An Optimizing Model of U.S. Wage and Price Dynamics

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Abstract

The objective of this paper is to provide an optimizing model of wage and price setting consistent with U.S. data. I first investigate the predictions of an optimizing labor supply model for the aggregate nominal wage, taking as given the evolution of prices and quantities. In this part I seek to determine whether a standard specification of consumption/leisure preferences is consistent with the data, and to what extent nominal or real rigidities in the wage setting process improve the fit with the data. Then I combine the evolution of wages predicted by this model with the evolution of prices predicted by staggered-price models to provide a model of the joint determination of prices and wages, given the evolution of real quantities. The paper thus supplies a "Phillips curve" specification that is consistent with intertemporal optimization and rational expectations.

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1. Introduction

This paper is an attempt to model the joint behavior of prices and wages in a way consistent with intertemporal optimization and rational expectations. Its ultimate goal is to construct a 'Phillips curve' specification that is consistent both with U.S. data and with optimizing behavior, to respond to the well known "Lucas critique".

The Phillips curve relationship has undergone a fruitful re-exploration in recent years. The effort has been devoted to explain the relation between nominal and real variables in rigorously specified general equilibrium, optimizing models[1]. For example, the so-called "New Keynesian" Phillips Curve (NKPC), which describes current inflation as a function of expected future inflation and a measure of output gap, is derived in the context of a general equilibrium, optimizing model, that allows some form of nominal rigidities, either by assuming staggered price-setting (for example, in the style of Calvo (1983) model), or by assuming staggered wage-setting, or both (for ex. Erceg et al. 1999)[2].

Models with nominal rigidities have been explored mostly in the context of monetary policy analysis. Providing a channel for real effects of monetary disturbances, staggered wage and price settings are in fact a suitable framework to investigate issues such as the optimality of alternative monetary policies.

However, the standard NKPC model predicts counterfactual comovement of output and inflation, unless there are large cyclical variations in potential output. For this reason, there have been some attempts to dismiss altogether the particular model of price-setting that lies at the heart of the model.

Some recent studies, in particular, have questioned the importance of the forward-looking component in pricing behavior, by focusing on empirical estimates of the implied inflation-output equation. For example, Fuhrer's (1997) empirical results point to a negligible role of future inflation in an estimated inflation-output relationship, specified in a way that

[1] See the contributions in the special issue of the Journal of Monetary Economics (1999).

[2] An early estimation of such a curve is in Roberts (1995).

is intended to nest the 'New Keynesian' Phillips Curve specification, the more complex variant proposed by Fuhrer and Moore (1995), and purely backward-looking Phillips Curve specifications. Roberts (1997, 1998) argues instead that the New Keynesian Phillips Curve fits reasonably well when survey measures are used to approximate inflation expectations, but that it does not fit well under the hypothesis of rational expectations. He thus proposes a model with an important backward-looking component in inflation expectations, which amounts to weakening the weight put on the forward-looking terms in his aggregate supply relation.

Some other recent work has shown, however, that, unlike tests of the standard NKPC model, tests of the pricing equation alone, derived from a staggering price model, seem to fit inflation data quite well, providing empirical support for the hypothesis of nominal price rigidity, and for the importance of forward-looking determinants of price-setting behavior[3]. In particular, Sbordone (1998) shows that, taking as given the evolution of unit labor costs, the dynamic of inflation predicted by sticky price models tracks actual data very closely, and implies a degree of price stickiness very much in line with that found through survey evidence.

But if one accepts the hypothesis that the evolution of inflation is well described by the evolution of future labor costs, then one should argue that the empirical failure of NKPC models is not due to a fault of the pricing mechanism, but to either the additional assumptions needed to obtain proportionality between marginal costs and real output, or to the used measure of output gap, or both.

In this paper I therefore seek to develop a more accurate optimizing model of the dynamics of unit labor costs: I first investigate, using a partial equilibrium approach, the predictions of an optimal labor supply model for the aggregate nominal wage, taking as given the evolution of prices and quantities. Together with the evolution of productivity, this model yields a quantitative model of the evolution of unit labor costs.

[3]See Sbordone (1998, 1999) and Gali and Gertler (1999). Both contributions use unit labor costs to proxy for variation in nominal marginal costs, but follow different estimation procedures.

Then, combining the predictions of this model with the predictions of an optimizing price-setting model for the evolution of the aggregate price level, I provide a joint model of price and wage dynamics, taking as given the evolution of real quantities.

In developing the wage model, I first analyze the fit of the baseline optimizing model used in standard RBC models, where a representative household chooses hours of work to maximize an expected lifetime utility function. The optimality condition for labor supply gives a desired real wage as a function of consumption and hours. Then I consider the hypothesis that the actual real wage adjusts only sluggishly to the desired wage, and compare the prediction of models with perfectly flexible wages to those of models with different kinds of wage rigidity[4].

The price-setting side of the model has one sector of production, monopolistic competition, and nominal price rigidity: these assumptions deliver the evolution of the price level as a function of expected future labor costs. The optimizing model of wage dynamics with wage rigidities, combined with the staggering price model, provides a complete optimizing model of wage-price dynamics.

The rest of the paper is organized as follows. In section 2 I discuss the inadequacy of the New Keynesian Phillips curve, and motivate my investigation of the behavior of labor costs. In section 3 I analyze the predictions of a baseline model of wage setting, and in Section 4 I study the implications of removing the flexible-wage assumption. I first introduce a model of nominal wage rigidity, then show how to nest it in more general models of indexed nominal wages. Section 5 contains the central result of the paper: I discuss the joint modeling of wage and price dynamics, and present the fit of price and wage dynamics obtained with a set of calibrated parameters. Section 6 concludes.

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[4] Although sticky wages are often postulated in theoretical models, the recent optimizing models with sticky wages have not yet been subject to much empirical testing. One recent piece of evidence for these models is Amato and Laubach (1999). Their empirical strategy is based on matching the impulse response functions to monetary shocks generated by the model with those estimated in the data.

2. The Inadequacy of the New Keynesian Phillips Curve

An optimization based Phillips curve relationships results from the combination of the price setting behavior of the firms, which links the evolution of prices to the evolution of marginal costs, and the wage setting behavior of the households, which links the evolution of wages to the evolution of consumption and hours.

In the wage-setting sector, a representative household chooses hours of work to maximize an expected lifetime utility

$$E_0\{\Sigma_{t=0}^{\infty}\ \beta^t\ U(C_t, H_t; \xi_t)\}$$

subject to an intertemporal budget constraint

$$\sum_{t=0}^{\infty} E_0\{R_{0,t}C_t\} \leq \sum_{t=0}^{\infty} E_0\{R_{0,t}\omega_t H_t\} + a_0$$

where β is a subjective discount factor, ξ_t is a stochastic disturbance to household's preferences, ω_t is the real wage, a_0 is initial wealth, and $R_{t,T}$ is the product of stochastic discount factors. The first order condition for optimal labor supply gives a *desired* real wage, which I will denote throughout the paper by v_t

$$v_t \ = -\frac{U_H}{U_C}(C_t, H_t; \xi_t) \equiv w(C_t, H_t; \xi_t) \tag{2.1}$$

In the price-setting sector, a continuum of monopolistic firms, indexed by i, produce differentiated goods, also indexed by i, and face a demand curve for their product of the form:

$$Y_{it} = (P_{it}/P_t)^{-\theta}\, Y_t \tag{2.2}$$

where θ is the Dixit-Stiglitz elasticity of substitution among differentiated goods, and Y_t is the aggregator function defined as $Y_t = \left[\int_0^1 Y_{it}^{(\theta-1)/\theta} di\right]^{\theta/(\theta-1)}$. The production technology of each firm i is of the form:

$$Y_{it} = f(\Theta_t H_{it}) \tag{2.3}$$

where capital is assumed as being allocated to each firm in a fixed amount, so that I consider labor H_t as the only factor of production, and Θ_t is a stochastic labor augmenting technological factor.

To obtain a Phillips Curve in this optimization based model, NKPC models assume that not all firms adjust prices in full every period. According to the Calvo (1983) model of random intervals between price changes,[5] in every period, a fraction $(1-\alpha)$ of the firms can set a new price, independently of the past history of price changes, which will then be kept fixed until the next time the firm is drawn to change prices again.[6] The expected time between price changes is therefore $\frac{1}{1-\alpha}$.

The pricing problem of a firm that revises its price in period t is to choose its price, X_{it}, to maximize its expected stream of profits

$$E_t\{\Sigma_{j=0}^{\infty} R_{t,t+j}\Pi_{it+j}\}$$

The solution to this problem leads to an optimal pricing condition of the form

$$\Sigma_{j=0}^{\infty}\ \alpha^j E_t \left\{ R_{t,t+j}Y_{t+j}\left(\frac{X_t}{P_{t+j}}\right)^{-\theta}\left[X_t - \frac{\theta}{\theta-1}\ S_{t+j,t}\right]\right\} = 0$$

where the subscript i on X_{it} is suppressed, since all the firms that change price solve the same problem, and $S_{t+j,t}$ denotes the marginal cost of producing , at date $t+j$, goods whose price was set at time t ($S_{t+j,t} \equiv \frac{1}{1-a}\frac{W_{t+j}H_{it+j}}{Y_{it+j}}$) Dividing by P_t, and defining $x_t \equiv X_t/P_t$ and $s_{t+j,t} \equiv S_{t+j,t}/P_{t+j}$, one can rewrite this expression as

$$\Sigma_{j=0}^{\infty}\ \alpha^j E_t \left\{ R_{t,t+j}Y_{t+j}\left(\frac{X_t}{P_{t+j}}\right)^{-\theta}\left[x_t - \frac{\theta}{\theta-1}\ s_{t+j,t}\prod_{k=1}^{j}\pi_{t+k}\right]\right\} = 0 \tag{2.4}$$

[5] Alternatively, nominal price rigidity can be introduced by assuming that firms face some convex cost of adjusting prices (Rotemberg 1982) and therefore, although all firms are allowed to change prices at any time, it is not optimal to do so.

[6] By letting α vary between 0 and 1, the model nests a wide range of assumptions about the degree of price stickiness, from perfect flexibility ($\alpha = 0$) to complete price rigidity (the limit as $\alpha \to 1$).

Here $s_{t+j,t}$ is in general different from the average marginal cost s_{t+j} (which is equal to $\frac{1}{1-a}\frac{W_{t+j}H_{t+j}}{P_{t+j}Y_{t+j}}$), since capital cannot be reallocated across firms to equate the shadow price of capital services at all times, and is related to s_{t+j} by

$$s_{t+j,t} \equiv \frac{1}{1-a}\frac{W_{t+j}H_{it+j}}{P_{t+j}Y_{it+j}} = s_{t+j} * \left[\left(\frac{X_t}{P_{t+j}}\right)^{-\theta}\right]^{\frac{a}{1-a}} \tag{2.5}$$

The optimal pricing condition (2.4), combined with the distribution of aggregate prices at any point in time

$$P_t = \left[(1-\alpha)X_t^{1-\theta} + \alpha P_{t-1}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{2.6}$$

allows one to describe the path of aggregate prices and inflation as a function of real marginal costs, shifted by expected inflation.

Specifically, combining the log-linear approximation of equations (2.4) and (2.6) around steady state values $x^*(\equiv 1)$, $s^*(\equiv \frac{\theta-1}{\theta})$, and $\pi^*(\equiv 1)$, with a log linear approximation of the equation (2.5), one obtains that the dynamics of inflation (deviation of inflation from long-run equilibrium) is described by an equation of the form[7]

$$\widehat{\pi}_t = \alpha_1 E_t \widehat{\pi}_{t+1} + \zeta \widehat{s}_t \tag{2.7}$$

where the parameter ζ measures the degree of stickiness in the adjustment of prices[8], α_1 is a discount factor[9], and hat variables indicate deviations from steady state values.

Solving this equation forward one obtains that inflation is a function of expected future

[7] A more complete derivation of this equation can be found in Sbordone ('98).

[8] Further specifying the production technology as of the Cobb-Douglas form, one can show that ζ depends on the probability of changing prices (the fraction of firms that are allowed to change prices every period), on the elasticity of substitution among differentiated goods, and on the Cobb-Douglas output elasticity. Equation (2.7) can also be obtained under the assumption that the nominal rigidity stems from the existence of costs of adjusting prices: in this case the parameter ζ is the inverse of the curvature of the adjustment cost function.

[9] $\alpha_1 = R\gamma_y^*$, where R is the steady state value of the discount factor, and γ_y^* is the steady state growth rate of output.

real marginal costs; one can then estimate the inflation dynamics by proxying the unobservable marginal costs with unit labor cost u_t[10].

Using this methodology, Sbordone (1998) shows that the dynamic of inflation predicted by this model tracks very closely the actual dynamic of U.S. inflation, and the point estimate of ζ implies a degree of nominal price inertia consistent with survey evidence[11].

To obtain the "New Keynesian Phillips Curve" in the standard form of a relationship between inflation and output gap, where again expectations of future inflation are a shifting factor, one has to show that labor costs are proportional to output gap. A proportionality of this kind is derived in Woodford (2001). From the first order condition of the flexible wage model, using market clearing to substitute out C_t , and the production function to substitute out H_t, eq. (2.1) can be written as

$$v_t = v(Y_t; \xi_t, \Theta_t) \tag{2.8}$$

Moreover, labor productivity can be written as some function $g(Y_t; \Theta_t)$, implying that

$$s_t = \frac{v_t}{MPL_t} = \frac{v(Y_t; \xi_t, \Theta_t)}{f(Y_t; \Theta_t)} = s(Y_t; \xi_t, \Theta_t) \tag{2.9}$$

If one denotes by Y_t^p the level of output, at each time t, for which real marginal cost would remain at a constant level, then Y_t^p must solve

$$s(Y_t^p; \xi_t, \Theta_t) = \mu^{-1} \tag{2.10}$$

where, with standard notation, μ denotes the markup of prices over marginal costs. Using a

[10] This basic measure of marginal cost is correct if the production technology is CRS, and there are no other friction which might break the proportionality between average and marginal costs (for ex., the existence of costs of adjusting hours, or of overhead labor). See Sbordone ('99) for a discussion of the empirical implications of using alternative measures of marginal costs when estimating inflation dynamics.

[11] See also Gali-Gertler (1999), and Gali, Gertler, and Lopez-Salido (2000).

log-linear approximation to (2.10), the log-linear approximation of (2.9) gives

$$\widehat{s}_t = \varepsilon_{sy}\widehat{Y}_t + \varepsilon_{s\xi}\widehat{\xi}_t + \varepsilon_{s\xi}\widehat{\Theta}_t = \varepsilon_{sy}\left(\widehat{Y}_t - \widehat{Y}_t^p\right)$$

where ε_{sx} denotes the elasticity of the marginal cost function to component x. Since Y_t^p can be thought of as a measure of potential output, $(\widehat{Y}_t - \widehat{Y}_t^p)$ can be thought as a measure of output gap. One then obtains the NKPC

$$\widehat{\pi}_t = \alpha_1 E_t \widehat{\pi}_{t+1} + \gamma\left(\widehat{Y}_t - \widehat{Y}_t^p\right) \tag{2.11}$$

where $\gamma = \zeta\varepsilon_{sy}$.

Note, however, that in the above derivation potential output stands for the 'efficient' level of output, and therefore need not be a smooth trend; in particular, it depends on the stochastic disturbances (ξ_t, Θ_t). Empirical estimates of the NKPC curve, instead, routinely approximate potential output Y_t^p by some deterministic function of time (for ex., Roberts '95 uses a quadratic trend); this is equivalent to arbitrarily assuming that consumption and hours move exactly in proportion to output.

To analyze the problems of such a specification, I first solve equation (2.11) forward and obtain an expression of inflation as a function of current and expected future output gaps. Then, as in the standard NKPC approach, I define the output gap as the deviation of output from a quadratic trend, and compute expected future output gaps by the forecast of this component derived from a multivariate VAR[12]. The parameter γ is estimated to maximize the fit of the model (minimize the distance between actual inflation and inflation as predicted by the model). The results of this exercise are presented in figure 1. Graph a. compares actual inflation (in deviation from the mean - solid line) to inflation predicted by eq. (2.11) (dotted line). The ability of this model to predict inflation is clearly poor, as the figure

[12] This exercise is in the same spirit of the analysis of Sbordone (98) which evaluates the dynamics of prices driven by nominal unit labor costs. The output measure I use is gdp for the private, non farm business sector. See later for the details of all the data used. The forecasting VAR includes detrended output, real unit labor cost, and the rate of growth of nominal unit labor cost.

shows; predicted and actual inflation are in fact negatively correlated.

Panel b. of the figure shows a further dimension in which the model fails, by comparing the lead-lag correlations of inflation and output gap $[\rho(gap_t, \pi_{t+k})]$. While in the data output gap leads inflation (the highest correlation occurs at $k = 6$), in the model output gap lags inflation (the highest correlation occurs at $k = -3$). Overall, the dynamic cross correlations predicted by the model lay outside the standard deviation bands, and can therefore judged to be significantly different from those computed in the data.

By contrast, figure 2, in the lower left corner, shows that a much better approximation to actual inflation is obtained when inflation is predicted according to eq. (2.7), where real marginal cost is proxied by real unit labor cost.[13] To understand why inflation dynamics is well explained when real marginal cost is approximated by unit labor costs (the result shown in Sbordone (1998) and Gali-Gertler (1999)) but is not well modeled when marginal cost is approximated by output gap, I compare these two measures in figure 3. Output gap and unit labor cost are negatively correlated (-.34)[14]. Clearly U.S. data do not support the hypothesis that output gap should proxy the evolution of labor costs; as a result, if the sticky-price model is true, the NKPC cannot fit the data well.

This evidence suggests that the empirical problems of NKPC models are not due to a misspecification of the price setting mechanism, but to the incorrect assumption of proportionality between marginal costs and output. Output gap, measured as deviation from a deterministic trend, is not the correct forcing variable of the inflation process. A better approach to the construction of an empirical Phillips curve is to look for an appropriate measure of real unit labor cost. The task I am taking next.is therefore an investigation of the wage setting mechanism.[15]

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[13] This figure is obtained with the same methodology of the previous one, and a bivariate VAR including real unit labor cost and the rate of growth of nominal unit labor cost is used to forecast real ulc.

[14] A qualitatively similar, although less dramatic result, obtains if one alternatively measures output gap as deviation from a stochastic trend (as discussed below, this specification would seem more appropriate with the data used here). This measure of output gap has a smaller negative correlation with unit labor cost, -.08, but still misses the lead-lag correlation with inflation.

[15] An alternative approach would be to construct a measure of output gap consistent with the model, as

3. A More General Flexible Wage Model

Since in the baseline optimizing model (see eq. (2.1)) the desired real wage is a function of the marginal rate of substitution between leisure and consumption, to examine the prediction of this model I directly analyze the joint behavior of real wages, consumption, and hours. To overcome the problem of unobservability of the preference shock, I make the hypothesis that ξ_t is a random walk (i.e. I assume that there is no forecastable component in the preference shock), and derive the following log-linear approximation to eq. (2.1)

$$\widehat{v}_t = \lambda \widehat{c}_t + \nu \widehat{h}_t$$

I then denote the empirical counterpart of this equation as

$$v_t^{cyc} = \lambda c_t^{cyc} + \nu h_t^{cyc} \tag{3.1}$$

where the superscript 'cyc' indicates that I proxy $\widehat{v}_t$, $\widehat{c}_t$, and $\widehat{h}_t$ with the cyclical components, respectively, of real wage, consumption, and hours, which are in turn defined as the log deviation from their trend (as explained below, real wages and consumption share a stochastic trend, while hours are trend stationary). The parameters λ and ν, respectively the elasticity of the marginal rate of substitution with respect to consumption and hours, are then preference parameters to be estimated.[16]

The estimation of this equation consists of two steps. I first construct the cyclical components of real wages, consumption and hour; then, denoting by $\underline{\phi}$ the vector of parameters of interest, $\underline{\phi} = [\lambda \;\; \nu]'$, I define the distance between the model and the data as

$$\varepsilon_t^v = v_t^{\text{mod}}(\underline{\phi}) - v_t^{data}$$

defined by (2.10). An attempt in this direction is Neiss-Nelson (2001)

[16] $1/\lambda$ represents the elasticity of consumption to the real wage (holding hours constant), and $1/\nu$ represents the elasticity of hours to the real wage (holding consumption constant).

and compute the value of the parameters λ and ν that minimize (a square measure of) this distance

$$\underline{\widehat{\phi}} = \arg\min var(\varepsilon_t^v) \tag{3.2}$$

3.1. Constructing the cyclical components

I use U.S. data for the non-farm private business sector (NFB), published by the BLS. The price index is the implicit GDP deflator, nominal wage is hourly compensation, real wage is nominal compensation divided by the price index [17], output is value added, and hours is total hours of work. Consumption is the NIPA aggregate for nondurables and services[18].

To address the presence of stochastic trends, I tested for the presence of unit roots in all the variables of interest:[19] the unit root hypothesis is rejected for hours and for the labor share, and the consumption/output ratio is stationary around a small, (negative) deterministic trend. Consistently with these results, I decompose the nonstationary series into permanent and transitory components using the Beveridge-Nelson decomposition: the stochastic trend is defined as the forecasting profile and the cyclical component is obtained as deviation from this trend. The forecasting profile is constructed using a multivariate forecasting model which includes productivity, hours (output/productivity ratio), the consumption/output ratio, the labor share and inflation. Specifically, the forecasting model is

$$A(L)\ X_t = u_t$$

[17] Note that this is different from what is reported in the statistics as 'real compensation' in the same sector, which is instead obtained by deflating the nominal compensation by a consumer price index.

[18] Al the data are retrieved from the FRED database at the St. Louis Fed.

[19] I conducted univariate unit root tests on these variables, allowing for the presence of a deterministic trend. Specifically, I test the joint hypothesis of a zero coefficient on the deterministic trend, and a unit coefficient on the first lag, in a regression of the level of the variable on its lagged level and two lags of its first difference.

where the vector X_t is defined as[20]

$$X_t = [\Delta q_t \quad y_t - q_t \quad c_t - y_t \quad sh_t \quad \Delta p_t]' \tag{3.3}$$

The VAR matrix polynomial is $A(L) = I - A_1 L - A_2 L^2$, and u_t are i.i.d. innovations.

Figure 4 plots the cyclical components of productivity, real wage, consumption and hours. The cyclical component of the real wage (in the upper right corner of the figure) is the variable that the model of real wage below will try to approximate.

3.2. Parameter estimates

The criterion (3.2) leads to the estimates reported in the first row of table 1. The table also reports the correlation between the estimated cyclical component of the real wage and the cyclical component of the real wage predicted by the model ($corr(w^a, w^p)$), and the residual variance ($var(\epsilon_t)$). This variance is the criterion function for estimation, and is taken as a benchmark for evaluating whether wage rigidities may improve the fit with the data.

The fitted value of the cyclical wage, constructed using these parameter values, is plotted against the cyclical component of the actual real wage in fig. 5a. The model fits the data significantly well - the generated and actual series also share very similar serial correlation pattern (fig. 5b). However, as the last two panel of fig. 5 show, the implied growth rate of both real and nominal wage are more volatile than the actual growth rates (the standard deviations are, respectively, about 13% and 16% higher).This result suggests that it is worth attempting to incorporate some degree of inertia in the adjustment either of the real or the nominal wage, and examine whether allowing for such inertia improves the fit with the data: this task will be taken up in section 4.

[20] Lowercase letters denote the natural log of the corresponding upper case variables; Q_t is labor productivity, Y_t is real output, C_t is real consumption expenditures on non durables and services, $SH_t = \frac{W_t H_t}{P_t Y_t}$ is the labor share (ratio of total compensation to nominal output and Δ is the first difference operator. Output and productivity share the same stochastic trend (hours are trend stationary), as do consumption and output. That justifies the ratios in the VAR.

3.3. A Stronger Hypothesis on Preference Shock

The above analysis is conditional upon the assumption that the preference shock follows a random walk. However, since the VAR model contains a single real unit root, we may wish to interpret this single source of non stationarity as a technology shock, that should not affect preferences. Here therefore I explore the alternative assumption that ξ_t is a deterministic trend: in this case the model implies

$$v^{trend} = \lambda c^{trend} + \nu h^{trend} + trend$$

As a consequence, $(v_t - \lambda c_t)$ should be a trend stationary series, and the parameter λ can be determined from the cointegrating vector, without reference to cyclical components of the series. Since the estimated VAR model implies that the variable $(v_t - c_t)$ is trend stationary, this hypothesis about the preference shock requires that $\lambda = 1$.

I re-estimated therefore the model imposing this parameter restriction. The estimation still gives a negative value for the elasticity ν, although lower in absolute value ($\nu = -.465$ ($s.e.$ 0.05)); however, the restriction on λ is strongly rejected, and the resulting cyclical component of the real wage is approximated to a much lower degree, as fig. 6a shows. When the constraint on λ is imposed, the criterion function used for estimation is about three times as large.

Before turning to the interpretation of the estimated parameter values, I examine another possible restriction, a non-negativity constraint on the elasticity ν. Not surprisingly, the optimal value of ν under such a restriction is zero, and the estimated value for λ is reduced to 1.29 ($s.e.$ 0.05). Under this restriction as well, the fit of the model deteriorates significantly (see fig. 6b). The statistics for the two restricted models are reported in the second and third row of table 1.

3.4. Evaluating the Baseline Wage-Setting Model

3.4.1. Interpretation of the estimated parameter values

The estimates obtained, both in the unconstrained and the constrained case, imply that the elasticity of hours to wages, keeping consumption constant, is negative, and the elasticity of consumption to wages, keeping hours constant, is less than 1. One way to understand which kind of preferences are consistent with such values is to use the correspondence between the parameters of this 'cyclical wage' model and the more familiar Frisch elasticities.

Appropriately transforming the economy into a stationary one, one can solve the first order conditions of the consumer maximization problem of the transformed economy to obtain the Frisch demand for consumption and hours

$$\begin{aligned}\widetilde{C}_t &= C(\widetilde{v}_t, \widetilde{\mu}_t) \\ \widetilde{H}_t &= H(\widetilde{v}_t, \widetilde{\mu}_t)\end{aligned}$$

Here I denote stationary variables with a tilde, and denote by $\widetilde{\mu}_t$ the (transformed) marginal utility of income. Denoting by (v^*, μ^*, c^*, h^*) the steady state value of $(\widetilde{v}_t, \widetilde{\mu}_t, \widetilde{c}_t, \widetilde{h}_t)$, a log-linearization of the Frisch demands around the steady state values gives

$$\widehat{c}_t = \epsilon_{cw}\widehat{v}_t + \epsilon_{c\mu}\widehat{\mu}_t \tag{3.4a}$$

$$\widehat{h}_t = \epsilon_{Hw}\widehat{v}_t + \epsilon_{H\mu}\widehat{\mu}_t \tag{3.4b}$$

where ϵ_{ij} denote Frisch elasticities. Combining eqs. (3.4a) and (3.4b), the desired real wage can be expressed as a function of consumption and hours

$$\widehat{v}_t = \frac{\epsilon_{H\mu}}{\epsilon_{H\mu}\epsilon_{cw} - \epsilon_{c\mu}\epsilon_{Hw}}\widehat{c}_t - \frac{\epsilon_{c\mu}}{\epsilon_{H\mu}\epsilon_{cw} - \epsilon_{c\mu}\epsilon_{Hw}}\widehat{h}_t \tag{3.5}$$

The parameters λ and ν are therefore the following transformations of the Frisch elasticities

$$\lambda = \frac{\epsilon_{H\mu}}{\epsilon_{H\mu}\epsilon_{cw} - \epsilon_{c\mu}\epsilon_{Hw}}$$

$$\nu = -\frac{\epsilon_{c\mu}}{\epsilon_{H\mu}\epsilon_{cw} - \epsilon_{c\mu}\epsilon_{Hw}}$$

The concavity of the utility function requires that $\frac{\partial H}{\partial w} > 0$, which means that $\epsilon_{Hw} > 0$ as well. The assumption that consumption is a normal good requires that $\frac{\partial C}{\partial \mu} < 0$, which implies that $\epsilon_{c\mu} < 0$ as well. Therefore, in order for λ and ν to have opposite signs (as delivered by the estimation), since they share the same denominator, it must be the case that the denominator is negative, $(\epsilon_{H\mu}\epsilon_{cw} - \epsilon_{c\mu}\epsilon_{Hw}) < 0$, and $\epsilon_{H\mu} < 0$.

These theoretical restrictions suggest two major departures from standard parametrization of preferences. First, and most obvious, preferences should be non-separable in consumption and leisure. Were the utility function separable, ϵ_{cw} would be 0, and one would not obtain opposite signs for the two parameters; so work must increase the marginal utility of consumption. Moreover, from the above derivation, it results that leisure should be an inferior good[21].

One can then conclude that these empirical results are not consistent with the theoretical framework of a representative household for which both consumption and leisure are normal goods. Furthermore, the result that $\lambda \neq 1$ implies that preferences are not consistent with balanced growth, unless they have a secular drift in them - which I have assumed with both my alternative hypotheses about the preference shock ξ, that rule out any predictable component in the shock.

[21] Alternatively, a negative ν and a positive λ could be obtained by assuming that leisure is a normal good and consumption is the inferior good: in this case in fact $\epsilon_{H\mu} > 0$ and $\epsilon_{c\mu} > 0$ (in that case the denominator of the two parameters needs to be positive, and it is required that $(\epsilon_{H\mu}\epsilon_{cw} - \epsilon_{c\mu}\epsilon_{Hw}) > 0$).

3.4.2. Alternative interpretations

There are a number of ways, however, to rationalize these results. One alternative, more simplistic interpretation, is that part of consumers are 'rule of thumb' consumers. These consumers will tend to increase consumption when income increases; as a result, keeping consumption constant, all increases in hours must be accompanied by a decline in wages.

Alternatively, one can assume that the economy is populated by a number of heterogeneous households, with different preferences for consumption and leisure, but for whom both consumption and leisure are normal goods. One can then show that, at least in some particular cases, the aggregation of consumption and labor supply behavior of these heterogeneous agents may as well deliver the estimated signs of the parameters.[22]

Another alternative is to maintain the representative household framework, but specify its preferences as in the "high substitution economy" of King and Rebelo (2000), a generalized indivisible-labor model. In this economy, there is a stand-in representative agent whose preferences are

$$u(c,N) = \frac{1}{1-\sigma}\{c^{1-\sigma}v^*(1-N)^{1-\sigma} - 1\}$$

where

$$v^*(1-N) = \left[\frac{N}{H}v_1^{\left(\frac{1-\sigma}{\sigma}\right)} + \left(1-\frac{N}{H}\right)v_2^{\left(\frac{1-\sigma}{\sigma}\right)}\right]^{\frac{\sigma}{\sigma-1}}$$

where H is the shift length of those who work, N indicates the average hours of work in the economy, and $v_1 = v(1-H)$ and $v_2 = v(1)$ are respectively the utility of leisure of those who work and those who do not work. A log-linear approximation to the first order conditions of the consumer[23] can be written as

$$-\sigma\widehat{c}_t - (1-\sigma)\eta\widehat{N}_t = \widehat{\mu}_t \tag{3.6}$$

$$(1-\sigma)\,\widehat{c}_t + \frac{(1-\sigma)^2}{\sigma}\eta\widehat{N}_t = \widehat{\mu}_t + \widehat{w}_t \tag{3.7}$$

[22] An example is in Sbordone 2001, appendix A.

[23] These are eqs. (6.8) and (6.9) in King and Rebelo, rewritten as function of hours, as opposed to leisure.

where $\widehat{\mu}_t$ is the marginal utility of consumption, $\widehat{w}_t$ is the real wage, and $\eta = \frac{v^{*\prime}(1-N)}{v^{*}(1-N)}N^{*}$. Substituting (3.6) into (3.7) one gets

$$\widehat{w}_t = \lambda\widehat{c}_t + \nu\widehat{N}_t$$

It is clear then that $\nu = \frac{1-\sigma}{\sigma}\eta$ has a negative value for any $\sigma > 1$. This model is able to rationalize the empirical result that non separable preferences are a necessary condition to obtain a negative value for the parameter ν, but also implies that, contrary to the empirical result obtained here, λ should be equal to 1.

4. Introducing sluggish wage adjustment

To address the high volatility of wage growth implied by the baseline real wage model I consider here the possibility that the actual wage departs in some way from the 'desired' wage that would hold under perfectly flexible wages.

4.1. Nominal wage stickiness

I assume a wage setting structure of the kind described by Erceg et al. (2000), which is the analogue to the structure developed by Calvo to model price stickiness. The model features monopolistic competition among households with respect to the supply of labor: each household offers a differentiated type of labor services to the firms. I further assume that households stipulate wage contracts in nominal terms, and that at the stipulated wage they supply as many hours as are demanded. Total labor employed by any firm j is an aggregation of individual differentiated hours

$$H_t^j = \left[\int_0^1 h_{it}^{(\theta-1)/\theta} di\right]^{\theta/(\theta-1)} \tag{4.1}$$

where θ is the Dixit-Stiglitz elasticity of substitution among differentiated labor services ($\theta > 1$). The wage index is defined as

$$W_t = \left[\int_0^1 W_{it}^{1-\theta} di\right]^{1/(1-\theta)}$$

Household i faces the following demand function for her labor services from each firm j[24]

$$h_{it}^j = (W_{it}/W_t)^{-\theta} H_t^j \tag{4.2}$$

which, aggregated across firms, gives the total demand of labor hours h_{it} equal to

$$h_{it} = (W_{it}/W_t)^{-\theta} H_t \tag{4.3}$$

where $H_t = \left[\int_0^1 H_t^j dj\right]$.

To introduce staggered wage changes, I assume that at each point in time only a fraction $(1-\psi)$ of the households can set a new wage, which I denote by X_{it}, independently of the past history of wage changes, and this wage will then remain fixed until the next time the household is drawn to change wages again. Letting ψ vary between 0 and 1, the model nests a wide range of assumptions about the degree of wage inertia, from perfect wage flexibility ($\psi = 0$) to complete nominal wage rigidity ($\psi \longrightarrow 1$). The expected time between wage changes is $\frac{1}{1-\psi}$. I also assume (as in Erceg et al. (2000)), that households have access to a complete set of state contingent contracts for consumption; in this way, although workers that work different amount of time also have different consumption paths, in equilibrium they have the same marginal utility of consumption.

The wage setting problem is defined as the choice of the wage X_{it} that maximizes the expected stream of discounted utility from the new wage, defined as the difference between the gain (measured in terms of the marginal utility of consumption) derived from the hours

[24]This demand is obtained by solving firm $j's$ problem of allocating a given wage payment among the differentiated labor services, i.e. the problem of maximizing (4.1) for a given level of total wages to be paid.

worked at the new wage and the disutility of working the number of hours associated with the new wage

$$E_t \left\{ \Sigma_{j=0}^{\infty} (\beta\psi)^j \left[\frac{U_C(C_{it+j}, h_{t+j,t})}{P_{t+j}} (X_t h_{t+j,t} - P_{t+j} C_{it+j}) + U(C_{it+j}, h_{t+j,t}) \right] \right\} \tag{4.4}$$

Here $h_{t+j,t}$ denotes the hours worked at $t+j$ at the wage set at time t, and I eliminate the index i on X_t since all the households that change wage at t solve the same problem.

The first order condition for this problem can be written as

$$E_t \left\{ \Sigma_{j=0}^{\infty} \; (\beta\psi)^j \left(x_t \Pi_{k=1}^{j} \left(\pi_{t+k}^{w}\right)^{-1} \right)^{-\theta} H_{t+j} \left[x_t \omega_{t+j} \Pi_{k=1}^{j} \left(\pi_{t+k}^{w}\right)^{-1} - \frac{\theta}{\theta-1} mrs_{t+j,t} \right] \right\} = 0 \tag{4.5}$$

where $x_t \equiv X_t/W_t$, π_t^w is the wage inflation $\pi_t^w \equiv W_t/W_{t-1}$, and $mrs_{t+j,t}$ is the marginal rate of substitution between consumption and hours, evaluated at the level of hours $h_{t+j,t}$.[25].

A log-linear approximation of (4.5) around $x^*(\equiv 1)$, $mrs^*(\equiv \frac{\theta}{\theta-1})$, and $\pi^{w*}(\equiv 1)$, combined with a similar approximation to the distribution of aggregate real wages, allows to obtain the following equation for the Calvo model of adjustment of nominal wage contracts

$$\Delta w_t = \gamma((v_t + p_t) - w_t) + \beta E_t \Delta w_{t+1} \tag{4.6}$$

where v_t is the desired real wage, defined as before as the real wage at which the marginal benefit of an increase in real wage is zero, and whose cyclical component is determined according the model above in (2.1). The parameter $\gamma \equiv \frac{(1-\psi)(1-\beta\psi)}{\psi(1+\chi\theta)}$, which I will refer to as the *inertia* parameter, is a measure of the degree of stickiness in the nominal wage.[26]

[25] See section 7.1.1 of the appendix for a complete derivation of this expression.

[26] For the derivation, see again section 7.1.1 of the appendix.

The solution of this model can be written as

$$w_t = \lambda_1 w_{t-1} + (1-\lambda_1)(1-\lambda_2^{-1}) \sum_{j=0}^{\infty} \lambda_2^{-j} E_t(v_{t+j} + p_{t+j}) \tag{4.7}$$

where λ_1 and λ_2 ($|\lambda_1| < 1, |\lambda_2| > 1$) denote the roots of the polynomial associated with the difference equation (4.6), satisfying $\lambda_1 + \lambda_2 = \frac{1+\gamma+\beta}{\beta}$ and $\lambda_1\lambda_2 = \frac{1}{\beta}$.

The approach I use for estimation is to take as given the evolution of the real variables that determine the evolution of the desired real wage v_t, and the evolution of prices, and construct the path of expected future desired nominal wage.[27] The structural parameters λ and ν, and the roots λ_1 and λ_2, are then estimated by minimizing the distance between the model and the data. From these estimates, fixing the subjective discount factor at $\beta = .99$, one can then retrieve an estimate for the inertia parameter γ.

The estimated parameters are reported in row b. of table 1. They show a statistically significant degree of nominal wage inertia, although the estimated elasticities λ and ν are not statistically different from those of the flexible wage model. The last row of the table indicates the gain, in terms of goodness of fit, of removing the assumption of perfect wage flexibility: the model improves significantly over the flexible wage model, by reducing the discrepancy between actual and estimated cyclical wage by slightly more than 30%. The contemporaneous correlation between the two series is also slightly higher than in the flexible wage case (.96 vs. .93). The implied growth of nominal wage has virtually the same volatility of the actual nominal wage growth, and the two series have a correlation of .78. Assuming nominal wage rigidity also smooths real wage growth (the volatility of the projected series is about 85% of that of the actual series).

[27] Since the desired *real* wage is modeled, as before, as a function of consumption and hours, its expected future value is constructed using forecasts of hours and consumption according to the VAR model discussed above.

4.2. Indexation of Nominal Wages

While nominal wage stickiness is a standard hypothesis, one can as well assume that households may be able to negotiate their contracts in real terms, or at least be able to partially index their nominal contracts to the price level. In this case, a similar version of the Calvo model delivers the following equation for the evolution of wages

$$\Delta w_t - \vartheta \Delta p_t = \gamma(v_t - (w_t - p_t)) + \beta E_t(\Delta w_{t+1} - \vartheta \Delta p_{t+1})$$

where the parameter $\vartheta \ \epsilon \ [0, 1]$ represents the degree of indexation. Such a formulation nests the 'nominal' wage stickiness case discuss above ($\vartheta = 0$) and a case of stickiness in the 'real' wage ($\vartheta = 1$).[28] The solution to this model is

$$w_t - \vartheta p_t = \lambda_1 (w_{t-1} - \vartheta p_{t-1}) + (1 - \lambda_1)(1 - \lambda_2^{-1}) \sum_{j=0}^{\infty} \lambda_2^{-j} E_t(v_{t+j} + (1 - \vartheta)\, p_{t+j})$$

Estimates imposing $\vartheta = 1$ are reported in row c. of table 1, while in the last row I report the estimates corresponding to the best indexation coefficient, namely the one that, by improving about 35% over the flexible wage model, allows the best approximation of the cyclical component of the real wage. The partially indexed model marginally outperforms the strictly nominal or real stickiness cases in all the other dimensions considered in the table, increasing the correlation between predicted and actual cyclical real wage (.96), and the correlation of predicted nominal and real wage growth with actual data (respectively .81 and .64). Figure 7 shows the extent to which the partially indexed wage model approximate actual data. Comparing this figure with the previous one of the flexible wage case one sees the significant reduction in the volatility of both nominal and real wage growth. It's worth pointing out, however, that while the best fit is obtained with a partial indexation model, the estimated preference parameters are virtually the same under any degree of indexation, and they are also not statistically different from those of the flexible wage model. In particular,

[28] A complete derivation of a model with sticky real wages in the appendix of Sbordone (2001).

the hypotheses that $\lambda = 1$, and $\nu \geq 0$ are still strongly rejected.

4.3. Backward-looking wage indexation

Before looking at the implication of the estimated wage model for wage inertia, I want to consider a different type of indexation proposed in a recent contribution by Christiano, Eichenbaum and Evans (2001). They modify the Calvo wage-setting model by introducing a backward-looking indexation rule, with the objective of allowing for further inertia in inflation and greater persistence in output.[29] Instead of assuming, as we do here, that the wage of those households that are not allowed to reset their wage contracts remain constant until next adjustment,[30] they assume that the wage of household l at time t,if not adjusted, is indexed to the average inflation rate of the previous period:

$$W_t^l = \pi_{t-1} W_{t-1}^l$$

Here I consider this hypothesis in the slightly more general formulation of Woodford (2001), allowing for any intermediate degree of indexation ϱ, and reformulate their hypothesis as

$$W_t^l = \pi_{t-1}^{\varrho} W_{t-1}^l$$

This formulation is convenient because it allows to nest both the case of full indexation to past price changes considered by Christiano et al. ($\varrho = 1$), and the simple sticky nominal wage model with no indexation $(\varrho = 0)$. Backward-looking indexation determines two modifications to the model analyzed in section 4.1. First, the wage set at time t by the household which optimizes, X_t, is allowed to grow during the time in which the contract is in place, so

[29] The objective of their paper is to provide a general equilibrium model that accounts of the dynamic response of a number of endogenous variables to a monetary policy shock.

[30] More generally (see Erceg et al. 2000), those wages are indexed to the steady state of inflation, which in this paper I assumed to be zero.

that at time $t+j$ the wage is $X_t \Psi_{tj}$, where

$$\Psi_{tj} = \begin{bmatrix} 1 & \text{if } j = 0 \\ \prod_{k=0}^{j-1} \pi_{t+k}^{\varrho} & \text{if } \ j \geq 1 \end{bmatrix}$$

Second, the aggregate wage at any time t, which is an average of the wage set by the workers that optimize and those who do not, is now

$$W_t = \left[(1-\psi) (X_t)^{1-\theta} + \psi \left(\pi_{t-1}^{\varrho} W_{t-1} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{4.8}$$

The objective function is therefore modified as

$$E_t \left\{ \Sigma_{j=0}^{\infty} (\beta\psi)^j \left[\frac{U_C(C_{it+j}, h_{t+j,t})}{P_{t+j}} (X_t \Psi_{tj} h_{t+j,t} - P_{t+j} C_{it+j}) + U(C_{it+j}, h_{t+j,t}) \right] \right\}$$

and the first order condition is

$$E_t \left\{ \Sigma_{j=0}^{\infty} \ (\beta\psi)^j \left(x_t \Psi_{tj} \Pi_{k=1}^{j} \left(\pi_{t+k}^{w} \right)^{-1} \right)^{-\theta} H_{t+j} \left[x_t \Psi_{tj} \omega_{t+j} \Pi_{k=1}^{j} \left(\pi_{t+k}^{w} \right)^{-1} - \frac{\theta}{\theta - 1} mrs_{t+j,t} \right] \right\} = 0$$

With similar methodology one then derives the following wage adjustment equation

$$\Delta w_t - \varrho \Delta p_{t-1} = \gamma \left((v_t + p_t) - w_t \right) + \beta E_t \left(\Delta w_{t+1} - \varrho \Delta p_t \right)$$

where the parameter γ is defined exactly as in the simple Calvo model of section 4.1. Estimates for this model are reported in the last row of table 1. The estimated elasticities are again remarkably similar to those of the simple Calvo specification while the inertia parameter is quite higher. The fit of the model is however worse than that of the two models studied above with respect to the measures considered in the table The objective function, compared to the flexible wage model, is reduced by only 20.7%, and decreasing the degree of the indexation (i.e. moving towards the purely sticky nominal wage model) improves all dimensions of the fit. At the same time, when the value of ϱ moves from 1 to 0, the estimate of γ also monotonically decreases, implying that this variant has a moderately lower degree of wage rigidity than the Calvo model.

4.4. Interpretation of wage stickiness

The 'inertia' parameter γ is a combination of various structural parameters:

$$\gamma = \frac{(1-\psi)(1-\beta\psi)}{\psi(1+\chi\theta)}$$

where ψ is the parameter that drives the frequency of wage changes, θ is the elasticity of substitution among differentiated labor services, and χ is a parameter which depends upon the elasticity of the marginal rate of substitution between leisure and consumption

$$\chi = \frac{-U_{cH}H}{U_{cc}C}\eta_{mrs,c} + \eta_{mrs,h} = \frac{-U_{cH}H}{U_{cc}C}\lambda + \nu \tag{4.9}$$

Given χ, β and θ, a higher γ implies a lower degree of stickiness. Except for the case of backward indexation, a higher degree of indexation, quite intuitively, lowers the estimate valued of γ, increasing the expected time between wage adjustments. The value of the parameter χ, on the other hand, is not affected by the degree of indexation, since, as noted, the estimates of λ and ν are virtually the same in any sticky wage models. The value of χ instead increases with the degree of non-separability between consumption and hours.

To parametrize χ, I first consider a slight transformation of expression (4.9)

$$\chi = \frac{-U_{cH}U_c}{U_{cc}U_H}\left(\frac{U_H H}{U_c C}\right)\lambda + \nu \tag{4.10}$$

and then write the expression for λ as

$$\lambda = -\frac{U_{cc}C}{U_c} + \frac{U_{Hc}C}{U_H} = \sigma + \frac{U_{Hc}C}{U_H} = \sigma + \frac{U_{Hc}}{U_{cc}}\left(\frac{U_{cc}C}{U_c}\right)\frac{U_c}{U_H} = \sigma\left(1 - \frac{U_{Hc}}{U_{cc}}\frac{U_c}{U_H}\right) \tag{4.11}$$

where, with conventional notation, I indicate with σ the inverse of the intertemporal elasticity of substituition in consumption. Expression (4.11) implies that

$$\frac{U_{Hc}}{U_{cc}}\frac{U_c}{U_H} = \frac{\sigma - \lambda}{\sigma}$$

Substituting this result in (4.10), I obtain

$$\chi = \left(\frac{\sigma - \lambda}{\sigma} * \tau\right)\lambda + \nu$$

Therefore, the value of χ can be determined by assigning a value to σ, and to the ratio wH/C, which I have denoted by τ.

Table 2 reports various coefficients of inertia for the partially index model (panel a) and, as a comparison, also for the model with purely sticky nominal wage (panel b), and the model with full dynamic updating (panel c). The computations are based on three different assumptions about the value of the intertemporal elasticity of substitution in consumption (σ = 4, 5, or 10), and three possible values of the steady state wage mark-up (10%, 30% and 50%, respectively). Note that every value of σ implies in turn a different degree of non-separability in preferences.

Allowing the best degree of indexation to the current level of prices, the implied wage inertia ranges between 3 and 6 months. In particular, the estimates show that, for any given degree of wage mark up, a higher χ (i.e. a higher degree of non separability in preferences) is consistent with a lower degree of wage inertia. Comparing the estimates of this model with the purely sticky nominal wage model, one notes that eliminating indexation reduces slightly the interval between wage adjustment. The high estimate of γ in the model with backward indexation, instead, brings the average interval between wage changes around 3.5 months.

5. Wage-price dynamics with staggered wages and prices

The wage model discussed above provides the link between the evolution of real quantities and the evolution of marginal costs we were seeking. It therefore allows to go from a forward-looking theory of price determination to a well specified Phillips curve that describes the dynamic path of inflation as function of the path of output and productivity. Specifically, one obtains such a specification by combining the wage model (I choose here the partially indexed model)

$$\Delta w_t - \vartheta \Delta p_t = \gamma(v_t - (w_t - p_t)) + \beta E_t(\Delta w_{t+1} - \vartheta \Delta p_{t+1}) \tag{5.1}$$

with a price equation derived from a staggered price model of price determination, which I rewrite here as

$$\Delta p_t = \zeta((w_t - p_t) - q_t) + \alpha_1 E_t \Delta p_{t+1} \tag{5.2}$$

q_t denotes, as before, average labor productivity.[31] The desired real wage v_t is in turn the sum of a stochastic trend and a cyclical component which is, according to eq. (3.1), a function of the cyclical components of consumption and hours

$$v_t = v_t^{tr} + (\lambda c_t^{cyc} + \nu h_t^{cyc}) \tag{5.3}$$

Instead of specifying all the remaining equations of a fully general model, the evolution of the real variables is taken as given. Specifically, I assume that the evolution of productivity, consumption, and hours (the last two in turn determining the evolution of the desired real wage), is well described by the stochastic process

$$Z_t = \Gamma Z_{t-1} + \varepsilon_{zt} \tag{5.4}$$

[31] Although it has been shown that it is possible to improve moderately on the empirical specification by adding a backward looking component (for example by adding a backward-looking indexation to the price-setting model as in the cited paper by Christiano et al. (2001), I prefer here to consider the purely forward-looking model to make my point more clearly.

where $Z_t = [X_t \quad X_{t-1}]'$, and X_t is defined as in (3.3).

Equations (5.1), (5.2), and (5.4) form a system that can be solved for equilibrium processes $\{w_t, p_t\}$, given stochastic processes for $\{v_t, q_t\}$, and initial conditions $\{w_{-1}, p_{-1}\}$. This system can be written in the form of a first order expectational difference equation system. First, using the identities

$$\begin{aligned} q_t &= q_{t-1} + \Delta q_t \qquad (5.5) \\ w_t - p_t &= w_{t-1} - p_{t-1} + \Delta w_t - \Delta p_t \qquad (5.6) \end{aligned}$$

the wage equation and the price equation can be written respectively as

$$E_t \Delta w_{t+1} - \vartheta E_t \Delta p_{t+1} = \frac{1+\gamma}{\beta} \Delta w_t - \frac{\gamma+\vartheta}{\beta} \Delta p_t + \frac{\gamma}{\beta}(w_{t-1} - p_{t-1}) - \frac{\gamma}{\beta} v_t \qquad (5.7)$$

and

$$E_t \Delta p_{t+1} = \frac{1+\zeta}{\alpha_1} \Delta p_t - \frac{\zeta}{\alpha_1} \Delta w_t - \frac{\zeta}{\alpha_1}(w_{t-1} - p_{t-1}) + \frac{\zeta}{\alpha_1} q_{t-1} + \frac{\zeta}{\alpha_1} e_1' Z_t \qquad (5.8)$$

Then, from (5.3), using the definition of stochastic trend, and the model for the cyclical components of hours and consumption, v_t can be written as a function of the variables in Z_t

$$v_t = q_t + \Xi Z_t \qquad (5.9)$$

where $\Xi = ((1-\lambda)e_1'(I-\Gamma)^{-1}\Gamma + (\lambda+\nu)e_2' + \lambda e_3')$, and e_i' denotes a 10-dimensional row vector which has a 1 in the i-th position, and zeros elsewhere. Finally, substituting (5.8) and (5.9) into (5.7), the nominal wage becomes the following function of observables

$$\begin{aligned} E_t \Delta w_{t+1} = & \left(\frac{\alpha_1(1+\gamma) - \beta\vartheta\zeta}{\alpha_1\beta}\right) \Delta w_t + \left(\frac{\beta\vartheta(1+\zeta) - \alpha_1(\gamma+\vartheta)}{\alpha_1\beta}\right) \Delta p_t \\ & + \left(\frac{\alpha_1\gamma - \beta\vartheta\zeta}{\alpha_1\beta}\right)(w_{t-1} - p_{t-1}) + \left(\frac{\beta\zeta\vartheta - \alpha_1\gamma}{\alpha_1\beta}\right) q_{t-1} + \Psi Z_t \qquad (5.10) \end{aligned}$$

where $\Psi = \left(\frac{\beta\zeta\vartheta - \alpha_1\gamma}{\alpha_1\beta}\right) e_1' - \frac{\gamma}{\beta}\Xi$.

Defining

$$Y_t = [\Delta w_t \ \Delta p_t \ (w_{t-1} - p_{t-1}) \ q_{t-1} \ Z_t]',$$

the system of equations (5.10), (5.8), (5.4), and identities (5.5) and (5.6) can be written as

$$E_t Y_{t+1} = M Y_t \tag{5.11}$$

This system has a unique solution since the matrix M has exactly two unstable eigenvalues[32]. Letting μ_1 and μ_2 denote these two eigenvalues, and x_1 and x_2 denote respectively the eigenvectors associated with them, the solution is given by the two equations

$$x_1'[\Delta w_t \ \Delta p_t \ (w_{t-1} - p_{t-1}) \ \ q_{t-1} \ Z_t] = 0 \tag{5.12}$$

and

$$x_2'[\Delta w_t \ \Delta p_t \ (w_{t-1} - p_{t-1}) \ \ q_{t-1} \ Z_t] = 0 \tag{5.13}$$

5.1. A calibration exercise

To experiment with the ability of this model to reproduce the simultaneous dynamics of prices and wages observed in the data, I calibrate the parameters of the model on the basis of single equation estimates, and compute the series of wages and prices according to the solution (5.12)-(5.13). Specifically, I choose parameter values $\alpha_1 = .99$, $\gamma = .76$, $\beta = .99$, $\lambda = 2.36$, $\nu = -.996$, $\zeta = .039$[33], $\vartheta = .5$. Inflation and nominal wage growth predicted by the model are plotted against the corresponding actual U.S. series in fig. 7.

The fit of the inflation process appears quite good: although predicted inflation overstates actual inflation in the late '80s, and overstates as well the decline in inflation in the second half of the '90s, it does nonetheless reproduce the major inflation waves of the middle and late '70s. Actual and predicted inflation have a correlation of .85. The model seems to be

[32] The conditions for uniqueness are verified in the Appendix, sect. 7.3.

[33] This parameter is the coefficient of price inertia estimated in eq. (2.7) and used in fig. 2.

able to reproduce also quite closely the major features of the wage process, although slightly overpredicting wage growth in '74-'75 and before the '82 recession. The correlation between actual and predicted nominal wage growth is .78.

The interesting question is whether the inflation dynamics predicted by this model is able to match the comovement between inflation and the deterministic measure of output gap observed in the data. As discussed in section 2, and emphasized in fig. 1 of this paper, the "standard" NKPC, which assumed proportionality between real marginal costs and this deterministic measure of output gap fails dramatically in this dimension. Quite to the contrary, as panel b. of figure 8 shows, the model presented here succeeds in accounting for the lead of output over inflation observed in the data[34]. Estimated and actual dynamic correlations peak at about the same lag, and are overall statistically close. Furthermore, the predicted inflation series has a significant degree of persistence.

6. Conclusion

This paper provides some evidence that, for a given evolution of real variables, it is possible to reproduce quite closely the evolution of U.S. prices and wages with a fully microfounded model with staggered prices and wages.

I view the contribution of the paper as twofold. First, its result shows that it is indeed possible to fit to U.S. data a Phillips curve specification consistent both with rational expectations and with optimizing behavior. The task of a simultaneous estimation of the dynamic wage and price equation is taken up in a follow-up paper.

Secondly, the empirical investigation of the wage setting mechanism implies that the standard form of preferences used in business cycle literature is at odd with the data. Although this is not a new result, it has not been too much acknowledged in the business cycle literature. Early estimates of intertemporal substitution models (for ex. Mankiw, Rotemberg,

[34]Consistent with the assumption, made in estimating the VAR, that output contains a unit root, the appropriate measure of output gap used here is the deviation from a stochastic trend, and it is therefore indicated as y^{cyc}.

and Summers (1985), Eichenbaum, Hansen, and Singleton (1988)) show that, when fit to the data, these models imply that leisure should be an inferior good, both when preferences are imposed to be separable in consumption and leisure, and when non-separability is allowed.

Only recently a new line of research seems to be interested in addressing the theoretical consequences of alternative forms of preferences (for example, Baxter and Jermann ('99), King and Rebelo ('99)).

7. Appendix

7.1. Derivation of the sticky wage equations

In this section I first derive the first order condition (4.5) and then obtain the log-linearizations that lead to eq. (4.6). To be consistent with the empirical results of the flexible wage model, which implies that preferences should be non-separable in consumption and leisure, I allow the marginal utility of consumption to vary with hours of work.

7.1.1. Derivation of eq. (4.5)

To derive the first order condition for optimal wage, observe that, by (4.3), $h_{t+j,t} = \left(\frac{X_t}{W_{t+j}}\right)^{-\theta} H_{t+j}$, and therefore

$$\frac{\partial h_{t+j,t}}{\partial X_t} = -\frac{\theta}{X_t}\left(\frac{X_t}{W_{t+j}}\right)^{-\theta} H_{t+j} = -\frac{\theta}{X_t} h_{t+j,t}$$

Using $U_{A,t+j}$ as short notation for $U_A(C_{it+j}, h_{t+j,t})$, $A = C, H$, the derivative of the terms in square brackets of the objective function (4.4) with respect to X_t is

$$\frac{\partial[.]}{\partial X_t} = h_{t+j,t}\frac{U_{C,t+j}}{P_{t+j}} + h_{t+j,t}\frac{X_t}{P_{t+j}}\frac{\partial U_{C,t+j}}{\partial X_t} + X_t\frac{U_{C,t+j}}{P_{t+j}}\frac{\partial h_{t+j,t}}{\partial X_t} - C_{it+j}\frac{\partial U_{C,t+j}}{\partial X_t} + U_{H,t+j}\frac{\partial h_{t+j,t}}{\partial X_t}$$

Efficient risk sharing implies that the marginal utility of consumption is the same across households, and therefore $\frac{\partial U_{C,t+j}}{\partial X_t} = 0$, so

$$\begin{aligned}
\frac{\partial [.]}{\partial X_t} &= h_{t+j,t}\frac{U_{C,t+j}}{P_{t+j}} + \frac{\partial h_{t+j,t}}{\partial X_t}(X_t \frac{U_{C,t+j}}{P_{t+j}} + U_{H,t+j}) = h_{t+j,t}\left[(1-\theta)\frac{U_{C,t+j}}{P_{t+j}} - \theta\frac{U_{H,t+j}}{X_t}\right] \\
&= h_{t+j,t}\left[\frac{X_t}{P_{t+j}} - \frac{\theta}{\theta-1}\left(\frac{-U_{H,t+j}}{U_{C,t+j}}\right)\right]
\end{aligned}$$

where $\frac{\theta}{\theta-1}$ denotes a wage mark up.

The first order condition can therefore be written as

$$E_t\left\{\Sigma_{j=0}^{\infty}\ (\beta\psi)^j\left(\frac{X_t}{W_{t+j}}\right)^{-\theta} H_{t+j}\left[\frac{X_t}{P_{t+j}} - \frac{\theta}{\theta-1}\left(\frac{-U_{H,t+j}}{U_{C,t+j}}\right)\right]\right\} = 0 \tag{7.1}$$

which has the usual interpretation that the optimal wage sets the discounted sum of labor income equal to the discounted expected sum of future marginal rates of substitution between consumption and leisure.

Defining now the variables $x_t \equiv X_t/W_t$, $\pi_t^w \equiv W_t/W_{t-1}$, $\omega_t = W_t/P_t$, and letting $mrs_{t+j,t} \equiv \frac{-U_{H,t+j}}{U_{C,t+j}}$, one obtains eq. (4.5) in the text by noting that $\frac{X_t}{W_{t+j}} = \frac{X_t}{W_t}\frac{W_t}{W_{t+j}} = x_t \Pi_{k=1}^{j}\left(\pi_{t+k}^w\right)^{-1}$, and $\frac{X_t}{P_{t+j}} = \frac{X_t}{W_{t+j}}\frac{W_{t+j}}{P_{t+j}}$.

7.1.2. Derivation of eq. (4.6)

Taking a log-linear approximation of (4.5) around $x^*(\equiv 1)$, $mrs^*(\equiv \frac{\theta}{\theta-1})$, and $\pi^{w*}(\equiv 1)$, one obtains

$$\Sigma_{j=0}^{\infty}\ (\beta\psi)^j\left(\widehat{x}_t - \Sigma_{k=1}^{j}\widehat{\pi^w}_{t+k} + \widehat{\omega}_{t+j}\right) = \Sigma_{j=0}^{\infty}(\beta\psi)^j\ E_t\widehat{mrs}_{t+j,t}$$

This gives

$$\frac{1}{1-\beta\psi}\widehat{x}_t = \Sigma_{j=0}^{\infty}(\beta\psi)^j\ E_t\left(\widehat{mrs}_{t+j,t}\ + \Sigma_{k=1}^{j}\widehat{\pi^w}_{t+k} - \widehat{\omega}_{t+j}\right)$$

or

$$\widehat{x}_t = (1-\beta\psi)\ \Sigma_{j=0}^{\infty}(\beta\psi)^j\ E_t\left(\widehat{mrs}_{t+j,t}\ - \widehat{\omega}_{t+j} + \Sigma_{k=1}^{j}\widehat{\pi^w}_{t+k}\right) \tag{7.2}$$

Solve for $mrs_{t+j,t}$ in terms of the marginal rate of substitution evaluated at *average* aggregate consumption and hours, mrs_{t+j} (which is the desired real wage in the baseline model, $=\frac{-U_H}{U_C}(c_{t+j}, h_{t+j})$). To do that, rewrite $mrs_{t+j,t}$ as

$$mrs_{t+j,t} \equiv \frac{-U_H}{U_C}(c_{t+j,t}, h_{t+j,t}) = \frac{\frac{-U_H}{U_C}(c_{t+j,t}, h_{t+j,t})}{\frac{-U_H}{U_C}(c_{t+j}, h_{t+j})} * \frac{-U_H}{U_C}(c_{t+j}, h_{t+j}) \tag{7.3}$$

A log-linearization of (7.3) gives therefore

$$\widehat{mrs}_{t+j,t}\ = \eta_{mrs,c}(\widehat{c}_{t+j,t} - \widehat{c}_{t+j}) + \eta_{mrs,h}\left(\widehat{h}_{t+j,t} - \widehat{H}_{t+j}\right) + \widehat{mrs}_{t+j} \tag{7.4}$$

where $\eta_{mrs,x}$ $(x = c, h)$ indicates the elasticity of the marginal rate of substitution with respect to x, evaluated at the steady state.

By the assumption that changes in consumption occur in a way to maintain the marginal utility of consumption equal across households, $\widehat{c}_{t+j,t}$ and $\widehat{c}_{t+j}$ are respectively function of $\widehat{h}_{t+j,t}$ and $\widehat{h}_{t+j}$. Moreover, since $h_{t+j,t} = \left(\frac{X_t}{W_{t+j}}\right)^{-\theta} H_{t+j}$,

$$\widehat{h}_{t+j,t} = -\theta\left(\widehat{x}_t - \Sigma_{k=1}^{j}\widehat{\pi^w}_{t+k}\right) + \widehat{h}_{t+j}$$

so that (7.4) becomes

$$\widehat{mrs}_{t+j,t}\ = -\chi\ \theta\left(\widehat{x}_t - \Sigma_{k=1}^{j}\widehat{\pi^w}_{t+k}\right) + \widehat{mrs}_{t+j} \tag{7.5}$$

where $\chi = \frac{-U_{cH}H}{U_{cc}C}\eta_{mrs,c} + \eta_{mrs,h}$.

Consider now that the distribution of nominal wages at time t is a mixture of the distribution of wages of the previous period (since all previous wages have the same probability of being changed), with weight ψ, and the new wage X_t, with weight $(1-\psi)$

$$W_t = \left[(1-\psi)X_t^{1-\theta} + \psi W_{t-1}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{7.6}$$

Dividing both sides by W_t, a log linear approximation of this expression is:

$$0 = (1-\psi)\widehat{x}_t - \psi\widehat{\pi}_t^w$$

or

$$\widehat{x}_t = \frac{\psi}{1-\psi}\ \widehat{\pi}_t^w \tag{7.7}$$

Substituting (7.7) and (7.5) into (7.2) one gets

$$\frac{\psi(1+\chi\ \theta)}{1-\psi}\widehat{\pi}_t^w = (1-\beta\psi)\ \Sigma_{j=0}^{\infty}(\beta\psi)^j E_t\left(\widehat{mrs}_{t+j}\ + (1+\chi\ \theta)\Sigma_{k=1}^{j}\widehat{\pi}_{t+k}^w - \widehat{\omega}_{t+j}\right)$$

so that

$$\widehat{\pi}_t^w = \gamma\ \Sigma_{j=0}^{\infty}(\beta\psi)^j\ E_t\left(\widehat{mrs}_{t+j}\ + (1+\chi\ \theta)\Sigma_{k=1}^{j}\widehat{\pi}_{t+k}^w - \widehat{\omega}_{t+j}\right) \tag{7.8}$$

where $\gamma \equiv \frac{(1-\psi)(1-\beta\psi)}{\psi(1+\chi\theta)}$.

I compute now $(\beta\psi)E_t\widehat{\pi}_{t+1}^w$ (by evaluating expression (7.8) at $t+1$, pre-multiplying it by $(\beta\psi)$), and taking expectations as of time t), and subtract the resulting expression from (7.8), to obtain

$$\widehat{\pi}_t^w - (\beta\psi)E_t\widehat{\pi}_{t+1}^w = \frac{(1-\psi)(1-\beta\psi)}{\psi(1+\chi\theta)}J_t \tag{7.9}$$

where

$$\begin{aligned}
J_t &= \Sigma_{j=0}^{\infty}(\beta\psi)^j\ E_t\left(\widehat{mrs}_{t+j}\ +(1+\chi\theta)\Sigma_{k=1}^{j}\widehat{\pi}_{t+k}^{w}-\widehat{\omega}_{t+j}\right)- \\
&\quad \Sigma_{j=0}^{\infty}(\beta\psi)^{j+1}\ E_t\left(\widehat{mrs}_{t+1+j}\ +(1+\chi\theta)\Sigma_{k=1}^{j}\widehat{\pi}_{t+1+k}^{w}-\widehat{\omega}_{t+j+1}\right) \\
&= \Sigma_{j=0}^{\infty}(\beta\psi)^j E_t\left[\begin{array}{c}(\widehat{mrs}_{t+j}\ -(\beta\psi)\widehat{mrs}_{t+1+j})+(1+\chi\theta)\left(\Sigma_{k=1}^{j}\widehat{\pi}_{t+k}^{w}-(\beta\psi)\Sigma_{k=1}^{j}\widehat{\pi^{w}}_{t+1+k}\right) \\ -(\widehat{\omega}_{t+j}-(\beta\psi)\widehat{\omega}_{t+j+1})\end{array}\right] \\
&= (\widehat{mrs}_t-\widehat{\omega}_t)+\frac{\beta\psi(1+\chi\theta)}{1-\beta\psi}E_t\widehat{\pi}_{t+1}^{w}
\end{aligned}$$

Expression (7.9) becomes then

$$\widehat{\pi}_t^w-(\beta\psi)E_t\widehat{\pi}_{t+1}^w=\frac{(1-\psi)(1-\beta\psi)}{\psi(1+\chi\theta)}(\widehat{mrs}_t-\widehat{\omega}_t)+\beta(1-\psi)E_t\widehat{\pi}_{t+1}^w$$

so that wage inflation is

$$\widehat{\pi}_t^w=\beta E_t\widehat{\pi}_{t+1}^w+\frac{(1-\psi)(1-\beta\psi)}{\psi(1+\chi\theta)}(\widehat{mrs}_t-\widehat{\omega}_t) \tag{7.10}$$

Finally, using the fact that $\widehat{mrs}_t=\widehat{v}_t$, we obtain the wage equation (4.6) of the text

$$\widehat{\pi}_t^w=\beta E_t\widehat{\pi}_{t+1}^w+\frac{(1-\psi)(1-\beta\psi)}{\psi(1+\chi\theta)}(\widehat{v}_t-\widehat{\omega}_t)$$

7.1.3. Solving for the optimal path of nominal wage

I first write explicitly the wage inflation equation as

$$w_t-w_{t-1}=\gamma(v_t+p_t)-\gamma w_t+\beta E_t w_{t+1}-\beta w_t$$

so that

$$
\begin{aligned}
v_t + p_t &= \frac{1+\gamma+\beta}{\gamma} w_t - \frac{1}{\gamma} w_{t-1} - \frac{\beta}{\gamma} E_t w_{t+1} = -\frac{\beta}{\gamma} E_t \left[1 - \frac{1+\gamma+\beta}{\beta} L + \frac{1}{\beta} L^2 \right] w_{t+1} \\
&= -\frac{\beta}{\gamma} E_t \left[L^2 P(L^{-1}) \right] w_{t+1} = -\frac{\beta}{\gamma} E_t \left[(1-\lambda_1 L)(1-\lambda_2 L) \right] \ w_{t+1}
\end{aligned}
$$

where $P(L^{-1}) = L^{-2} - \frac{1+\gamma+\beta}{\beta} L^{-1} + \frac{1}{\beta}$ has real roots λ_1, λ_2 satisfying $0 < \lambda_1 < 1$, and $\lambda_2 > \beta^{-1} \geq 1$.

Then, defining $x_{t+1} = (1 - \lambda_1 L) \ w_{t+1}$, I rewrite $v_t + p_t$ as

$$
v_t + p_t = -\frac{\beta}{\gamma} E_t (1 - \lambda_2 L) \ x_{t+1} = -\frac{\beta}{\gamma} E_t \ x_{t+1} + \frac{\beta \lambda_2}{\gamma} x_t
$$

from which

$$
x_t = \frac{\gamma}{\beta \lambda_2} (v_t + p_t) + \lambda_2^{-1} E_t x_{t+1}
$$

Solving forward

$$
x_t = \frac{\gamma}{\beta \lambda_2} \sum_{j=0}^{\infty} \lambda_2^{-j} E_t \left(v_{t+j} + p_{t+j} \right) = (1-\lambda_1)(1-\lambda_2^{-1}) \sum_{j=0}^{\infty} \lambda_2^{-j} E_t \left(v_{t+j} + p_{t+j} \right)
$$

where the equality $\frac{\gamma}{\beta \lambda_2} = (1-\lambda_1)(1-\lambda_2^{-1})$ follows from the fact that $\lambda_1 + \lambda_2 = \frac{1+\gamma+\beta}{\beta}$ and $\lambda_1 \lambda_2 = 1/\beta$. Finally, from the definition of x_t, I obtain

$$
w_t = \lambda_1 w_{t-1} + (1-\lambda_1)(1-\lambda_2^{-1}) \sum_{j=0}^{\infty} \lambda_2^{-j} E_t \left(v_{t+j} + p_{t+j} \right)
$$

which is expression (4.7) in the text.

7.2. Solution of the system (5.11)

For the system (5.11) to have a unique solution, the matrix M, which is

$$M = \begin{bmatrix} \frac{\alpha_1(1+\gamma)-\beta\vartheta\zeta}{\alpha_1\beta} & \frac{\beta\vartheta(1+\zeta)-\alpha_1(\gamma+\vartheta)}{\alpha_1\beta} & \frac{\alpha_1\gamma-\beta\vartheta\zeta}{\alpha_1\beta} & -\frac{\alpha_1\gamma-\beta\vartheta\zeta}{\alpha_1\beta} & \Psi \\ -\frac{\zeta}{\alpha_1} & \frac{(1+\zeta)}{\alpha_1} & -\frac{\zeta}{\alpha_1} & \frac{\zeta}{\alpha_1} & \frac{\zeta}{\alpha_1}e_1' \\ 1 & -1 & 1 & 0 & \underline{0} \\ 0 & 0 & 0 & 1 & e_1' \\ 0 & 0 & 0 & 0 & \Gamma \end{bmatrix}$$

must have two eigenvalues with modulus strictly bigger than one. One can see that it is enough to check the eigenvalues of the upper left 3x3 matrix, call it $\widetilde{M}$, which solve

$$P(\mu) =\mid \widetilde{M} - \mu I \mid= \mu^3 + \mu^2 M_2 + \mu M_1 + M_0 = 0$$

where

$$\begin{aligned} M_2 &= -(1 + \tfrac{1+\gamma}{\beta} + \tfrac{1}{\alpha_1}(1+\zeta(1-\vartheta))) \\ M_1 &= \tfrac{1}{\alpha_1} + \tfrac{1}{\beta} + \tfrac{1}{\alpha_1\beta}(1+\gamma+\zeta(1-\vartheta)) \\ M_0 &= -\tfrac{1}{\alpha_1\beta} \end{aligned}$$

The coefficients (M_0, M_1, M_2) satisfy the following necessary and sufficient conditions for determinacy

$$\begin{aligned} &i. && 1 + M_2 + M_1 + M_0 > 0 \\ &ii. && -1 + M_2 - M_1 + M_0 < 0 \end{aligned}$$

and either

$$iii. \quad M_0^2 - M_0 M_2 + M_1 - 1 > 0$$

or[35]

$$\begin{aligned} &iv. && M_0^2 - M_0 M_2 + M_1 - 1 < 0 \\ &v. && |M_2| > 3 \end{aligned}$$

[35] These conditions are stated in Woodford (2000).

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TABLE 1
Estimates of wage models†

	λ	ν	γ	ϑ	$\rho(\omega^{a}, \omega^{p})$	$var(\varepsilon_t^{w})$ $[*e^{-5}]$	% $var\ red$
a. Flexible wage	2.15 (.066)	-.84 (.037)			.93	8.5	
$\lambda-$ restricted	1	-.465 (.030)			.92	25	-195
$\nu-$ restricted	1.29 (.054)	0			.68	36	-328
b. Sticky nominal wage	2.32 (.077)	-.987 (.045)	1.699 (.26)	0	.96	5.8	31.6
c. Sticky real wage	2.40 (.058)	-.988 (.058)	.266 (.079)	1	.95	6.6	22.3
d. Partial wage indexation	2.32 (.077)	-.996 (.047)	.76 (.14)	.5	.96	5.5	34.8
e. Backward-looking wage indexation	2.30 (.072)	-.965 (.045)	4.19 (.74)		.94	6.7	20.7

† Standard errors are in parenthesis.$\rho(\omega^{a}, \omega^{p})$ indicates the correlation between the cyclical component of the real wage estimated from the data (ω^{a}), and the one predicted by each model (ω^{p}).

TABLE 2
Estimated average time between wage changes (months)

(a) Partially indexed wages

	low wage mark-up ($\mu^{w*} = 1.1$)	mid wage mark-up ($\mu^{w*} = 1.3$)	high wage mark-up ($\mu^{w*} = 1.5$)
$\tau = 1$; $\sigma = 4$ (low non-sep.)	5.6	5.4	5.3
$\tau = 1$; $\sigma = 5$ (mid non-sep.)	3.8	4.3	4.5
$\tau = 1$; $\sigma = 10$ (high non-sep.)	3.4	3.7	3.9

(b) Sticky nominal wages

	low wage mark-up ($\mu^{w*} = 1.1$)	mid wage mark-up ($\mu^{w*} = 1.3$)	high wage mark-up ($\mu^{w*} = 1.5$)
$\tau = 1$; $\sigma = 4$ (low non-sep.)	4.4	4.3	4.3
$\tau = 1$; $\sigma = 5$ (mid non-sep.)	3.4	3.7	3.8
$\tau = 1$; $\sigma = 10$ (high non-sep.)	3.2	3.3	3.4

(c) Sticky nominal wages with backward indexation

	low wage mark-up ($\mu^{w*} = 1.1$)	mid wage mark-up ($\mu^{w*} = 1.3$)	high wage mark-up ($\mu^{w*} = 1.5$)
$\tau = 1$; $\sigma = 4$ (low non-sep.)	3.6	3.6	3.6
$\tau = 1$; $\sigma = 5$ (mid non-sep.)	3.2	3.3	3.3
$\tau = 1$; $\sigma = 10$ (high non-sep.)	3.1	3.1	3.2

Fig. 1a – Inflation predicted by output gap

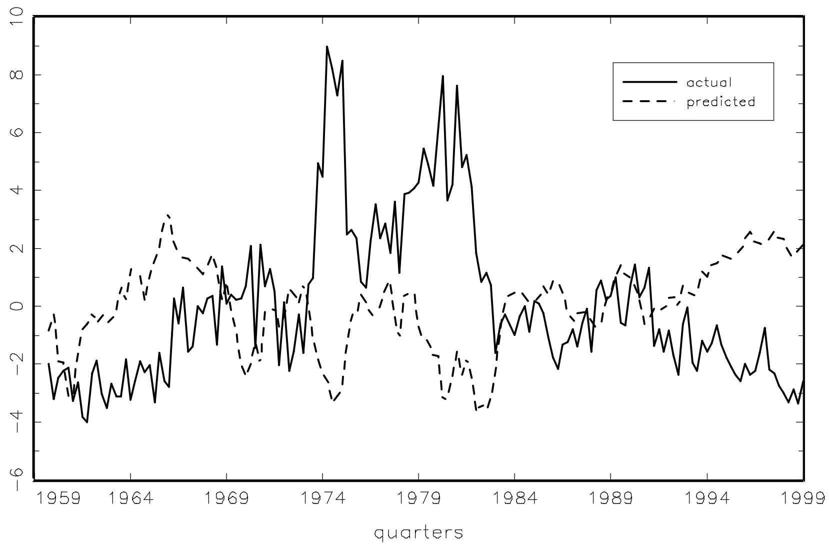


Fig. 1b – Dynamic cross-correlations $\rho(gap_t, \pi_{t+k})$

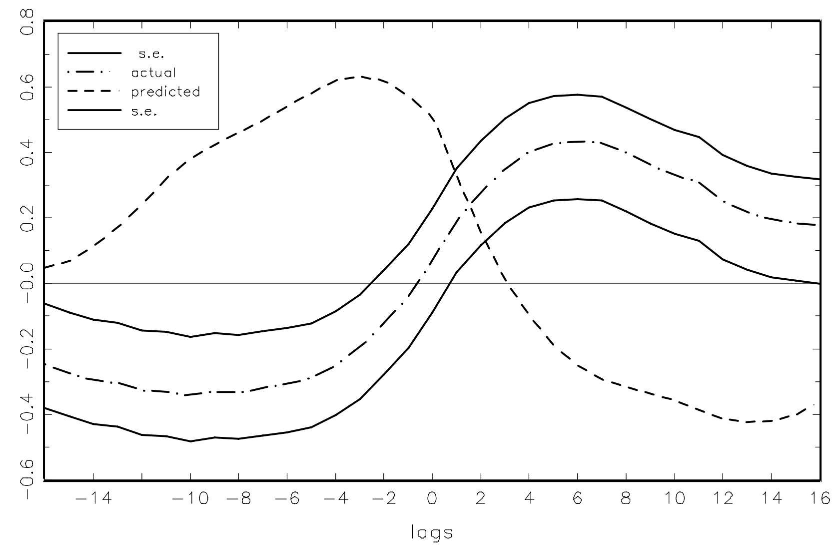


Fig. 2 – Inflation predicted by real ulc

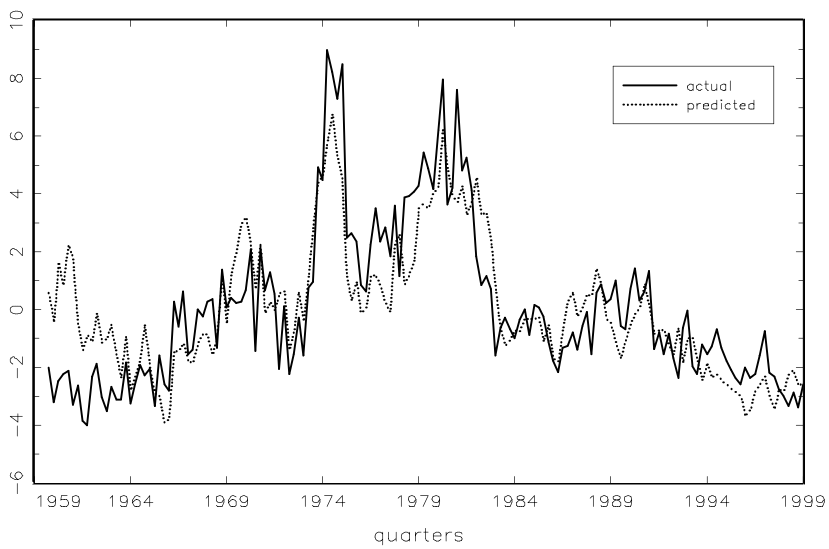


Fig. 3 – Output Gap and Real Unit labor Cost

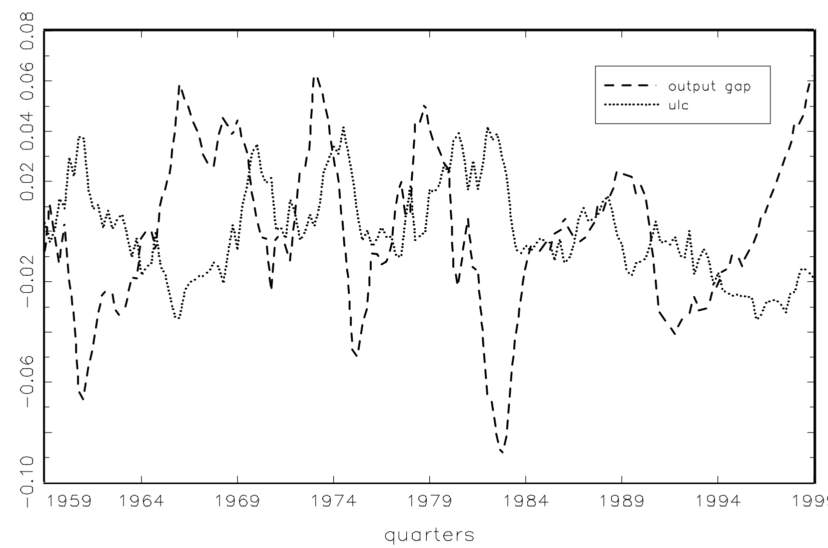
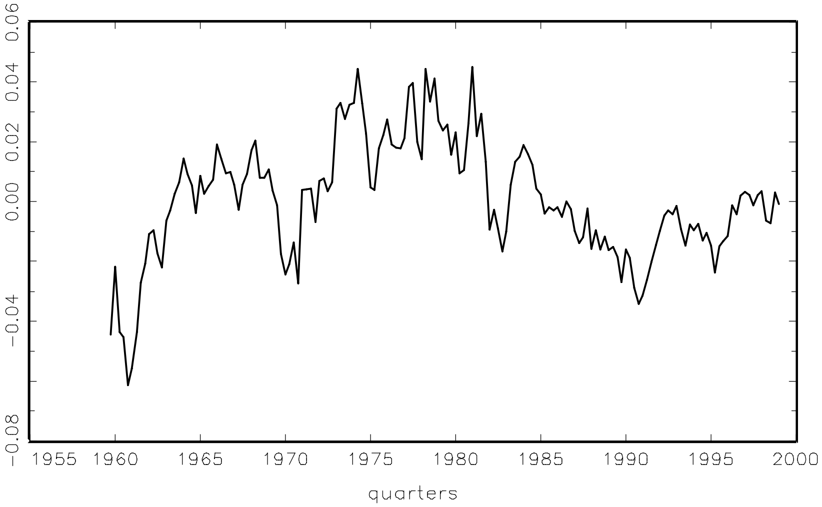
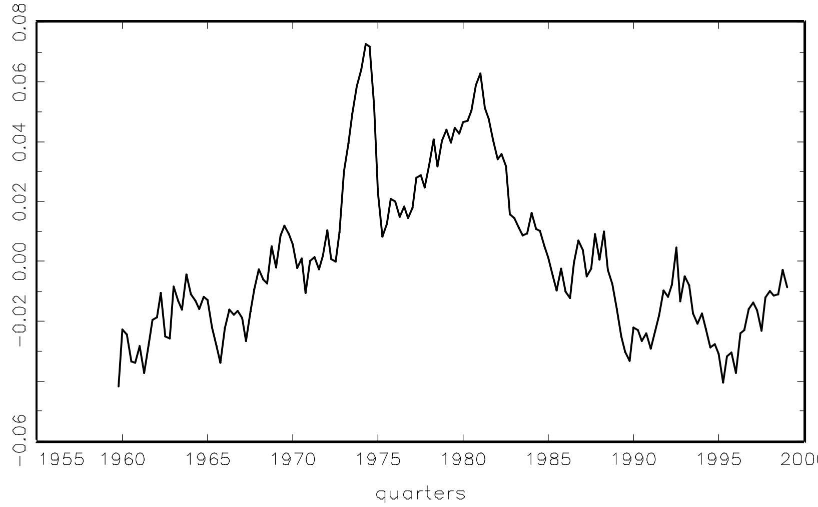


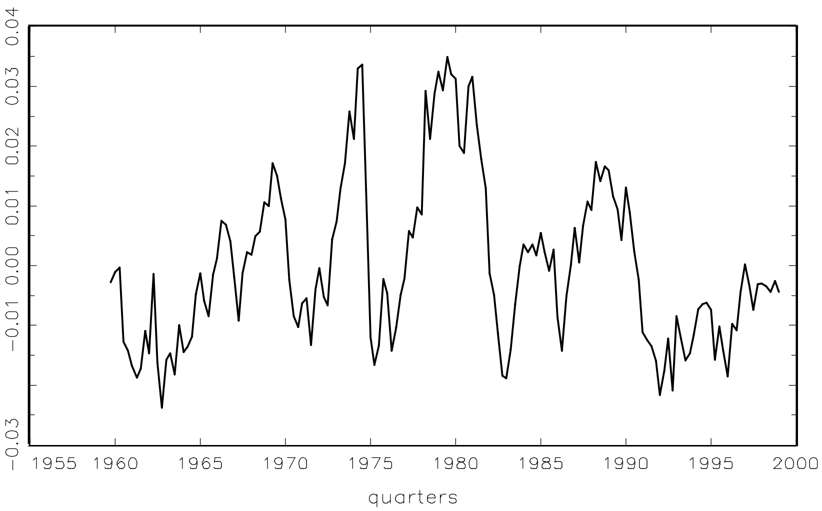
Fig. 4 – a. Cyclical Component of Productivity
VAR(2): (dq, y–q, c–y, sh, dp)', 59:4–99:1



b. Cyclical Component of Real Wage
VAR(2): (dq, y–q, c–y, sh, dp)', 59:4–99:1



c. Cyclical Component of Consumption
VAR(2): (dq, y–q, c–y, sh, dp)', 59:4–99:1



d. Cyclical Component of Hours
VAR(2): (dq, y–q, c–y, sh, dp)', 59:4–99:1

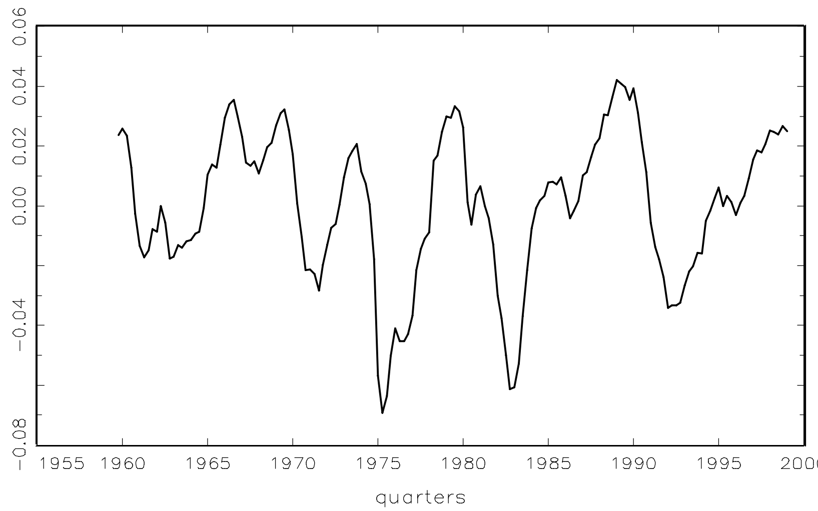


Fig. 5 — Flexible real wage model

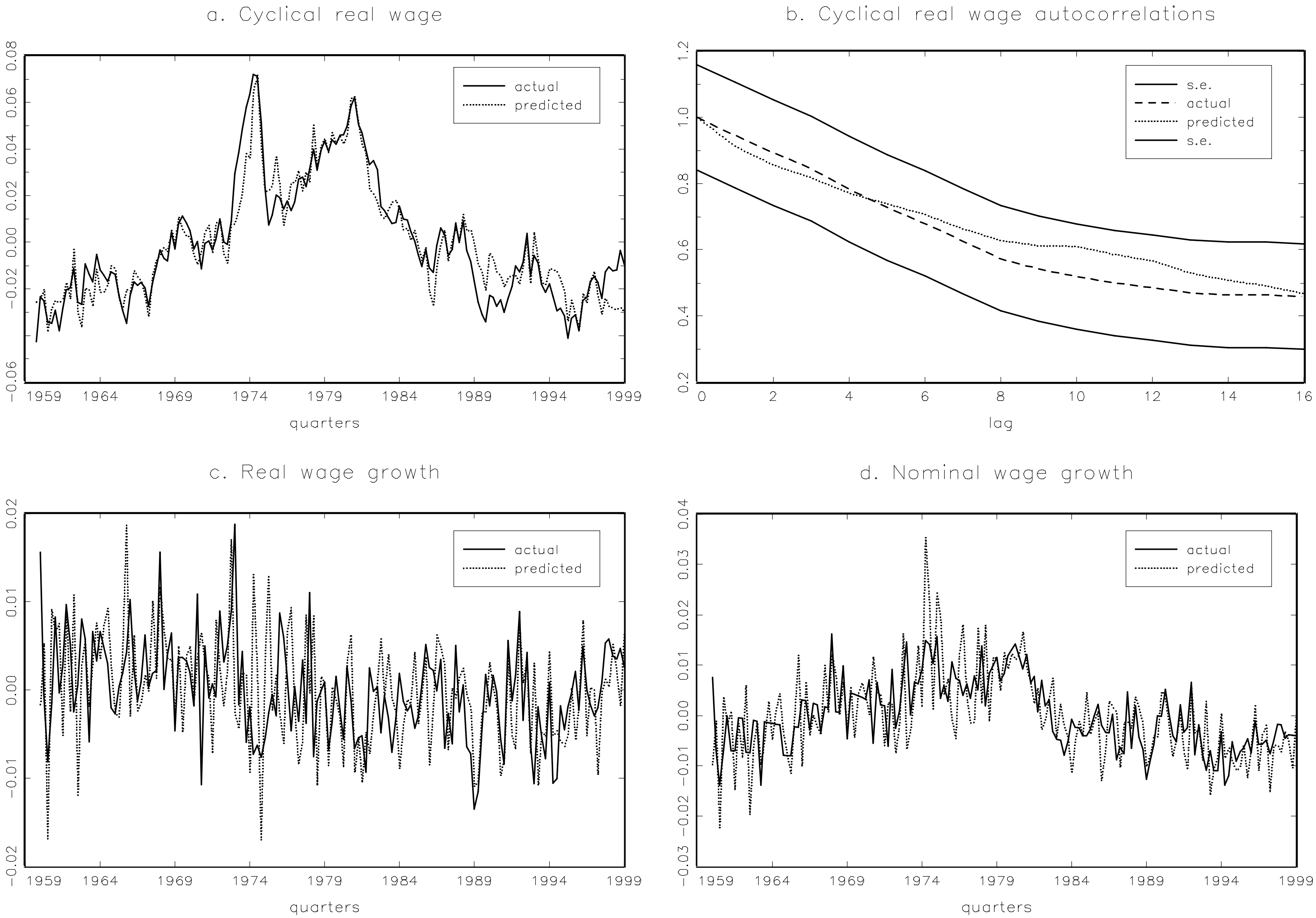
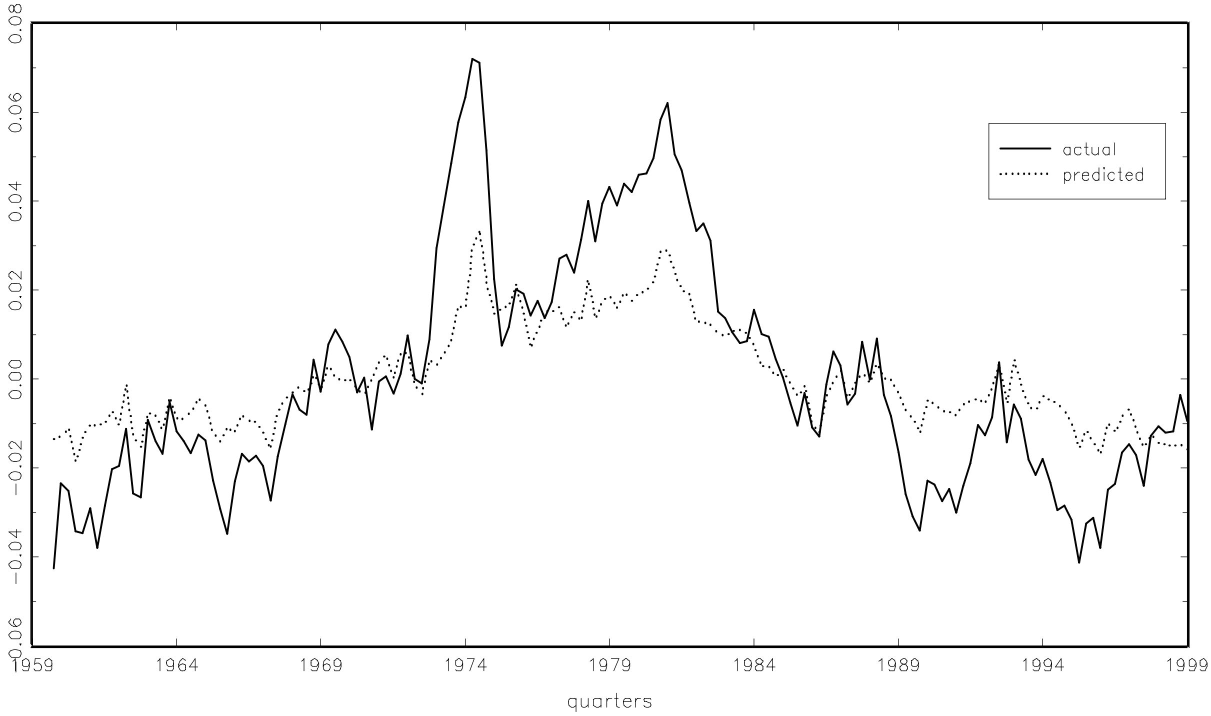


Fig. 6 – Restricted flexible wage model

a. Cyclical real wage, λ-restricted ($\lambda^r=1$, $\nu^r=-.465$)



b. Cyclical real wage, ν-restricted ($\lambda^r=1.29$, $\nu^r=0$)

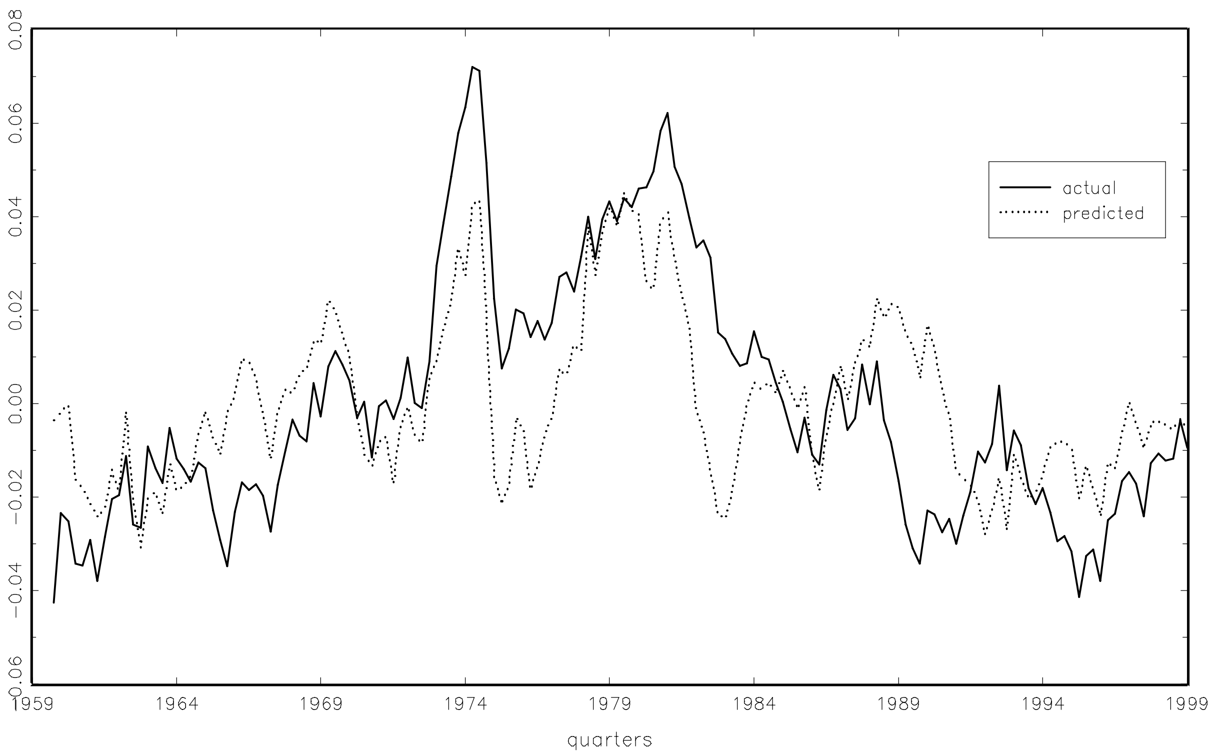


Fig. 7 — Partially indexed wage model

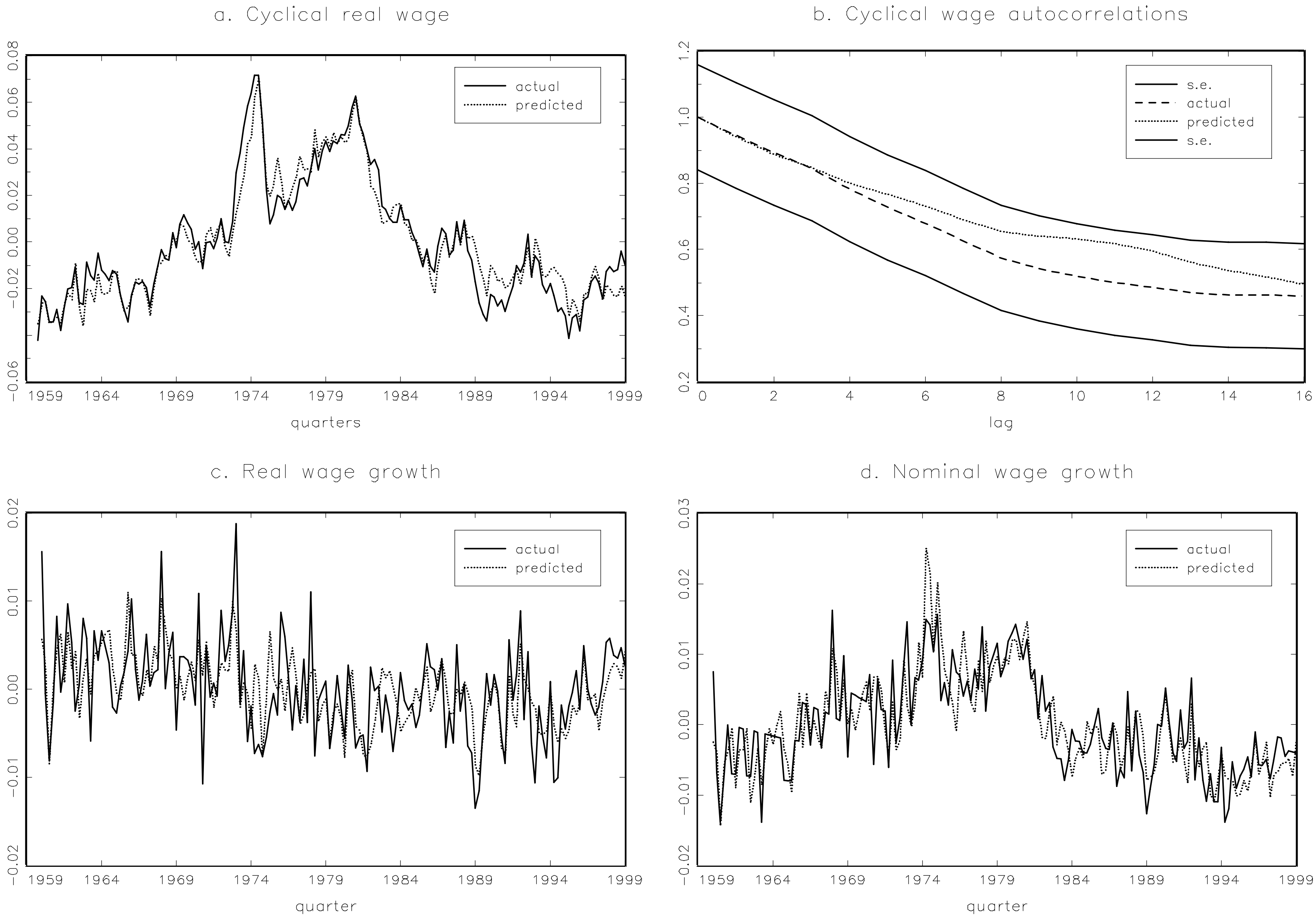


Fig. 8 — Simultaneous Wage and Price Dynamics