presents the same analysis with the dependent variable being log total length of utility lines (most frequently these are power lines, but also included are telephone lines and pipelines). The results are even more striking because they show that poorer sides of borders have more utility lines than richer sides of borders do, the result being statistically significant. Lower-growth sides of borders also have more utility lines than do higher-growth sides of borders, though this result is not statistically significant. This finding is particularly surprising given that the electricity used to generate the lights almost certainly is going through the power lines being measured. A reconciliation of this finding with the baseline result may be that the power lines are used with different intensity on different sides of borders, with more (and more energy-intensive) houses and factories using the power lines on richer sides of borders than on poorer sides.

It is instructive to look at maps of road density for the regions in which we observed significant discontinuities in the level and growth rate of light density per capita. In contrast, these regions appear to have continuous road density across borders. Superimposed on a road map of the Korean peninsula, the North Korea-South Korea border is imperceptible. In Eastern Europe, road networks rarefy in the sparsely populated Carpathian mountain region that straddles Ukraine and Romania, but have the same density on either side of the Ukraine-Poland or the Romania-Moldova borders. Hence, it is plausible that road networks are continuous across borders more generally, and hence, that discontinuities in economic activity at borders are not accounted for by variation in road networks.

Figure 8 (8)



Road density on the Korean Peninsula and in Eastern Europe

Finally, I estimate the baseline regression with levels of public goods being included as independent variables. Rows 2 and 4 Table 5 present the results for my baseline specification with the different varieties of public goods included as independent variables, either by themselves (in row 2) or together with geographical and climate controls (row 1). The coefficients on the country-level output variables are very similar in magnitude and significance to the baseline in Table 3. Therefore, variation in local public goods is unlikely to account for border discontinuities in economic activity. However, variation in the national level of public goods still may be important if the main benefits from public goods are global rather than local (e.g. a high national level of public goods enriches everyone sufficiently that places with low local public goods benefit from having richer trading partners).

5 Potential Mechanisms

I now turn to exploring the association between discontinuities in the growth rate of lights per capita at borders and national-level variables of the bordering countries that are commonly hypothesized to be determinants of economic growth. This analysis will not use any further sources of identification and will be much closer to a correlational study of determinants of growth. However, because areas close to national borders can be considered as having been "randomly assigned" vectors of determinants of growth, whereas countries as a whole obviously have developed these vectors endogenously over the course of history, an important source of endogeneity will be absent. If an exhaustive list of determinants of growth existed, the identification problem in trying to explain discontinuities in economic activity at the border would be solved. While such a list does not exist, accounting for its most important components is conceivable.

The past decade has been extremely fruitful in investigating how government activity may affect economic growth. Acemoglu et al. (2000, 2001) argued that the degree of property rights protection and the constraints faced by the country's executive have a strong causal impact on economic activity. La Porta et al. (1998) and, less formally, de Soto (2000) have suggested that contracting institutions such as the simplicity of procedures to enforce contracts and trade property improve economic growth. However, Acemoglu and Johnson (2005) argue that controlling for property rights institutions, contracting institutions do not matter for growth. Glaeser et al. (2004) and more recently La Porta et al. (2008) contend that an important determinant of economic growth is human capital, which in almost all countries is substantially affected by government policies. Algan and Cahuc (2010) find a causal effect of trust on growth, and a line of thought stretching back to Weber (1910) posits that cultural attitudes formed by religion affect economic growth. Since government policies tend to homogenize religion and culture within their borders, these variables may also be discontinuous across borders, and thus, may be potential channels through which border discontinuities in economic activity arise.

While the literature has identified institutions of various kinds to be a fundamental cause of economic growth, it has not yet fully explained the channels through which these institutions bring about growth. One strand of thought, dating back to the founding of economics as a science, has considered that security of property enables citizens to optimize their well-being by trading and producing in the market, bringing about prosperity without much further action from the government except to enforce the property rights system. In the words of Adam Smith (1776) : "Little else is required to carry a state to the highest degree of affluence from the lowest barbarism but peace, easy taxes, and a tolerable administration of justice; all the rest being brought about by the natural course of things." An alternative theory, however, may be that secure property rights and tight constraints on the executive enable citizens to control their government and ensure that it provides adequate amounts of high-quality public goods, the latter being necessary for development and long-run growth. While I am unaware of recent work postulating this theory, the literature on economic growth and political economy has established both theoretically and empirically that a causal channel of institutions on growth that runs through public goods is conceivable. Dell (2010) and

Huillery (2009) document the importance of public goods for positive economic outcomes. Acemoglu (2005) presents a model in which "consensually strong states," in which the citizens are strong relative to the government, collect higher taxes and spend more on public goods (as the citizens desire) than states in which government is not checked by citizens.

The regression of interest is

$$\tilde{g}_{i,b}^{w} = \alpha_b + \gamma_g^w g_i^w + Z_i' \Delta + \varepsilon_{i,b}^w \tag{7}$$

where Z_i is a vector of country-level determinants of growth and Δ is a vector of their coefficients, which, together with γ_g^w, the correlation between differences in growth at the border and differences in national growth, form the variables of interest. All other variables are defined as in equation (6). I use the 20-year growth rate of Penn World Table 7.1 GDP as my independent variable for the growth rate.[12]

The determinants of growth that I consider are as follows: the measure of the rule of law obtained from the World Governance Indicators, which are maintained by the World Bank; fraction of roads paved (a public goods variable motivated by Gennaioli and Rainer [2007]) obtained from the World Bank's World Development Indicators (WDI) for 1990 or the closest year; the amount of time necessary to enforce a contract (a measure of contracting institutions), also from the WDI for 1990; an index of political freedom based on the Freedom House measure, obtained from Acemoglu, Johnson, Robinson and Yared (2008); average years of education in 1990, obtained from Barro and Lee (2010); and the proportion of respondents from the given country answering that "most people can be trusted" in the World Values Survey, from La Porta et al. (2008). The governance measures, public goods and contracting institutions variables span nearly the whole sample of countries, while other variables tend to be missing for many countries, which removes borders from analysis.

[12] The results are not robust to using the nationwide 20-year growth rate of light density in place of the 20-year Penn World Tables growth rate. However, if the climate and local public goods controls are included, the results are qualitatively similar to the GDP-based results, although none of the coefficients except for the 20-year nationwide lights growth rate are significant.

A key variable in the analysis will be the rule of law variable from WGI, which is defined as "the extent to which agents have confidence in and abide by the rules of society, and in particular the quality of contract enforcement, property rights, the police, and the courts, as well as the likelihood of crime and violence." This variable (as well as the other WGI indicators) has been constructed using an unobserved components model using expert evaluations and surveys of businesses, NGOs and other agencies. A list of the questions in the surveys that were used to construct this variable is available at

http://info.worldbank.org/governance/wgi/pdf/rl.pdf

The questions from these surveys used to construct the rule of law variable ask about security of property, the willingness of the government to honor its contractual obligations and follow its own laws, and the independence of the judiciary, as well as the ease of using the judicial system to enforce private contracts, which makes the rule of law variable a composite of a measure of private property protection and of a measure of contracting institutions. However, since most of the questions used do not concern the enforcement of private contracts, I consider that the rule of law variable is much closer to a measure of private property institutions than it is to contracting institutions. I use the rule of law variable in this analysis rather than other measures of property rights institutions such as protection against expropriation risk from Political Risk Services or the property rights variable from the Fraser Institute because the rule of law variable is available for many more countries than either of these two variables. In Appendix Tables A1 and A2, I estimate the specifications in (8) using modified versions of these measures (with missing values for one measure predicted using the other measure) and obtain very similar results.

Table 8 presents results from estimating equation (7) for several versions of the vector of determinants Z_i using the two-step local linear procedure described in Section 4. For comparability purposes, all covariates (except the World Bank growth rate) have been standardized before regression, so the unit of each covariate is a standard deviation.

Column 1 reproduces the baseline estimate of γ_g, the baseline association between border and national differences of growth, for comparability purposes. Column 2 includes the rule of law as a covariate. We see that the coefficient on the rule of law is positive and statistically significant, with a 1 standard deviation increase in the difference of bordering countries' expropriation protections being associated with an increase in the difference of their average annualized 20-year growth rates at the border of 2.5 percentage points. The coefficient on the difference in national growth rates is halved (to 0.44) and becomes only marginally statistically significant.[13] Column 3 leaves out the rule of law, but includes public goods (percent of roads paved). We see that public goods with a coefficient of about 1.8, hence a 1 standard deviation increase in the difference between two countries' fractions of roads paved increases the difference of their 20-year growth rates at the border by 1.8 percentage points. However, this coefficient is not statistically significant at conventional levels, and the coefficient on the national growth rate, γ_g, remains significant, with nearly the same magnitude as in the baseline specification (0.74). Column 4 includes both rule of law and public goods. We now see that the coefficient on the rule of law remains very close to the one from the second specification (2.31) and is significant at 1%, while the coefficient on public goods shrinks markedly to 0.67. The coefficient on the national growth rate shrinks to a marginally significant 0.44. Thus, controlling for the rule of law, public goods do not appear to matter for growth. Column 5 adds the climate and local public goods covariates at borders from Table 5. The estimate of the rule of law coefficient is unaffected, but the paved roads coefficient turns negative (though it remains insignificant) and the coefficient on the countrywide 20-year growth rate becomes slightly larger (0.54) and significant at 5%. In subsequent columns I retain the climate and local public goods controls to reduce the standard error of the regression; my results do not change if these controls are excluded.

[13] The magnitudes of the two coefficients are not comparable because expropriation protection has been standardized while the national growth rate has not. I estimate this equation using beta coefficients, and find a beta coefficient of 0.1 for the national growth rate and a beta coefficient of 0.25 for expropriation protection. Therefore, the magnitude of the association between expropriation protection and border growth, conditional on border fixed effects, is about 2.5 times larger than the magnitude between the association between national growth and border growth when both are expressed in like units

These results, especially column 4, show that of two ways that institutions that can matter for growth – creating a rule of law to protects private property in the market, and creating a consensual state to provide public goods – only the rule of law appears to matter for economic growth at national borders, in accordance with Adam Smith. Moreover, the rule of law is such an important variable that once differences in the respect for the rule of law are accounted for, differences in national growth rates are of limited use in predicting differences in border growth rates.

The additional columns of Table 8 present further checks of the importance of property rights protection compared with other determinants of growth. Column 6 compares private property institutions and contracting institutions by including the amount of time necessary to enforce a contract as a covariate; the coefficient is positive (counter to expectations) and statistically insignificant. The coefficient on the rule of law variable remains unchanged. Another hypothesis can be that property rights protection is proxying for political freedom. Column 7 replaces the contracting institutions variable with average value of the Freedom House Political Rights Index between 1990 and 1999. We see that, if anything, political freedom appears to decrease growth rates at the border once economic freedom is controlled for.[14] The coefficient on the rule of law variable rises by over 50% to 3.9. However, the Freedom House Political Rights Index is not available for some countries, which reduces my sample of borders by about one-third. In Column 10, I add the estimate of average years of education from Barro and Lee (2010) to the specification in Column 5. Like the Freedom House measure, the Barro-Lee education measure is not available for some borders, which reduces my sample. The coefficient on the rule of law variable rises somewhat compared to Column 5 (to 3.3), and the coefficient on the Barro-Lee variable is negative, large in magnitude, and statistically significant at 5% (-2.7). Finally, column 9 includes a variable for trust measured by the World Values Survey.[15] Unfortunately, data on trust from the WVS

[14] The coefficient on the Freedom House variable is negative and statistically significant at 5%, but is only significant at 10% if the climate and local public goods covariates are excluded.

[15] The exact text of the relevant WVS question is: "Generally speaking, would you say that most people can be trusted or that you need to be very careful in dealing with people?" The two responses are "Most

is available for only 80 countries, which severely restricts the sample. Therefore, I construct an augmented trust variable by predicting trust using region dummies for countries without trust data. Given that cultural variables tend to be similar within regions (see the map by Ingelhart and Welzel, http://www.worldvaluessurvey.org/), this approach appears to be reasonable. The coefficient on the rule of law remains very close to the baseline, and the coefficient on trust is negative and insignificant. In all the above specifications, the coefficient on the fraction of roads paved is statistically insignificant, and frequently negative. Hence, the rule of law appears to be resilient to controlling for other potential determinants of growth.

Border Permeability

An important concern for interpreting border discontinuities as estimates of the impact of government activity upon the economy is that national borders act as discontinuous increases in transaction costs. Even in a world with countries having identical institutions, public goods provision and other political structures, we could then still see discontinuities in economic activity at borders because local economic shocks would not transmit to neighboring countries but would remain contained in the country of origin. If borders were completely impermeable to trade, different market equilibria would prevail in each country, and shocks that affect one country's equilibrium would have no effect on its neighboring country, thus creating a discontinuity in prices, wages, quantities, and most likely economic activity and growth across the countries' common border.

I test whether the transaction costs channel is important by looking at whether the presence of trade mitigates discontinuities in economic activity at borders, and whether accounting for trade changes the ways in which the covariates discussed in this section affect discontinuities at borders. Since the transaction costs channel should manifest itself through differences in trade flows, if the amount of trade across the border (suitably normalized to

people can be trusted," and "Can't be too careful."

account for obvious determinants of trade) does not affect whether or not border discontinuities are present, then it is unlikely that borders pose sufficient barriers to trade to explain the existence of border discontinuities. For the key independent variable of this part of the analysis, I obtain data on bilateral trade for all borders in 2000 from the IMF's Direction of Trade Statistics. As my trade variable, I use the log volume of trade, normalized by the product of the bordering countries' GDPs to take into account gravity effects. This trade variable is obviously at the border level, and hence is captured by border fixed effects, so I use it only to create interaction terms in regressions.

The regression of interest becomes

$$\tilde{y}_{i,b,t} = \alpha^w_{b,t} + \left(\gamma^w + t^w_\beta \times T_b\right) y^w_{i,t} + Z'_i \left(\Delta + \tau_Z \times T_b\right) + \varepsilon^w_{i,b,t} \tag{8}$$

where T_b is the trade measure, and $t^{d,w}_g$ and τ are coefficients on the interactions of this trade measure with the national growth measure and with the covariates, respectively. All other variables are as in equations (6) and (7). For ease of interpretation, I standardize T_b to have mean zero and variance 1.

Table 9 presents some key specifications from the previous sections estimated using the two-step local linear regression methodology in Section 4. The first four columns present results for reestimating the baseline equation (6) with trade interactions. The coefficient on national output per capita or national growth in output per capita decreases, and the interaction with trade enters negatively for the regressions of log output per capita (the coefficient being insignificant if lights data is used to construct the output measure and marginally significant at -0.23 if GDP data is used), which suggests that some of the discontinuities in output per capita might be explained by trade. However, the trade interaction coefficients are statistically insignificant, small and positive for the growth rate regressions, which contradicts the hypothesis that trade effaces border discontinuities, and hence, that border discontinuities can be explained by trade. The subsequent four columns replicate columns 2, 3, and 4 of Table 8, the analysis of the rule of law and public goods. We see that nothing important changes: the magnitudes of the interaction coefficients are tiny, and the stylized facts of the rule of law explaining border discontinuities and of public goods not explaining them as well remain.

In theory, the volume of trade should be a sufficient statistic for economic interaction between two countries, regardless of the source of the barriers to such interactions. However, trade flows are not a good proxy for economic interactions between two countries because of the difficulty of valuing trade in services, or because some trade might be informal and not recorded in official statistics. Appendix Tables A3 through A5 therefore provide estimates of equation 8 in which the trade measure T_b is constructed using average tariffs between two countries (obtained from the World Bank), migration between the two countries normalized by their populations (again, from the World Bank) and genetic distance between the two countries (from Spolaore and Wacziarg 2009). Note that the tariff barriers and genetic distance measures should be negatively correlated with border permeability, so positive rather than negative interaction coefficients suggest that higher border permeability decreases discontinuities at borders. We observe that discontinuities in growth rates, and their relationship to differences in the rule of law and public goods provision on different sides of borders, are not affected by accounting for differential border permeability (in fact, for the trade barriers measure, lower tariffs at borders appear to increase rather than decrease the magnitude of the discontinuities). The border permeability effects on discontinuities in levels are small (or the wrong sign) for some measures, but larger for others. One potential explanation may be that the economic channel for creating discontinuities at borders operates over a longer time horizon than does the political channel, since economic shocks may propagate slowly over a country, while changes in incentives brought about by changes in political institutions may be immediate. Hence, border permeability may not affect discontinuities in economic growth but might contribute to discontinuities in levels of economic development.

6 Conclusion

This paper uses satellite data on nighttime lights to find large and statistically significant discontinuities in economic activity at national borders. More surprising than the discontinuities in the levels of economic activity is the finding that there are equally large and significant discontinuities in growth rates of output over a 20-year period. Furthermore, I derive the properties of local polynomial estimators when the data generating process remains bounded but increases in resolution asymptotically, and use the derived variance formulae to compute local polynomial estimates of border discontinuities. In addition, I propose a novel procedure for removing overglow from nighttime lights data in a way that does not respect national borders and implement it to improve the performance of local linear estimation.

I interpret the estimated discontinuities in economic activity at the border as measures of the absolute effect of government activity (both short-run through policy and long-run through institutions) on economic activity. An alternative explanation for border discontinuities could be that they arise because borders discretely raise transaction costs, thus creating discontinuities between prices, wages and other nonpolitical variables across the border. Using data on trade flows between countries, I show that such an explanation is implausible because the magnitudes of border discontinuities do not seem to respond to differences in the flow of trade across the borders. I present a correlational analysis of potential determinants of border discontinuities, and find that the discontinuities in economic growth across borders can be explained by differences in the rule of law. In particular, once the rule of law is accounted for, differences in national growth no longer have a statistically significant association with differences in growth at the border, and other potential determinants of growth such as public goods, education, contracting institutions, political freedom and interpresonal trust either do not matter or do not account for the impact of the rule of law on growth.

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7 Tables

Table 1

Descriptive Statistics				
Panel 1: Lights on the Border and Nationwide				
	Mean, Overall	Mean, Low Ntwide Side	Mean, High Ntwide Side	Number of Borders
Log Lights per Capita, Border	4.73	4.52	4.93	270
	(1.74)	(1.78)	(1.67)	
Log Lights per Capita Nationwide	8.00	7.58	8.41	270
	(1.40)	(1.47)	(1.19)	
Growth in Lights per Capita, Border	2.70	1.37	4.13	270
	(4.54)	(4.09)	(4.56)	
Growth in Lights per Capita, Nationwide	3.38	1.94	4.92	270
	(3.28)	(2.84)	(3.02)	
Log Lights per Capita, Border	4.73	4.59	4.86	270
	(1.74)	(1.76)	(1.70)	
Log GDP per Capita, WB	7.87	7.60	8.15	270
	(1.21)	(1.23)	(1.14)	
Growth in Lights per Capita, Border	2.70	1.95	3.47	270
	(4.54)	(4.18)	(4.76)	
Growth in GDP per Capita, WB	2.48	1.57	3.43	270
	(1.91)	(1.70)	(1.62)	
	Panel 2: Covariates			
	Mean, Overall	Mean, Low Ntwide Growth GDP Side	Mean, High Ntwide Growth GDP Side	Number of Borders
Log Roads in 30-km Border Neighborhood	3.02	2.96	3.09	270
	(1.67)	(1.74)	(1.60)	
Log Population	16.06	15.99	16.14	270
	(2.05)	(2.11)	(1.97)	
Rule of Law, WB	-.34	-.49	-.19	266
	(.91)	(.90)	(.89)	
Percent of Roads Paved, WDI	-.26	-.30	-.22	264
	(.90)	(1.00)	(.77)	
Average years of Education, BL	-.57	-.63	-.51	186
	(.88)	(.85)	(.91)	
Trust, WVS	.19	.03	.32	98
	(.83)	(.76)	(.86)	

(1)

Descriptive Statistics. Standard deviations are in parentheses. There are two observations per border piece: one for the poorer (or lower-growing) side, and one for the richer (or higher-growing) side. Data for lights at border and their growth rate are for 70-km neighborhoods around the border. Data for roads in 70-km neighborhood of border from Digital Chart of the World. Data for rule of law and fraction of roads paved from the World Bank and WGI. Data for education from Barro-Lee (2010). Data for trust from WVS.

Table 2

Overglow into Wasteland			
Dep. Var. is Light Density in Wasteland			
	(1)	(2)	(3)
Light Density, 10 km Away	.23***	.29***	.35***
	(.02)	(.05)	(.05)
Light Density, 20 km Away		-.03*	-.06*
		(.02)	(.03)
Light Density, 30 km Away			.02
			(.01)
No. Observations	2949	1996	1123
No. Squares	888	768	631
R^2	.55	.59	.63
P-value higher lags are 0		.1	.17

(2)

Overglow Correction. Each observation corresponds to a 1-degree grid square on a world map, split up into 10-km wide bands that are parallel to the frontier between wasteland and non-wasteland in that square. Wasteland areas are defined according to CIESIN as shown in Figure 7.

Table 3

Local Linear Estimates of Border Discontinuities								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dep. Var.	Log Lights p/c	Log Lights p/c	Log Lights p/c	Log Lights p/c	Growth, Lights p/c	Growth, Lights p/c	Growth, Lights p/c	Growth, Lights p/c
Indep. Var.	Dummy Log Lights p/c	Log Lights p/c	Dummy Log GDP p/c	Log GDP p/c	Dummy Growth, Lights p/c	Growth, Lights p/c	Dummy Growth, GDP p/c	Growth, GDP p/c
Baseline Estimates (H=51)	.58**	.65***	.40*	.63**	3.56***	1.29***	2.63**	.88***
	(.23)	(.23)	(.23)	(.29)	(.98)	(.30)	(1.07)	(.34)
Baseline Slope	.04	.04	.05	.10	-.20	-.07	-.12	-.03
	(.08)	(.08)	(.08)	(.11)	(.31)	(.09)	(.31)	(.10)
Estimates with BW=30 km	.28	.46**	.14	.56**	4.10***	1.49***	2.39**	.97***
	(.20)	(.22)	(.18)	(.28)	(.92)	(.28)	(1.02)	(.31)
Placebo Estimates at -30 km	.16	.05	.14	-.09	-.15	-.09	.37	.06
	(.25)	(.24)	(.25)	(.28)	(.70)	(.22)	(.67)	(.24)
Placebo Estimates at +30 km	-.02	-.01	-.04	-.07	-.12	.15	.40	.12
	(.19)	(.15)	(.19)	(.19)	(1.13)	(.33)	(1.09)	(.45)
Baseline Estimates, No Correction	.25*	.23*	.12	.23	1.11*	.46***	1.26**	.44**
	(.13)	(.13)	(.12)	(.14)	(.60)	(.15)	(.57)	(.20)
Baseline Slope, No Correction	.31***	.37***	.33***	.45***	.63***	.21***	.38*	.16*
	(.08)	(.08)	(.08)	(.10)	(.20)	(.06)	(.21)	(.08)
Baseline Estimates, 10 km off	.53***	.58***	.39***	.61***	2.41***	.93***	2.53***	.89***
	(.15)	(.16)	(.14)	(.17)	(.73)	(.21)	(.68)	(.28)
Baseline Slope, 10 km off	.18**	.19**	.20**	.26**	.00	-.01	-.20	-.01
	(.09)	(.09)	(.09)	(.11)	(.23)	(.08)	(.23)	(.11)
Border-Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
No. Observations	6760	6760	6760	6760	1352	1352	1352	1352
No. Borders	270	270	270	270	270	270	270	270
R^2 of first row	.02	.03	.02	.03	.00	.00	.00	.00
							(3)	

Data on lights and population available from the NOAA and CIESIN, respectively, for 1990 (1992 for lights), 1995, 2000, 2005 and 2010. Observation unit is a country-border piece-year. Robust standard errors clustered on border and taking into account infill asymptotics in parentheses. Each observation weighted by population in the respective buffer piece.

Table 4

Border Discontinuities by Continent					
	(1)	(2)	(3)	(4)	(5)
Continent	OECD	Post-Soviet	Asia	Africa	America
Indicator, Log Lights per Capita	.30	.67***	.84**	.06	.75*
	(.22)	(.23)	(.42)	(.29)	(.42)
Log Lights per Capita	1.24*	1.30***	.80***	-.03	.70**
	(.66)	(.37)	(.31)	(.28)	(.35)
Indicator, Log GDP per Capita	.09	.34	.84**	-.30	.30
	(.26)	(.27)	(.38)	(.27)	(.34)
Log GDP per Capita	.67	.66*	1.14***	-.25	.66*
	(1.07)	(.38)	(.42)	(.27)	(.40)
Indicator, Growth Lights per Capita	.89*	10.44***	5.31***	-.25	2.16
	(.48)	(2.92)	(1.33)	(1.36)	(1.40)
Growth Lights per Capita	.93***	2.54***	1.66***	.19	1.93*
	(.32)	(.56)	(.45)	(.27)	(1.08)
Indicator, Growth GDP per Capita	-.61	6.53*	3.79**	1.11	-.18
	(.48)	(3.45)	(1.66)	(1.32)	(1.54)
Growth GDP per Capita	-.32	3.22**	1.16**	.14	1.42
	(1.13)	(1.62)	(.52)	(.33)	(1.10)
Border-Year Fixed Effects	Yes	Yes	Yes	Yes	Yes
No. Observations	122	174	420	442	240
No. Borders	23	45	75	93	36
R^2 of first row	.00	.14	.08	.00	.04

(4)

Data on lights and population available from the NOAA and CIESIN, respectively, for 1990 (1992 for lights), 1995, 2000, 2005 and 2010. Observation unit is a country-border piece-year. Robust standard errors clustered on border and taking into account infill asymptotics in parentheses. Each observation weighted by population in the respective buffer piece.

Table 5

Border Discontinuities with Controls								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dep. Var.	Log Lights p/c	Log Lights p/c	Log Lights p/c	Log Lights p/c	Growth, Lights p/c	Growth, Lights p/c	Growth, Lights p/c	Growth, Lights p/c
Indep. Var.	Dummy Log Lights p/c	Log Lights p/c	Dummy Log GDP p/c	Log GDP p/c	Dummy Growth, Lights p/c	Growth, Lights p/c	Dummy Growth, GDP p/c	Growth, GDP p/c
Baseline Estimates	.58**	.65***	.40*	.63**	3.56***	1.29***	2.63**	.88***
	(.23)	(.23)	(.23)	(.29)	(.98)	(.30)	(1.07)	(.34)
Climate Controls	.70***	.90***	.47**	.69***	3.62***	1.28***	2.74***	.92***
	(.19)	(.20)	(.18)	(.25)	(.83)	(.26)	(.97)	(.28)
Local Public Goods Ctrls.	.60**	.68***	.40*	.62**	3.69***	1.30***	2.54***	.83***
	(.23)	(.25)	(.23)	(.30)	(.88)	(.27)	(.95)	(.31)
All Controls	.72***	.92***	.45***	.66***	3.68***	1.25***	2.53***	.84***
	(.18)	(.21)	(.17)	(.25)	(.75)	(.23)	(.90)	(.25)
Border-Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
No. Observations	6760	6760	6760	6760	1352	1352	1352	1352
No. Borders	270	270	270	270	270	270	270	270
R^2 of first row	.04	.08	.02	.04	.12	.17	.06	.04

(5)

Data on lights and population available from the NOAA and CIESIN, respectively, for 1990 (1992 for lights), 1995, 2000, 2005 and 2010. Observation unit is a country-border piece-year. Robust standard errors clustered on border and taking into account infill asymptotics in parentheses. All control variables rescaled to have mean 0 and variance 1, and are described in the text. Each observation weighted by population in the respective buffer piece.

Table 6

Behavior of Geographical Covariates across Borders							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Dep. Var.	Log Population	Log Altitude	Log Slope	Mean Temperature	Log SD Temp.	Log Precipitation	Fraction Cropland
Indicator, Log Lights	-.01	.01	.12**	-.01	-.01***	.00	-.02
	(.04)	(.02)	(.05)	(.01)	(.00)	(.00)	(.03)
Indicator, Log GDP	-.02	.01	.09*	-.01*	-.00*	.00	-.03
	(.03)	(.02)	(.05)	(.00)	(.00)	(.00)	(.02)
Indicator, Growth Lights	-.01	.00	.06	-.00	-.00	-.00	-.03
	(.05)	(.03)	(.07)	(.01)	(.00)	(.01)	(.03)
Indicator, Growth GDP	-.05	.01	.09	-.01	-.00	.00	.00
	(.05)	(.03)	(.07)	(.01)	(.00)	(.01)	(.03)
Border-Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
No. Observations	1352	1352	1352	1352	1352	1352	1352
No. Borders	270	270	270	270	270	270	270
R^2 of last row	.00	.00	.07	.02	.01	.00	.00

6

Data on lights and population available from NOAA and CIESIN. Data on altitude and slope obtained from SRTM. Data on climatic variables available from WorldClimate (Hijmans et al. 2005). All climate variables rescaled to have mean 0 and variance 1. Observation unit is a country-border piece-year. The last (10 km) observation on each side of the border is excluded to minimize overglow. Robust standard errors clustered on border and taking into account infill asymptotics in parentheses. Each observation weighted by population in the respective buffer piece.

Table 7

Behavior of Local Public Goods across Borders				
	(1)	(2)	(3)	(4)
Dep. Var.	Log Roads	Fraction Roads	Log Railroads	Log Utilities
Indicator, Log Lights	-.01	-.00	.12	-.64***
	(.06)	(.01)	(.14)	(.21)
Indicator, Log GDP	.03	-.01	.13	-.48**
	(.05)	(.01)	(.13)	(.21)
Indicator, Growth Lights	.07	-.01	.16	-.14
	(.08)	(.01)	(.16)	(.26)
Indicator, Growth GDP	.01	.00	.29*	-.25
	(.08)	(.01)	(.16)	(.25)
Border-Year Fixed Effects	Yes	Yes	Yes	Yes
No. Observations	1352	1352	1352	1352
No. Borders	270	270	270	270
R^2 of last row	.00	.00	.02	.00

7

Data on lights and population available from NOAA and CIESIN. Data on roads, railroads and road type available from the Digital Chart of the World for 2003. Observation unit is a country-border piece-year. All dependent variables rescaled to have mean 0 and variance 1. Robust standard errors clustered on border and taking into account infill asymptotics in parentheses. Each observation weighted by population in the respective buffer piece.

Table 8

Correlates of Border Discontinuities *Dep. Var. is 20-Year Growth Rate of Light per Capita*									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
20-Yr. Growth, GDP	.88***	.44*	.74***	.44*	.54**	.54	.35	.28	.52**
	(.34)	(.26)	(.26)	(.25)	(.25)	(.33)	(.37)	(.38)	(.26)
Rule of Law, WB		2.54***		2.23***	2.31***	2.34***	3.93***	3.32***	2.35**
		(.85)		(.69)	(.59)	(.60)	(1.12)	(.69)	(.63)
Frac. of Roads Paved, WB			1.80	.67	-.56	-.52	1.09	-.69	-.54
			(1.11)	(.91)	(.86)	(.91)	(.91)	(.90)	(.87)
Time to Enforce Contract						.47			
						(.79)			
Freedom House Score							-3.32**		
							(1.60)		
Schooling, Barro-Lee								-2.66**	
								(1.25)	
Predicted Trust, WVS									-.22
									(.62)
Border-Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Clim. and Pub. Gds. Ctrls.	No	No	No	No	Yes	Yes	Yes	Yes	Yes
No. Observations	1352	1342	1336	1336	1336	1282	952	968	1336
No. Borders	270	266	264	264	264	254	180	184	264
R^2	.04	.10	.07	.10	.28	.29	.43	.38	.28

(8)

Data on lights and population available from NOAA and CIESIN. Data on determinants of growth described in the text. All covariates except 20-year growth normalized to have mean 0 and variance 1. Robust standard errors clustered on border and taking into account infill asymptotics in parentheses. Each observation weighted by population in the respective buffer piece.

Table 9

Permeability: Trade							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Dep. Var.	Log Lights	Log Lights	Growth, Lights	Growth, Lights	Growth, Lights	Growth, Lights	Growth, Lights
Nat. Output / Growth	.54**	.44*	1.33***	.87**	.46	.73**	.40
	(.23)	(.25)	(.32)	(.40)	(.34)	(.36)	(.35)
Nat. Output / Growth X Trade	-.15	-.28*	.10	-.00	.07	.13	.15
	(.12)	(.15)	(.17)	(.15)	(.15)	(.17)	(.15)
Rule of Law					2.62***		2.39***
					(.89)		(.76)
Rule of Law X Trade					-.36		-.24
					(.55)		(.48)
Fraction of Roads Paved						1.83	.53
						(1.16)	(1.00)
Fraction of Roads Paved X Trade						-.75	-.86
						(.80)	(.71)
Output Source	Lights	GDP	Lights	GDP	GDP	GDP	GDP
Border-Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
No. Observations	6760	6760	1352	1352	1342	1336	1336
No. Borders	270	270	270	270	266	264	264
R^2	.09	.06	.17	.04	.10	.07	.11

(9)

Data on lights and population available from NOAA and CIESIN. Data on trade is from IMF's Direction of Trade Statistics for 2000. Trade volume normalized by product of bordering country GDPs and to have mean zero and unit variance. Observation unit is a country-border piece-year. Robust standard errors clustered on border and taking into account infill asymptotics in parentheses. Each observation weighted by population in the respective buffer piece.

8 Appendix I: Additional Tables

Table A1

Correlates of Border Discontinuities: Expropriation Risk as Rule of Law Measure								
Dep. Var. is 20-Year Growth Rate of Light per Capita								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
20-Yr. Growth, GDP	.88***	.50*	.74***	.49*	.58**	.70**	.68**	.41
	(.34)	(.26)	(.26)	(.25)	(.24)	(.31)	(.34)	(.36)
Avg. Protection ctr. Exp. AJR		2.54***		2.44***	2.55***	2.67***	3.32***	3.63***
		(.89)		(.92)	(.82)	(.87)	(.87)	(.96)
Fraction of Roads Paved, WB			1.80	.34	-.88	-.75	.96	-1.24
			(1.11)	(.87)	(.93)	(.95)	(.78)	(1.06)
Time to Enforce Contract						.30		
						(.81)		
Freedom House Score							-1.84*	
							(1.11)	
Schooling, Barro-Lee								-1.97*
								(1.06)
Predicted Trust, WVS								
Border-Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Climate and Public Goods Ctrls.	No	No	No	No	Yes	Yes	Yes	Yes
No. Observations	1352	1342	1336	1336	1336	1282	952	968
No. Borders	270	266	264	264	264	254	180	184
R^2	.04	.14	.07	.14	.31	.32	.46	.43

(A1)

Data on lights and population available from NOAA and CIESIN. Data on determinants of growth described in the text. All covariates except 20-year growth normalized to have mean 0 and variance 1. Robust standard errors clustered on border and taking into account infill asymptotics in parentheses. Each observation weighted by population in the respective buffer piece.

Table A2

Correlates of Border Discontinuities: Alternative Rule of Law Measure									
Dep. Var. is 20-Year Growth Rate of Light per Capita									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(
20-Yr. Growth, GDP	.88***	.69**	.74***	.64**	.71***	.76**	.75*	.59	.7
	(.34)	(.27)	(.26)	(.25)	(.24)	(.32)	(.40)	(.37)	(.
Property Rights, Fraser Institute		2.00**		1.70**	1.78***	1.92***	1.91***	2.65***	1.7
		(.82)		(.69)	(.56)	(.59)	(.65)	(.65)	(.
Fraction of Roads Paved, WB			1.80	.97	-.28	-.22	1.97*	-.31	-
			(1.11)	(.92)	(.88)	(.92)	(1.09)	(.93)	(.
Time to Enforce Contract						.55			
						(.80)			
Freedom House Score							-1.16		
							(1.25)		
Schooling, Barro-Lee								-1.99*	
								(1.16)	
Predicted Trust, WVS									.
									(.
Border-Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Y
Climate and Public Goods Ctrls.	No	No	No	No	Yes	Yes	Yes	Yes	Y
No. Observations	1352	1342	1336	1336	1336	1282	952	968	13
No. Borders	270	266	264	264	264	254	180	184	2
R^2	.04	.09	.07	.10	.27	.28	.40	.38	.

(A2)

Data on lights and population available from NOAA and CIESIN. Data on determinants of growth described in the text. All covariates except 20-year growth normalized to have mean 0 and variance 1. Robust standard errors clustered on border and taking into account infill asymptotics in parentheses. Each observation weighted by population in the respective buffer piece.

Table A3

Permeability: Migration							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Dep. Var.	Log Lights	Log Lights	Growth, Lights	Growth, Lights	Growth, Lights	Growth, Lights	Growth, Lights
Nat. Output / Growth	.66***	.68**	1.40***	.99**	.59*	.87***	.59*
	(.24)	(.30)	(.30)	(.39)	(.30)	(.32)	(.30)
Nat. Output / Growth X Migration	.01	.11	.21	.15	.20	.17	.20
	(.11)	(.13)	(.13)	(.14)	(.16)	(.14)	(.15)
Rule of Law					2.86***		2.53***
					(.91)		(.77)
Rule of Law X Migration					.07		.07
					(.55)		(.55)
Fraction of Roads Paved						2.01*	.71
						(1.20)	(1.00)
Fraction of Roads Paved X Migration						.03	-.04
						(.70)	(.72)
Output Source	Lights	GDP	Lights	GDP	GDP	GDP	GDP
Border-Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
No. Observations	6760	6760	1352	1352	1342	1336	1336
No. Borders	270	270	270	270	266	264	264
R^2	.08	.04	.18	.05	.11	.07	.11

(A3)

Data on lights and population available from NOAA and CIESIN, respectively, for 1990, 1995, 2000, 2005 and 2010. Data on migration from the World Bank. Migration normalized by product of bordering country populations and to have mean zero and unit variance. Observation unit is a country-border piece-year. Robust standard errors clustered on border and taking into account infill asymptotics in parentheses. Each observation weighted by population in the respective buffer piece.

Table A4

Permeability: Tariffs							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Dep. Var.	Log Lights	Log Lights	Growth, Lights	Growth, Lights	Growth, Lights	Growth, Lights	Growth, Lights
Nat. Output / Growth	.63***	.60*	1.47***	1.05**	.60*	.90**	.59*
	(.24)	(.31)	(.30)	(.45)	(.35)	(.36)	(.34)
Nat. Output / Growth X Barriers	.11	.06	-.66**	-.34	-.33	-.36	-.34
	(.22)	(.26)	(.26)	(.34)	(.30)	(.31)	(.30)
Rule of Law					2.76***		2.44***
					(.98)		(.86)
Rule of Law X Barriers					.00		-.07
					(.82)		(.75)
Fraction of Roads Paved						1.99*	.75
						(1.17)	(.97)
Fraction of Roads Paved X Barriers						.43	.49
						(.91)	(.85)
Output Source	Lights	GDP	Lights	GDP	GDP	GDP	GDP
Border-Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
No. Observations	6620	6620	1324	1324	1316	1310	1310
No. Borders	261	261	261	261	258	256	256
R^2	.09	.04	.21	.05	.11	.08	.11

(A4)

Data on lights and population available from NOAA and CIESIN, respectively, for 1990, 1995, 2000, 2005 and 2010. Data on tariff barriers from the World Bank. Tariff barriers normalized to have mean zero and unit variance. Observation unit is a country-border piece-year. Robust standard errors clustered on border and taking into account infill asymptotics in parentheses. Each observation weighted by population in the respective buffer piece.

Table A5

Trade Channel: Genetic Distance							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Dep. Var.	Log Lights	Log Lights	Growth, Lights	Growth, Lights	Growth, Lights	Growth, Lights	Growth, Lights
Nat. Output / Growth	.59***	.53**	1.29***	.90***	.47*	.77***	.45*
	(.20)	(.24)	(.28)	(.34)	(.25)	(.26)	(.25)
Nat. Output / Growth X Gen. Dist.	-.24**	-.31*	-.25	.22	.33**	.24	.32**
	(.11)	(.17)	(.21)	(.18)	(.15)	(.16)	(.16)
Rule of Law					2.69***		2.52***
					(.94)		(.77)
Rule of Law X Gen. Dist.					-.32		-.31
					(.55)		(.58)
Fraction of Roads Paved						1.80	.58
						(1.17)	(1.07)
Fraction of Roads Paved X Gen. Dist.						-.55	.19
						(.52)	(.51)
Output Source	Lights	GDP	Lights	GDP	GDP	GDP	GDP
Border-Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
No. Observations	6760	6760	1352	1352	1342	1336	1336
No. Borders	270	270	270	270	266	264	264
R^2	.11	.06	.18	.05	.11	.08	.11

(A5)

Data on lights and population available from NOAA and CIESIN, respectively, for 1990, 1995, 2000, 2005 and 2010. Data on genetic distance is from Spolaore and Wacziarg. Genetic distance normalized to have mean zero and unit variance. Observation unit is a country-border piece-year. Robust standard errors clustered on border and taking into account infill asymptotics in parentheses. Each observation weighted by population in the respective buffer piece.

9 Appendix II: Proof of Proposition

Setup

I consider the properties of the standard local polynomial estimator in an infill asymptotics context. The core result is that the local polynomial estimator has smaller standard errors if the random shocks of the stochastic process are correlated than if they are not, and that a feasible and consistent estimator exists to estimate these standard errors.

To present the results formally, define the following notation: let

$$G = \left\{ \frac{u}{N} \right\}_{u=-N}^{N}$$

be a sequence with resolution N. Let s denote a generic element of G. Note that the domain of G remains bounded, and is contained in $[-1, 1]$, which is an arbitrary interval in R up to a normalization. The process is assumed to have a discontinuity at $s = 0$. In particular, we are interested in using the sequence

$$\left\{ y \left(\frac{u}{N} \right) \right\}_{u=-N}^{N}$$

where $y(s)$ is a scalar, to predict the value

$$y_+ (0) := \lim_{u \downarrow 0} y (u)$$

The value y_- is defined analogously:

$$y_- (0) := \lim_{u \uparrow 0} y (u)$$

The discontinuity at 0 is defined to be

$$\bar{\Delta} = y_+ (0) - y_- (0)$$

Estimating $y_+ (0)$ and $y_- (0)$ is of central interest in this document.

Define the local polynomial estimator of $y_+ (0)$ by

$$\hat{\alpha}_N^+ = e_1' \left(\sum_{u=1}^{N} \left(\frac{1}{Nh} \right) k \left(\frac{u}{Nh} \right) X \left(\frac{u}{Nh} \right) X \left(\frac{u}{Nh} \right)' \right)^{-1} \left(\sum_{u=1}^{N} \left(\frac{1}{Nh} \right) k \left(\frac{u}{Nh} \right) X \left(\frac{u}{Nh} \right) y \left(\frac{u}{N} \right) \right)$$

where $X()$ is a $K \times 1$ vector of polynomials, and $k\left(\frac{u}{Nh}\right)$ is a positive kernel. Define the local polynomial estimator of $y_- (0)$ similarly by $\hat{\alpha}_N^-$. For convenience, define $D_N = \sum_{u=1}^{N} \left(\frac{1}{Nh}\right) k \left(\frac{u}{Nh}\right) X \left(\frac{u}{Nh}\right) X \left(\frac{u}{Nh}\right)'$, the denominator of this expression and note that it is nonstochastic and converges to $D = \int_0^\infty k(u) X(u) X(u)' du$. Then, the local polynomial estimator of the discontinuity at zero is given by

$$\hat{\Delta}_N = \hat{\alpha}_N^+ - \hat{\alpha}_N^-$$

Finally, I define the *increment* of a stochastic process $y(s)$ to be given by

$$y_{N,s} = y(s) - y\left(s - \frac{1}{N}\right)$$

Assumptions on the Bandwidth and Kernel

B1. Define the bandwidth $h(N)$. Then, $\lim_{N\to\infty} h(N) = 0$, $\lim_{N\to\infty} Nh(N) = \infty$ and $\lim_{N\to\infty} Nh(N)^2 = 0$

B2. The kernel $k()$ satisfies $\int_0^\infty \left(\int_u^\infty k(v) v^p dv \right) du < \infty$ for all $p \leq K$.

Assumptions on $y(s)$

We assume that we can decompose $y(s)$ as

$$y(s) = F(s) + v(s) + e(s)$$

Consider some assumptions on the components of $y(s)$:

1. $F(s) \in C^1[0,1]$ is a deterministic function.

2. $v(s)$ is a random shock that is independent across realizations: $E(v(s)) = 0$, $E(v(s)v(t)) = \bar{V}(s) \cdot 1(s = t)$, with $\bar{V}(s) \in C[0,1]$, and $E\left(v(s)^{2+\delta}\right) \leq K < \infty$.

2' $v(s)$ is identically zero.

3. $e(s)$ is a random shock such that $E(e(s)) = 0$, $E(e(s)v(t)) = 0$, but

$$E(e(s)e(t)) = C(s,t)$$

for some function $C(s,t)$ that a) belongs to $C^2\left\{(s,t) \in [0,1]^2 : s \neq t\right\}$, b) satisfies $\lim_{t \to s^+} C(s,t) = \lim_{t \to s^-} C(s,t)$, and c) satisfies $V(s) = C(s,s) \in C^1([0,1])$. Moreover,

$$\operatorname{cov}\left(Ne_{N,u}^2, Ne_{N,u'}^2\right) = O\left(\operatorname{cov}\left(\sqrt{N}e_{N,u}, \sqrt{N}e_{N,u'}\right)^2\right)$$

3' $e(s)$ is defined as in Assumption 3, but with an additional assumption.

$$\sigma(s) := V'(s) - 2\lim_{N \to \infty} C_1\left(s, s - \frac{1}{N}\right) \geq 0,\ \forall s \in [0,1]$$

4. The increments of the error process $e_{N,u} = e\left(\frac{u}{N} + \frac{1}{N}\right) - e\left(\frac{u}{N}\right)$ are associated if N is large enough.

It is insightful to compare these assumptions with standard assumptions from the geostatistics literature, which deals with spatially correlated processes. A very general assumption in that literature is that the zero-mean stochastic process $y(s)$ has a stationary variogram, which is continuous everywhere except at zero. Hence,

$$E(y(s)y(t)) = \frac{1}{2}[V(s) + V(t)] - \gamma(|s-t|)$$

where γ is a continuous function with $\gamma(0) = 0$ and $\lim_{s \to 0}\gamma(s) =: c_0 > 0$. Defining $\tilde{\gamma}(s) = \gamma(s) - c_0$, we can then write

$$E(y(s)y(t)) = \frac{1}{2}[V(s) + V(t)] - \tilde{\gamma}(|s-t|) + c_0 \cdot (s = t)$$

Now, under Assumptions 1, 2 and 3, we have

$$E((v(s) + e(s))(v(t) + e(t))) = C(s,t) + \bar{V}(s) \cdot (s = t)$$

where $C(s,t)$ is any positive definite, continuous function satisfying some smoothness conditions, and $\bar{V}(s)$ is also a continuous function. Hence, any sufficiently smooth (except at zero) variogram and variance function satisfy these assumptions. It is immediate that any random field with $c_0 = 0$ satisfies assumptions 1, 2' and 3. Moreover, the geostatistics literature tends to work with Gaussian stochastic processes (or Gaussian random fields), and it is known that jointly Gaussian random variables x and y satisfy $\operatorname{cov}\left(x^2, y^2\right) = 2\operatorname{cov}(x,y)^2$ by Isserlis's Theorem. Therefore, any Gaussian random field with a sufficiently smooth variogram and variance function satisfies Assumptions 1, 2 and 3'. Therefore, the set of assumptions considered is extremely general.

Propositions

We then have the following propositions (with $V(0)$ being the variance of $e(0)$ and $\bar{V}(0)$ being the variance of $v(0)$)

Proposition 1 *Suppose that Assumptions 1, 2 and 3 hold. Then, $\hat{\alpha}_N^+$ consistently estimates $y_+(0) - v_+(0)$, and*

$$\begin{aligned}\sqrt{Nh}\left(\hat{\alpha}_N^+ - \left(y_+(0) - v_+(0)\right)\right) \quad &\rightarrow \quad {}^d N\left(0, V_1\right) \\ V_1 \quad : &= \bar{V}(0)\, e_1' D^{-1}\left(\int_0^{\infty}\left(k(u)\right)^2 X(u) X(u)' du\right) D^{-1} e_1\end{aligned}$$

Moreover, a feasible and consistent estimator of V_1 is

$$\hat{V}_{1,N} = \frac{1}{2} e_1' D_N^{-1}\left[\sum_{u=1}^{N} \hat{e}_{N,u}^2 \frac{1}{Nh} k^2\left(\frac{u}{Nh}\right) X\left(\frac{u}{Nh}\right) X\left(\frac{u}{Nh}\right)'\right] D_N^{-1} e_1$$

Proposition 2 *Suppose that Assumptions 1, 2 and 3 hold. Then the White estimator of the variance of $\hat{\alpha}_N^+$ asymptotically overestimates the true variance in expectation. Specifically,*

$$NhE\left(\hat{V}_{OLS}\right) \rightarrow \left(\bar{V}(0) + V(0)\right) e_1' D^{-1}\left(\int_0^{\infty}\left(k(u)\right)^2 X(u) X(u)' du\right) D^{-1} e_1$$

Proposition 3 *Suppose that Assumptions 1, 2' , 3' and 4 hold. Then, $\hat{\alpha}_N^+$ consistently estimates $y_+(0)$ and*

$$\begin{aligned}\left(\sqrt{h}\right)^{-1}\left(\alpha_N^+ - y_+(0)\right) \quad &\rightarrow \quad {}^d N\left(0, V_2\right) \\ V_2 \quad : &= \sigma(0)\, e_1' D^{-1}\left[\left(\int_0^{\infty}\left(\int_u^{\infty} k(v) X(v)\, dv\right)\left(\int_u^{\infty} k(v) X(v)\, dv\right)' du\right)\right] D^{-1} e_1\end{aligned}$$

Moreover, a feasible and consistent estimator of V_2 is

$$\begin{aligned}\hat{V}_2 \quad = \quad & \frac{1}{2} e_1' D_N^{-1}\left[\sum_{u=1}^{N} \frac{1}{Nh} k\left(\frac{u}{Nh}\right) X\left(\frac{u}{Nh}\right) y_{N,u}^2\right] \times \\ & e_1' D_N^{-1}\left[\frac{1}{h(N)} \sum_{u=1}^{N}\left(\sum_{v=u}^{N} \frac{1}{Nh} k\left(\frac{v}{Nh}\right) X\left(\frac{v}{Nh}\right)\right)\left(\sum_{v=u}^{N} \frac{1}{Nh} k\left(\frac{v}{Nh}\right) X\left(\frac{v}{Nh}\right)\right)'\right] D_N^{-1} e_1\end{aligned}$$

Proof of Proposition 1

Suppose that Assumptions 1, 2 and 3 hold. Consider

$$\begin{aligned}Z_N^+ \quad &= \quad \alpha_N^+ - y_+(0) \\ &= \quad e_1' D_N^{-1}\left(\sum_{u=1}^{N}\left(\frac{1}{Nh}\right) k\left(\frac{u}{Nh}\right) X\left(\frac{u}{Nh}\right)\left(F\left(\frac{u}{N}\right) - F_+(0)\right)\right) \\ &\quad + e_1' D_N^{-1}\left(\sum_{u=1}^{N}\left(\frac{1}{Nh}\right) k\left(\frac{u}{Nh}\right) X\left(\frac{u}{Nh}\right)\left(v\left(\frac{u}{N}\right) - v_+(0)\right)\right) \\ &\quad + e_1' D_N^{-1}\left(\sum_{u=1}^{N}\left(\frac{1}{Nh}\right) k\left(\frac{u}{Nh}\right) X\left(\frac{u}{Nh}\right)\left(e\left(\frac{u}{N}\right) - e_+(0)\right)\right)\end{aligned}$$

I first show that α_N^+ is asymptotically unbiased: $\lim_{N\rightarrow\infty} E\left(Z_N^+\right) = 0$. To do this, I present a very simple lemma that will be useful in further analysis.

Lemma 4 *: Suppose* $\lim\limits_{N\to\infty}\sum_{u=1}^{N}\left(\frac{1}{Nh}\right)S\left(\frac{u}{Nh}\right)=\int_0^\infty S(u)\,du$ *exists in* R*, and* $F(x)$ *is a continuous function on* $[0,1]$*. Then,*

$$\lim_{N\to\infty}\sum_{u=1}^{N}\left(\frac{1}{Nh}\right)S\left(\frac{u}{Nh}\right)F\left(\frac{u}{N}\right)=F(0)\int_0^\infty S(u)\,du$$

Proof.

$$\begin{aligned}\sum_{u=1}^{N}\left(\frac{1}{Nh}\right)S\left(\frac{u}{Nh}\right)\left|F\left(\frac{u}{N}\right)-F(0)\right| \quad &\leq \quad \sup_{u\in[0,\tau]}|F(u)-F(0)|\sum_{u=1}^{N}\left(\frac{1}{Nh}\right)S\left(\frac{u}{Nh}\right)\\ &\quad +\sup_{u\in[0,1]}|F(u)-F(0)|\sum_{u=\tau N}^{\tau N}\left(\frac{1}{Nh}\right)S\left(\frac{u}{Nh}\right)\end{aligned}$$

Since F is continuous, we have $\lim\limits_{\tau\to 0}\sup\limits_{u\in[0,\tau]}|F(u)-F(0)|=0$. Moreover, since $\sum_{u=1}^{N}\left(\frac{1}{Nh}\right)S\left(\frac{u}{Nh}\right)$ is a Riemann sum, it is immediate that

$$\lim_{N\to\infty}\sum_{u=1}^{\tau N}\left(\frac{1}{Nh}\right)S\left(\frac{u}{Nh}\right)=\int_0^\infty S(u)\,du \text{ and } \lim_{N\to\infty}\sum_{u=\tau N}^{N}\left(\frac{1}{Nh}\right)S\left(\frac{u}{Nh}\right)=0,\ \forall\tau>0$$

Therefore, for any ε, we can find a $\tau(\varepsilon)$ small enough that $\sup\limits_{u\in[0,\tau(\varepsilon)]}|F(u)-F(0)|<\varepsilon$, and an N large enough that $\sum_{u=1}^{\tau(\varepsilon)N}\left(\frac{1}{Nh}\right)S\left(\frac{u}{Nh}\right)=(1-\varepsilon)\sum_{u=1}^{N}\left(\frac{1}{Nh}\right)S\left(\frac{u}{Nh}\right)$.

Hence, $\sum_{u=1}^{N}\left(\frac{1}{Nh}\right)S\left(\frac{u}{Nh}\right)\left|F\left(\frac{u}{N}\right)-F(0)\right|\leq\varepsilon\left[(1-\varepsilon)+\sup\limits_{u\in[0,1]}|F(u)-F(0)|\right]$, and taking the limit as ε goes to zero completes the proof. ■

To prove $\lim\limits_{N\to\infty}E\left(Z_N^+\right)=0$, I apply Lemma 4 (because F is continuous):

$$\lim_{N\to\infty}E\left(Z_N^+\right)=\lim_{N\to\infty}e_1'D_N^{-1}\left(\sum_{u=1}^{N}\left(\frac{1}{Nh}\right)k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)\left(F\left(\frac{u}{N}\right)-F_+(0)\right)\right)=0$$

Now, I show that $\lim\limits_{N\to\infty}\operatorname{var}\left(Z_N^++v_+(0)\right)=0$

$$\begin{aligned}\operatorname{var}\left(Z_N^++v_+(0)\right) &= E\left(\left(Z_N^++v_+(0)\right)^2\right)-E\left(Z_N^++v_+(0)\right)^2\\ &= E\left(\left[e_1'D_N^{-1}\left(\sum_{u=1}^{N}\left(\frac{1}{Nh}\right)k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)v\left(\frac{u}{N}\right)\right)\right]^2\right)\ \text{(I)}\\ &\quad +E\left(\left[e_1'D_N^{-1}\left(\sum_{u=1}^{N}\left(\frac{1}{Nh}\right)k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)\left(e\left(\frac{u}{N}\right)-e_+(0)\right)\right)\right]^2\right)\ \text{(II)}\end{aligned}$$

Consider term (I). We have

$$E\left(\left[e_1'D_N^{-1}\left(\sum_{u=1}^{N}\left(\frac{1}{Nh}\right)k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)v\left(\frac{u}{N}\right)\right)\right]^2\right)=\frac{1}{Nh}e_1'D_N^{-1}\left(\sum_{u=1}^{N}\left(\frac{1}{Nh}\right)k^2\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)'\bar{V}\left(\frac{u}{N}\right)\right)D_N^{-1}e_1$$

where $\bar{V}\left(\frac{u}{N}\right)=E\left(v\left(\frac{u}{N}\right)^2\right)$. Now, since $\lim\limits_{N\to\infty}\sum_{u=1}^{N}\left(\frac{1}{Nh}\right)k^2\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)'=\int_0^\infty k^2(u)X(u)X(u)'\,du$ and $\bar{V}(u)$ is continuous, we can appeal to Lemma 4 to argue that

$$\lim_{N\to\infty} NhE\left(\left[e_1' D_N^{-1}\left(\sum_{u=1}^{N}\left(\frac{1}{Nh}\right)k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)v\left(\frac{u}{N}\right)\right)\right]^2\right) \to \bar{V}_+(0)\, e_1' D^{-1}\left(\int_0^\infty k^2(u)\,X(u)\,X(u)'\,du\right)D^{-1}e_1 =: V_1$$

because

$$\lim_{N\to\infty}\sum_{u=1}^{N}\left(\frac{1}{Nh}\right)k^2\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)'\left|\bar{V}\left(\frac{u}{N}\right)-\bar{V}_+(0)\right| = 0$$

Now, consider term (II). We can rewrite it in terms of increments $e_{N,u} = e\left(\frac{u}{N}\right) - e\left(\frac{u}{N}-\frac{1}{N}\right)$ as the following:

$$\text{(II)} = \left(e_1' D_N^{-1}\sum_{u=1}^{N}\left(\sum_{v=u}^{N}\frac{1}{Nh}k\left(\frac{v}{Nh}\right)X\left(\frac{v}{Nh}\right)\right)e_{N,u}\right)^2$$

by noting that

$$e\left(\frac{v}{N}\right) - e_+(0) = \sum_{u=1}^{v}\left(e\left(\frac{u}{N}\right) - e\left(\frac{u}{N}-\frac{1}{N}\right)\right) = \sum_{u=1}^{v} e_{N,u}$$

We can further break down term (II) by noting that

$$\begin{aligned}
\text{(II)} \quad = \quad & e_1' D_N^{-1}\sum_{u=1}^{N}\left(\sum_{v=u}^{N}\frac{1}{Nh}k\left(\frac{v}{Nh}\right)X\left(\frac{v}{Nh}\right)\right)\left(\sum_{v=u}^{N}\frac{1}{Nh}k\left(\frac{v}{Nh}\right)X\left(\frac{v}{Nh}\right)\right)' E\left(e_{N,u}^2\right)D_N^{-1}e_1 \quad (1)\\
& + e_1' D_N^{-1}\sum_{u=1}^{N}\sum_{\substack{u'=1\\ u'\neq u}}^{N}\left(\sum_{v=u}^{N}\frac{1}{Nh}k\left(\frac{v}{Nh}\right)X\left(\frac{v}{Nh}\right)\right)\left(\sum_{v=u'}^{N}\frac{1}{Nh}k\left(\frac{v}{Nh}\right)X\left(\frac{v}{Nh}\right)\right)' E\left(e_{N,u}e_{N,u'}\right)D_N^{-1}e_1 \quad (2)
\end{aligned}$$

Now, by Assumption 3, we have

$$\begin{aligned}
E\left(e_{N,u}e_{N,u'}\right) &= C(s,t) - C\left(s,t-\frac{1}{N}\right) - C\left(s-\frac{1}{N},t\right) + C\left(s-\frac{1}{N},t-\frac{1}{N}\right)\\
&= \left(C(s,t) - C\left(s,t-\frac{1}{N}\right)\right) - \left(C\left(s-\frac{1}{N},t\right) - C\left(s-\frac{1}{N},t-\frac{1}{N}\right)\right)\\
&\to \frac{1}{N^2}\frac{\partial^2}{\partial s\partial t}C(s,t) = O\left(\frac{1}{N^2}\right)
\end{aligned}$$

and

$$\begin{aligned}
E\left(e_{N,u}^2\right) &= V(s) - 2C\left(s,s-\frac{1}{N}\right) + V\left(s-\frac{1}{N}\right)\\
&= \left(V(s) - V\left(s-\frac{1}{N}\right)\right) - 2\left(C\left(s,s-\frac{1}{N}\right) - C\left(s-\frac{1}{N},s-\frac{1}{N}\right)\right)\\
&= \frac{1}{N}\left[\Delta V(s) - 2\Delta_1 C\left(s,s-\frac{1}{N}\right)\right] \to \frac{1}{N}\left[V'(s) - 2\lim_{N\to\infty}C_1\left(s,s-\frac{1}{N}\right)\right] = O\left(\frac{1}{N}\right)
\end{aligned}$$

if $C(s,t)$ is not twice differentiable at $s=t$ with uniformly bounded second derivatives. (If it is twice differentiable at $s=t$, then $E\left(e_{N,u}^2\right) = O\left(\frac{1}{N^2}\right)$).

Under the boundedness conditions on $\frac{\partial^2}{\partial s\partial t}C(s,t)$, $C_1\left(s,s-\frac{1}{N}\right)$ and $V'(s)$, the convergence can be taken as uniform, and

$$\sup_{(s,t)\in[0,1]^2}\frac{\partial^2}{\partial s\partial t}C(s,t) \le K_2 < \infty$$

and

$$\sup_{(s,t)\in[0,1]^2}\left[V'(s)-2\lim_{N\to\infty}C_1\left(s,s-\frac{1}{N}\right)\right]\leq K_1<\infty$$

Therefore,

$$\begin{aligned}\limsup_{N\to\infty}(\mathrm{II}) &\leq hK_1e_1'D^{-1}\left(\int_0^\infty\left(\int_u^\infty k(v)X(v)\right)\left(\int_u^\infty k(v)X(v)\right)'du\right)D^{-1}e_1\\ &+h^2K_2e_1'D^{-1}\left[\left(\int_0^\infty\left(\int_u^\infty k(v)X(v)\right)du\right)\left(\int_0^\infty\left(\int_u^\infty k(v)X(v)\right)du\right)'\right]D^{-1}e_1\end{aligned}$$

and hence,

$$(\mathrm{II})=O(h)$$

Therefore,

$$\operatorname{var}\left(Z_N^+ + v_+(0)\right)=\frac{1}{Nh}V_1+O\left(Nh^2\right)\to 0 \text{ as } N\to\infty$$

and consistency is proved.

Asymptotic Distribution

To find the asymptotic distribution of α_N^+, we note that

$$\begin{aligned}\sqrt{Nh}\left(\alpha_N^+-(y_+(0)-v_+(0))\right) &= \sqrt{Nh}e_1'D_N^{-1}\left(\sum_{u=1}^N\left(\frac{1}{Nh}\right)k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)\left(F\left(\frac{u}{N}\right)-F_+(0)\right)\right) \quad (1)\\ &+\sqrt{Nh}e_1'D_N^{-1}\left(\sum_{u=1}^N\left(\frac{1}{Nh}\right)k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)v\left(\frac{u}{N}\right)\right) \quad (2)\\ &+\sqrt{Nh}e_1'D_N^{-1}\left(\sum_{u=1}^N\left(\frac{1}{Nh}\right)k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)\left(e\left(\frac{u}{N}\right)-e_+(0)\right)\right) \quad (3)\end{aligned}$$

By decomposing Term 1 into increments as in the proof of consistency, we see that

$$(1)=\sqrt{Nh}e_1'D_N^{-1}\sum_{u=1}^N\left(\sum_{v=u}^N\left(\frac{1}{Nh}\right)k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)\right)F_{N,u}$$

Since $F(s)$ is continuously differentiable by Assumption 1, we have that $F_{N,u}=\frac{1}{N}\Delta F_{N,u}\to\frac{1}{N}F'\left(\frac{u}{N}\right)$. Under the boundedness conditions of Assumption 1, we therefore have

$$\sup_{s\in[0,1]}F'(s)\leq K<\infty$$

Therefore,

$$\begin{aligned}(1) &\leq K\sqrt{Nh}e_1'D_N^{-1}\sum_{u=1}^N\frac{1}{N}\left(\sum_{v=u}^N\left(\frac{1}{Nh}\right)k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)\right)\\ &= Kh\sqrt{Nh}e_1'D_N^{-1}\sum_{u=1}^N\frac{1}{Nh}\left(\sum_{v=u}^N\left(\frac{1}{Nh}\right)k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)\right)\end{aligned}$$

and

$$(1)\to K\left(\lim_{N\to\infty}h\sqrt{Nh}\right)e_1'D^{-1}\left(\int_0^\infty\left(\int_u^\infty k(v)X(v)\,dv\right)du\right)$$

Hence, if $\lim_{N\to\infty} h\sqrt{Nh} = 0$, the asymptotic bias is zero. This is implied by the bandwidth assumption $\lim_{N\to\infty} Nh^2 = 0$.

By the variance results from the consistency proof, term (2) is $O_p(1)$, while term (3) is $o_p(1)$. Therefore, we are interested only in the asymptotic distribution of term (2). Since $v(s)$ is independent of $v(t)$ for all t and s, term (2) is a sum of independent random variables. To satisfy the hypotheses of the Liapunov Central Limit Theorem, we must prove that

$$\lim_{N\to\infty} \sum_{u=1}^{N} \text{var}\left(\sqrt{Nh} e_1' D_N^{-1} \left(\frac{1}{Nh}\right) k\left(\frac{u}{Nh}\right) X\left(\frac{u}{Nh}\right) v\left(\frac{u}{N}\right)\right) < \infty$$

and

$$\lim_{N\to\infty} \sum_{u=1}^{N} E\left[\left(\sqrt{Nh} e_1' D_N^{-1} \left(\frac{1}{Nh}\right) k\left(\frac{u}{Nh}\right) X\left(\frac{u}{Nh}\right) v\left(\frac{u}{N}\right)\right)^{2+\delta}\right] = 0 \ \exists \delta > 0$$

The first condition follows from the computation of the variance. The second condition follows because

$$\begin{aligned}
& \lim_{N\to\infty} \sum_{u=1}^{N} E\left[\left(\sqrt{Nh} e_1' D_N^{-1} \left(\frac{1}{Nh}\right) k\left(\frac{u}{Nh}\right) X\left(\frac{u}{Nh}\right) v\left(\frac{u}{N}\right)\right)^{2+\delta}\right] \\
= & \lim_{N\to\infty} \frac{1}{(Nh)^{\delta/2}} \sum_{u=1}^{N} \left(\frac{1}{Nh}\right) \left[\left(e_1' D_N^{-1} X\left(\frac{u}{Nh}\right)\right)^{2+\delta}\right] k^{2+\delta}\left(\frac{u}{Nh}\right) E\left(v^{2+\delta}\left(\frac{u}{N}\right)\right) \\
= & \lim_{N\to\infty} \frac{1}{(Nh)^{\delta/2}} \cdot \int_0^{\infty} \left[\left(e_1' D^{-1} X(u)\right)^{2+\delta}\right] k^{2+\delta}(u) E\left(v^{2+\delta}(u)\right) du = 0
\end{aligned}$$

since $E\left(v^{2+\delta}(u)\right) \leq K < \infty$. Therefore, the hypothesis of the Liapunov CLT are satisfied, and we have

$$\sqrt{Nh}\left(\hat{\alpha}_N^{+} - \left(y_{+}(0) - v_{+}(0)\right)\right) \to^{d} N\left(0, V_1\right)$$

as desired.

Feasible Estimation

Finally, I show that a feasible and consistent estimator of V_1 is

$$\hat{V}_{1,N} = \frac{1}{2} e_1' D_N^{-1} \left[\sum_{u=1}^{N} \hat{e}_{N,u}^2 \frac{1}{Nh} k^2\left(\frac{u}{Nh}\right) X\left(\frac{u}{Nh}\right) X\left(\frac{u}{Nh}\right)'\right] D_N^{-1} e_1$$

which replaces the residuals in the OLS estimator with the residual increments $e_{N,u}^2$.

To prove this, we show that $\hat{V}_{1,N}^{A} = Nh\frac{1}{2}\sum_{u=1}^{N} \hat{e}_{N,u}^2 \left(\frac{1}{Nh} k\left(\frac{u}{Nh}\right) X\left(\frac{u}{Nh}\right)\right)\left(\frac{1}{Nh} k\left(\frac{u}{Nh}\right) X\left(\frac{u}{Nh}\right)\right)'$ is a consistent estimator of $V_{1,N}^{A} = \bar{V}(0)\int_0^{\infty} k^2(u) X(u) X(u)' du$

First, we see that $\hat{V}_{1,N}^{A}$ is asymptotically unbiased for $V_{1,N}^{A}$:

$$\begin{aligned}
E\left(\hat{V}_{1,N}^{A}\right) &= \frac{1}{2}\sum_{u=1}^{N} \frac{1}{Nh}\left(k\left(\frac{u}{Nh}\right) X\left(\frac{u}{Nh}\right)\right)\left(k\left(\frac{u}{Nh}\right) X\left(\frac{u}{Nh}\right)\right)' E\left(\hat{e}_{N,u}^2\right) \\
&= \frac{1}{2} e_1' D_N^{-1} \left[\left(\begin{array}{c}
\sum_{u=1}^{N} \frac{1}{Nh}\left(k\left(\frac{u}{Nh}\right) X\left(\frac{u}{Nh}\right)\right)\left(k\left(\frac{u}{Nh}\right) X\left(\frac{u}{Nh}\right)\right)' \times \\
E\left(v\left(\frac{u}{N}\right) - v\left(\frac{u-1}{N}\right)\right)^2 + E\left(\hat{e}_{N,u}^2\right) \\
+E\left(\left(\frac{1}{N}\Delta F_{N,u} - \frac{1}{N}\Delta X\left(\frac{u}{N}\right)' E\left(\hat{\beta}\right) + \frac{1}{N}\Delta X\left(\frac{u}{N}\right)'\left(E\left(\hat{\beta}\right) - \beta\right)\right)^2\right) \\
+E\left(\left(\frac{1}{N}\Delta F_{N,u} - \frac{1}{N}\Delta X\left(\frac{u}{N}\right)' E\left(\hat{\beta}\right) + \frac{1}{N}\Delta X\left(\frac{u}{N}\right)'\left(E\left(\hat{\beta}\right) - \beta\right)\right)\left(v_{N,u} + e_{N,u}\right)\right)
\end{array}\right)\right] \\
&= \frac{1}{2} e_1' D_N^{-1} \left[\sum_{u=1}^{N} \frac{1}{Nh}\left(k\left(\frac{u}{Nh}\right) X\left(\frac{u}{Nh}\right)\right)\left(k\left(\frac{u}{Nh}\right) X\left(\frac{u}{Nh}\right)\right)'\left(\left(\bar{V}\left(\frac{u}{N}\right) + \bar{V}\left(\frac{u-1}{N}\right)\right) + o(1)\right)\right]
\end{aligned}$$

Now, invoking Lemma 4, it is clear that $\lim_{N\to\infty}\left[\sum_{u=1}^{N}\frac{1}{Nh}\left(k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)\right)\left(k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)\right)'\bar{V}\left(\frac{u}{N}\right)\right]=\bar{V}(0)\int_0^\infty k^2(u)\,X(u)\,X(u)'$ since $\bar{V}\left(\frac{u}{N}\right)$ is continuous. The analysis for $\sum_{u=1}^{N}\frac{1}{Nh}\left(k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)\right)\left(k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)\right)'\bar{V}\left(\frac{u}{N}\right)$ is similar. Hence, $E\left(\hat{V}_{1,N}\right)\to V_1$.

To compute the variance of $\hat{V}_{1,N}^{A}$, note that

$$\begin{aligned}
E\left(\hat{e}_{N,u}^2\hat{e}_{N,u'}^2\right) &= E\left(v_{N,u}^2v_{N,u'}^2\right)+E\left(e_{N,u}^2e_{N,u}^2\right)+E\left(v_{N,u}^2\right)E\left(e_{N,u'}^2\right)+E\left(v_{N,u'}^2\right)E\left(e_{N,u}^2\right)+O\left(\frac{1}{N^2}\right)\\
&= E\left(v_{N,u}^2v_{N,u'}^2\right)+o(1)
\end{aligned}$$

Moreover,

$$E\left(v_{N,u}^2v_{N,u'}^2\right)-E\left(v_{N,u}^2\right)E\left(v_{N,u'}^2\right)=\begin{cases}
\left[E\left(v\left(\frac{u-1}{N}\right)^4\right)-\left(E\left(v\left(\frac{u-1}{N}\right)^2\right)\right)^2\right]=:S_1\left(\frac{u}{N}\right)\ \text{ if } |u-u'|=1\\
\left[\begin{array}{c}\left(E\left(v\left(\frac{u}{N}\right)^4\right)-\left(E\left(v\left(\frac{u}{N}\right)^2\right)\right)^2\right)\\ +\left(E\left(v\left(\frac{u-1}{N}\right)^4\right)-\left(E\left(v\left(\frac{u-1}{N}\right)^2\right)\right)^2\right)\\ +4E\left(v\left(\frac{u}{N}\right)^2\right)E\left(v\left(\frac{u-1}{N}\right)^2\right)\end{array}\right]=:S_0\left(\frac{u}{N}\right)\ \text{ if } u=u'\\
0,\ \text{if }\ |u-u'|>1
\end{cases}$$

Therefore,

$$\begin{aligned}
(Nh)^2E\left(\left(\hat{V}_{1,N}^{A}\right)^2\right) &= \left[\sum_{u=1}^{N}\sum_{u'=1}^{N}\left(\frac{1}{Nh}\right)^2k^2\left(\frac{u}{Nh}\right)k^2\left(\frac{u'}{Nh}\right)X\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)'X\left(\frac{u'}{Nh}\right)X\left(\frac{u'}{Nh}\right)'E\left(e_{N,u}^2e_{N,u'}^2\right)\right]\\
&= \left[\sum_{u=1}^{N}\sum_{u'=1}^{N}\left(\frac{1}{Nh}\right)^2k^2\left(\frac{u}{Nh}\right)k^2\left(\frac{u'}{Nh}\right)X\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)'X\left(\frac{u'}{Nh}\right)X\left(\frac{u'}{Nh}\right)'E\left(e_{N,u}^2e_{N,u'}^2\right)\right]+O\left(\frac{1}{N}\right)\\
&= \left[E\left(\hat{V}_{1,N}^{A}\right)\right]^2+O\left(\frac{1}{N}\right)+\left[\sum_{u=1}^{N}\left(\frac{1}{Nh}\right)^2k^4\left(\frac{u}{Nh}\right)\left(X\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)'\right)^2S_0\left(\frac{u}{N}\right)\right]\\
&\quad+\left[\sum_{u=1}^{N}\left(\frac{1}{Nh}\right)^2k^4\left(\frac{u}{Nh}\right)\left(X\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)'\right)^2S_1\left(\frac{u}{N}\right)\right]
\end{aligned}$$

Finally, note that

$$\begin{aligned}
&\sum_{u=1}^{N}\left(\frac{1}{Nh}\right)^2k^4\left(\frac{u}{Nh}\right)\left(X\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)'\right)^2S_0\left(\frac{u}{N}\right)\\
\leq\ &\left\{\sup_u S\left(\frac{u}{N}\right)\right\}\frac{1}{Nh}\sum_{u=1}^{N}\left(\frac{1}{Nh}\right)k^4\left(\frac{u}{Nh}\right)\left(X\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)'\right)^2\to 0.
\end{aligned}$$

and similarly for $\sum_{u=1}^{N}\left(\frac{1}{Nh}\right)^2k^4\left(\frac{u}{Nh}\right)\left(X\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)'\right)^2S_1\left(\frac{u}{N}\right)$. Therefore,

$$E\left(\hat{V}_{1,N}^{A}\right)\to\left[E\left(V_1^{A}\right)\right]^2$$

A straightforward application of the Slutsky theorem is sufficient to show that $\hat{V}_{1,N}$ is a consistent estimator of V_1.

Proof of Proposition 2

The White estimator of the variance of the local polynomial estimator is given by

$$V_N^{OLS} = e_1 D_N^{-1}\left(\sum_{u=1}^{N}\hat{e}^2\left(\frac{u}{N}\right)\frac{1}{Nh}k^2\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)'\right)D_N^{-1}e_1$$

where

$$\hat{e}^2(u) = \left(y\left(\frac{u}{N}\right) - X\left(\frac{u}{N}\right)'\hat{\beta}\right)^2$$

and $\hat{\beta}$ is the local polynomial estimator of the entire vector of derivatives of $y(s)$ at zero. Therefore, $X(0)'\hat{\beta} = \alpha_N^+$. Note from the previous proof that $\hat{\beta}$ is a consistent estimator of its expected value, and in particular, that $\sqrt{Nh}\left(\hat{\beta} - E\left(\hat{\beta}\right)\right) = O_p(1)$.

Therefore,

$$\hat{e}(u) = \left(\left(F\left(\frac{u}{N}\right) - X\left(\frac{u}{N}\right)'E\left(\hat{\beta}\right) + v\left(\frac{u}{N}\right) + e\left(\frac{u}{N}\right)\right) + X\left(\frac{u}{N}\right)'\left(\hat{\beta} - E\left(\hat{\beta}\right)\right)\right)$$

and

$$\begin{aligned}
E\left(V_N^{OLS}\right) &= e_1 D_N^{-1}\left(\sum_{u=1}^{N}\left(\frac{1}{Nh}k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)\right)\left(\frac{1}{Nh}k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)\right)'\bar{V}\left(\frac{u}{N}\right)\right)D_N^{-1}e_1 \quad (1)\\
&+e_1 D_N^{-1}\left(\sum_{u=1}^{N}\left(\frac{1}{Nh}k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)\right)\left(\frac{1}{Nh}k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)\right)'V\left(\frac{u}{N}\right)\right)D_N^{-1}e_1 \quad (2)\\
&+e_1 D_N^{-1}\left(\sum_{u=1}^{N}\left(\frac{1}{Nh}k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)\right)\left(\frac{1}{Nh}k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)\right)'\left(F\left(\frac{u}{N}\right) - X\left(\frac{u}{N}\right)'E\left(\hat{\beta}_N\right)\right)^2\right)D_N^{-1}e_1 \quad (3)\\
&+2e_1 D_N^{-1}\left(\begin{array}{c}\sum_{u=1}^{N}\left(\frac{1}{Nh}k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)\right)\left(\frac{1}{Nh}k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)\right)'X\left(\frac{u}{N}\right)'\left(\hat{\beta}_N - E\left(\hat{\beta}_N\right)\right)\times\\ \left(F\left(\frac{u}{N}\right) - X\left(\frac{u}{N}\right)'E\left(\hat{\beta}_N\right) + v\left(\frac{u}{N}\right) + e\left(\frac{u}{N}\right)\right)\end{array}\right)D_N^{-1}e_1 \quad (4)\\
&+e_1 D_N^{-1}\left(\sum_{u=1}^{N}\left(\frac{1}{Nh}k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)\right)\left(\frac{1}{Nh}k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)\right)'X\left(\frac{u}{N}\right)'\operatorname{var}\left(\hat{\beta}_N\right)X\left(\frac{u}{N}\right)\right)D_N^{-1}e_1 \quad (5)
\end{aligned}$$

It is clear that $Nh\cdot(1)$ and $Nh\cdot(2)$ converge to

$$\bar{V}(0)\,e_1 D^{-1}\left(\int_0^\infty k^2(u)\,X(u)\,X(u)'\,du\right)D^{-1}e_1$$

and

$$V(0)\,e_1 D^{-1}\left(\int_0^\infty k^2(u)\,X(u)\,X(u)'\,du\right)D^{-1}e_1$$

respectively. Since $\lim_{N\to\infty} X\left(\frac{u}{N}\right)'E\left(\hat{\beta}_N\right) = F_+(0)$, we have that $Nh\cdot(3)\to 0$. Now, we know from the proof of Proposition 1 that $\sqrt{Nh}\left(\hat{\beta}_N - E\left(\hat{\beta}_N\right)\right)$ converges to an $O_p(1)$ random variable, and therefore, so does $(Nh)^{3/2}\cdot(4)$. Hence, $Nh\cdot(4)\to 0$. Finally, we know that $Nh\operatorname{var}\left(\hat{\beta}_N\right)\to O(1)$, so $(Nh)^2\cdot(5)\to O(1)$ and hence, $Nh\cdot(5)\to 0$.

Therefore,

$$NhV^{OLS}\to\left(\bar{V}(0)+V(0)\right)e_1 D^{-1}\left(\int_0^\infty k^2(u)\,X(u)\,X(u)'\,du\right)D^{-1}e_1$$

which is strictly larger than the true asymptotic variance of the local polynomial estimator. Intuitively, the White estimator assumes that the errors around the trend are independent, and fails to account for the fact that the local polynomial estimator exploits correlations between errors to improve its predictive power. Only the "independent" part of the error contributes to the asymptotic variance; the "correlated" part of the error can ultimately be perfectly predicted as the resolution of the data becomes infinite.

Proof of Proposition 3

For this proof, we assume that $v(s) = 0$ for all s

Consistency

We have already shown that α_N^+ is consistent for $y_+(0) - v_+(0)$ in proposition 1. It therefore remains to compute its asymptotic variance.

Under assumption 2', the leading term of the variance in Proposition 1 is zero. Therefore, we consider term (II) from the proof of Proposition 1:

$$\begin{aligned}
(\mathrm{II}) &= e_1' D_N^{-1} \sum_{u=1}^{N} \left(\sum_{v=u}^{N} \frac{1}{Nh} k\left(\frac{v}{Nh}\right) X\left(\frac{v}{Nh}\right) \right) \left(\sum_{v=u}^{N} \frac{1}{Nh} k\left(\frac{v}{Nh}\right) X\left(\frac{v}{Nh}\right) \right)' E\left(e_{N,u}^2\right) D_N^{-1} e_1 \quad (1) \\
&\quad + e_1' D_N^{-1} \sum_{u=1}^{N} \sum_{\substack{u'=1 \\ u' \neq u}}^{N} \left(\sum_{v=u}^{N} \frac{1}{Nh} k\left(\frac{v}{Nh}\right) X\left(\frac{v}{Nh}\right) \right) \left(\sum_{v=u'}^{N} \frac{1}{Nh} k\left(\frac{v}{Nh}\right) X\left(\frac{v}{Nh}\right) \right)' E\left(e_{N,u} e_{N,u'}\right) D_N^{-1} e_1 \quad (2)
\end{aligned}$$

Now, following the proof of Proposition 1,

$$\begin{aligned}
&\sum_{u=1}^{N} \left(\sum_{v=u}^{N} \frac{1}{Nh} k\left(\frac{v}{Nh}\right) X\left(\frac{v}{Nh}\right) \right) \left(\sum_{v=u}^{N} \frac{1}{Nh} k\left(\frac{v}{Nh}\right) X\left(\frac{v}{Nh}\right) \right)' \\
= &\lim_{N \to \infty} \sum_{u=1}^{N} \frac{1}{Nh} \left(\sum_{v=u}^{N} \frac{1}{Nh} k\left(\frac{v}{Nh}\right) X\left(\frac{v}{Nh}\right) \right) \left(\sum_{v'=u}^{N} \frac{1}{Nh} k\left(\frac{v'}{Nh}\right) X\left(\frac{v'}{Nh}\right) \right)' \left[\Delta V'\left(\frac{u}{N}\right) - 2\Delta_1 C\left(\frac{u}{N}, \frac{u}{N} - \frac{1}{N}\right) \right] \\
= &\sigma(0) \left(\int_0^\infty \left(\int_u^\infty k(v) X(v)\, dv \right) \left(\int_u^\infty k(v) X(v)\, dv \right)' du \right) \left(\lim_{N \to \infty} h \right)
\end{aligned}$$

so

$$(1) = O(h(N))$$

and

$$\begin{aligned}
&\sum_{u=1}^{N} \sum_{\substack{u'=1 \\ u' \neq u}}^{N} \left(\sum_{u=1}^{N} \frac{1}{Nh} \left(\sum_{v=u}^{N} \frac{1}{Nh} k\left(\frac{v}{Nh}\right) X\left(\frac{v}{Nh}\right) \right) \right) \left(\sum_{u'=1}^{N} \frac{1}{Nh} \left(\sum_{v=u'}^{N} \frac{1}{Nh} k\left(\frac{v}{Nh}\right) X\left(\frac{v}{Nh}\right) \right) \right)' E\left(e_{N,u} e_{N,u'}\right) \\
\leq &\sup_{u \leq N} E\left(e_{N,u} e_{N,u'}\right) \sum_{u=1}^{N} \sum_{u'=1}^{N} \left(\sum_{u=1}^{N} \frac{1}{Nh} \left(\sum_{v=u}^{N} \frac{1}{Nh} k\left(\frac{v}{Nh}\right) X\left(\frac{v}{Nh}\right) \right) \right) \left(\sum_{u'=1}^{N} \frac{1}{Nh} \left(\sum_{v=u'}^{N} \frac{1}{Nh} k\left(\frac{v}{Nh}\right) X\left(\frac{v}{Nh}\right) \right) \right)' \\
= &h(N)^2 O(1) \left(\sum_{u=1}^{N} \frac{1}{Nh} \left(\sum_{v=u}^{N} \frac{1}{Nh} k\left(\frac{v}{Nh}\right) X\left(\frac{v}{Nh}\right) \right) \right) \left(\sum_{u=1}^{N} \frac{1}{Nh} \left(\sum_{v=u}^{N} \frac{1}{Nh} k\left(\frac{v}{Nh}\right) X\left(\frac{v}{Nh}\right) \right) \right)' = O\left(h(N)^2\right)
\end{aligned}$$

Therefore,

$$\frac{1}{h} \operatorname{var}\left(\alpha_N^+\right) \to \sigma(0) \left(\int_0^\infty \left(\int_u^\infty k(v) X(v)\, dv \right) \left(\int_u^\infty k(v) X(v)\, dv \right)' du \right) =: V_2$$

since the second term is $O\left(h^2\right)$.

Asymptotic Distribution

I now consider the asymptotic distribution of $\frac{1}{\sqrt{h}}\left(\alpha_N^+ - y_+(0)\right)$:

$$\begin{aligned}\frac{1}{\sqrt{h}}\left(\alpha_N^+ - y_+(0)\right) &= \frac{1}{\sqrt{h}} e_1' D_N^{-1}\left(\sum_{u=1}^{N}\left(\frac{1}{Nh}\right) k\left(\frac{u}{Nh}\right) X\left(\frac{u}{Nh}\right)\left(F\left(\frac{u}{N}\right) - F_+(0)\right)\right) \quad (1)\\ &+ \frac{1}{\sqrt{h}} e_1' D_N^{-1}\left[\sum_{u=1}^{N}\left(\sum_{v=u}^{N}\left(\frac{1}{Nh}\right) k\left(\frac{v}{Nh}\right) X\left(\frac{v}{Nh}\right)\right) e_{N,u}\right] \quad (2)\end{aligned}$$

Term (1), the bias, is $O\left(\sqrt{h}\right)$, and therefore goes to zero. Term (2) is a sum of mean-zero random variables:

$$(2) = \sum_{u=1}^{N} A_N(u) + o_p(1)$$

where

$$A_N(u) = \frac{1}{\sqrt{h}}\left(e_1' D_N^{-1}\sum_{v=u}^{N}\left(\frac{1}{Nh}\right) k\left(\frac{v}{Nh}\right) X\left(\frac{v}{Nh}\right)\right) e_{N,u}$$

Now, by Assumption 4, the increments $e_{N,u}$, and hence, $A_N(u)$ are associated. For associated random variables (Charles M. Newman, 1984), we have the following condition:

$$\left|E\left(\exp\left(it\sum_{v=1}^{N} A_N(v)\right)\right) - \prod_{v=1}^{N} E\left(\exp\left(itA_N(v)\right)\right)\right| \le t^2 \sum_{v=1}^{N}\sum_{\substack{v'=1\\ v\neq v'}}^{N} \operatorname{cov}\left(A_N(v), A_N(v')\right)$$

and $\sum_{v=1}^{N}\sum_{\substack{v'=1\\ v\neq v'}}^{N} \operatorname{cov}\left(A_N(v), A_N(v')\right)$ can be expressed as

$$\frac{1}{h} e_1' D_N^{-1}\sum_{u=1}^{N}\sum_{\substack{u'=1\\ u'\neq u}}^{N}\left(\sum_{v=u}^{N}\left(\frac{1}{Nh}\right) k\left(\frac{v}{Nh}\right) X\left(\frac{v}{Nh}\right)\right)\left(\sum_{v=u'}^{N}\left(\frac{1}{Nh}\right) k\left(\frac{v}{Nh}\right) X\left(\frac{v}{Nh}\right)\right)' E\left(e_{N,u} e_{N,u'}\right) D_N^{-1} e_1 = O(h(N))$$

Since

$$\prod_{v=1}^{N} E\left(\exp\left(itA_N(v)\right)\right) \to_N \Phi\left(t, 0, V_2\right)$$

where $\Phi\left(z, \mu, \sigma^2\right)$ is the Gaussan distribution function with mean μ and variance σ^2, we have

$$\left|E\left(\exp\left(it\sum_{v=1}^{N} A_N(v)\right)\right) - \Phi\left(t, 0, \operatorname{var}(A_N)\right)\right| \le o(1) + O(h(N))\, t^2 \to_N 0 \text{ for each } t.$$

Hence,

$$\frac{1}{\sqrt{h(N)}}\left(\alpha_N^+ - y_+(0)\right) \to^d N\left(0, V_2\right)$$

Feasible Estimation

The formula

$$V := \sigma(0)\, e_1' D^{-1}\left[\left(\int_0^\infty\left(\int_u^\infty k(v) X(v)\, dv\right)\left(\int_u^\infty k(v) X(v)\, dv\right)' du\right)\right] D^{-1} e_1$$

contains the unknown constant $\sigma(0)$ that must be estimated. I argue that the estimator:

$$\bar{V}_N = e_1' D_N^{-1} \left[\frac{1}{h} \sum_{u=1}^{N} k\left(\frac{u}{Nh}\right) X\left(\frac{u}{Nh}\right) y_{N,u}^2 \right]$$

is a feasible and consistent estimator of $\sigma(0)$ under Assumption 2'.

To prove this, we compute the moments of $\bar{V}_N$ and show that $E\left(\bar{V}_N\right) \to \bar{V}(0)$, while $E\left(\bar{V}_N^2\right) \to \left[E\left(\bar{V}_N\right)\right]^2$. First, we see that $\bar{V}_N$ is asymptotically unbiased for $\sigma(0)$:

$$\begin{aligned}
E\left(\bar{V}_N\right) &= e_1' D_N^{-1} \left[\sum_{u=1}^{N} \frac{1}{h} k\left(\frac{u}{Nh}\right) X\left(\frac{u}{Nh}\right) E\left[\left(y\left(\frac{u}{N}\right) - y\left(\frac{u-1}{N}\right) \right)^2 \right] \right] \\
&= e_1' D_N^{-1} \left[\sum_{u=1}^{N} \frac{1}{h} k\left(\frac{u}{Nh}\right) X\left(\frac{u}{Nh}\right) \left(F_{N,u}^2 + E\left(e_{N,u}^2\right) \right) \right] \\
&= e_1' D_N^{-1} \left[\sum_{u=1}^{N} \frac{1}{Nh} k\left(\frac{u}{Nh}\right) X\left(\frac{u}{Nh}\right) E\left(N e_{N,u}^2\right) \right] + O\left(\frac{1}{N}\right)
\end{aligned}$$

Now, invoking Lemma 4, it is clear that $\lim_{N\to\infty} e_1' D_N^{-1} \left[\sum_{u=1}^{N} \frac{1}{Nh} k\left(\frac{u}{Nh}\right) X\left(\frac{u}{Nh}\right) E\left(N e_{N,u}^2\right) \right] = E\left(N e_{N,0}^2\right) = \sigma(0)$, since $\sigma(s)$ is continuous. Hence, $E\left(\bar{V}_N\right) \to \bar{V}(0)$.

To compute the variance of $\bar{V}_N$, note that

$$N^2 E\left(y_{N,u}^2 y_{N,u'}^2\right) = F_{N,u}^2 F_{N,u'}^2 + E\left(e_{N,u}^2 e_{N,u}^2\right) = N^2 E\left(e_{N,u}^2 e_{N,u'}^2\right) + O\left(\frac{1}{N^2}\right)$$

Therefore, we need to show only that

$$\begin{aligned}
& e_1' D_N^{-1} \left[\sum_{u=1}^{N} \sum_{u'=1}^{N} \left(\frac{1}{Nh} k\left(\frac{u}{Nh}\right) X\left(\frac{u}{Nh}\right) \right) \left(\frac{1}{Nh} k\left(\frac{u'}{Nh}\right) X\left(\frac{u'}{Nh}\right) \right)' N^2 E\left(e_{N,u}^2 e_{N,u'}^2\right) \right] D_N^{-1} e_1 \\
\to \quad & \left(e_1' D_N^{-1} \left[\sum_{u=1}^{N} \frac{1}{Nh} k\left(\frac{u}{Nh}\right) X\left(\frac{u}{Nh}\right) E\left(N e_{N,u}^2\right) \right] \right)^2
\end{aligned}$$

Under the assumption

$$\operatorname{cov}\left(N e_{N,u}^2, N e_{N,u'}^2\right) = O\left(\operatorname{cov}\left(\sqrt{N} e_{N,u}, \sqrt{N} e_{N,u'}\right)^2 \right)$$

(satisfied for a Gaussian process) we have

$$E\left(N e_{N,u}^2 N e_{N,v}^2\right) = O\left(\operatorname{cov}\left(\sqrt{N} e_{N,u}, \sqrt{N} e_{N,u'}\right)^2 \right) + E\left(N e_{N,u}^2\right) E\left(N e_{N,v}^2\right)$$

Therefore,

$$
\begin{aligned}
& e_1' D_N^{-1}\left[\sum_{u=1}^{N}\sum_{u'=1}^{N}\left(\frac{1}{Nh}k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)\right)\left(\frac{1}{Nh}k\left(\frac{u'}{Nh}\right)X\left(\frac{u'}{Nh}\right)\right)' N^2 E\left(e_{N,u}^2 e_{N,u'}^2\right)\right] D_N^{-1} e_1 \\
& -\left(e_1' D_N^{-1}\left[\sum_{u=1}^{N}\frac{1}{Nh}k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)E\left(Ne_{N,u}^2\right)\right]\right)^2 \\
= & \; e_1' D_N^{-1}\left[\sum_{u=1}^{N}\sum_{u'=1}^{N}\left(\frac{1}{Nh}k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)\right)\left(\frac{1}{Nh}k\left(\frac{u'}{Nh}\right)X\left(\frac{u'}{Nh}\right)\right)' O\left(\operatorname{cov}\left(\sqrt{N}e_{N,u},\sqrt{N}e_{N,u'}\right)^2\right)\right] D_N^{-1} e_1 \\
= & \; e_1' D_N^{-1}\left[\sum_{u=1}^{N}\sum_{u'=1}^{N}\left(\frac{1}{Nh}k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)\right)\left(\frac{1}{Nh}k\left(\frac{u'}{Nh}\right)X\left(\frac{u'}{Nh}\right)\right)' O\left(\frac{1}{N^2}\right)\right] D_N^{-1} e_1 \to 0.
\end{aligned}
$$

since $\operatorname{cov}\left(\sqrt{N}e_{N,u},\sqrt{N}e_{N,u'}\right) = E\left(Ne_{N,u}e_{N,u'}\right) = O\left(\frac{1}{N}\right)$. Consistency is proved.

Covariance of α_N^+ and α_N^-

Under Assumption 2, it was obvious that the leading stochastic term of α_N^+ was composed of independent random variables, and that the covariance between α_N^+ and α_N^- is zero. However, with Assumption 2', the leading stochastic term of α_N^+ involves correlated random variables. Here, we check that the asymptotic covariance between α_N^+ and α_N^- remains zero.

The above equation is valid so long as $\operatorname{cov}\left(\alpha_N^+,\alpha_N^-\right) = o\left(h\left(N\right)\right)$. To prove this, I extend Assumption 3', so that for $s>0$ and $t<0$, we have the following covariance structure:

$$
E\left(e\left(s\right)e\left(t\right)\right) = C\left(s,t\right)
$$

where $C\left(s,t\right)$ satisfies the hypotheses of Assumption 3'. Then, define

$$
C_N^0\left(u\right) = \sum_{u=1}^{N}\left(\frac{1}{Nh}k\left(\frac{u}{Nh}\right)X\left(\frac{u}{Nh}\right)\right)
$$

We have

$$
\begin{aligned}
\operatorname{cov}\left(\alpha_N^+,\alpha_N^-\right) &= E\left(Z_N^+ Z_N^-\right) = E\left(e_1' D_N^{-1}\sum_{u=1}^{N}\sum_{u'=1}^{N} C_N^0\left(u\right) C_N^0\left(u'\right)' y_{N,u}^+ y_{N,u'}^- D_N^{-1} e_1\right) \\
&= e_1' D_N^{-1}\sum_{u=1}^{N}\sum_{u'=1}^{N} C_N^0\left(u\right) C_N^0\left(u'\right)' E\left(e_{N,u}^+ e_{N,u'}^-\right) D_N^{-1} e_1 \quad (1') \\
&\quad + e_1' D_N^{-1}\sum_{u=1}^{N}\sum_{u'=1}^{N} C_N^0\left(u\right) C_N^0\left(u'\right)' F_{N,u}^+ F_{N,u'}^- D_N^{-1} e_1 \quad (2')
\end{aligned}
$$

It is obvious that $(2') = O\left(h\left(N\right)^2\right)$, since $F_{N,u}^+ = O\left(1/N\right)$, and $(1') = O\left(h\left(N\right)^2\right)$, because it contains only covariance terms.

Implementation

We are ultimately interested in running regressions of the form

$$
y_{i,b}\left(0\right) = \alpha_b + X_{i,b}\gamma + \varepsilon_{i,b}
$$

Therefore, under the assumption that $v_+(0) = v_-(0) = v(0)$, the independent term goes into the fixed effect and does not need to be predicted. Hence, I can compute the variance of $y_{i,b}(0)$ as $V_{i,b} = \bar{V}_N$. I then compute the variance of γ as the appropriate submatrix of

$$\bar{V} = \frac{N}{N-K} \left(\hat{X}'W\hat{X}\right)^{-1} \left(\hat{X}'W\left(\mathrm{diag}\left(\hat{\varepsilon}^2\right) + \bar{V}\right)W'\hat{X}\right) \left(\hat{X}'W\hat{X}\right)^{-1}$$

where K is the number of regressors (including fixed effects), $\mathrm{diag}\left(\hat{\varepsilon}^2\right)$ is a diagonal matrix of the squared residuals, W is a weight matrix, $\hat{X}$ is the matrix of regressors including the fixed effects. and $\bar{V}$ is a diagonal matrix of the estimated variances.